EL2450 – Assignment I

1 Question 3

The reference signal is a step of 10 units from time 100 seconds, with an offset of 40 units, as seen in figure 1.

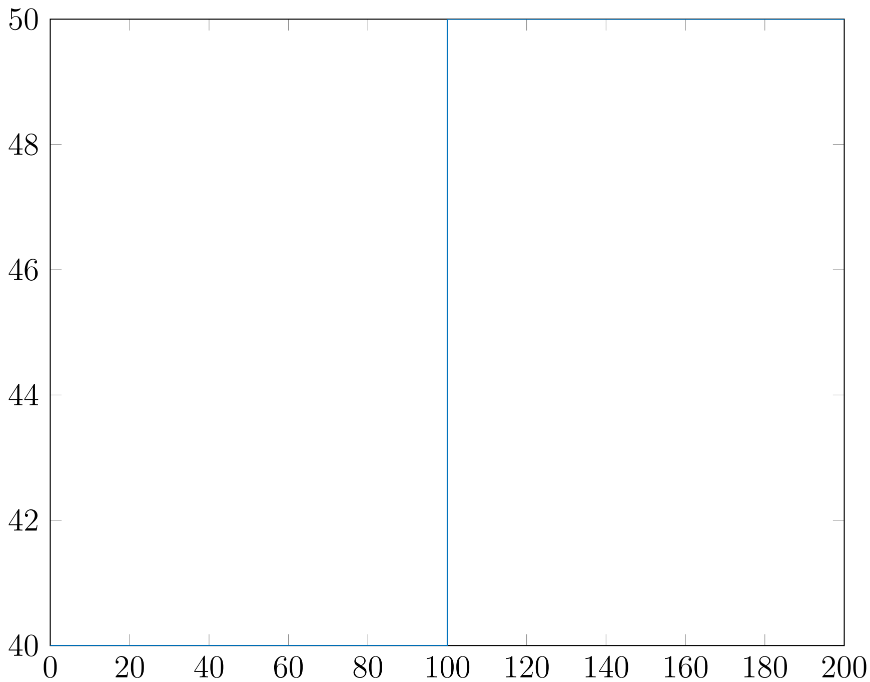

Figure 1: The reference signal.

2 Question 4

Table 1 illustrates the corresponding values of the K, T_I, T_D and N coefficients for set χ, ζ and ω_0.

χ	ζ	ω_0	K	T_I	T_D	M
0.5	0.7	0.1	2.6062	14.4445	5.5143	0.9791
0.5	0.7	0.2	5.9243	9.3823	3.1938	1.1191
0.5	0.8	0.2	6.3325	10.3873	3.1523	1.1591

Table 1: Coefficients of the PID controller per set χ, ζ and ω_0 values.

3 Question 5

Table 2 illustrates the rise time, overshoot and settling time for set values of χ, ζ and ω_0.

χ	ζ	ω_0	T_r	M	T_s
0.5	0.7	0.1	8.2	14.40	39.0
0.5	0.7	0.2	5.0	34.67	23.7
0.5	0.8	0.2	4.95	31.72	24.25

Table 2: Rise time (T_r) in seconds, overshoot (M) as a percentage of the output's steady state value, and settling time T_s in seconds for set values of χ, ζ and ω_0.

Due to our step response requirements, the best control performance is given by the third set of (χ, ζ, ω_0) parameters. All three requirements are fulfilled, as opposed to the case of the first set, and in comparison to the case of the second set, the rise time and overshoot are less, while their settling times are comparable.

Figures 2, 3 and 4 depict the step response for the three sets of (χ, ζ, ω_0) parameters.

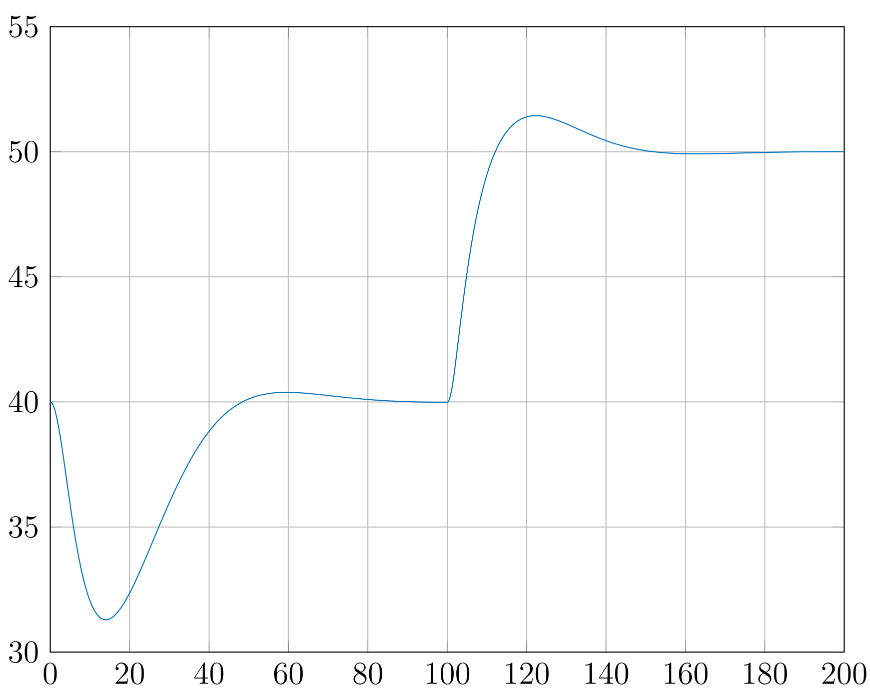

Figure 2: Step response for $(\chi, \zeta, \omega_0) \equiv (0.5, 0.7, 0.1)$.

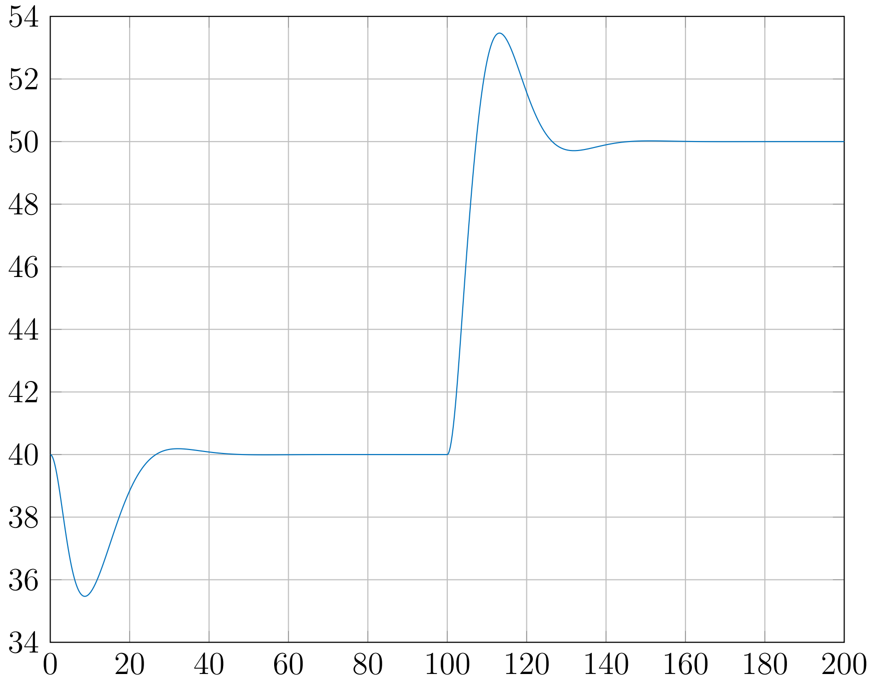

Figure 3: Step response for $(\chi, \zeta, \omega_0) \equiv (0.5, 0.7, 0.2)$.

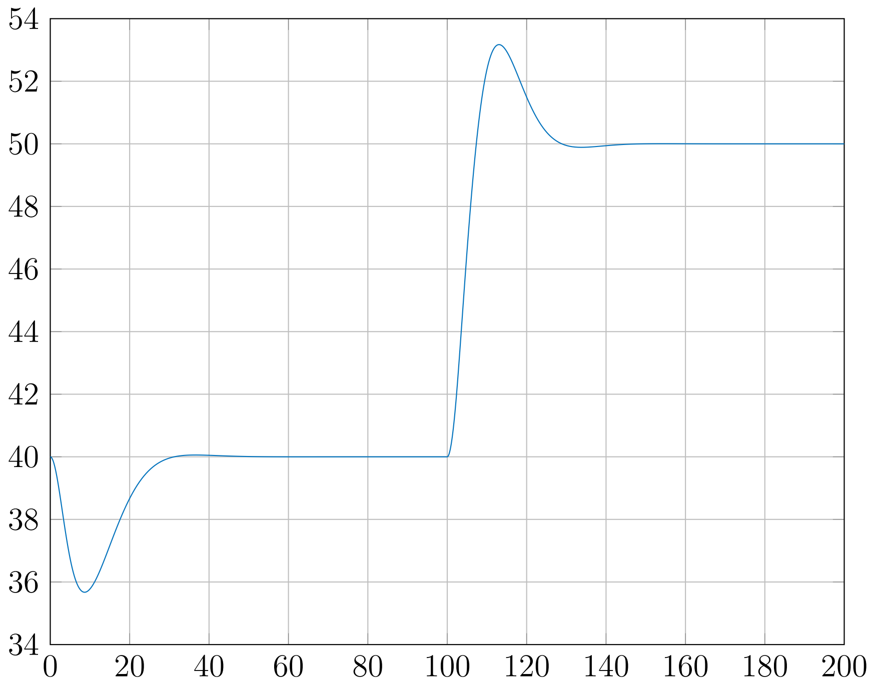

Figure 4: Step response for $(\chi, \zeta, \omega_0) \equiv (0.5, 0.8, 0.2)$.

4 Question 6

The open-loop transfer function is equal to the product of the transfer function of the controller $F(s)$ and that of the process $G(s)$. The crossover frequency ω_c is the frequency at which the magnitute of $F(j\omega)G(j\omega)$ is 1.0.

In practice, we were able to derive the crossover frequency by using MATLAB's `margin()` function, with argument the open-loop transfer function.

Table 3 illustrates the crossover frequencies in rad/s for set values of the χ, ζ and ω_0 parameters.

χ	ζ	ω_0	ω_c
0.5	0.7	0.1	0.2239
0.5	0.7	0.2	0.3426
0.5	0.8	0.2	0.3619

Table 3: Crossover frequencies depending on the set values of χ, ζ and ω_0.

5 Question 7

Figure 5 shows the root locus of the open-loop transfer function without the use of a zero-order hold, while figure 6 shows exacly the same, but with the addition of a zero-order hold between the controller and the process, with sampling time of $h = 1$ sec. It is apparent that without the zero-order hold, the system is stable, since all poles have negative real values. However, the time delay and the pole at zero that the zero-order hold introduces deliver a reduction in the degree of stability.

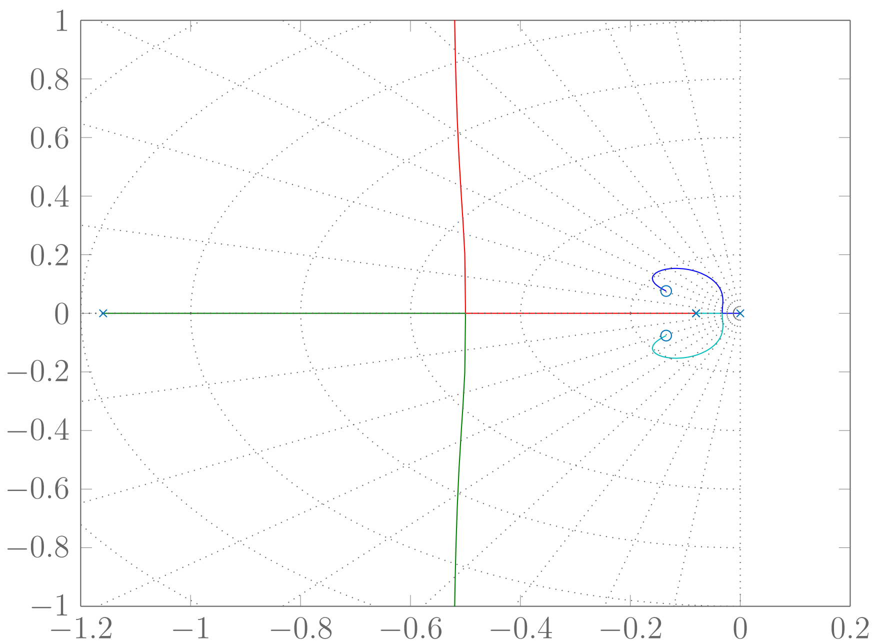

Figure 5: Root locus of the open-loop system without the use of a zero-order hold.

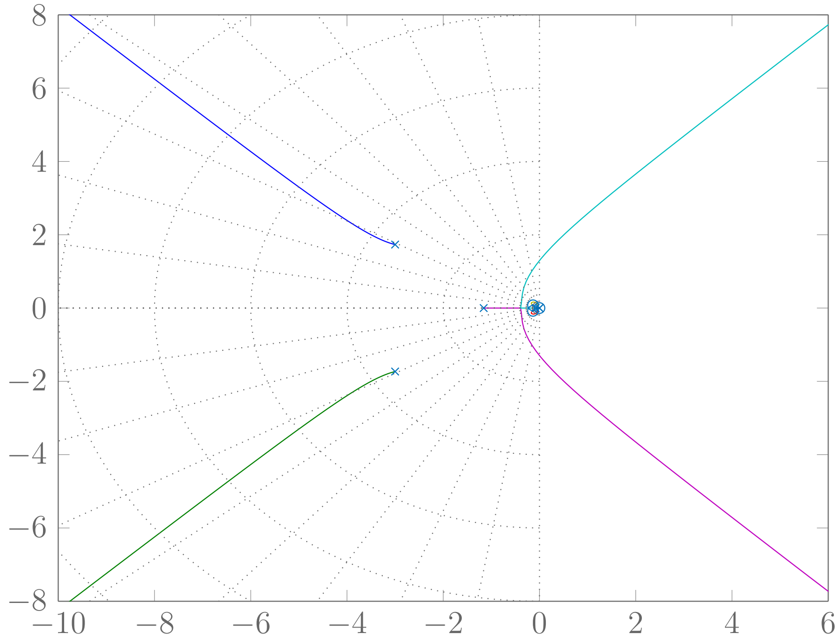

Figure 6: Root locus of the open-loop system with the use of a zero-order hold.

Figures 7-14 illustrate the step response when between the continuous controller and the plant a zero-order hold has been inserted, for sampling time varying between 1 and 8 seconds. The values of the χ, ζ and ω_0 parameters were chosen to be the ones giving the best performance among the three sets, hence $(\chi, \zeta, \omega_0) \equiv (0.5, 0.8, 0.2)$.

Here, up until $h = 5$ sec, as the sampling time increases, so do the rise time, settling time and overshoot. However, increasing h beyond 7 sec make the system critically stable, since two conjugate poles are approaching 0.

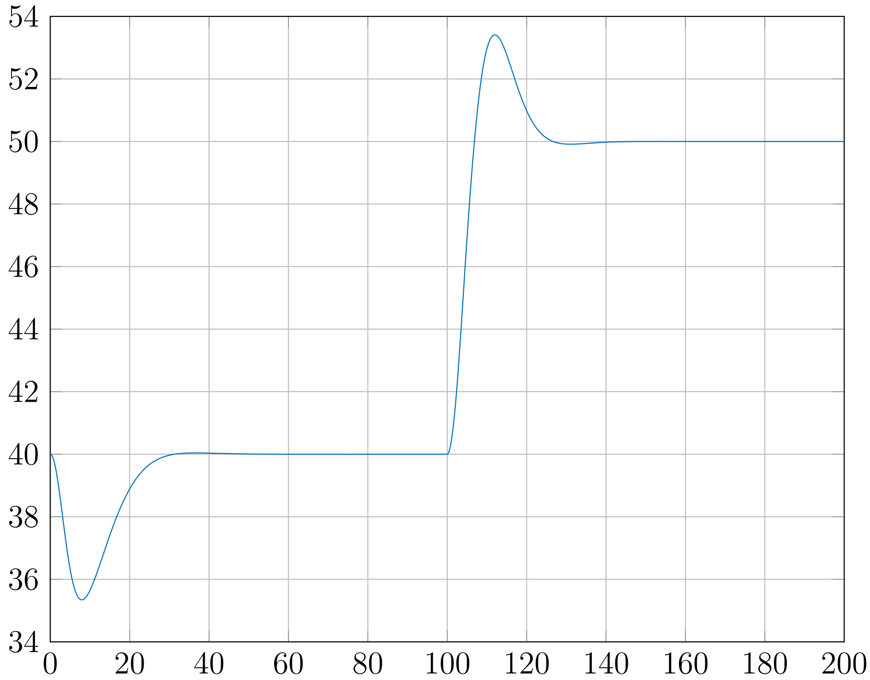

Figure 7: Step response using a zero-order hold of sample time 1 sec.

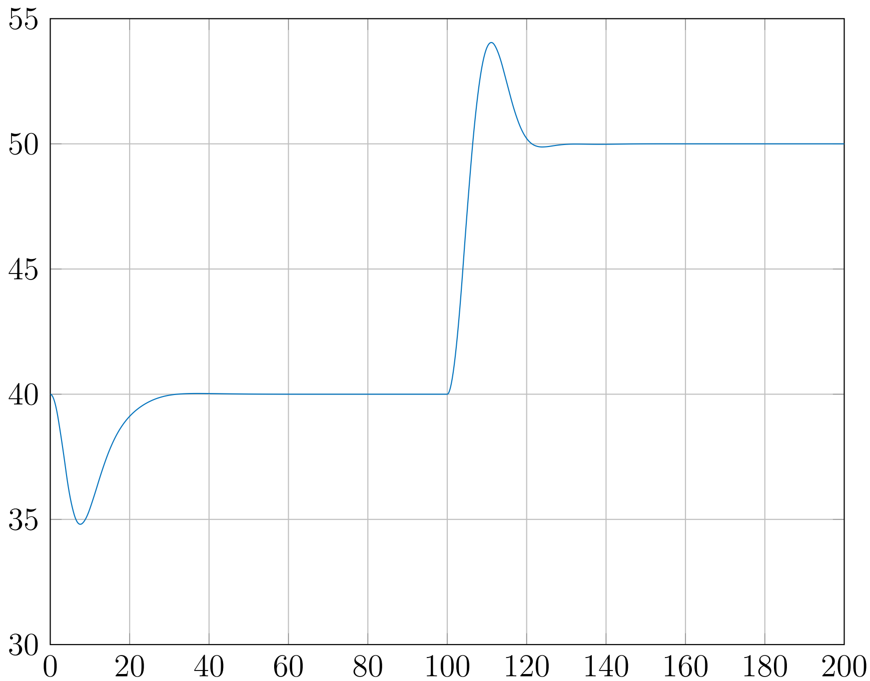

Figure 8: Step response using a zero-order hold of sample time 2 sec.

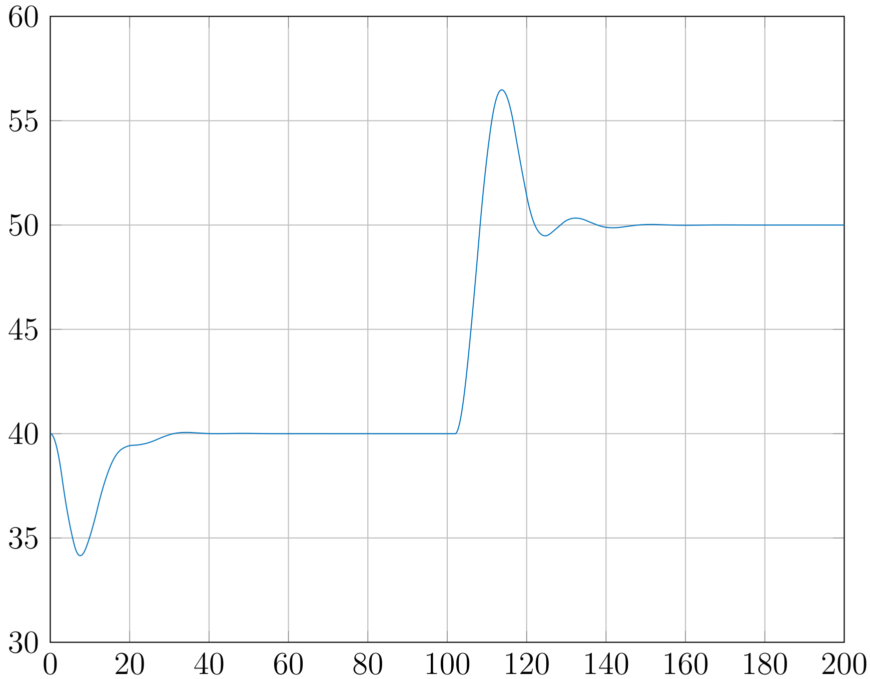

Figure 9: Step response using a zero-order hold of sample time 3 sec.

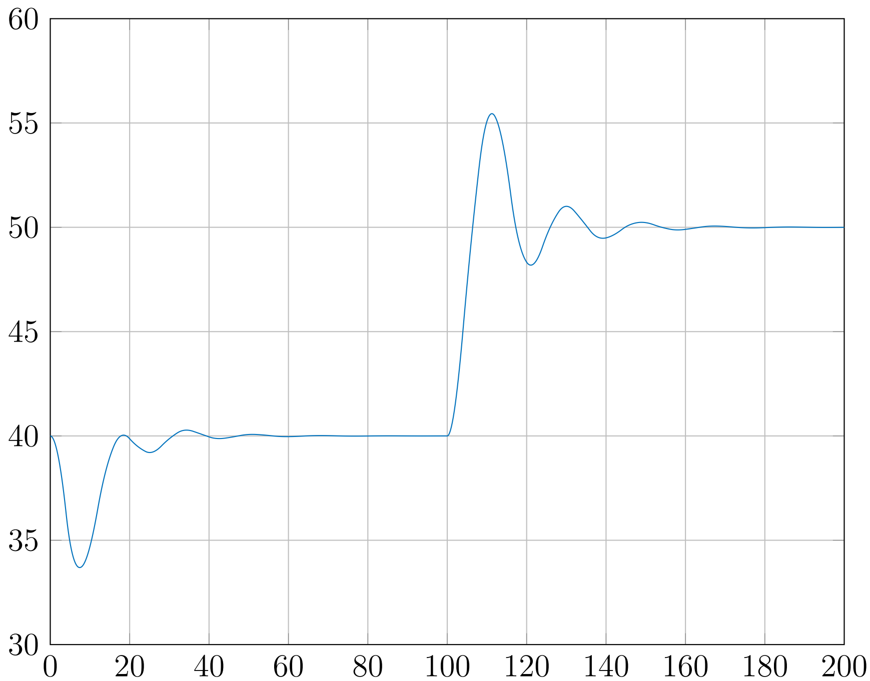

Figure 10: Step response using a zero-order hold of sample time 4 sec.

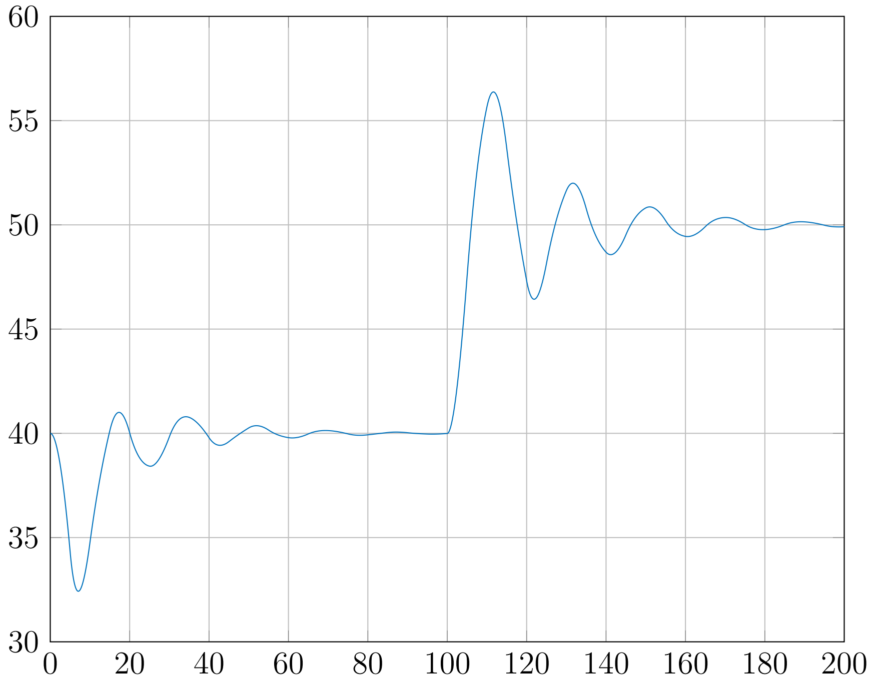

Figure 11: Step response using a zero-order hold of sample time 5 sec.

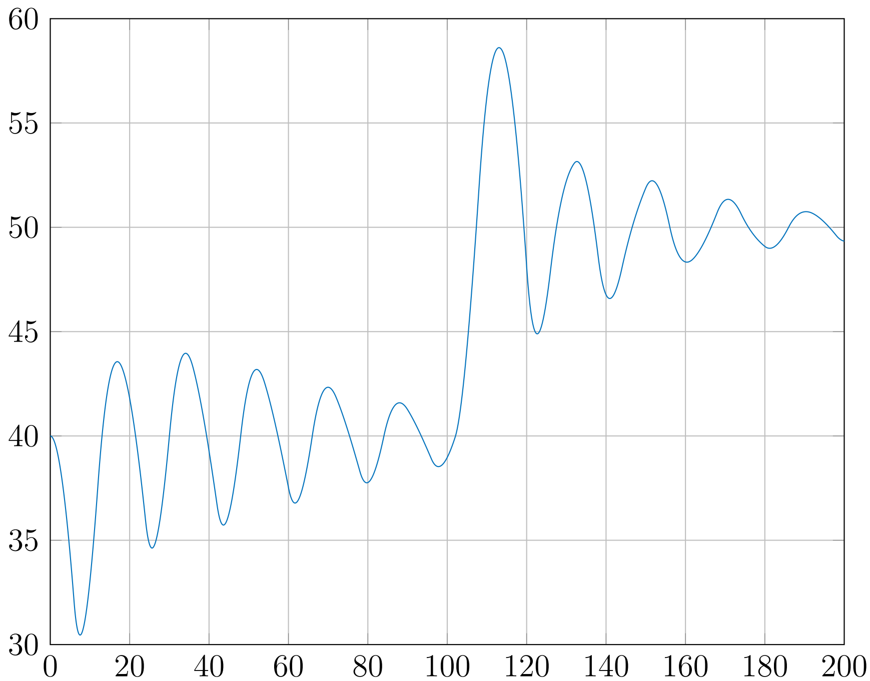

Figure 12: Step response using a zero-order hold of sample time 6 sec.

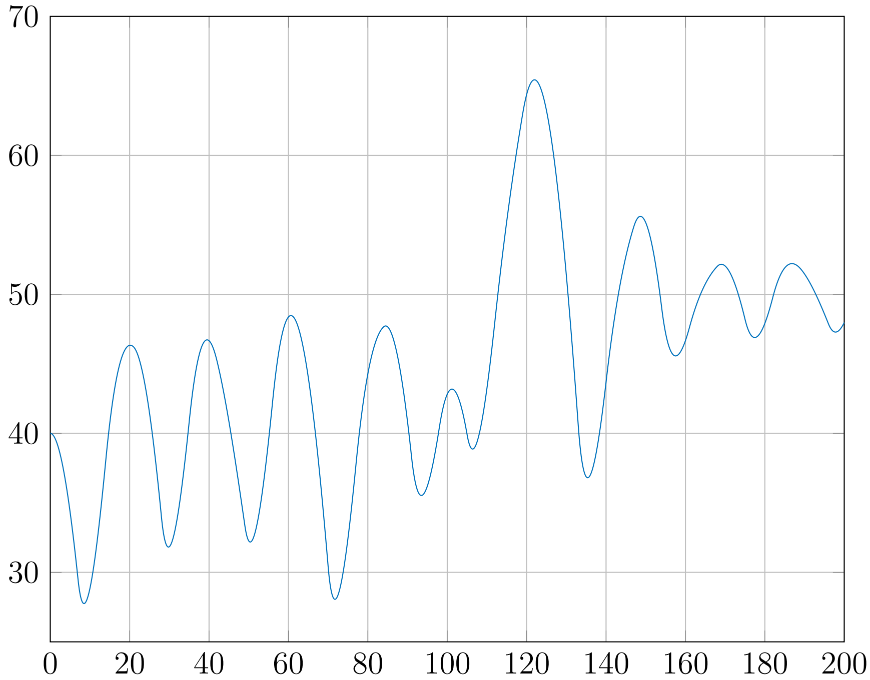

Figure 13: Step response using a zero-order hold of sample time 7 sec.

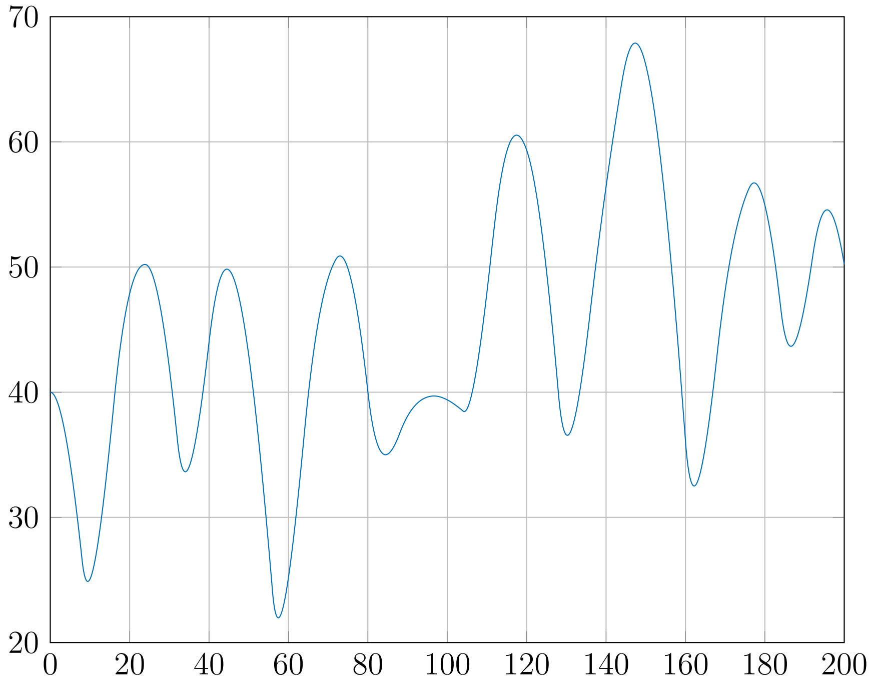

Figure 14: Step response using a zero-order hold of sample time 8 sec.

6 Question 8

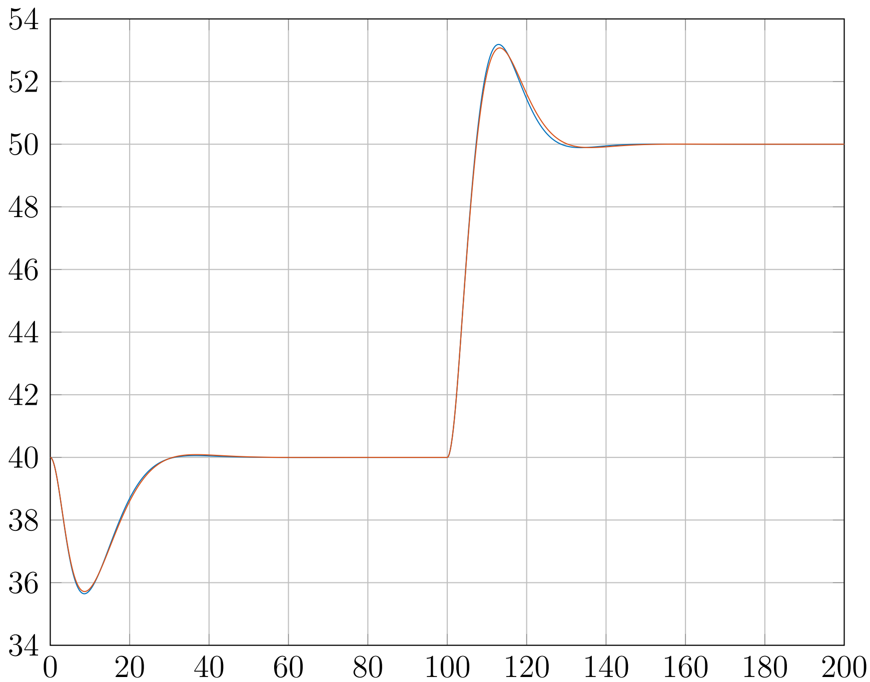

Figure 15: Step response using a discrete controller (red) with sampling time $T_s = 0.1$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 0.1$.

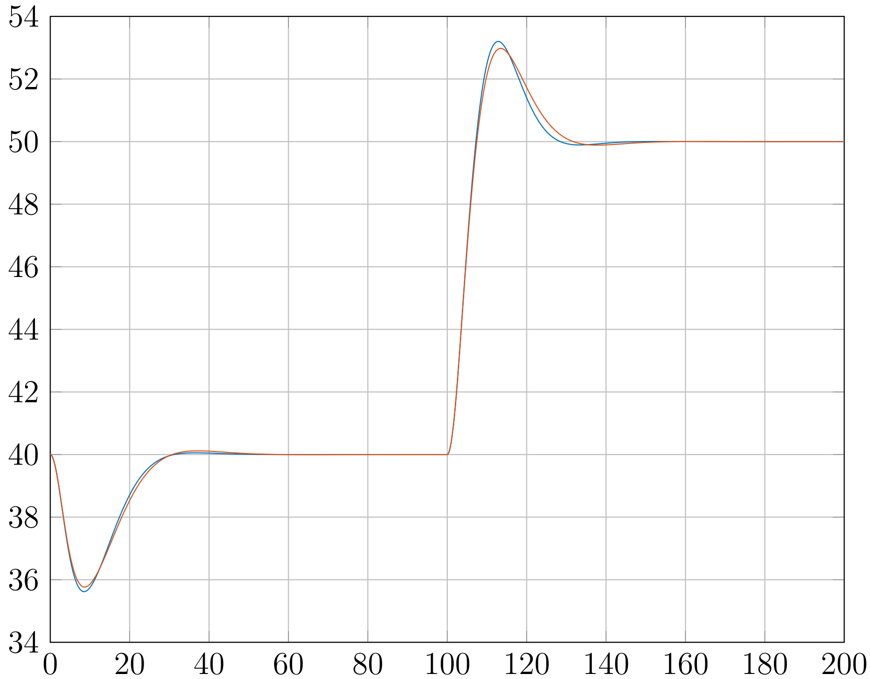

Figure 16: Step response using a discrete controller (red) with sampling time $T_s = 0.2$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 0.2$.

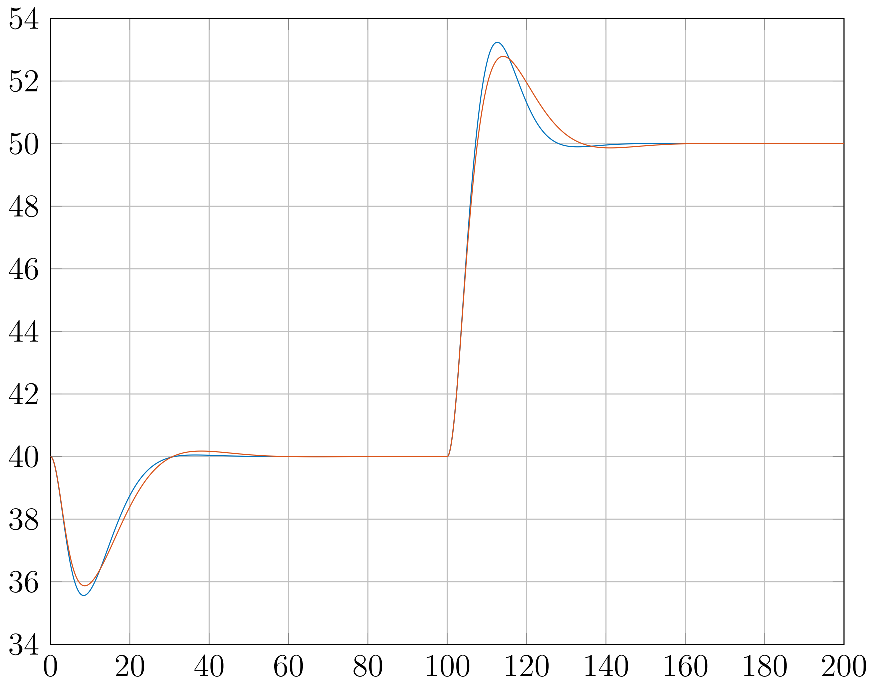

Figure 17: Step response using a discrete controller (red) with sampling time $T_s = 0.4$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 0.4$.

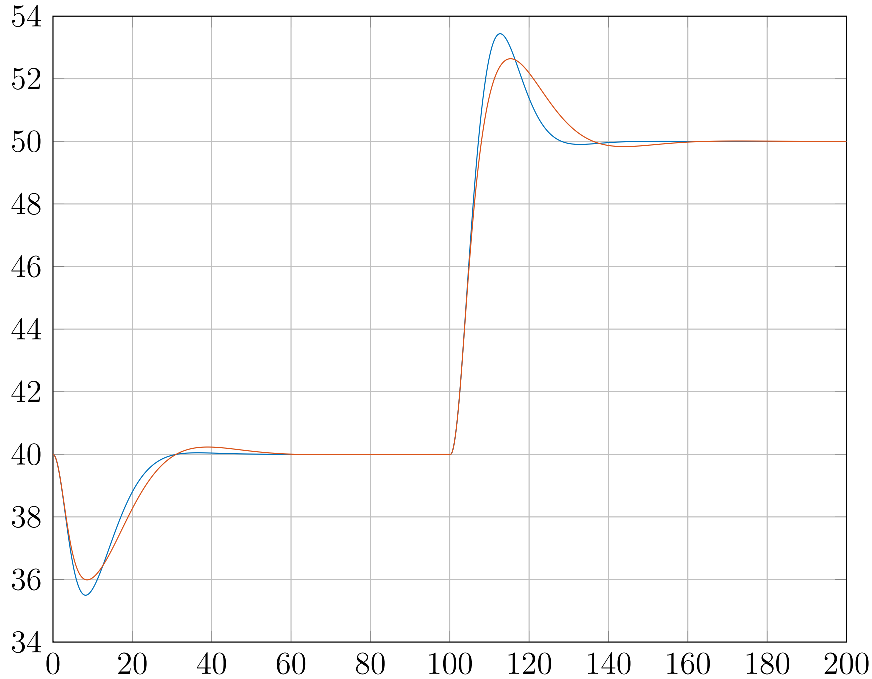

Figure 18: Step response using a discrete controller (red) with sampling time $T_s = 0.6$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 0.6$.

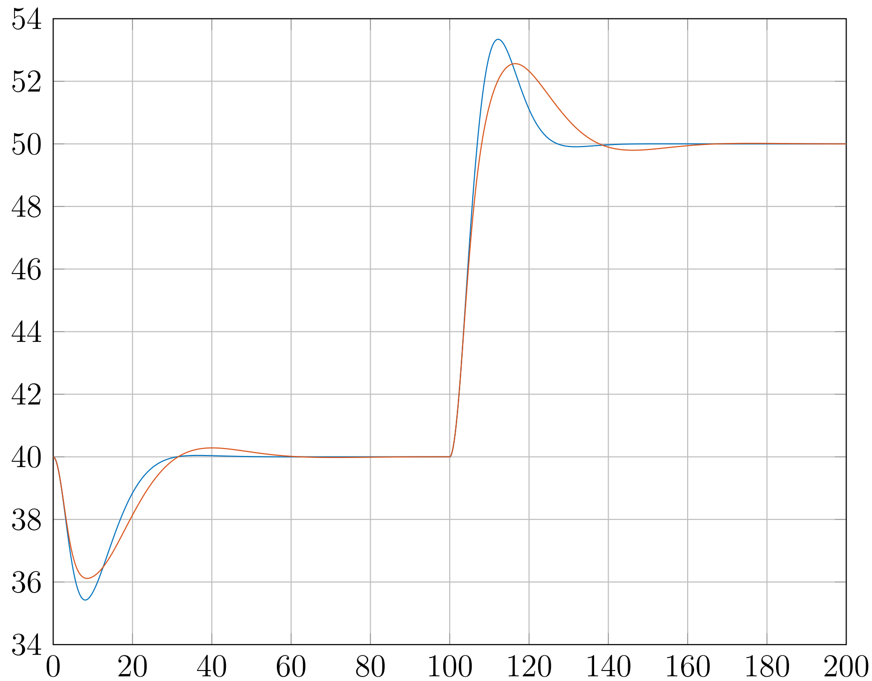

Figure 19: Step response using a discrete controller (red) with sampling time $T_s = 0.8$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 0.8$.

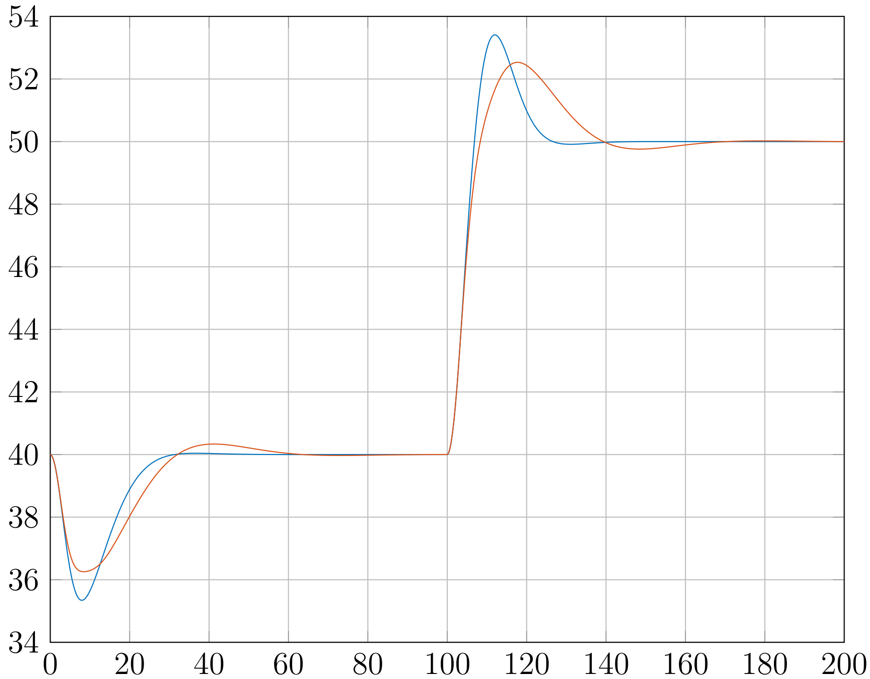

Figure 20: Step response using a discrete controller (red) with sampling time $T_s = 1$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 1$.

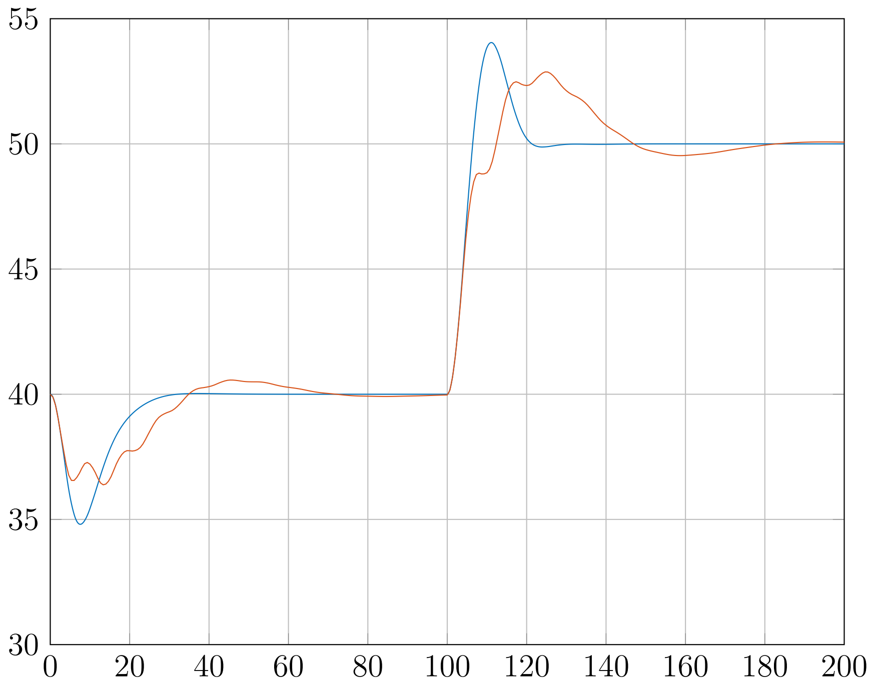

Figure 21: Step response using a discrete controller (red) with sampling time $T_s = 2$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 2$.

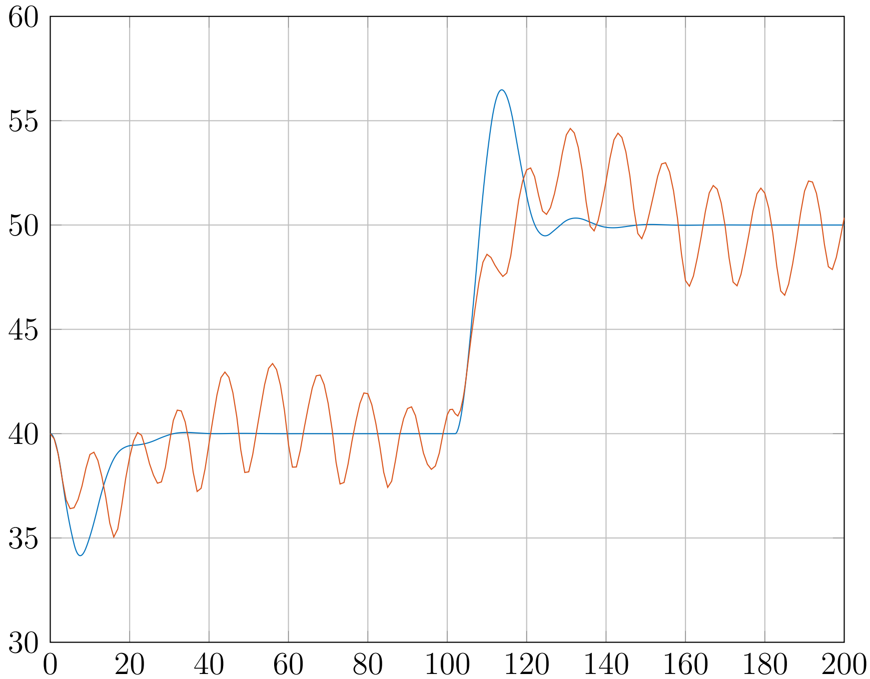

Figure 22: Step response using a discrete controller (red) with sampling time $T_s = 3$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 3$.

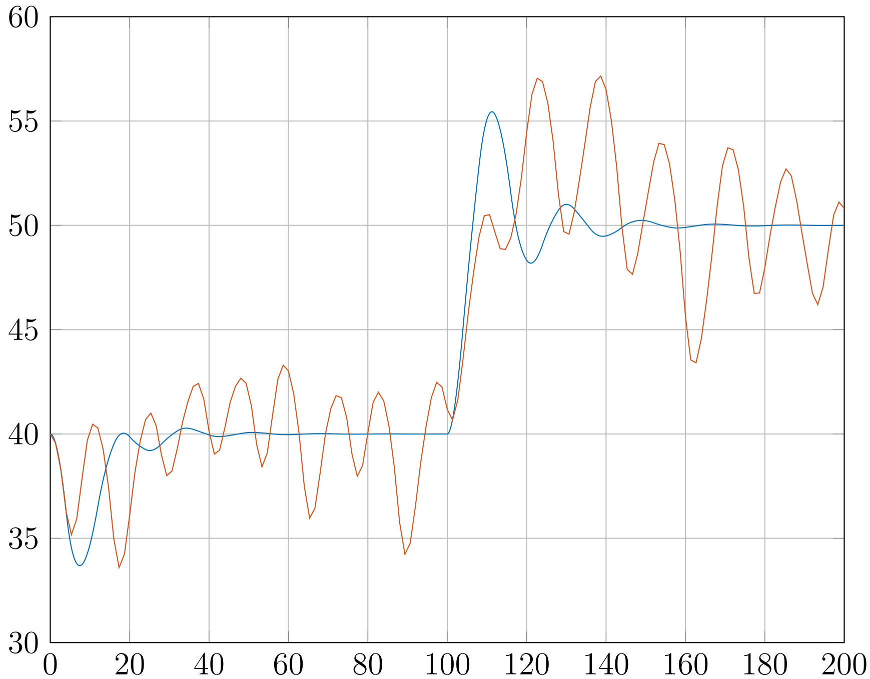

Figure 23: Step response using a discrete controller (red) with sampling time $T_s = 4$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 4$.

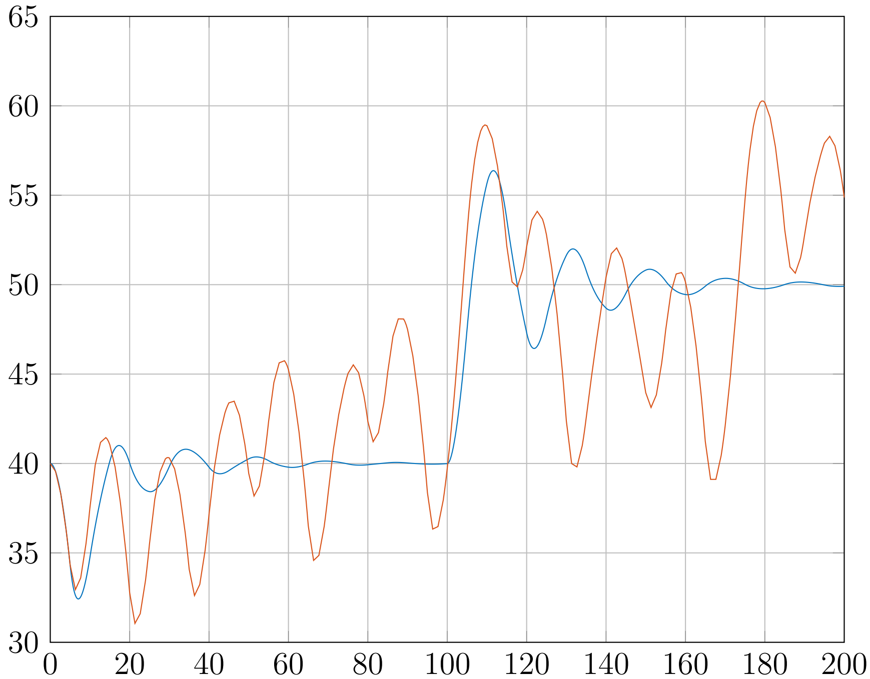

Figure 24: Step response using a discrete controller (red) with sampling time $T_s = 5$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 5$.

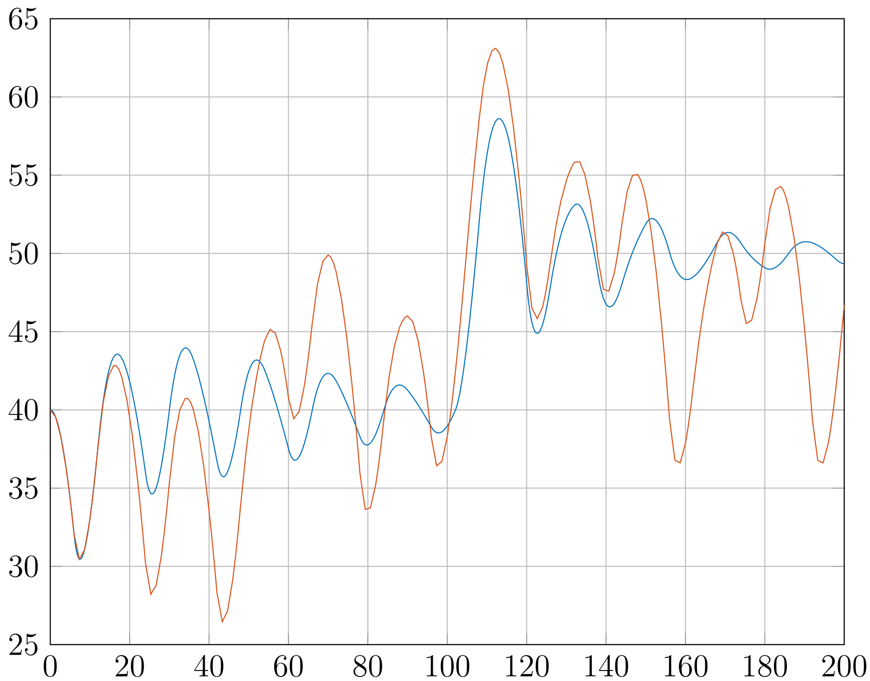

Figure 25: Step response using a discrete controller (red) with sampling time $T_s = 6$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 6$.

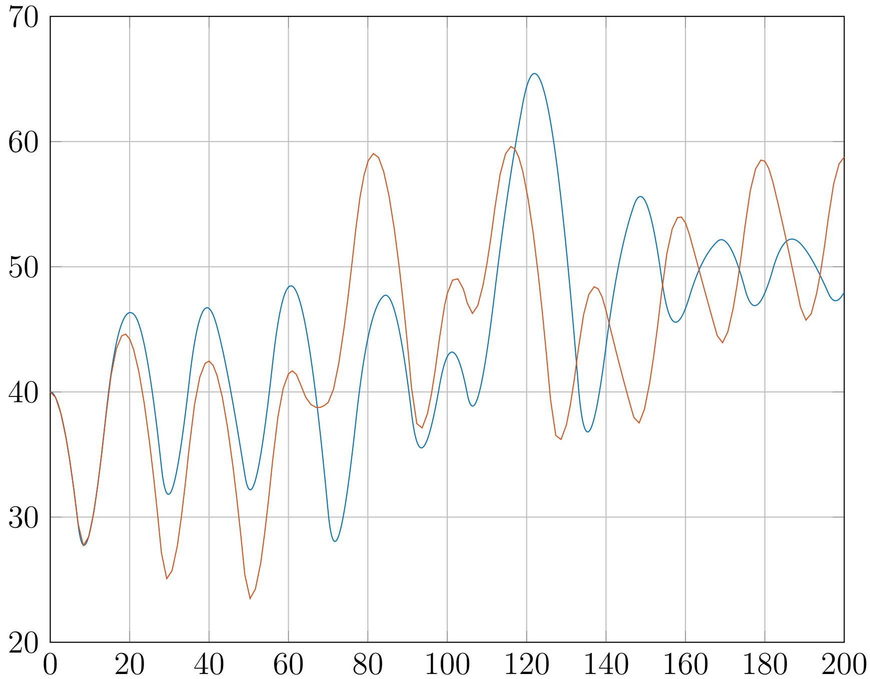

Figure 26: Step response using a discrete controller (red) with sampling time $T_s = 7$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 7$.

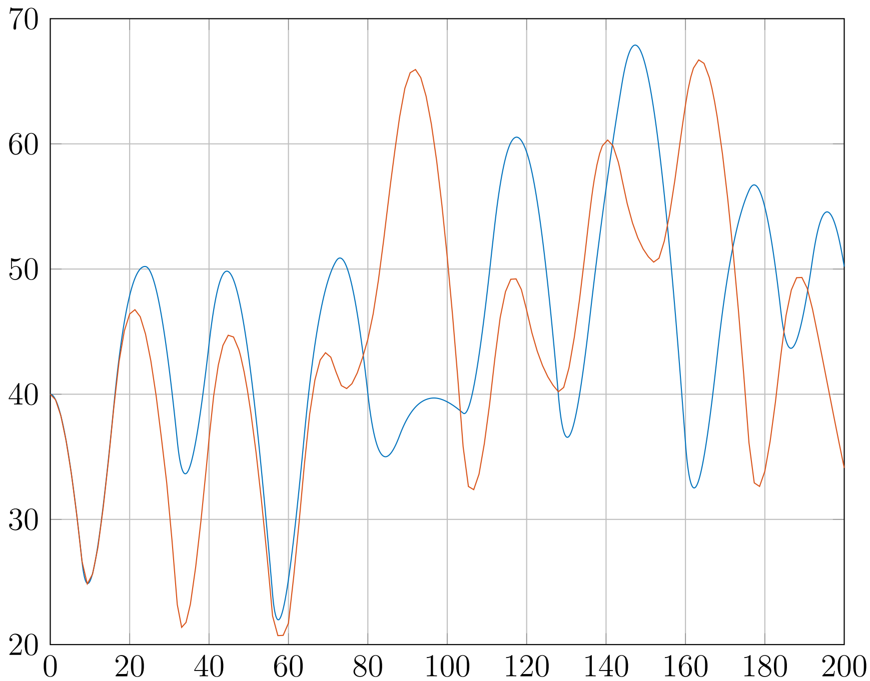

Figure 27: Step response using a discrete controller (red) with sampling time $T_s = 8$ sec and a continuous one with a zero-order hold (blue) with sampling time $h = 8$.

7 Question 9

In theory[1], as a general rule of thumb, the sampling period should be selected so that

$$0.08 < T_s\omega_c < 0.3$$

where ω_c is the crossover frequency of the open-loop system, hence with $\omega_c = 0.3619$ rad/s

$$0.220 < T_s < 0.829 \ sec$$

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A second way one could look at this is to choose a sampling time such that there are 4 to 10 samples per rise time T_r. In this case where $T_r = 4.95$ sec, the resulting range of acceptable values for the sampling time is

$$0.495 < T_s < 1.2375 \ sec$$

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Another way one could look at this is to select a sampling frequency that is much higher than twice the bandwidth of the closed-loop system. Given our setting, the bandwidth $\omega_0 = 0.5819$ rad/s and

$$10\omega_0 < \omega_s < 30\omega_0$$

$$0.36 < T_s < 1.08 \ sec$$

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References

[1] William Levine et al. The Control Handbook. CRC Press, 1996.