# EL2450 – Assignment I

## 1 Question 3

The reference signal is a step of 10 units from time 100 seconds, with an offset of 40 units, as seen in figure 1.

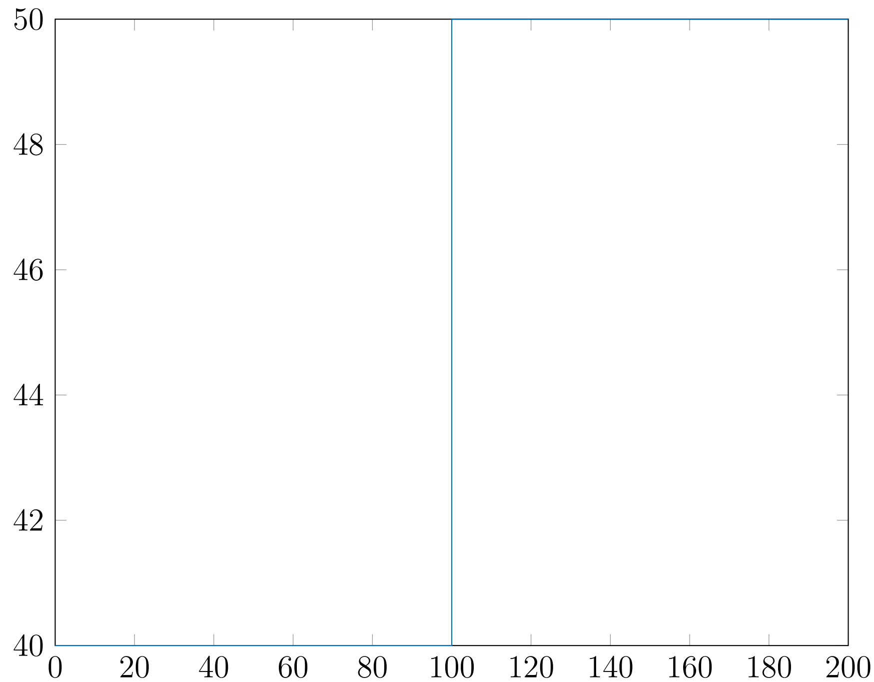


Figure 1: The reference signal.

## 2 Question 4

Table 1 illustrates the corresponding values of the $K$, $T_I$, $T_D$ and $N$ coefficients for set $\chi$, $\zeta$ and $\omega_0$.

| $\chi$ | $\zeta$ | $\omega_0$ | $K$ | $T_I$ | $T_D$ | $M$ |
|---|---|---|---|---|---|---|
| 0.5 | 0.7 | 0.1 | 2.6062 | 14.4445 | 5.5143 | 0.9791 |
| 0.5 | 0.7 | 0.2 | 5.9243 | 9.3823 | 3.1938 | 1.1191 |
| 0.5 | 0.8 | 0.2 | 6.3325 | 10.3873 | 3.1523 | 1.1591 |

Table 1: Coefficients of the PID controller per set $\chi$, $\zeta$ and $\omega_0$ values.

# 3 Question 5

Table 2 illustrates the rise time, overshoot and settling time for set values of $\chi$, $\zeta$ and $\omega_0$.

| $\chi$ | $\zeta$ | $\omega_0$ | $T_r$ | $M$ | $T_s$ |
|---|---|---|---|---|---|
| 0.5 | 0.7 | 0.1 | 8.2 | 14.40 | 39.0 |
| 0.5 | 0.7 | 0.2 | 5.0 | 34.67 | 23.7 |
| 0.5 | 0.8 | 0.2 | 4.95 | 31.72 | 24.25 |

Table 2: Rise time ($T_r$) in seconds, overshoot ($M$) as a percentage of the output's steady state value, and settling time $T_s$ in seconds for set values of $\chi$, $\zeta$ and $\omega_0$.

Due to our step response requirements, the best control performance is given by the third set of $(\chi, \zeta, \omega_0)$ parameters. All three requirements are fulfilled, as opposed to the case of the first set, and in comparison to the case of the second set, the rise time and overshoot are less, while their settling times are comparable.

Figures 2, 3 and 4 depict the step response for the three sets of $(\chi, \zeta, \omega_0)$ parameters.

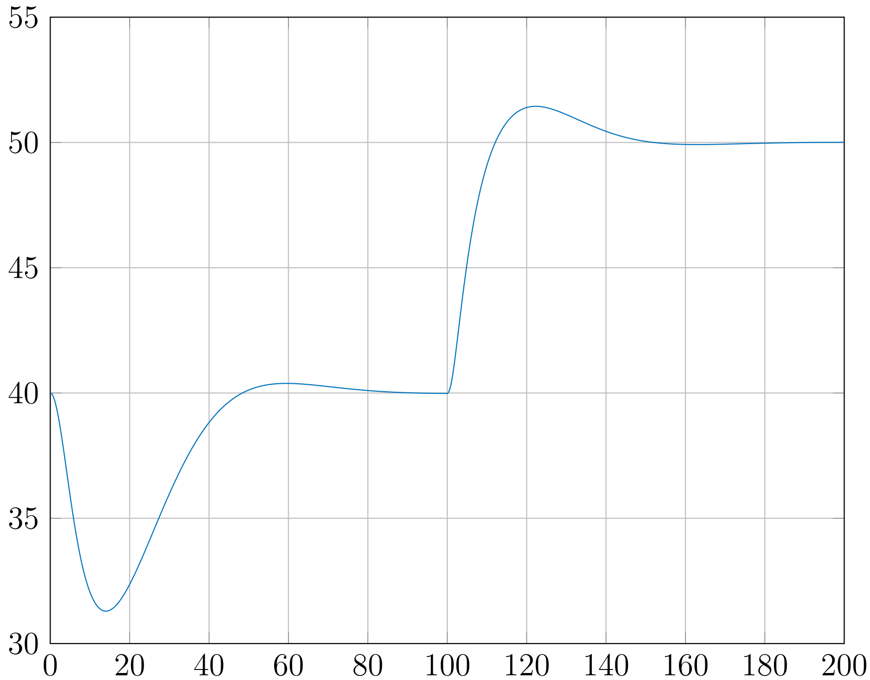


Figure 2: Step response for $(\chi, \zeta, \omega_0) \equiv (0.5, 0.7, 0.1)$.

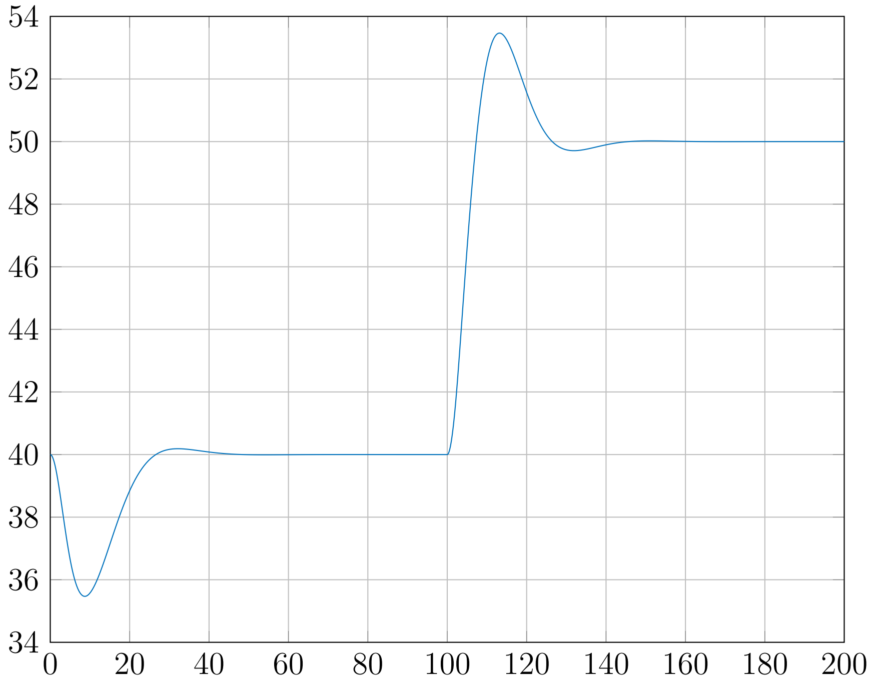


Figure 3: Step response for $(\chi, \zeta, \omega_0) \equiv (0.5, 0.7, 0.2)$.

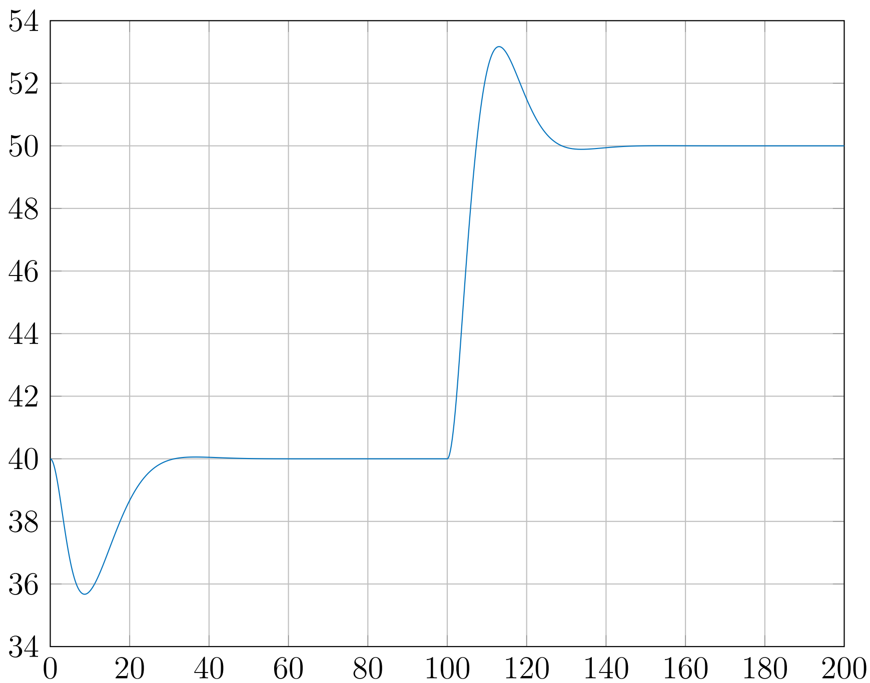


Figure 4: Step response for $(\chi, \zeta, \omega_0) \equiv (0.5, 0.8, 0.2)$.

# 4 Question 6

The open-loop transfer function is equal to the product of the transfer function of the controller $F(s)$ and that of the process $G(s)$. The crossover frequency $\omega_c$ is the frequency at which the magnitude of $F(j\omega)G(j\omega)$ is 1.0.

In practice, we were able to derive the crossover frequency by using MATLAB's `margin()` function, with argument the open-loop transfer function.

Table 3 illustrates the crossover frequencies in rad/s for set values of the $\chi$, $\zeta$ and $\omega_0$ parameters.

| $\chi$ | $\zeta$ | $\omega_0$ | $\omega_c$ |
|---|---|---|---|
| 0.5 | 0.7 | 0.1 | 0.2239 |
| 0.5 | 0.7 | 0.2 | 0.3426 |
| 0.5 | 0.8 | 0.2 | 0.3619 |

Table 3: Crossover frequencies depending on the set values of $\chi$, $\zeta$ and $\omega_0$.

# 5 Question 7

Figures 5-12 illustrate the step response when between the continuous controller and the plant a zero-order hold has been inserted, for sampling time varying between 1 and 8 seconds. The values of the $\chi$, $\zeta$ and $\omega_0$ parameters were chosen to be the ones giving the best performance among the three sets, hence $(\chi, \zeta, \omega_0) \equiv (0.5, 0.8, 0.2)$.

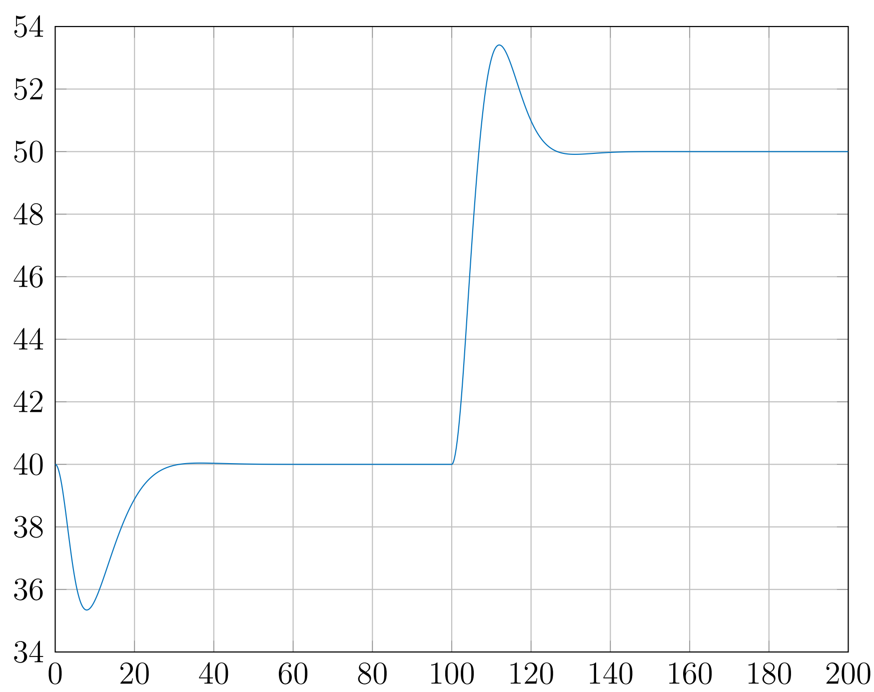


Figure 5: Step response using a zero-order hold of sample time 1 sec.

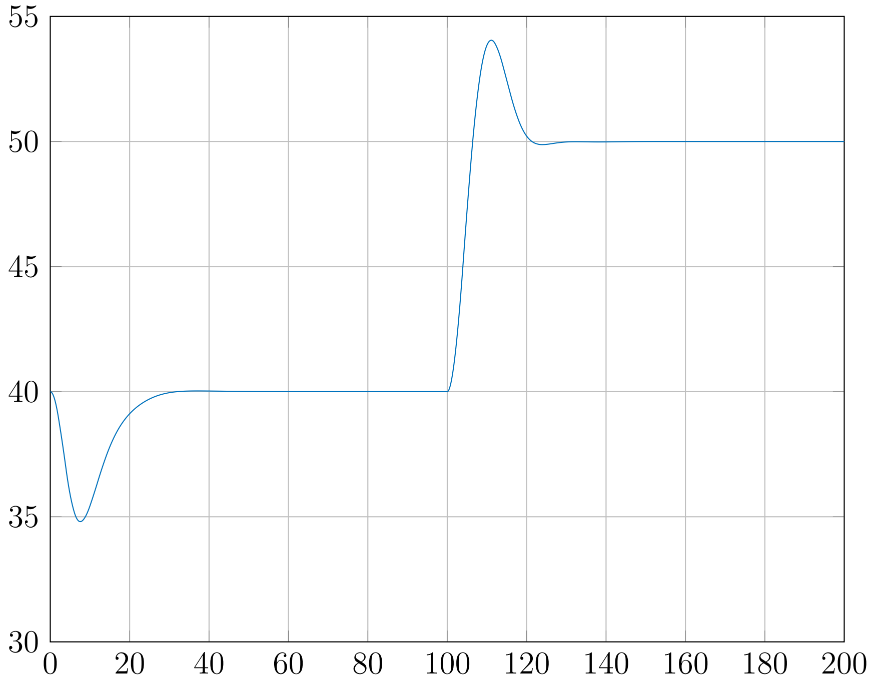


Figure 6: Step response using a zero-order hold of sample time 2 sec.

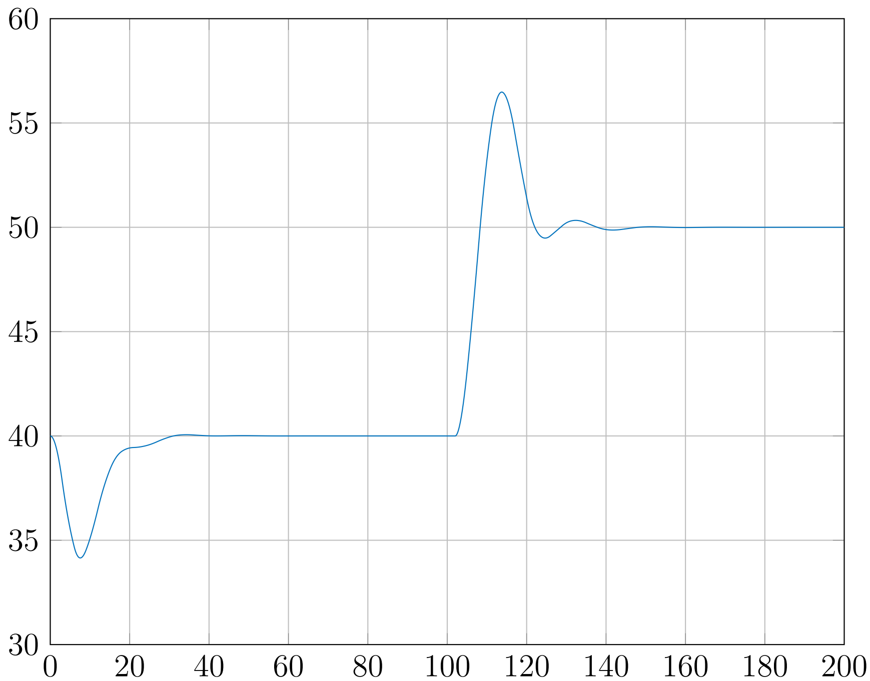


Figure 7: Step response using a zero-order hold of sample time 3 sec.

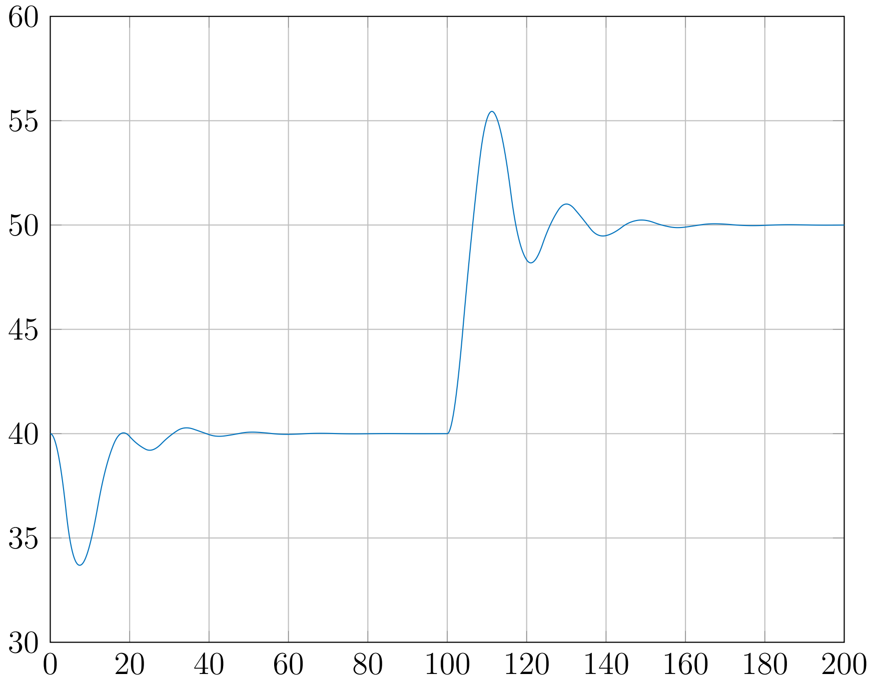


Figure 8: Step response using a zero-order hold of sample time 4 sec.

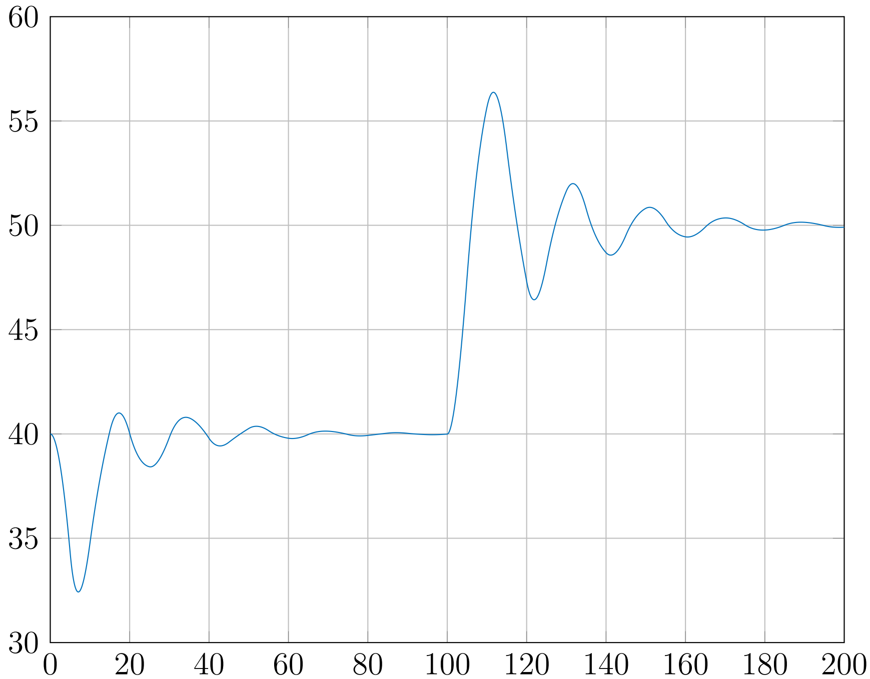


Figure 9: Step response using a zero-order hold of sample time 5 sec.

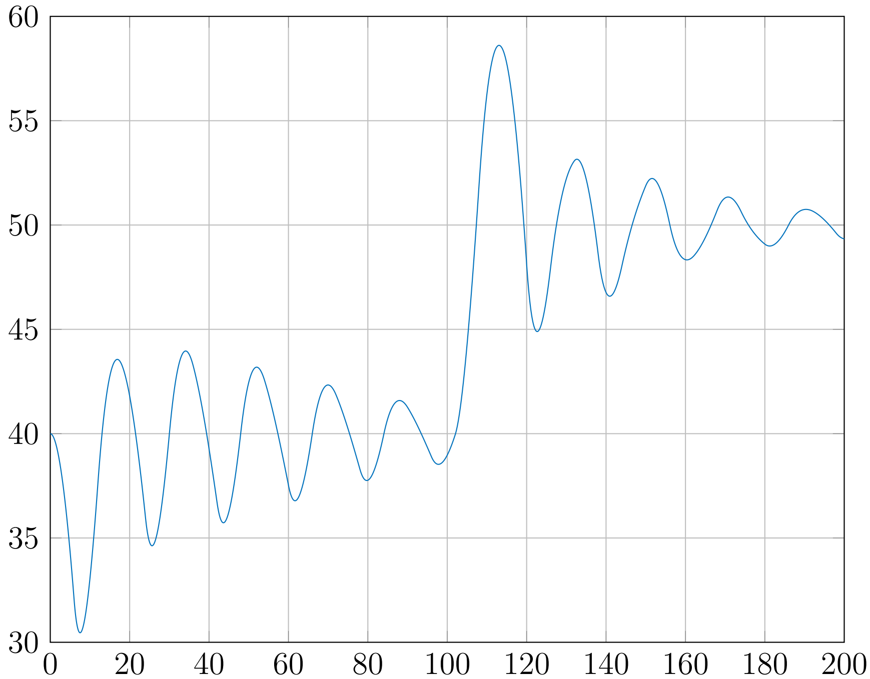


Figure 10: Step response using a zero-order hold of sample time 6 sec.

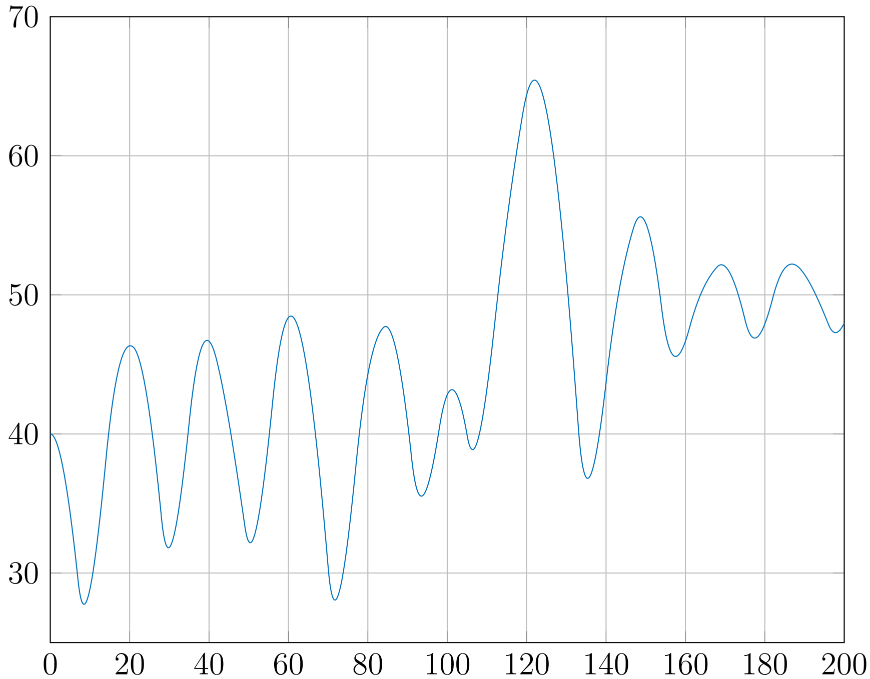


Figure 11: Step response using a zero-order hold of sample time 7 sec.

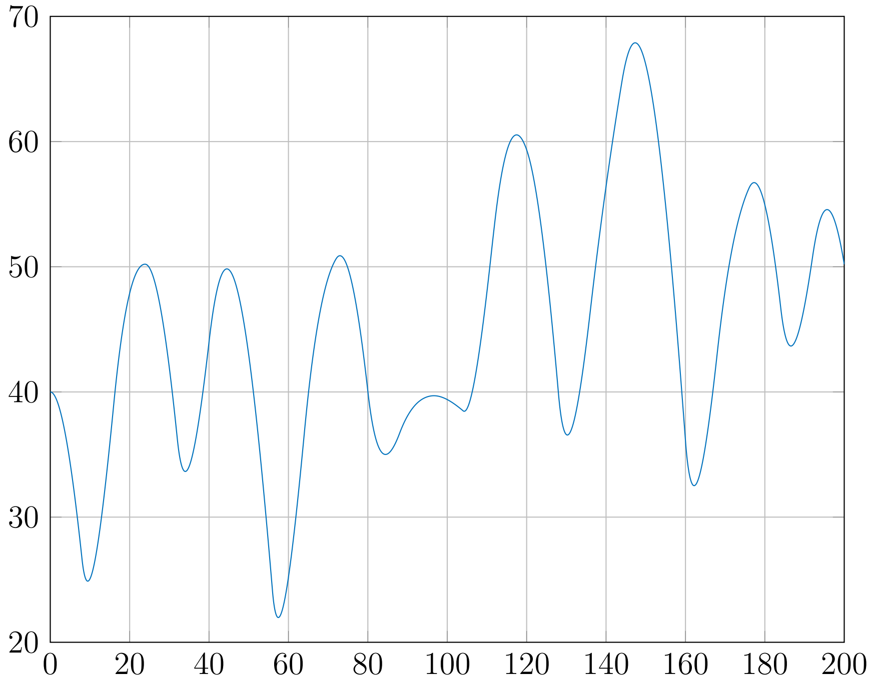


Figure 12: Step response using a zero-order hold of sample time 8 sec.

# 6 Question 8

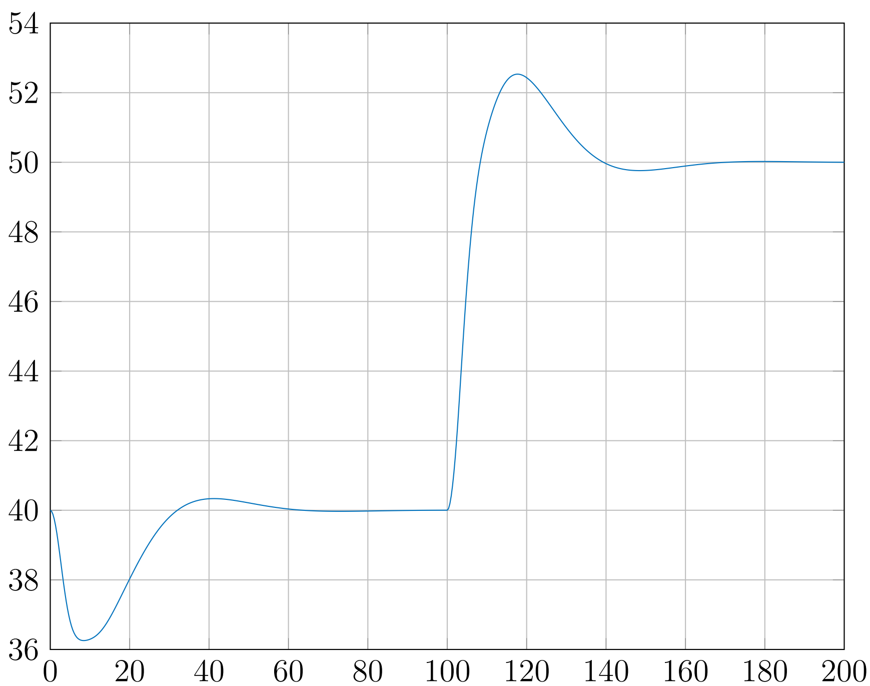


Figure 13: Response using a discrete controller with sampling time $T_s = 1$ sec.

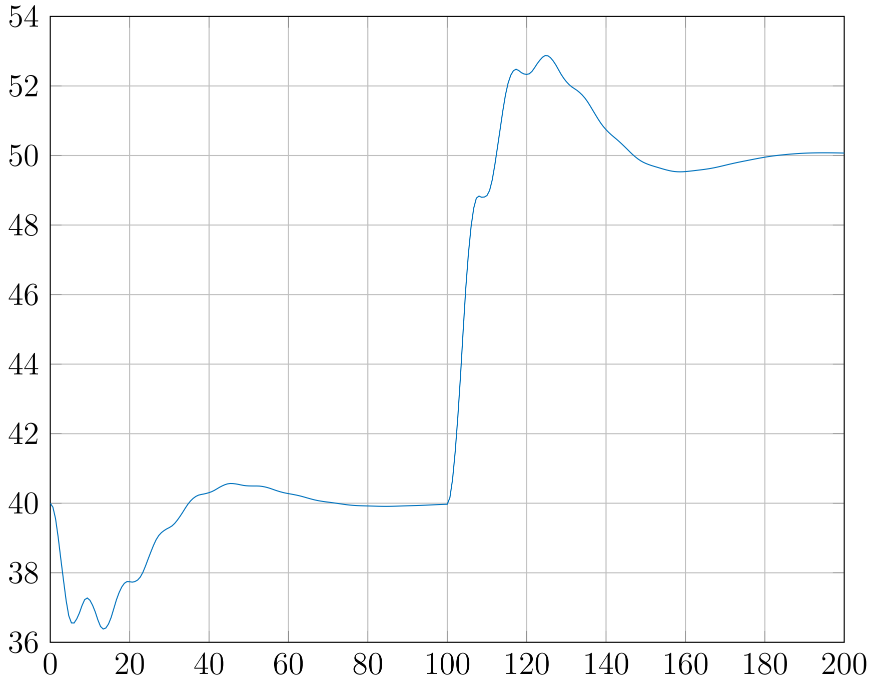


Figure 14: Response using a discrete controller with sampling time $T_s = 2$ sec.

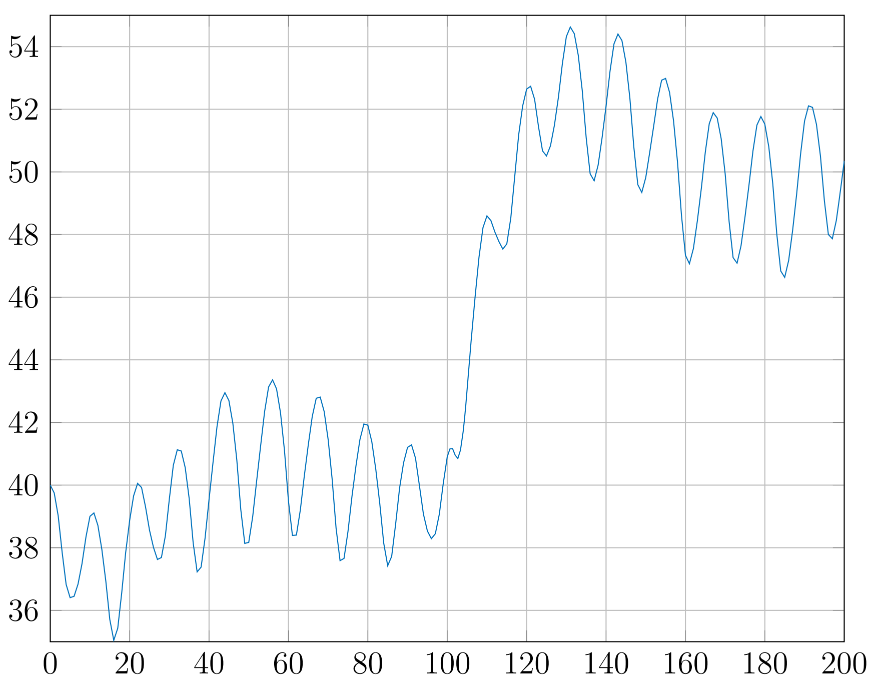


Figure 15: Response using a discrete controller with sampling time $T_s = 3$ sec.

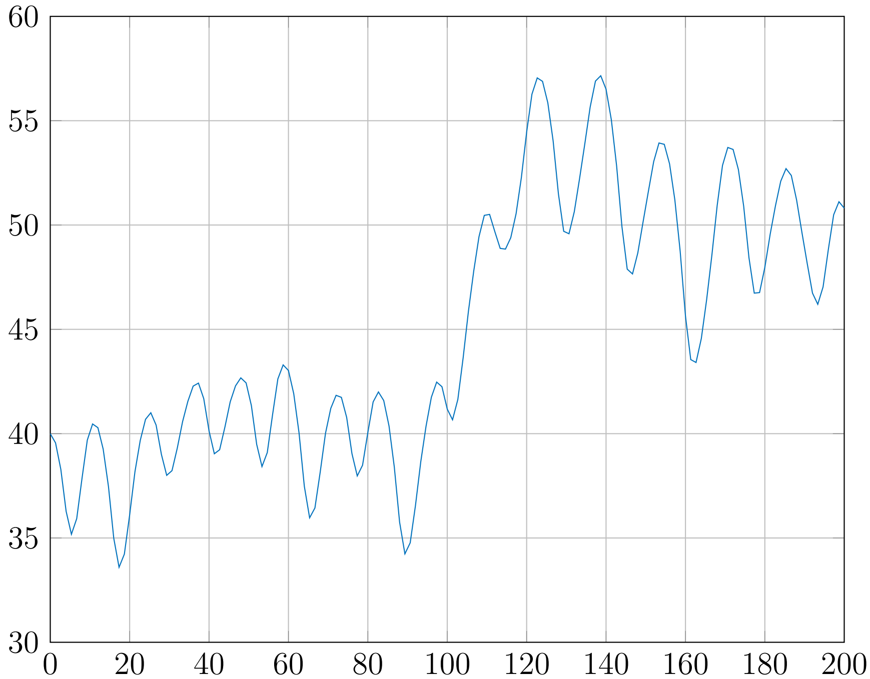


Figure 16: Response using a discrete controller with sampling time $T_s = 4$ sec.

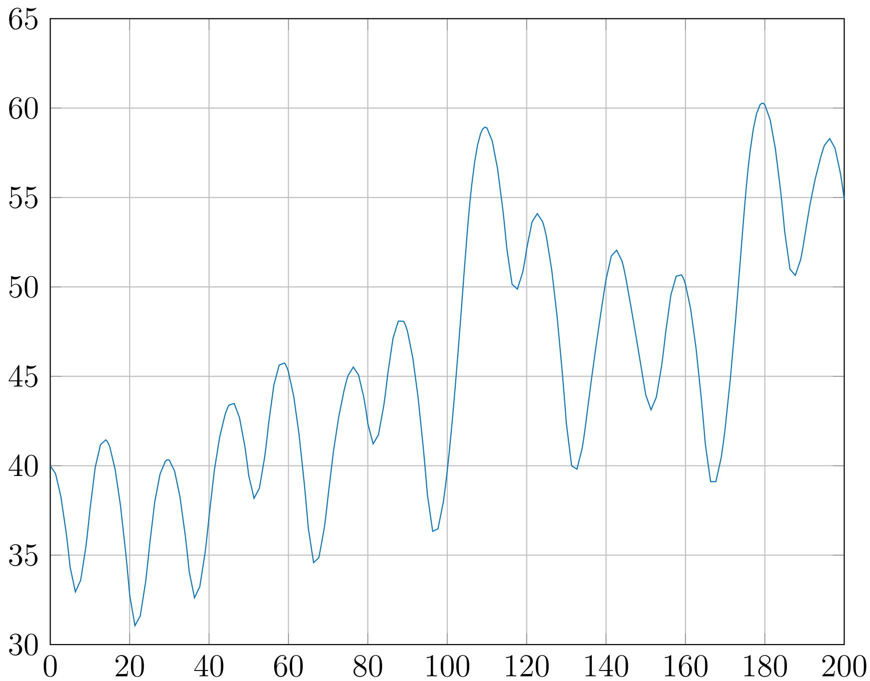


Figure 17: Response using a discrete controller with sampling time $T_s = 5$ sec.

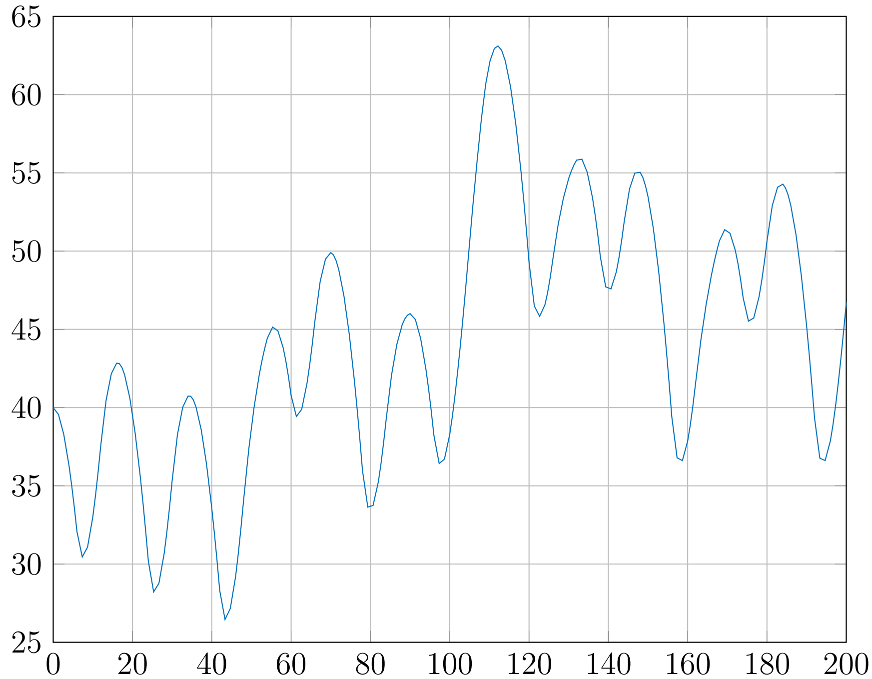


Figure 18: Response using a discrete controller with sampling time $T_s = 6$ sec.

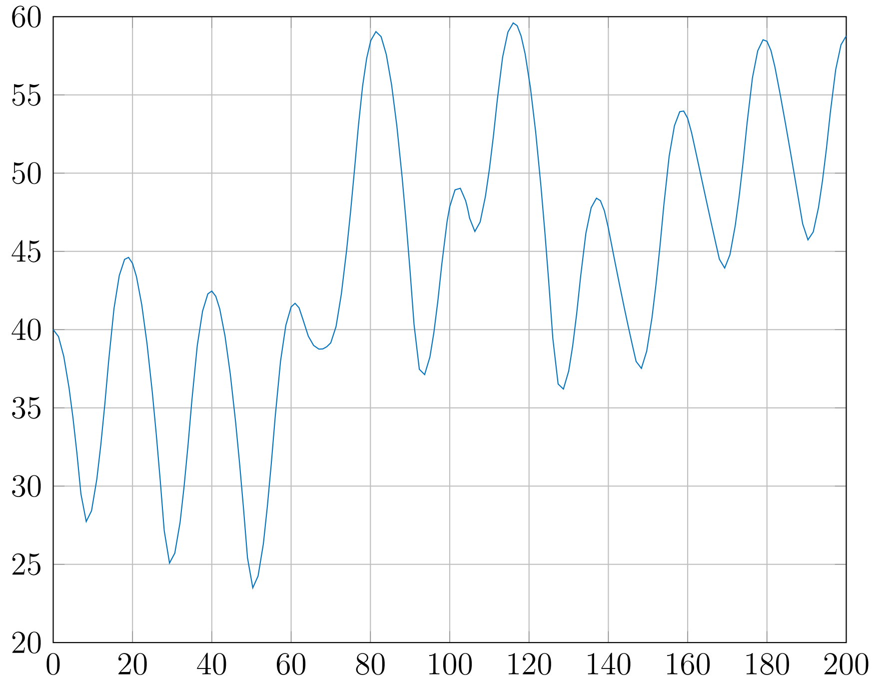


Figure 19: Response using a discrete controller with sampling time $T_s = 7$ sec.

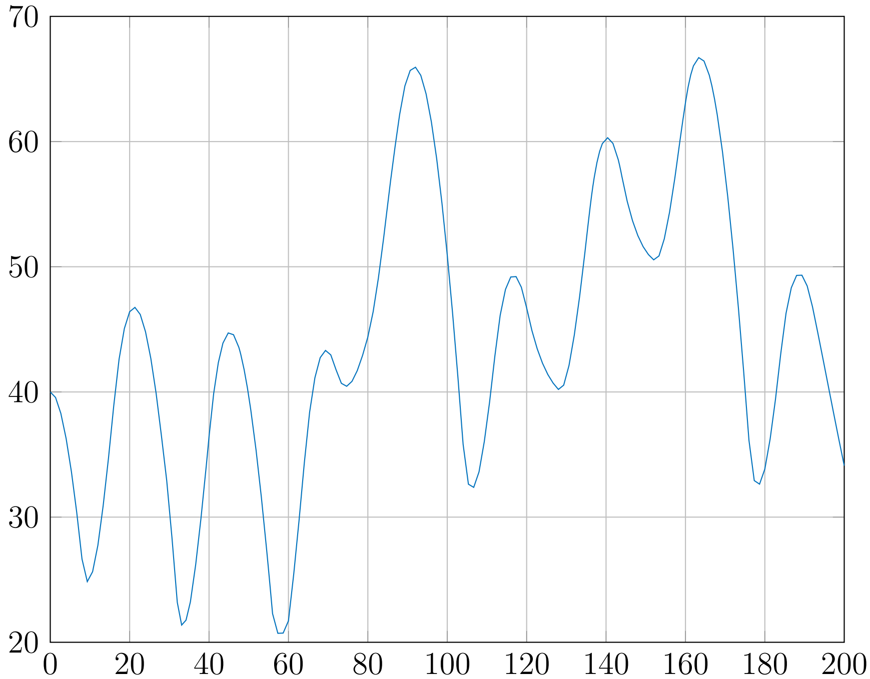


Figure 20: Response using a discrete controller with sampling time $T_s = 8$ sec.

# 7 Question 9

In theory[1], as a general rule of thumb, the sampling period should be selected so that

$$0.08 < T_s\omega_c < 0.3$$

where $\omega_c$ is the crossover frequency of the open-loop system, hence with $\omega_c = 0.3619$ rad/s

$$0.220 < T_s < 0.829 \ sec$$

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A second way one could look at this is to choose a sampling time such that there are 4 to 10 samples per rise time $T_r$. In this case where $T_r = 4.95$ sec, the resulting range of acceptable values for the sampling time is

$$0.495 < T_s < 1.2375 \ sec$$

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Another way one could look at this is to select a sampling frequency that is much higher than twice the bandwidth of the closed-loop system. Given our setting, the bandwidth $\omega_0 = 0.5819$ rad/s and

$$10\omega_0 < \omega_s < 30\omega_0$$

$$0.36 < T_s < 1.08 \ sec$$

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# References

[1] William Levine et al. The Control Handbook. CRC Press, 1996.