

# EL2450 – Assignment I

## 1 Question 3

The reference signal is a step of 10 units from time 100 seconds, with an offset of 40 units, as seen in figure 1.

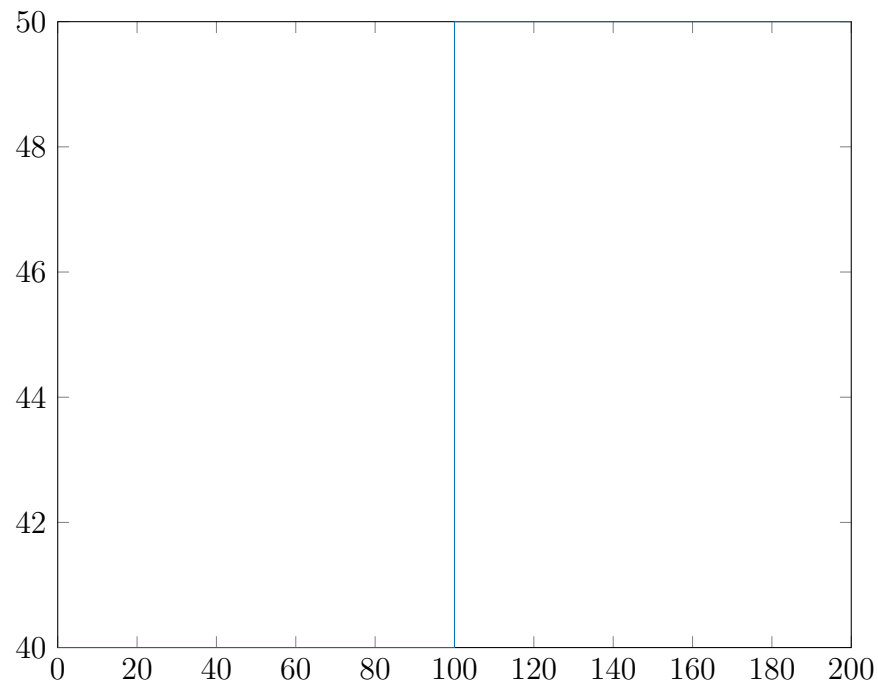


Figure 1: The reference signal.

## 2 Question 4

Table 1 illustrates the corresponding values of the  $K$ ,  $T_I$ ,  $T_D$  and  $N$  coefficients for set  $\chi$ ,  $\zeta$  and  $\omega_0$ .

$\chi$	$\zeta$	$\omega_0$	$K$	$T_I$	$T_D$	$M$
0.5	0.7	0.1	2.6062	14.4445	5.5143	0.9791
0.5	0.7	0.2	5.9243	9.3823	3.1938	1.1191
0.5	0.8	0.2	6.3325	10.3873	3.1523	1.1591

Table 1: Coefficients of the PID controller per set  $\chi$ ,  $\zeta$  and  $\omega_0$  values.

### 3 Question 5

Table 2 illustrates the rise time, overshoot and settling time for set values of  $\chi$ ,  $\zeta$  and  $\omega_0$ .

$\chi$	$\zeta$	$\omega_0$	$T_r$	$M$	$T_s$
0.5	0.7	0.1	8.2	14.40	39.0
0.5	0.7	0.2	5.0	34.67	23.7
0.5	0.8	0.2	4.95	31.72	24.25

Table 2: Rise time ( $T_r$ ) in seconds, overshoot ( $M$ ) as a percentage of the output's steady state value, and settling time  $T_s$  in seconds for set values of  $\chi$ ,  $\zeta$  and  $\omega_0$ .

Due to our step response requirements, the best control performance is given by the third set of  $(\chi, \zeta, \omega_0)$  parameters. All three requirements are fulfilled, as opposed to the case of the first set, and in comparison to the case of the second set, the rise time and overshoot are less, while their settling times are comparable.

Figures 2, 3 and 4 depict the step response for the three sets of  $(\chi, \zeta, \omega_0)$  parameters.

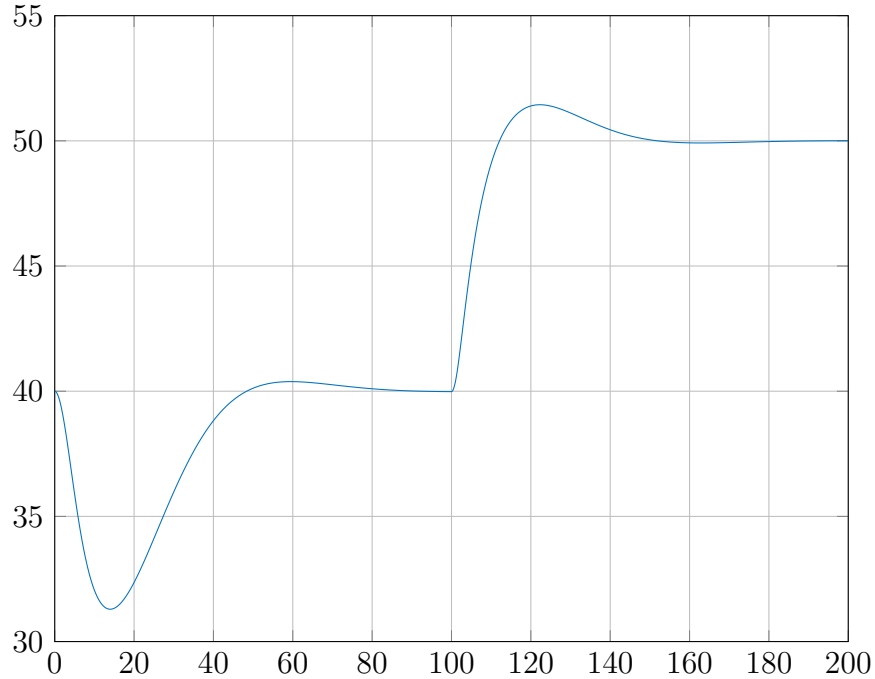


Figure 2: Step response for  $(\chi, \zeta, \omega_0) \equiv (0.5, 0.7, 0.1)$ .

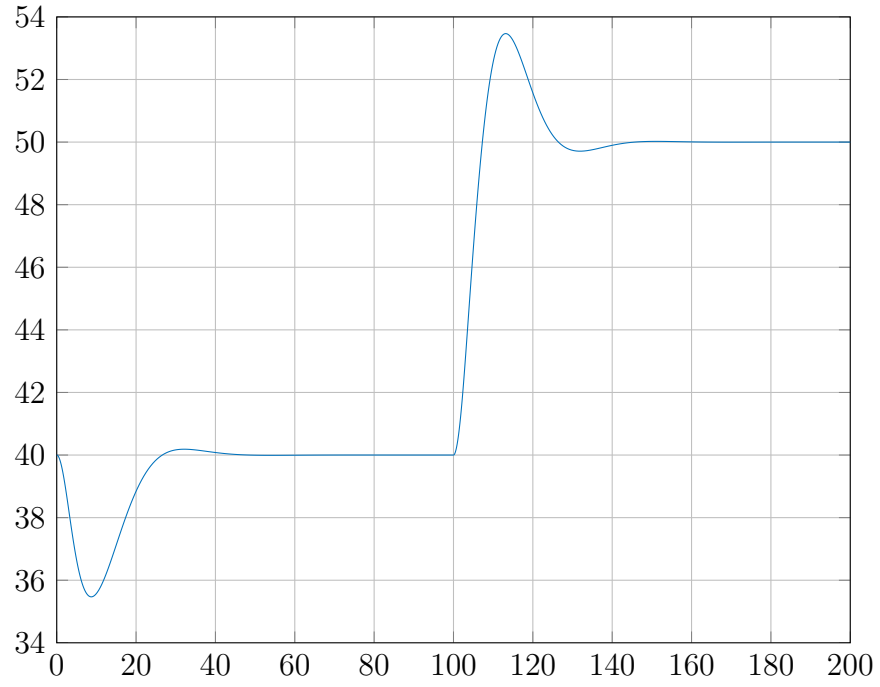


Figure 3: Step response for  $(\chi, \zeta, \omega_0) \equiv (0.5, 0.7, 0.2)$ .

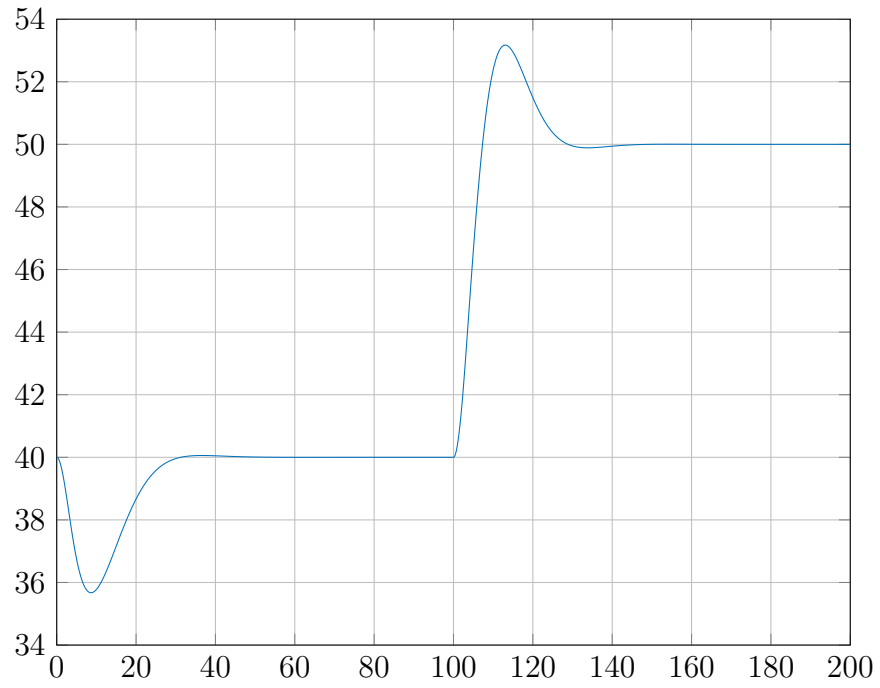


Figure 4: Step response for  $(\chi, \zeta, \omega_0) \equiv (0.5, 0.8, 0.2)$ .

## 4 Question 6

The open-loop transfer function is equal to the product of the transfer function of the controller  $F(s)$  and that of the process  $G(s)$ . The crossover frequency  $\omega_c$  is the frequency at which the magnitude of  $F(j\omega)G(j\omega)$  is 1.0.

In practice, we were able to derive the crossover frequency by using MATLAB's `margin()` function, with argument the open-loop transfer function.

Table 3 illustrates the crossover frequencies in rad/s for set values of the  $\chi$ ,  $\zeta$  and  $\omega_0$  parameters.

$\chi$	$\zeta$	$\omega_0$	$\omega_c$
0.5	0.7	0.1	0.2239
0.5	0.7	0.2	0.3426
0.5	0.8	0.2	0.3619

Table 3: Crossover frequencies depending on the set values of  $\chi$ ,  $\zeta$  and  $\omega_0$ .

## 5 Question 7

Figures 5-12 illustrate the step response when between the continuous controller and the plant a zero-order hold has been inserted, for sampling time varying between 1 and 8 seconds. The values of the  $\chi$ ,  $\zeta$  and  $\omega_0$  parameters were chosen to be the ones giving the best performance among the three sets, hence  $(\chi, \zeta, \omega_0) \equiv (0.5, 0.8, 0.2)$ .

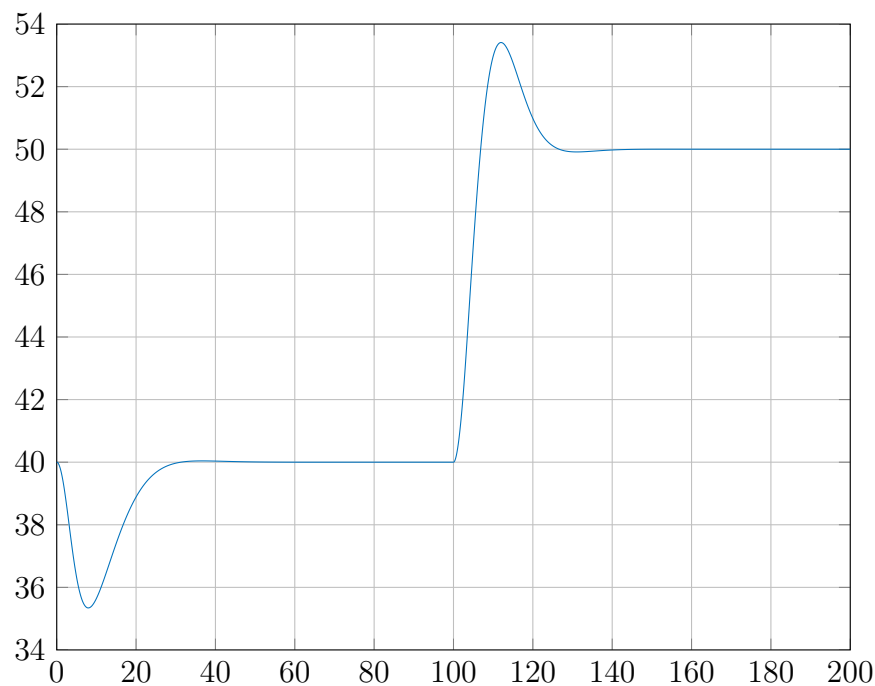


Figure 5: Step response using a zero-order hold of sample time 1 sec.

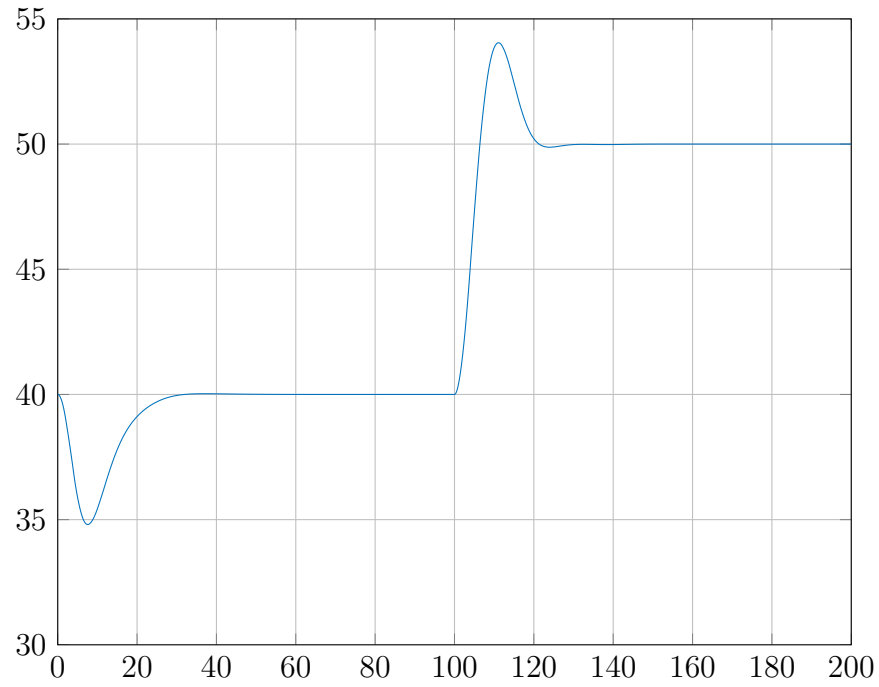


Figure 6: Step response using a zero-order hold of sample time 2 sec.

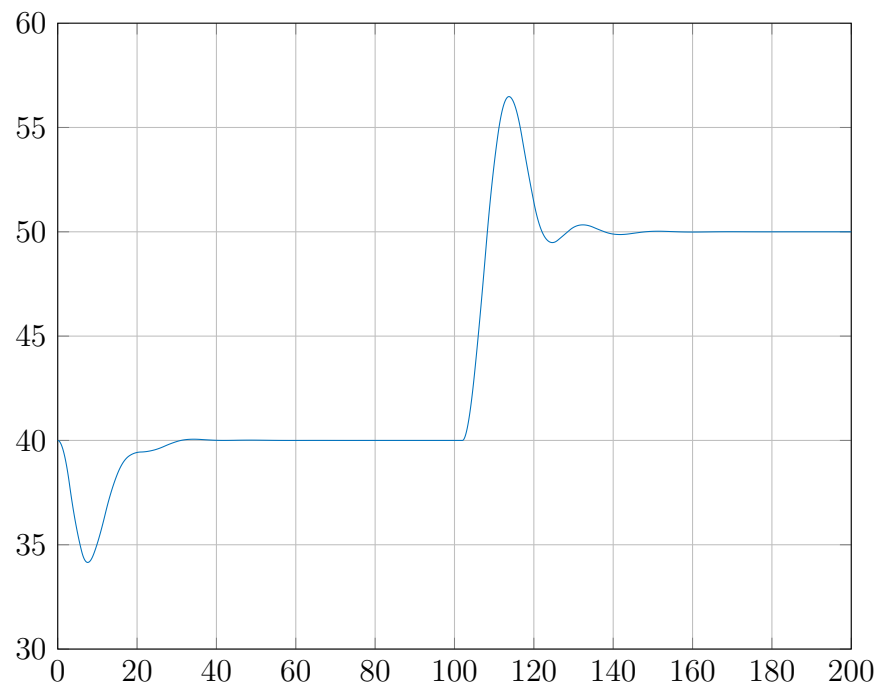


Figure 7: Step response using a zero-order hold of sample time 3 sec.

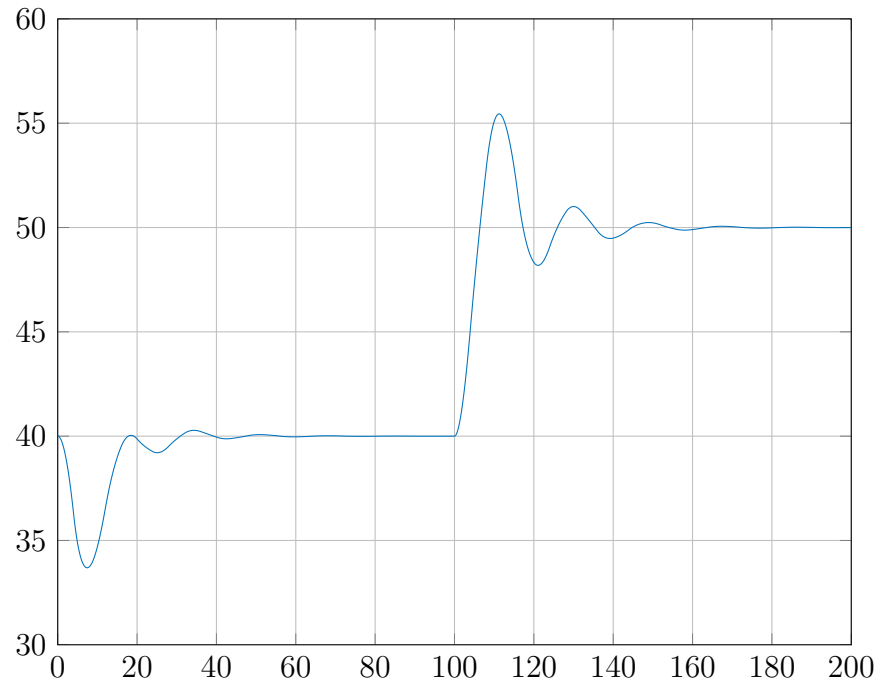


Figure 8: Step response using a zero-order hold of sample time 4 sec.

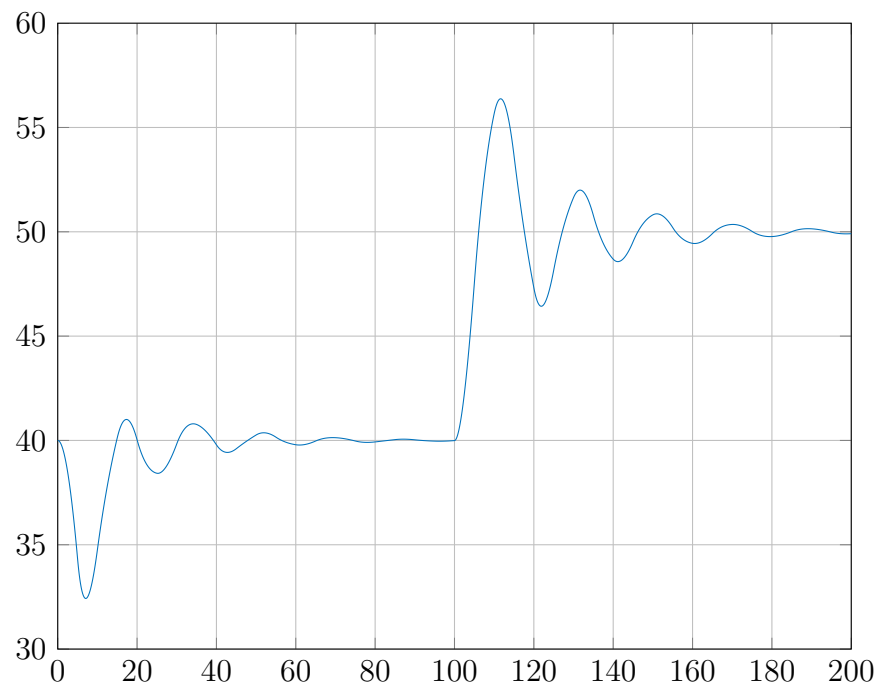


Figure 9: Step response using a zero-order hold of sample time 5 sec.

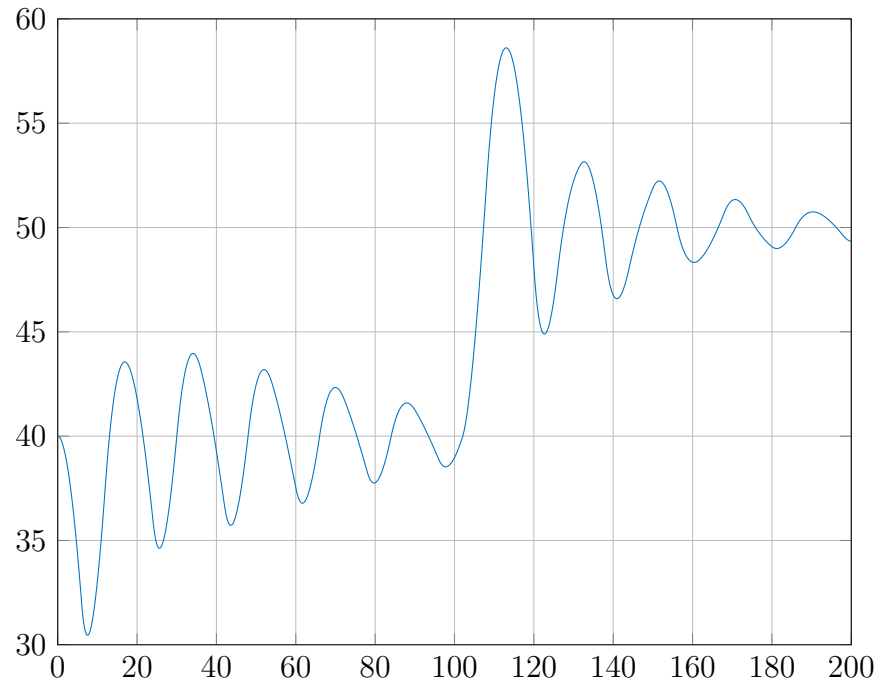


Figure 10: Step response using a zero-order hold of sample time 6 sec.

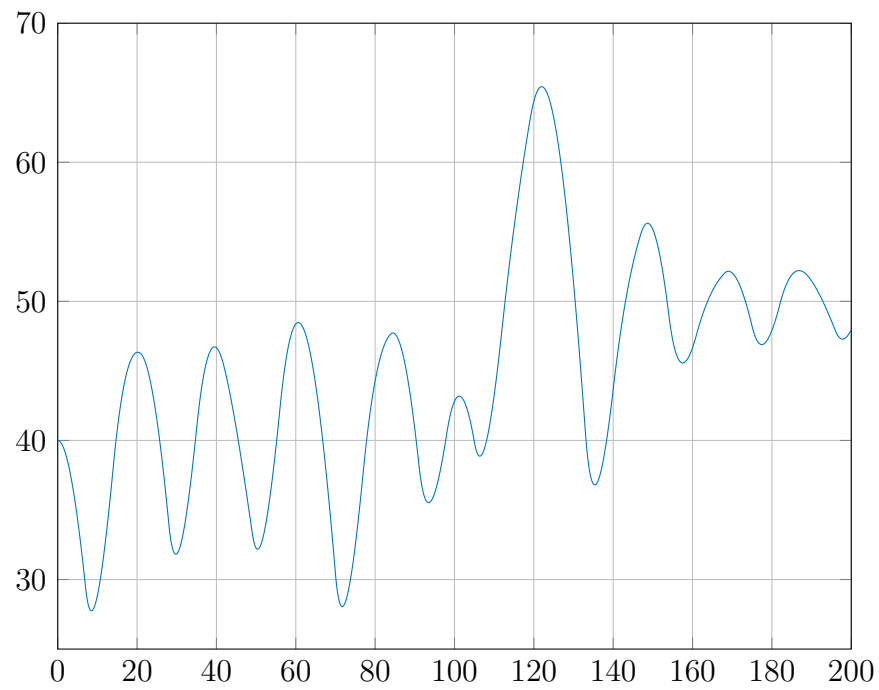


Figure 11: Step response using a zero-order hold of sample time 7 sec.

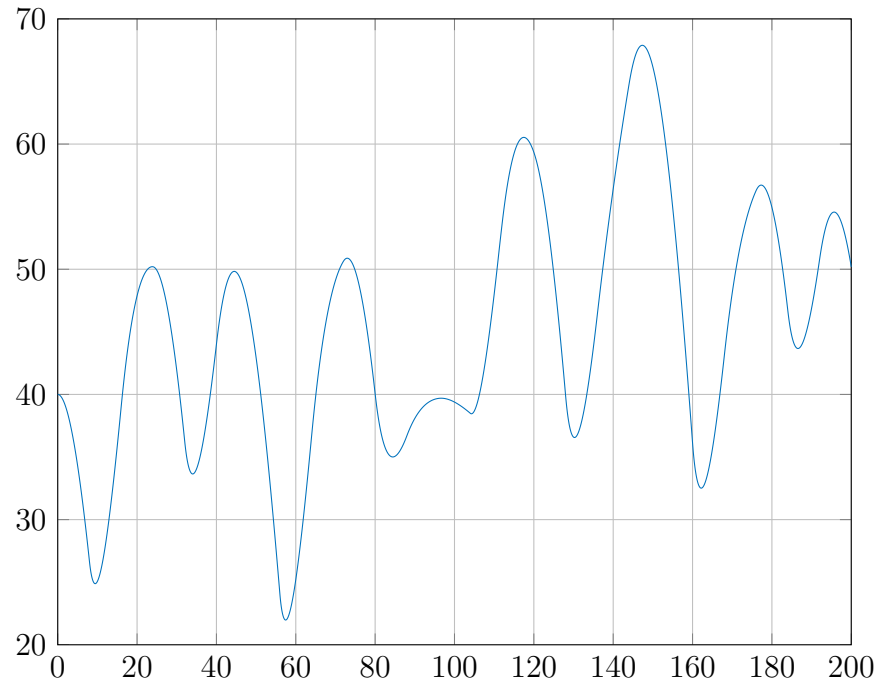


Figure 12: Step response using a zero-order hold of sample time 8 sec.

## 6 Question 8

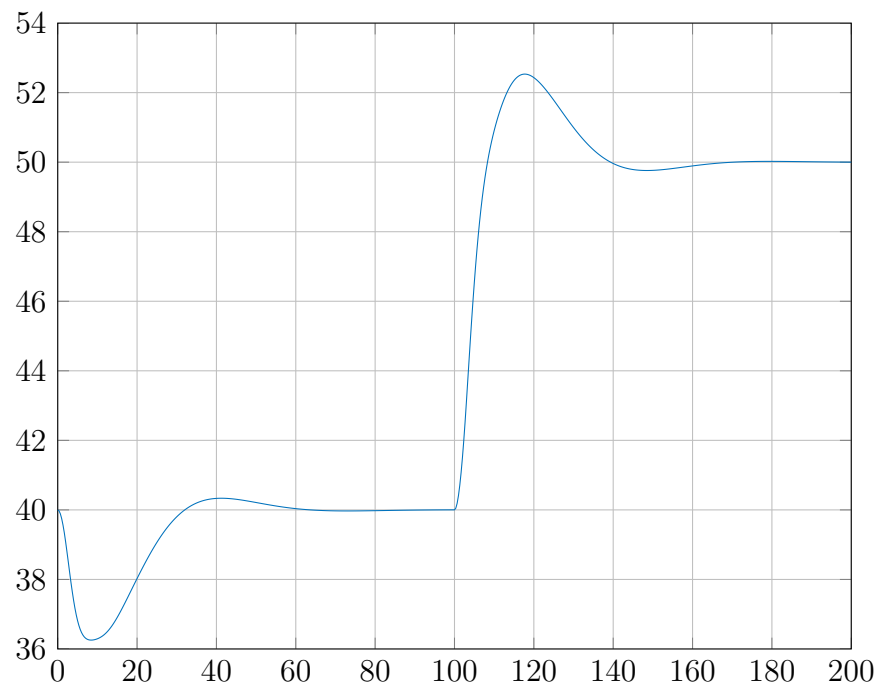


Figure 13



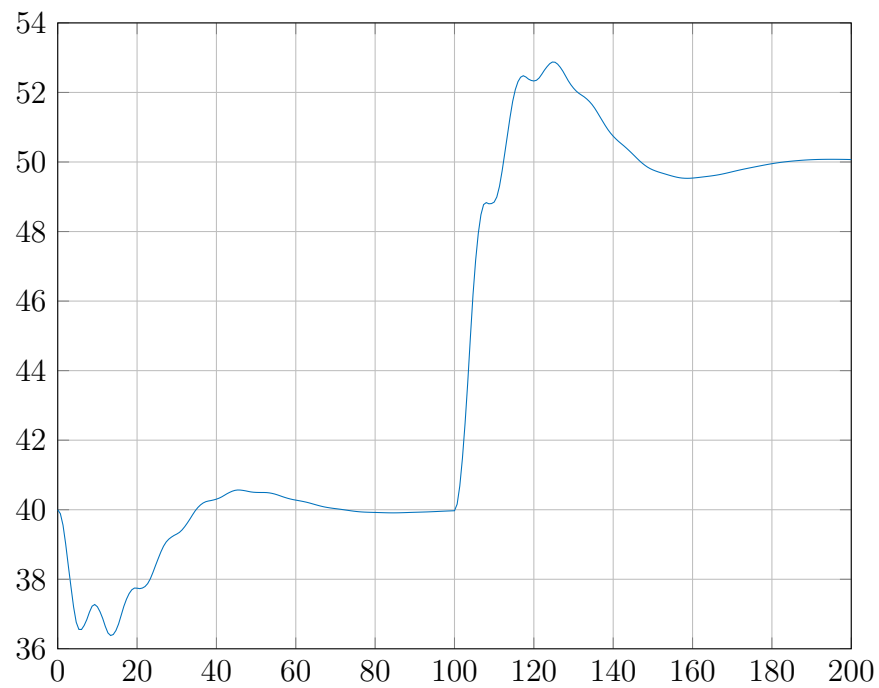


Figure 14

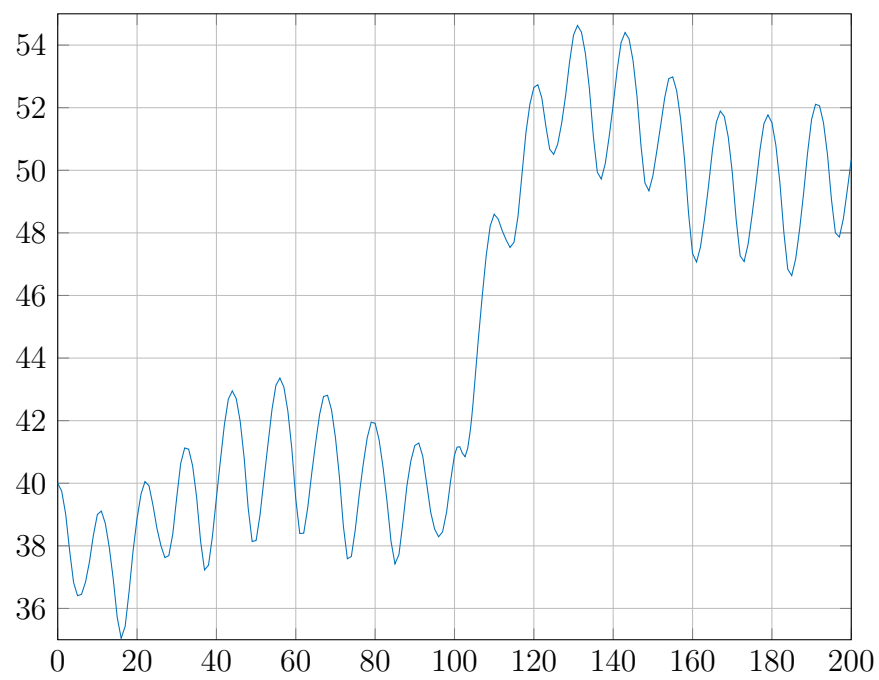


Figure 15

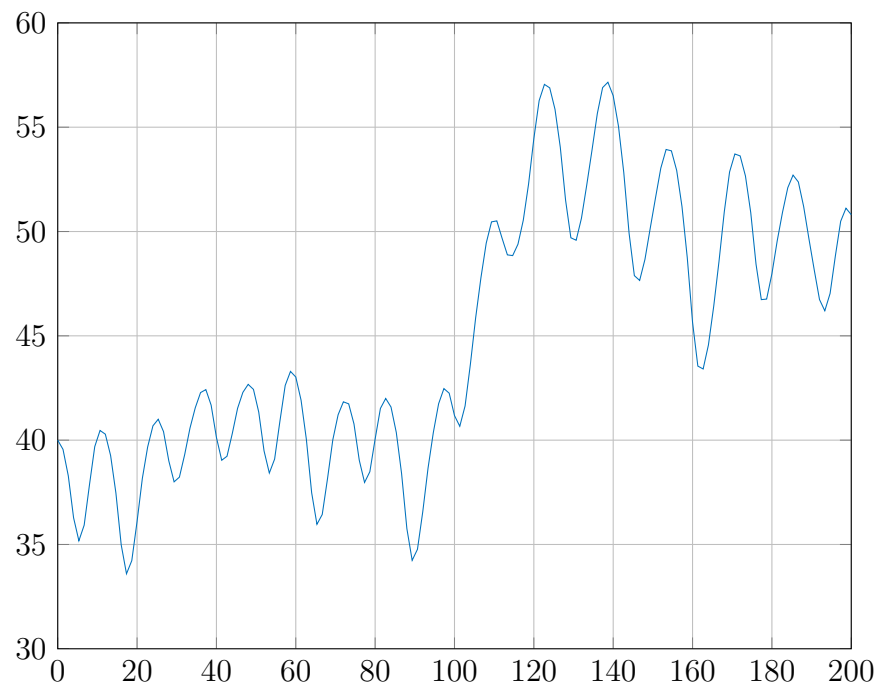


Figure 16

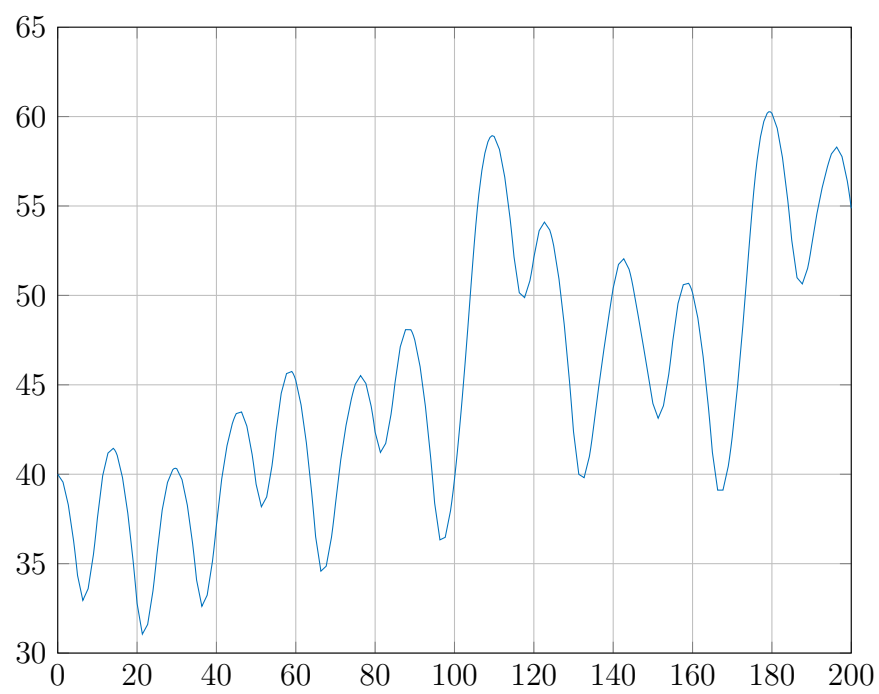


Figure 17

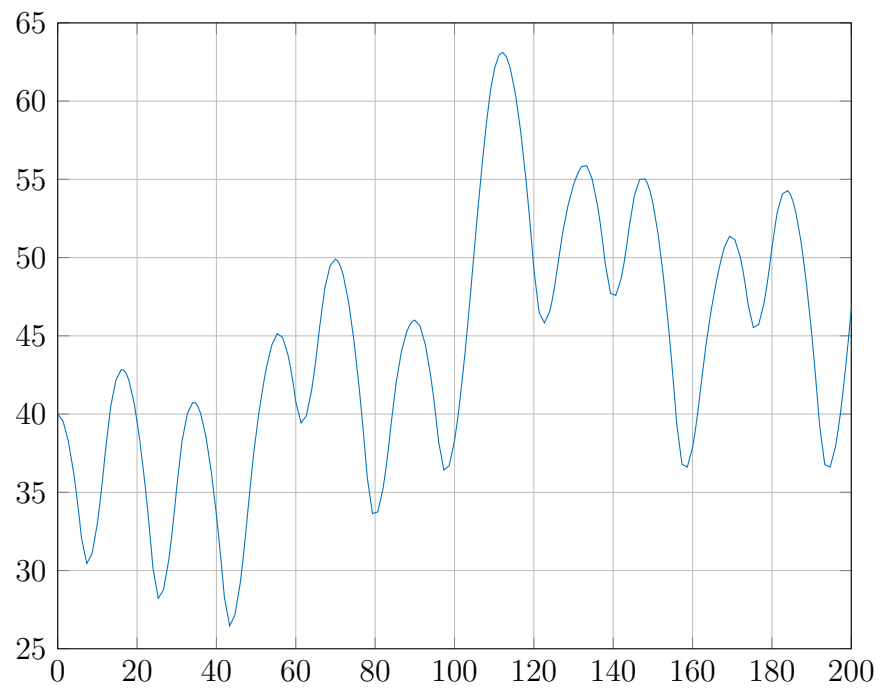


Figure 18

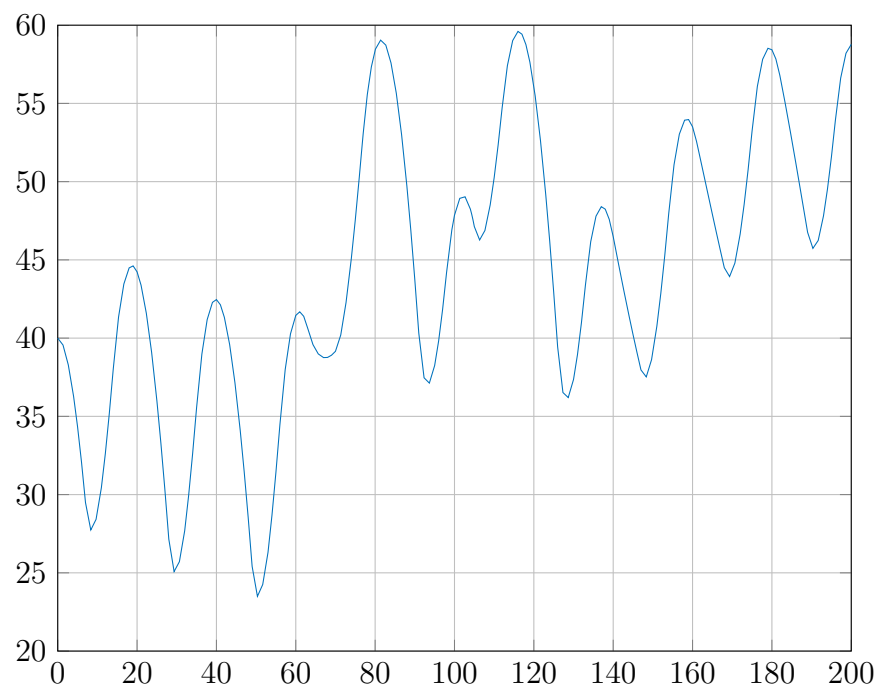


Figure 19

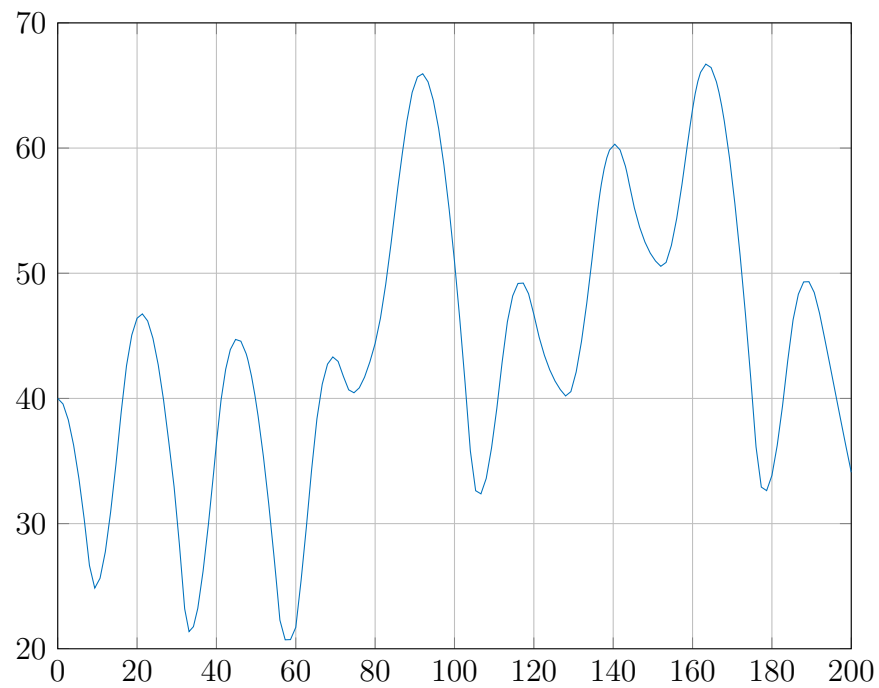


Figure 20