Homework 3 in EL2450 Hybrid and Embedded Control Systems

First name1 Last name1
person number
email

First name2 Last name2
person number
email

First name3 Last name3
person number
email

First name4 Last name4
person number
email

Task 1

Since

$$
\begin{aligned}
u_\omega &= \frac{u_r + u_l}{2} \\
u_\Psi &= u_r - u_l
\end{aligned}
$$

$$\Leftrightarrow$$

$$
\begin{aligned}
u_l &= u_\omega - \frac{u_\Psi}{2} \\
u_r &= u_\omega + \frac{u_\Psi}{2}
\end{aligned}
$$

Task 2

Task 3

Task 4

Task 5

Task 6

Task 7

$$\begin{aligned}\theta[k+1] &= \theta[k] + \frac{T_s R}{L} u_\Psi[k] \\ &= \theta[k] + \frac{T_s R}{L} K_\Psi^R (\theta^R - \theta[k]) \\ &= \theta[k](1 - \frac{T_s R}{L} K_\Psi^R) + \frac{T_s R}{L} K_\Psi^R \theta^R\end{aligned}$$

By subtracting θ^R from both sides one gets

$$\begin{aligned}\theta[k+1] - \theta^R &= \theta[k](1 - \frac{T_s R}{L} K_\Psi^R) + (\frac{T_s R}{L} K_\Psi^R - 1)\theta^R \\ &= (\theta[k] - \theta^R)(1 - \frac{T_s R}{L} K_\Psi^R) \qquad (1)\end{aligned}$$

We now define state θ' as

$$\theta'[k] = \theta[k] - \theta^R$$

Then, equation 1 becomes

$$\theta'[k+1] = \theta'[k](1 - \frac{T_s R}{L} K_\Psi^R)$$

In order for this system to be stable, that is, $\theta \to \theta^R$, the solution of the equation inside the parentheses should lie inside the unit circle:

$$\begin{aligned}\left|1 - \frac{T_s R}{L} K_\Psi^R\right| &< 1 \\ -1 < 1 - \frac{T_s R}{L} K_\Psi^R &< 1 \\ -2 < -\frac{T_s R}{L} K_\Psi^R &< 0 \\ 0 < \frac{T_s R}{L} K_\Psi^R &< 2 \\ 0 < K_\Psi^R &< \frac{2L}{R T_s} \qquad (2)\end{aligned}$$

Hence, the maximum value K_{Ψ}^{R} can take is $K_{\Psi,max}^{R} = \dfrac{2L}{RT_s}$

Theoretically, the value of K_{Ψ}^{R} can be chosen to be any value inside the interval defined in inequality 2. However, first, it would be wise to choose a value that is far enough from the maximum value so as to avoid overshoot, but close enough to it, so that convergence happens in reasonable time. Hence, in practice, it is reasonable that one would need to experiment with different values and choose one that results in balancing a small angular error, a minimal overshoot, if any, and a quick enough settling time.

Eventually, the choice made for K_{Ψ}^{R} was $K_{\Psi}^{R} = 0.5K_{\Psi,max}^{R}$.

Task 8

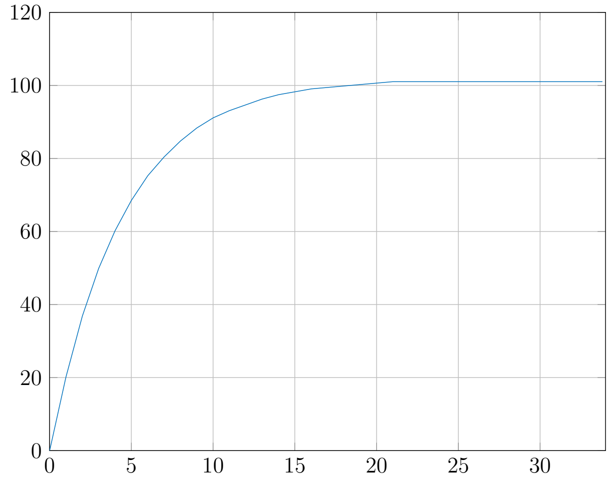

Figure 1: The orientation of the robot over time for $K_{\Psi}^{R} = 0.1K_{\Psi,max}^{R}$

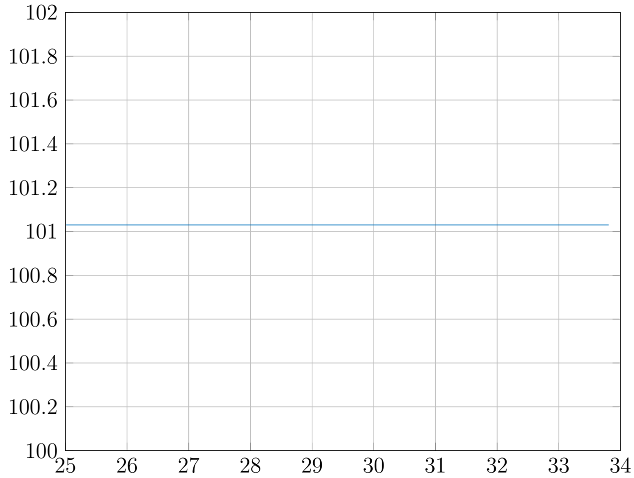

Figure 2: The steady state orientation of the robot for $K_{\Psi}^{R} = 0.1K_{\Psi,max}^{R}$

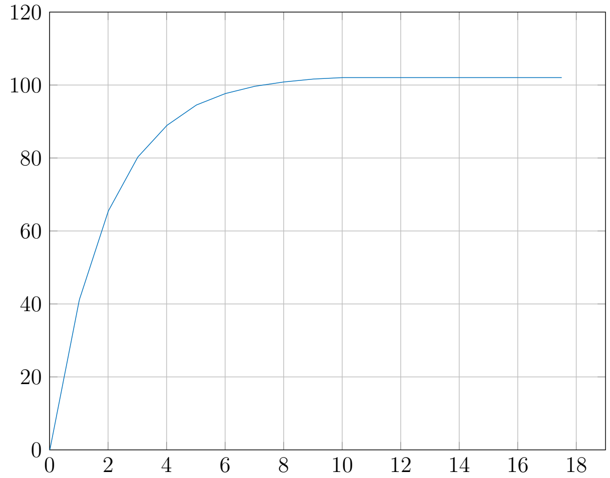

Figure 3: The orientation of the robot over time for $K_{\Psi}^{R} = 0.2K_{\Psi,max}^{R}$

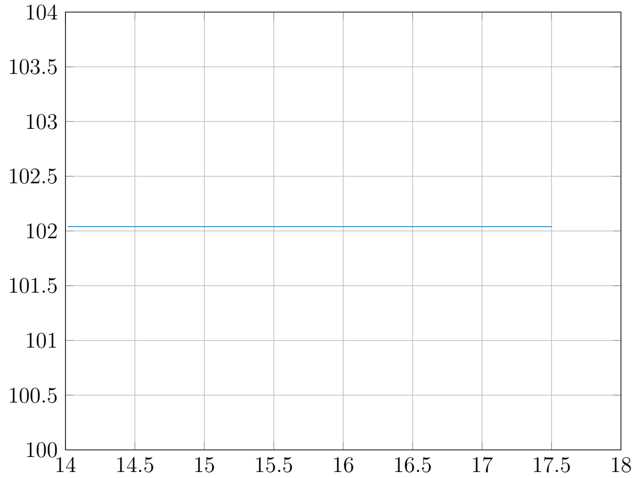

Figure 4: The steady state orientation of the robot for $K_{\Psi}^{R} = 0.2K_{\Psi,max}^{R}$

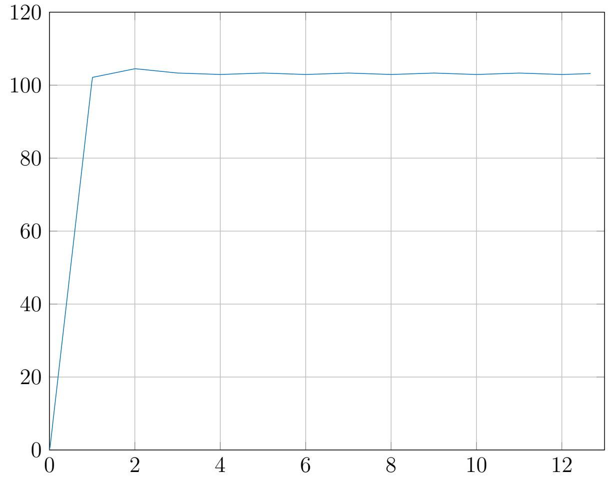

Figure 5: The orientation of the robot over time for $K_{\Psi}^{R} = 0.5K_{\Psi,max}^{R}$

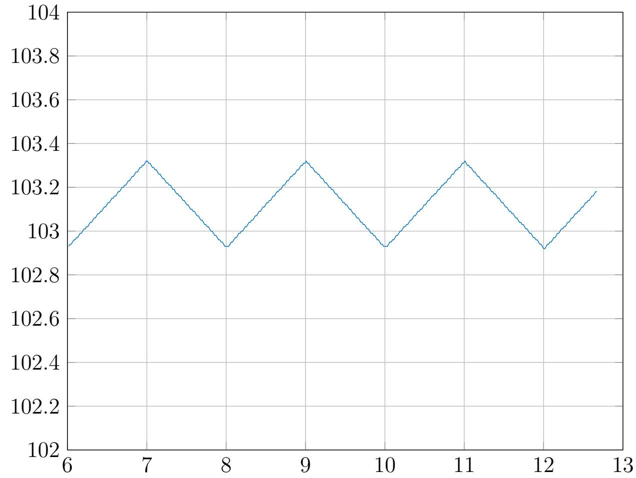

Figure 6: The steady state orientation of the robot for $K_{\Psi}^{R} = 0.5K_{\Psi,max}^{R}$

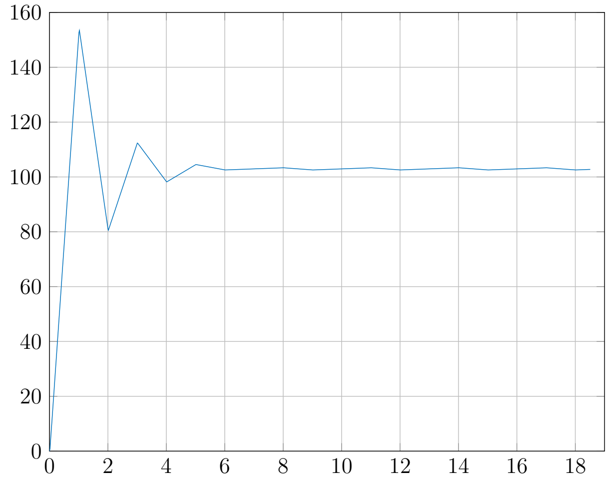

Figure 7: The orientation of the robot over time for $K_{\Psi}^{R} = 0.75K_{\Psi,max}^{R}$

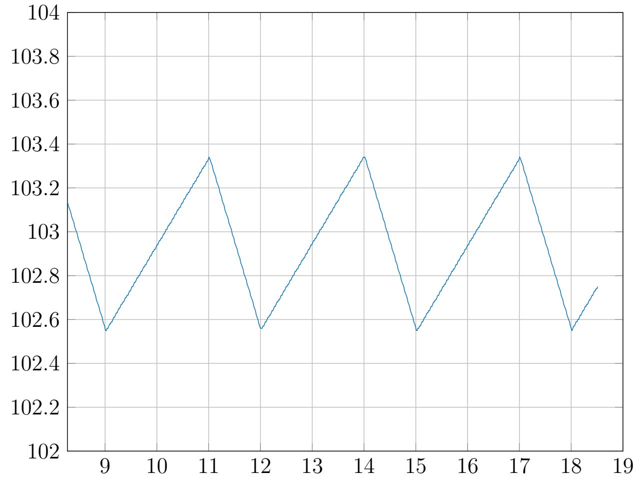

Figure 8: The steady state orientation of the robot for $K_{\Psi}^{R} = 0.75K_{\Psi,max}^{R}$

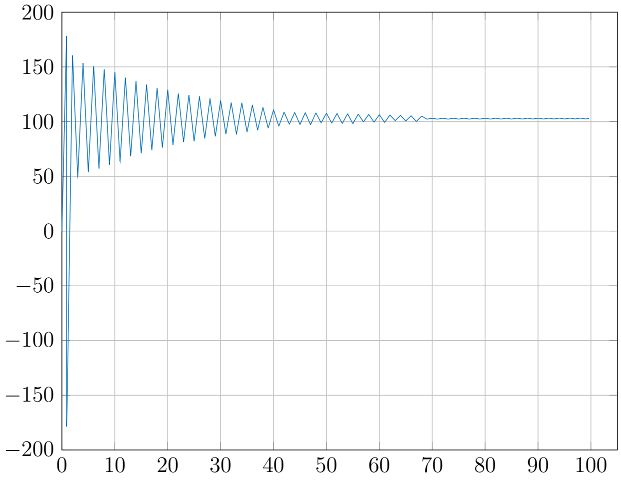

Figure 9: The orientation of the robot over time for $K_{\Psi}^{R} = K_{\Psi,max}^{R}$. This is the upper limit value of K_{Ψ}^{R} before the system becomes unstable

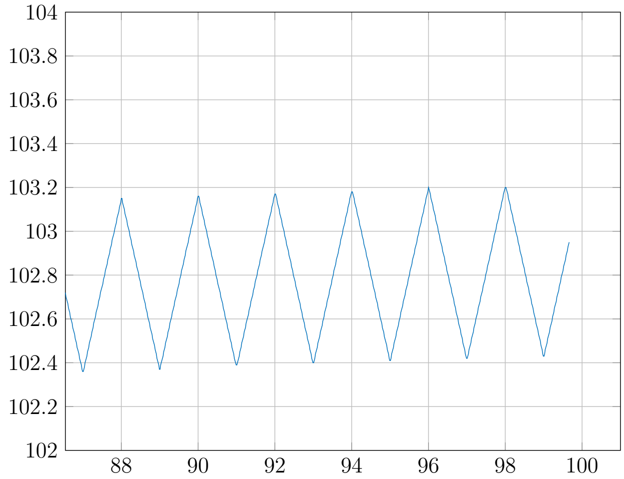

Figure 10: The steady state orientation of the robot for $K_{\Psi}^{R} = K_{\Psi,max}^{R}$

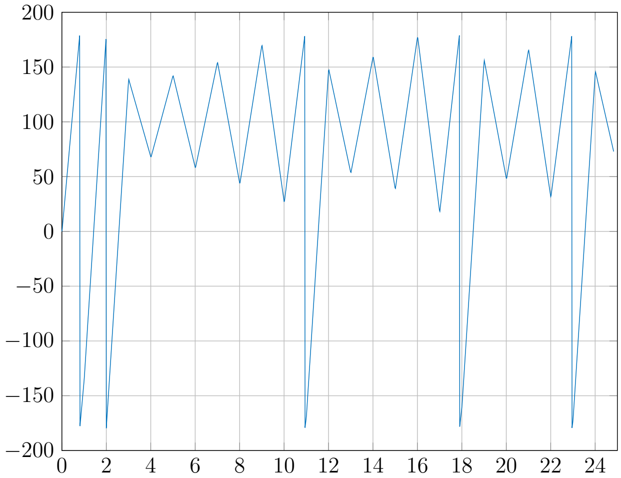

Figure 11: The orientation of the robot over time for $K_{\Psi}^{R} = K_{\Psi,max}^{R} + 1$. The system is indeed unstable

Task 9

Task 10

Task 11

Task 12

Task 13

Task 14

Task 15

Task 16

Task 17

Task 18

Task 19

Task 20

Task 21

Task 22

References

[1] Hassan K Khalil. *Nonlinear systems.* Prentice Hall, Upper Saddle river, 3. edition, 2002. ISBN 0-13-067389-7.

[2] Tobias Oetiker, Hubert Partl, Irene Hyna, and Elisabeth Schlegl. *The Not So Short Introduction to LaTeX2ε.* Oetiker, OETIKER+PARTNER AG, Aarweg 15, 4600 Olten, Switzerland, 2008. http://www.ctan.org/info/lshort/.

[3] Shankar Sastry. *Nonlinear systems: analysis, stability, and control*, volume 10. Springer, New York, N.Y., 1999. ISBN 0-387-98513-1.