Homework 3 in EL2450 Hybrid and Embedded Control Systems

First name1 Last name1
person number
email

First name2 Last name2
person number
email

First name3 Last name3
person number
email

First name4 Last name4
person number
email

Task 1

Since

$$u_\omega = \frac{u_r + u_l}{2}$$
$$u_\Psi = u_r - u_l$$

$$\Leftrightarrow$$

$$u_l = u_\omega - \frac{u_\Psi}{2}$$
$$u_r = u_\omega + \frac{u_\Psi}{2}$$

Task 2

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$$
\begin{aligned}
\theta[k+1] &= \theta[k] + \frac{T_s R}{L} u_\Psi[k] \\
&= \theta[k] + \frac{T_s R}{L} K_\Psi^R (\theta^R - \theta[k]) \\
&= \theta[k](1 - \frac{T_s R}{L} K_\Psi^R) + \frac{T_s R}{L} K_\Psi^R \theta^R
\end{aligned}
$$

By subtracting θ^R from both sides one gets

$$
\begin{aligned}
\theta[k+1] - \theta^R &= \theta[k](1 - \frac{T_s R}{L} K_\Psi^R) + (\frac{T_s R}{L} K_\Psi^R - 1)\theta^R \\
&= (\theta[k] - \theta^R)(1 - \frac{T_s R}{L} K_\Psi^R) \qquad (1)
\end{aligned}
$$

We now define state θ' as

$$\theta'[k] = \theta[k] - \theta^R$$

Then, equation 1 becomes

$$\theta'[k+1] = \theta'[k](1 - \frac{T_s R}{L} K_\Psi^R)$$

In order for this system to be stable, that is, $|\theta - \theta^R| \to 0$ as $k \to \infty$, the solution of the equation inside the parentheses should lie inside the unit circle:

$$
\begin{aligned}
\left|1 - \frac{T_s R}{L} K_\Psi^R\right| &< 1 \\
-1 < 1 - \frac{T_s R}{L} K_\Psi^R &< 1 \\
-2 < -\frac{T_s R}{L} K_\Psi^R &< 0 \\
0 < \frac{T_s R}{L} K_\Psi^R &< 2 \\
0 < K_\Psi^R &< \frac{2L}{R T_s} \qquad (2)
\end{aligned}
$$

Hence, the maximum value K_{Ψ}^R can take for the system to be marginally stable is $K_{\Psi,max}^R = \dfrac{2L}{RT_s}$

Theoretically, the value of K_{Ψ}^R can be chosen to be any value inside the interval defined in inequality 2. However, first, it would be wise to choose a value that is far enough from the maximum value so as to avoid overshoot, but close enough to it, so that convergence happens in reasonable time. Hence, in practice, it is reasonable that one would need to experiment with different values and choose one that results in balancing a small angular error, a minimal overshoot, if any, and a quick enough settling time.

Eventually, the choice made for K_{Ψ}^R was $K_{\Psi}^R = \frac{1}{2}K_{\Psi,max}^R$.

Task 8

For the purpose of simulating this part of the controller, the initial point of the robot was taken to be $I(x_0, y_0) \equiv (0,0)$. The goal was set to $G(x_g, y_g) \equiv (-0.37, 1.68)$, which is node 1 in the simulation environment. The angle between the line connecting I and G and the x-axis is hence $\theta_R = tan^{-1}(1.68/-0.37) = 102.42$ degrees.

Figures 2, 4, 6, 8, 10 show the angular response of the robot for different values of K_{Ψ}^R inside the interval set by inequality 2. Figure 12 verifies that the upper limit for K_{Ψ}^R is indeed $K_{\Psi,max}^R = \frac{2L}{RT_s}$ by showing that the angular response of the robot cannot converge for $K_{\Psi}^R > K_{\Psi,max}^R$.

Here, one can see that the smaller the value of K_{Ψ}^R is, the larger the settling time, the lower the rise time and the smoother the response is. However, as the value of K_{Ψ}^R increases, the steady-state response begins to oscillate, with the amplitude of this oscillation proportional to the value of K_{Ψ}^R.

Figures 3, 5, 7, 9 and 3 focus on the steady-state value of the aforementioned responses. As it is evident, none of the responses converge to the value $\theta_R = 102.42$. This is reasonable since with only a purely proportional control signal, as the angular error, i.e. $e(\theta) = \theta^R - \theta$, tends to zero, the product of K_{Ψ}^R and $e(\theta)$ isn't large enough to force the robot to rotate exactly θ^R degrees.

Another way to look at this is by looking at the steady-state response of the system for a step input of magnitude θ^R. Figure 1 shows the structure of the system. The z-transform of the input is then $R(z) = \dfrac{\theta^R}{1 - z^{-1}}$ and the equation of the closed-loop system is

$$Y(z) = \frac{K_{\Psi}^R G(z)}{1 + K_{\Psi}^R G(z)} R(z)$$

The steady-state response is

$$lim_{t\to\infty} y(t) = lim_{z\to 1}(1 - z^{-1})\frac{\theta^R}{1 - z^{-1}}\frac{K_{\Psi}^R G(z)}{1 + K_{\Psi}^R G(z)} = \theta^R \cdot lim_{z\to 1}\frac{K_{\Psi}^R G(z)}{1 + K_{\Psi}^R G(z)}$$

The steady-state response cannot reach exactly θ^R as the above limit cannot converge to 1 under our limitations for K_{Ψ}^R and the dynamics of $G(z)$.

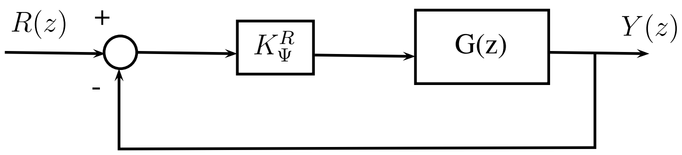

Figure 1: The structure of the system under rotational control.

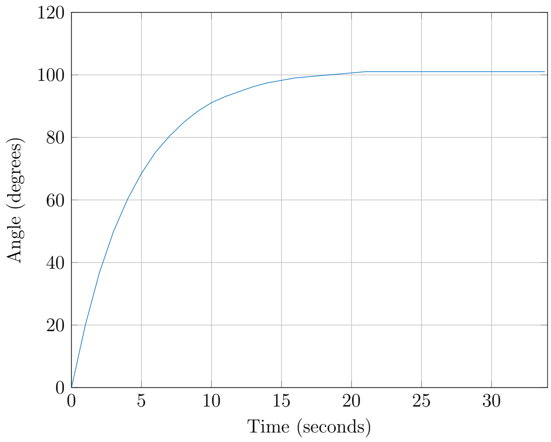

Figure 2: The orientation of the robot over time for $K_\Psi^R = 0.1K_{\Psi,max}^R$

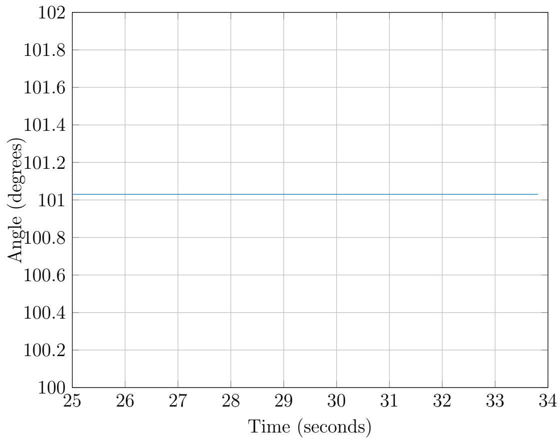

Figure 3: The steady state orientation of the robot for $K_\Psi^R = 0.1K_{\Psi,max}^R$

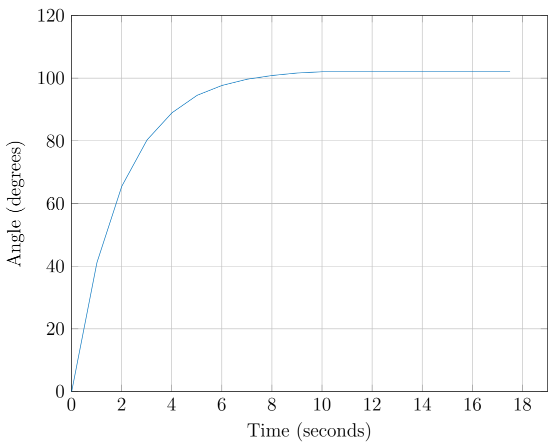

Figure 4: The orientation of the robot over time for $K_\Psi^R = 0.2K_{\Psi,max}^R$

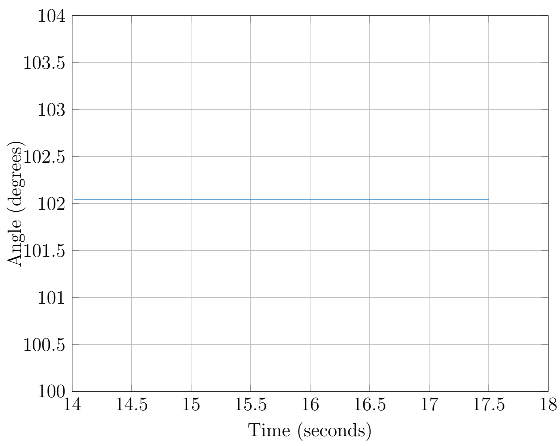

Figure 5: The steady state orientation of the robot for $K_\Psi^R = 0.2K_{\Psi,max}^R$

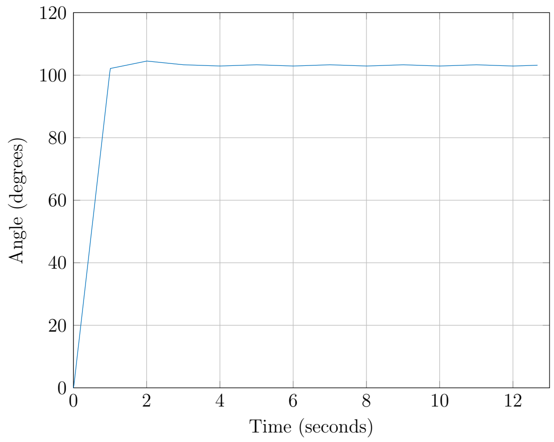

Figure 6: The orientation of the robot over time for $K^R_\Psi = 0.5K^R_{\Psi,max}$

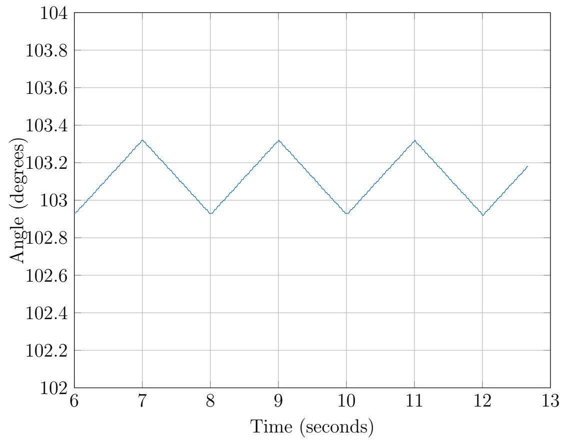

Figure 7: The steady state orientation of the robot for $K^R_\Psi = 0.5K^R_{\Psi,max}$

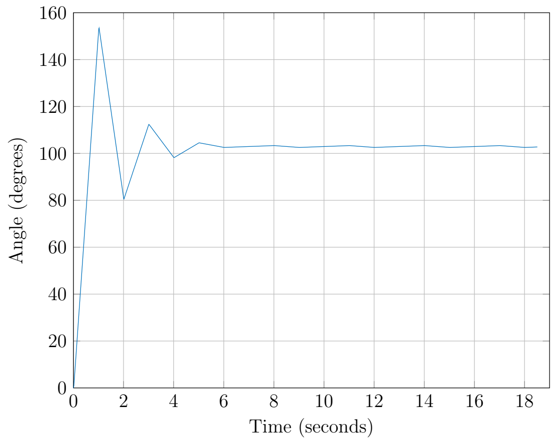

Figure 8: The orientation of the robot over time for $K^R_\Psi = 0.75K^R_{\Psi,max}$

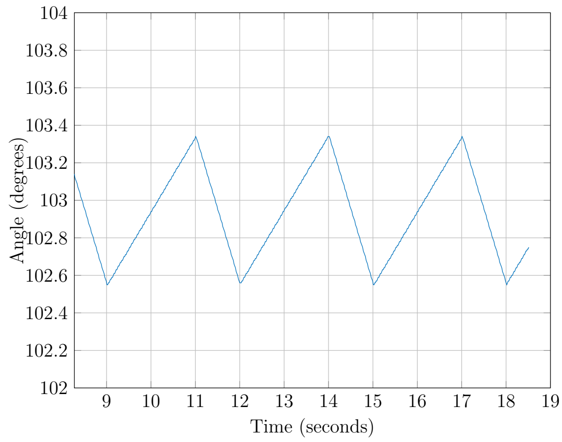

Figure 9: The steady state orientation of the robot for $K^R_\Psi = 0.75K^R_{\Psi,max}$

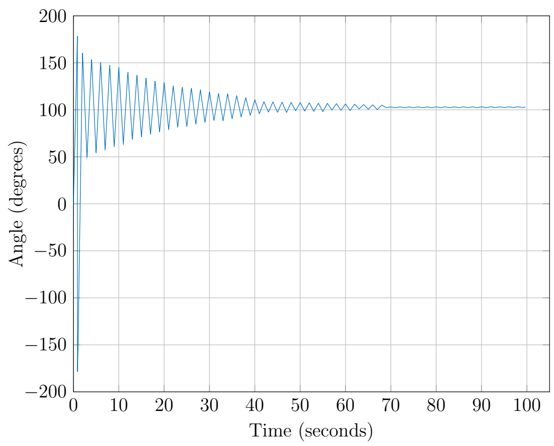

Figure 10: The orientation of the robot over time for $K^R_\Psi = K^R_{\Psi,max}$. This is the upper limit value of K^R_Ψ before the system becomes unstable

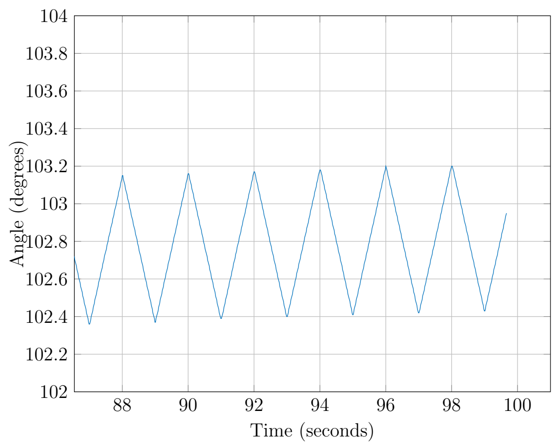

Figure 11: The steady state orientation of the robot for $K^R_\Psi = K^R_{\Psi,max}$

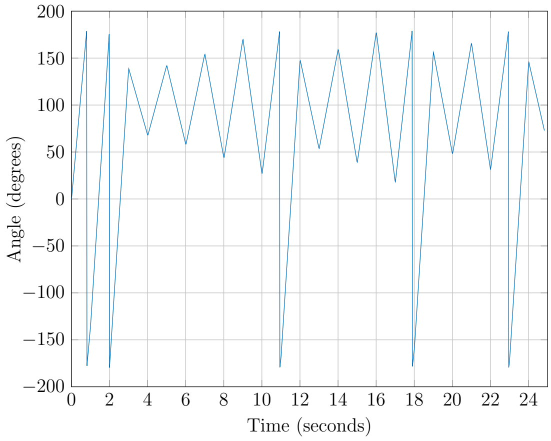

Figure 12: The orientation of the robot over time for $K^R_\Psi = K^R_{\Psi,max} + 1$. The system is indeed unstable

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Task 11

Task 12

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Task 20

Task 21

Task 22

References

[1] Hassan K Khalil. *Nonlinear systems.* Prentice Hall, Upper Saddle river, 3. edition, 2002. ISBN 0-13-067389-7.

[2] Tobias Oetiker, Hubert Partl, Irene Hyna, and Elisabeth Schlegl. *The Not So Short Introduction to LaTeX 2ε.* Oetiker, OETIKER+PARTNER AG, Aarweg 15, 4600 Olten, Switzerland, 2008. http://www.ctan.org/info/lshort/.

[3] Shankar Sastry. *Nonlinear systems: analysis, stability, and control*, volume 10. Springer, New York, N.Y., 1999. ISBN 0-387-98513-1.