VT16 – EP2200 – Home assignment II

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1 Problem 1

The first packet will wait, on average, the sum of the interarrival times between it and the 50^{th} packet. Since the packets arrive in a Poisson fashion, the interarrival times are exponentially distributed. Due to the memoryless property of the exponential distribution, the mean interarrival time between packets is

$$E[T] = \frac{1}{\lambda} = 1 \text{ ms}$$

Hence, the mean waiting time for the first packet is

$$\overline{T}_w = \sum_{i=1}^{50-1} E[T] = \frac{49}{\lambda} = 49 \text{ ms}$$

The probability that k packets are transmitted in a time interval of $\Delta t = 10$ ms is

$$P_k = \frac{(\lambda \Delta t)^k}{k!} e^{-\lambda \Delta t} = \frac{10^k}{k!} e^{-10}$$

The average number of packets in a block is

$$E[P] = \lambda \Delta t = 10 \text{ packets/block}$$

2 Problem 2

For the missing pieces of the diagonal of the state transition intensity matrix, the following equation holds:

$$q_{i,i} = -\sum_{i \neq j} q_{i,j}$$

Hence matrix Q is formed as

$$Q = \begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & -4 & 3 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 3 & 0 & -3 \end{bmatrix}$$

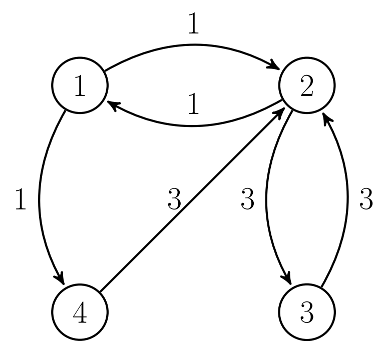

Figure 1: The Markovian chain corresponding to the above Q matrix

In steady state $[\pi_1\ \pi_2\ \pi_3\ \pi_4] \cdot \mathrm{Q} = 0$, where π_i is the probability of the radio being in state i. Solving the system of equations gives

$$[\pi_1\ \pi_2\ \pi_3\ \pi_4] = [\frac{3}{16}\ \frac{3}{8}\ \frac{3}{8}\ \frac{1}{16}]$$

Hence there is a probability of $\frac{3}{16} = 18.75\%$ that the radio is in stand-by, $\frac{3}{8} = 37.5\%$ that the radio is listening to the radio channel for incoming packets, or receiving a packet, and $\frac{1}{16} = 6.25\%$ that the radio is transmitting a packet.

This means that if we observe the state distribution of the radio over a period of time T, then as T approaches infinity, we will observe that the radio is in state 1 for $0.1875T$ time units, in state 2 for $0.375T$ time units, as is also the case for the time spent in state 3, and in state 4 for $0.0625T$ time units.

3 Problem 3

a) With Little's result, using N_{min} as the minimum number of simultaneous listeners, λ_{min} as the minimum request intensity and T the length of a song, then

$$N_{min} = \lambda_{min} T \Leftrightarrow \lambda_{min} = \frac{N_{min}}{T} = 5/3 \text{ requests per minute, or}$$

$$\lambda_{min} = \frac{5 \cdot 24 \cdot 60}{3} = 2400 \text{ requests per day}$$

b) With Little's result, using N as the number of people in the exhibition, T the average duration of their visit and λ the rate at which the tickets to exhibition are made available

$$N = \lambda T \Leftrightarrow \lambda = \frac{N}{T} = \frac{250}{40} \text{ available tickets per minute, or}$$

$$\lambda = \frac{250 \cdot 60}{40} = 375 \text{ available tickets per hour}$$

4 Problem 4

5 Problem 5