VT16 – EP2200
Project I - Error Control in Relay Networks

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Theoretic Analysis

1 AF relaying with end-to-end ARQ

1.1 End-to-end error probability

For this queueing network, the different error-free arrival rates for each station are:

$$
\begin{aligned}
\lambda_2 &= \lambda_1(1 - p_{e,1}) \\
\lambda_3 &= \lambda_2(1 - p_{e,2}) = \lambda_1(1 - p_{e,1})(1 - p_{e,2}) \\
&\dots \\
\lambda_{r+1} &= \lambda_r(1 - p_{e,r}) = \lambda_1 \prod_{k=1}^{r}(1 - p_{e,k}), \text{ and, hence} \\
\lambda_0 &= \lambda_1 \prod_{k=1}^{r+1}(1 - p_{e,k})
\end{aligned}
$$

The success rate, or end-to-end probability of successful reception of packets at the mobile station is hence

$$p_{s,e2e} = \prod_{k=1}^{r+1}(1 - p_{e,k})$$

and the end-to-end error probability is then

$$p_{e,e2e} = 1 - p_{s,e2e} = 1 - \prod_{k=1}^{r+1}(1 - p_{e,k})$$

1.2 Packet arrival rate at the first relay station

Then, at the first relay station, the packet arrival rate is

$$\begin{aligned}
\lambda_1 &= \lambda + \lambda_1 p_{e,e2e} \\
\lambda_1(1 - p_{e,e2e}) &= \lambda \\
\lambda_1 &= \frac{\lambda}{1 - p_{e,e2e}} \\
\lambda_1 &= \frac{\lambda}{1 - (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))} \\
\lambda_1 &= \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k})}
\end{aligned}$$

1.3 Average number of packets

Since there is no blocking of erroneous packets made in this queueing network, the arrival rate at every relay station and at the mobile station will be equal to $\lambda_i = \lambda_1$. The offered load at each relay station i and at the mobile station is then given by equations 1 and 2 respectively.

$$\rho_i^{AF} = \lambda_i \overline{x}_{AF} = \frac{\lambda_1}{\mu_{AF}} = \frac{\lambda}{\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{1}$$

$$\rho^{MS} = \lambda_{r+1} \overline{x}_{MS} = \frac{\lambda_1}{\mu_{MS}} = \frac{\lambda}{\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{2}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \frac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 3 and 4 respectively.

$$N_{i,q}^{AF} = \frac{\lambda^2}{\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k}) \Big(\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda \Big)} \tag{3}$$

$$N_q^{MS} = \frac{\lambda^2}{\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k}) \Big(\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda \Big)} \tag{4}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 1 and 2 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 5 and 6.

$$N_i^{AF} = \frac{\rho_i^{AF}}{1-\rho_i^{AF}} = \frac{\lambda}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} \tag{5}$$

$$N^{MS} = \frac{\rho^{MS}}{1-\rho^{MS}} = \frac{\lambda}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} \tag{6}$$

1.4 Average queueing delay

From Little's theorem, the average queueing delay is $W_i = \frac{N_{q,i}}{\lambda_i}$. Equations 7 and 8 give the average queueing delay at a relay station i and at the mobile station respectively.

$$W_i^{AF} = \frac{\lambda}{\mu_{AF}\Big(\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda\Big)} \tag{7}$$

$$W^{MS} = \frac{\lambda}{\mu_{MS}\Big(\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda\Big)} \tag{8}$$

1.5 Average end-to-end delay

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$T_i^{AF} = W_i^{AF} + \frac{1}{\mu_{AF}} = \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} \tag{9}$$

$$T^{MS} = W^{MS} + \frac{1}{\mu_{MS}} = \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} \tag{10}$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$\begin{aligned} T = T^{MS} + \sum_{i=1}^{r} T_i^{AF} &= \frac{N^{MS} + \sum_{i=1}^{r} N_i^{AF}}{\lambda} \\ &= \frac{1}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} + \sum_{i=1}^{r} \frac{1}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k}) - \lambda} \end{aligned}$$

2 DF relaying with end-to-end ARQ

2.1 Packet arrival rate at each DF and at the MS

For this queueing network, the different arrival rates for each station are:

$$\left.\begin{array}{ll} \lambda_2 & = \lambda_1(1-p_{e,1}) \\ \lambda_3 & = \lambda_2(1-p_{e,2}) = \lambda_1(1-p_{e,1})(1-p_{e,2}) \\ & \dots \\ \lambda_{r+1} & = \lambda_r(1-p_{e,r}) = \lambda_1 \prod\limits_{k=1}^{r}(1-p_{e,k}) \end{array}\right\} \tag{11}$$

From the structure of the queueing network:

$$\begin{aligned} \lambda_1 &= \lambda + \lambda_1 p_{e,1} + \lambda_2 p_{e,2} + \lambda_3 p_{e,3} + \dots + \lambda_{r+1} p_{e,r+1} \\ &= \lambda + \lambda_1 p_{e,1} + \lambda_1(1-p_{e,1})p_{e,2} + \lambda_1(1-p_{e,1})(1-p_{e,2})p_{e,3} + \dots + \lambda_1 \prod_{i=1}^{r}(1-p_{e,i})p_{e,r+1} \\ &= \lambda + \lambda_1 \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right) \end{aligned}$$

Hence

$$\lambda_1 = \frac{\lambda}{1-\sum\limits_{k=0}^{r}\left(p_{e,k+1}\prod\limits_{i=1}^{k}(1-p_{e,i})\right)}$$

and from the set of equations 11, we obtain the packet arrival rates λ_i and λ_{r+1} at every relay station i and at the mobile station:

$$\lambda_i = \frac{\lambda \prod\limits_{k=1}^{i-1}(1-p_{e,k})}{1-\sum\limits_{k=0}^{r}\left(p_{e,k+1}\prod\limits_{i=1}^{k}(1-p_{e,i})\right)} \tag{12}$$

$$\lambda_{r+1} = \frac{\lambda \prod\limits_{k=1}^{r}(1-p_{e,k})}{1-\sum\limits_{k=0}^{r}\left(p_{e,k+1}\prod\limits_{i=1}^{k}(1-p_{e,i})\right)} \tag{13}$$

2.2 Average number of packets

The offered load at each relay station i and at the mobile station is then given by equations 14 and 15.

$$\rho_i^{DF} = \lambda_i \overline{x}_{DF} = \frac{\lambda_1}{\mu_{DF}} = \frac{\lambda \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF}\left(1 - \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right)} \tag{14}$$

$$\rho^{MS} = \lambda_{r+1} \overline{x}_{MS} = \frac{\lambda_1}{\mu_{MS}} = \frac{\lambda \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{MS}\left(1 - \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right)} \tag{15}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \dfrac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 16 and 17 respectively.

$$N_{i,q}^{DF} = \frac{(\rho_i^{DF})^2}{1-\rho_i^{DF}} \tag{16}$$

$$N_q^{MS} = \frac{(\rho^{MS})^2}{1-\rho^{MS}} \tag{17}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 14 and 15 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 18 and 19.

$$N_i^{DF} = \frac{\rho_i^{DF}}{1-\rho_i^{DF}} = \frac{\lambda \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF}\left(1 - \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right) - \lambda \prod_{k=1}^{i-1}(1-p_{e,k})} \tag{18}$$

$$N^{MS} = \frac{\rho^{MS}}{1-\rho^{MS}} = \frac{\lambda \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{MS}\left(1 - \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right) - \lambda \prod_{k=1}^{r}(1-p_{e,k})} \tag{19}$$

2.3 Average queueing delay

From Little's theorem, the average queueing delay is $W = \dfrac{N_q}{\lambda}$. Equations 20 and 21 give the average queueing delay at a relay station i and at the mobile station respectively.

$$W_i^{DF} = \frac{N_{i,q}^{DF}}{\lambda_i} = \frac{\lambda \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF}\left(\mu_{DF}\left(1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right)-\lambda\prod_{k=1}^{i-1}(1-p_{e,k})\right)} \tag{20}$$

$$W^{MS} = \frac{N_q^{MS}}{\lambda_{r+1}} = \frac{\lambda \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{MS}\left(\mu_{MS}\left(1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right)-\lambda\prod_{k=1}^{r}(1-p_{e,k})\right)} \tag{21}$$

2.4 Average end-to-end delay

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$T_i^{DF} = W_i^{DF} + \frac{1}{\mu_{DF}} \tag{22}$$

$$T^{MS} = W^{MS} + \frac{1}{\mu_{MS}} \tag{23}$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$\begin{aligned} T = T^{MS} + \sum_{i=1}^{r} T_i^{DF} &= \frac{N^{MS} + \sum_{i=1}^{r} N_i^{DF}}{\lambda} \\ &= \frac{1}{(1-p_{e,r+1})\mu_{MS} - \lambda} + \sum_{i=1}^{r} \frac{1}{\mu_{DF}(1-p_{e,i})^{r+2-i} - \lambda} \end{aligned}$$

3 DF relaying with hop-by-hop ARQ

3.1 Packet arrival rate at each DF and at the MS

For this queueing network, the arrival rate at the first relay station is

$$\lambda_1 = \lambda + \lambda_1 p_{e,1} \Leftrightarrow \lambda_1 = \frac{\lambda}{1-p_{e,1}}$$

From the structure of the queueing network, the arrival at the subsequent relay stations rate will be the sum of the error-free arrivals from its previous station and the erroneous packets that were processed at that specific station

$$\left.\begin{aligned} \lambda_2 &= \lambda_2 p_{e,2} + (1-p_{e,1})\lambda_1 \Leftrightarrow \lambda_2(1-p_{e,2}) = (1-p_{e,1})\lambda_1 \\ \lambda_3 &= \lambda_3 p_{e,3} + (1-p_{e,2})\lambda_2 \Leftrightarrow \lambda_3(1-p_{e,3}) = (1-p_{e,2})\lambda_2 \\ &\dots \\ \lambda_{r+1} &= \lambda_r p_{e,r} + (1-p_{e,r})\lambda_r \Leftrightarrow \lambda_{r+1}(1-p_{e,r+1}) = (1-p_{e,r})\lambda_r \end{aligned}\right\} \tag{24}$$

From this set of equations we observe that

$$\lambda_1(1-p_{e,1}) = \lambda_2(1-p_{e,2}) = \lambda_3(1-p_{e,3}) = \cdots = \lambda_{r+1}(1-p_{e,r+1})$$

Hence the arrival rates at each relay station i and at the mobile station are

$$\lambda_i = \frac{\lambda}{1-p_{e,i}} \tag{25}$$

$$\lambda_{r+1} = \frac{\lambda}{1-p_{e,r+1}} \tag{26}$$

3.2 Average number of packets

The offered load at each relay station i and at the mobile station is then given by equations 27 and 28.

$$\rho_i^{DF} = \lambda_i \overline{x}_i = \frac{\lambda}{\mu_{DF}(1-p_{e,i})} \tag{27}$$

$$\rho^{MS} = \lambda_{r+1}\overline{x}_{r+1} = \frac{\lambda}{\mu_{MS}(1-p_{e,r+1})} \tag{28}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \dfrac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 29 and 30 respectively.

$$N_{i,q}^{DF} = \frac{(\rho_i^{DF})^2}{1-\rho_i^{DF}} = \frac{\lambda^2}{\mu_{DF}(1-p_{e,i})\Big(\mu_{DF}(1-p_{e,i})-\lambda\Big)} \tag{29}$$

$$N_q^{MS} = \frac{(\rho^{MS})^2}{1-\rho^{MS}} = \frac{\lambda^2}{\mu_{MS}(1-p_{e,r+1})\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)} \tag{30}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 27 and 28 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 18 and 19.

$$N_i^{DF} = \frac{\rho_i^{DF}}{1-\rho_i^{DF}} = \frac{\lambda}{\mu_{DF}(1-p_{e,i})-\lambda} \tag{31}$$

$$N^{MS} = \frac{\rho^{MS}}{1-\rho^{MS}} = \frac{\lambda}{\mu_{MS}(1-p_{e,r+1})-\lambda} \tag{32}$$

3.3 Average queueing delay

From Little's theorem, the average queueing delay is $W = \dfrac{N_q}{\lambda}$. Equations 33 and 34 give the average queueing delay at a relay station i and at the mobile station respectively.

$$\begin{aligned} W_i^{DF} = \frac{N_{i,q}^{DF}}{\lambda_i} &= \frac{\dfrac{\lambda^2}{\mu_{DF}(1-p_{e,i})\Big(\mu_{DF}(1-p_{e,i})-\lambda\Big)}}{\dfrac{\lambda}{1-p_{e,i}}} \\ &= \frac{\lambda}{\mu_{DF}\Big(\mu_{DF}(1-p_{e,i})-\lambda\Big)} \end{aligned} \tag{33}$$

$$\begin{aligned} W^{MS} = \frac{N_q^{MS}}{\lambda_{r+1}} &= \frac{\dfrac{\lambda^2}{\mu_{MS}(1-p_{e,r+1})\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)}}{\dfrac{\lambda}{1-p_{e,r+1}}} \\ &= \frac{\lambda}{\mu_{MS}\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)} \end{aligned} \tag{34}$$

3.4 Average end-to-end delay

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$\begin{aligned} T_i^{DF} &= W_i^{DF} + \frac{1}{\mu_{DF}} \\ &= \frac{\lambda}{\mu_{DF}\Big(\mu_{DF}(1-p_{e,i})-\lambda\Big)} + \frac{1}{\mu_{DF}} \\ &= \frac{1-p_{e,i}}{\mu_{DF}(1-p_{e,i})-\lambda} \end{aligned}$$

$$\begin{aligned} T^{MS} &= W^{MS} + \frac{1}{\mu_{MS}} \\ &= \frac{\lambda}{\mu_{MS}\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)} + \frac{1}{\mu_{MS}} \\ &= \frac{1-p_{e,r+1}}{\mu_{MS}(1-p_{e,r+1})-\lambda} \end{aligned}$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$
\begin{aligned}
T &= T^{MS} + \sum_{i=1}^{r} T_i^{DF} \\
&= \frac{1 - p_{e,r+1}}{\mu_{MS}(1 - p_{e,r+1}) - \lambda} + \sum_{i=1}^{r} \frac{1 - p_{e,i}}{\mu_{DF}(1 - p_{e,i}) - \lambda}
\end{aligned}
$$

Numerical Evaluation

4 Stability region

4.1 AF relaying with end-to-end ARQ

Given that $p_{e,i} = 0.1, i = \{1, 2 \ldots r+1\}$ and $\mu_{AF} = \mu_{MS} = 2$, the arrival rates at each relay station i and at the mobile station are:

$$\lambda_i = \lambda_{MS} = \lambda_1 = \frac{\lambda}{(1-0.1)^{r+1}} = \lambda\ 0.9^{-r-1}$$

Since (a) the entire network is stable when all queues are stable, (b) every station is a M/M/1 system, and (c) such a system is considered to be stable when $\rho_i = \dfrac{\lambda_i}{\mu_i} < 1$, then the network is stable if

$$\lambda \cdot 0.9^{-r-1} < 2 \Leftrightarrow \lambda < 2 \cdot 0.9^{r+1}$$

Figure 1 plots function $\lambda_{max}(r) = 2 \cdot 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is defined as the region where $\lambda < \lambda_{max}(r)$.

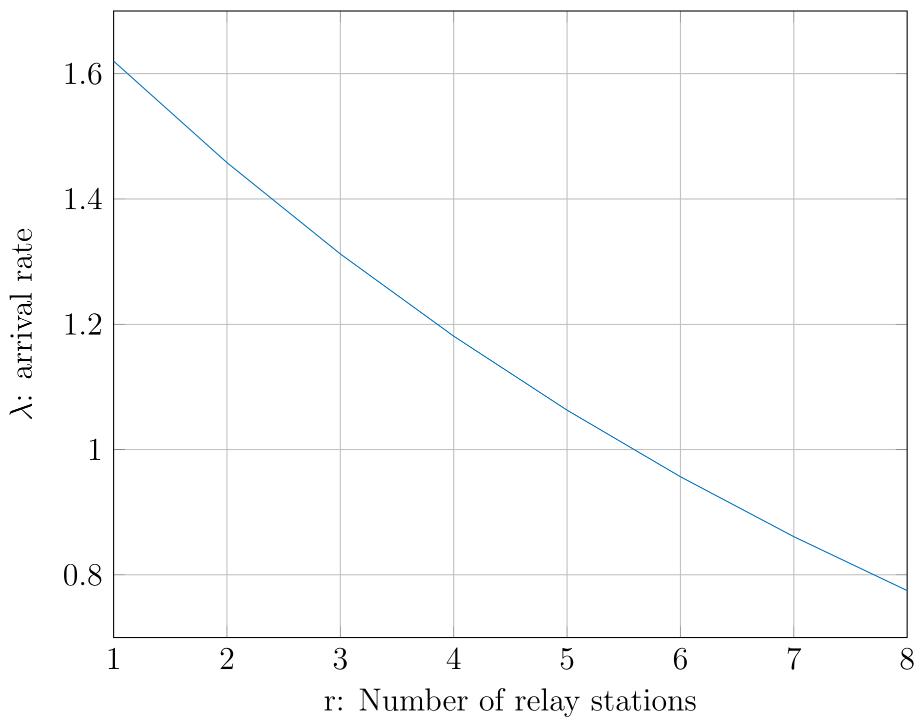

Figure 1: Function $\lambda_{max}(r) = 2 \cdot 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is the area under the curve.

As $r \to \infty$, the stability region gets smaller and smaller in size, until it is obliterated, since $\lambda_{max}(r) \to 0$ as $r \to \infty$.

This is reasonable, since, as r increases, the number of relay stations increase, and each one of them carries a probability of erroneous transmission with it. Each erroneous packet has to be retransmitted from the base station, and the end-to-end error probability goes to 1 as $r \to \infty$:

$$\lim_{r\to\infty} p_{e,e2e} = \lim_{r\to\infty}(1 - \prod_{k=1}^{r+1} 0.9) = 1 - 0 = 1$$

4.2 DF relaying with end-to-end ARQ

With $p_{e,i} = 0.1, i = \{1, 2 \ldots r + 1\}$, the arrival rates at each relay station and at the mobile station, given by equations 12 and 13, become

$$\lambda_i = \frac{\lambda}{1 - \sum_{k=0}^{r}\left(0.1\prod_{i=1}^{k}(1-0.1)\right)}\prod_{k=1}^{i-1}(1-0.1) \tag{35}$$

$$\lambda_{r+1} = \frac{\lambda}{1 - \sum_{k=0}^{r}\left(0.1\prod_{i=1}^{k}(1-0.1)\right)}\prod_{k=1}^{r}(1-0.1) \tag{36}$$

The first part of the right-hand side of the above 2 equations does not change over each station, and is equal to

$$\frac{\lambda}{1 - \sum_{k=0}^{r}\left(0.1\prod_{i=1}^{k}(1-0.1)\right)} = \frac{\lambda}{1 - 0.1\sum_{k=0}^{r} 0.9^k} = \frac{\lambda}{1 - 0.1\dfrac{1-0.9^{r+1}}{1-0.9}} = \lambda \cdot 0.9^{-r-1}$$

Hence, equations 35 and 36 become

$$\lambda_i = \lambda \cdot 0.9^{-r-1}\prod_{k=1}^{i-1}(1-0.1) = \lambda \cdot 0.9^{-r-1} \cdot 0.9^{i-1} = \lambda \cdot 0.9^{i-r-2}$$

$$\lambda_{r+1} = \lambda \cdot 0.9^{-r-1}\prod_{k=1}^{r}(1-0.1) = \lambda \cdot 0.9^{-r-1} \cdot 0.9^{r} = \lambda \cdot 0.9^{-1}$$

Since (a) the entire network is stable when all queues are stable, (b) every station is a M/M/1 system, and (c) such a system is considered to be stable when $\rho_k = \dfrac{\lambda_k}{\mu_k} < 1$, then, the network is stable if all of the following inequalities hold:

$$\lambda_i < \mu_{DF}$$
$$\lambda_{r+1} < \mu_{MS}$$

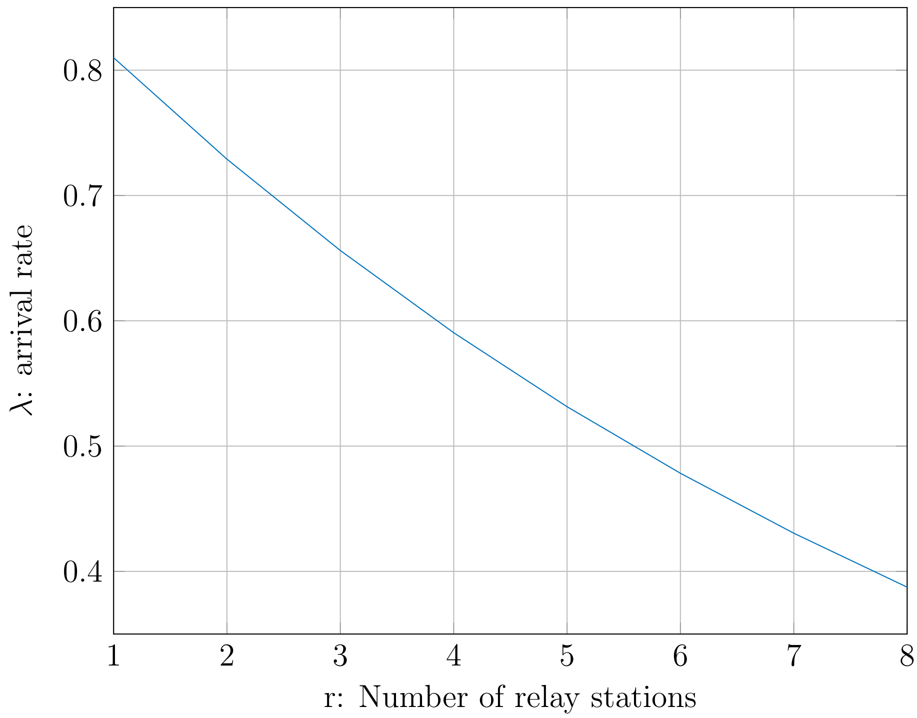

Figure 2: Function $\lambda_{max}(r) = 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is the area under the curve.

or

$$\lambda < 0.9^{r+1}$$
$$\lambda < 0.9^{r}$$
$$\lambda < 0.9^{+r-1}$$
$$\dots$$
$$\lambda < 0.9^{2}$$
and
$$\lambda < 2 \cdot 0.9$$

From these conditions, since $r \geq 1$, we can see that the most restrictive condition is the first one, i.e. if the first one holds, then, inevitably, all the following conditions will be true. Figure 2 plots function $\lambda_{max}(r) = 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is defined as the region where $\lambda < \lambda_{max}(r)$.

As $r \to \infty$, the stability region gets smaller and smaller in size, until it is obliterated, since $\lambda_{max}(r) \to 0$ as $r \to \infty$.

This is reasonable, since, as r increases, the number of relay stations increase, and each one of them carries a probability of erroneous transmission with it. Each erroneous packet has to be retransmitted from the base station and, hence, the load that the first relay station has to serve determines the overall stability of the network.

4.3 DF relaying with hop-by-hop ARQ

With $p_{e,i} = 0.1, i = \{1, 2 \dots r+1\}$, the arrival rates at each relay station and at the mobile station, given by equations 25 and 26, become

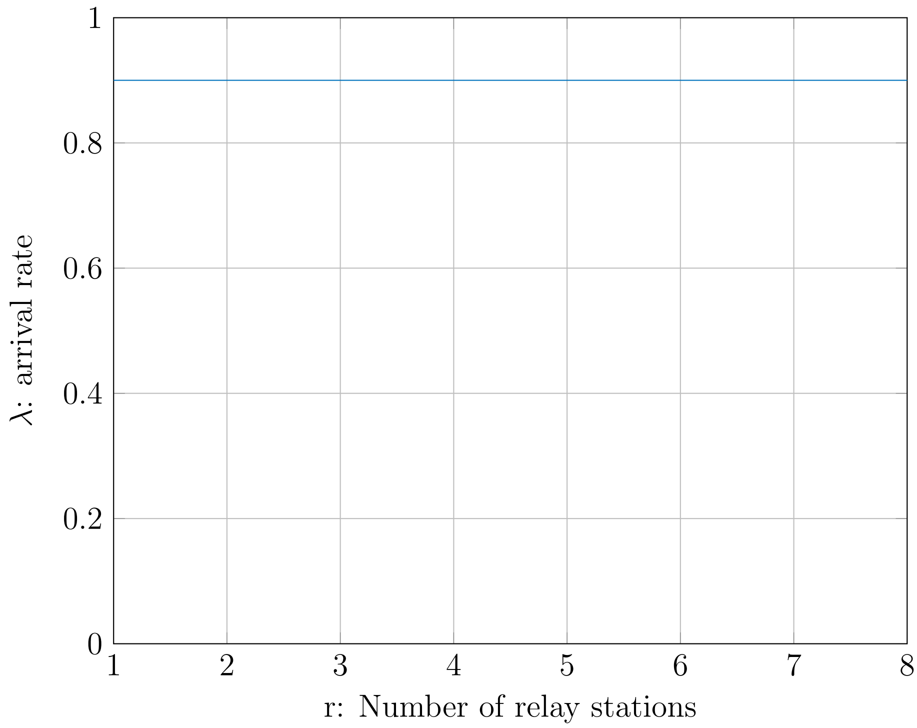

Figure 3: Function $\lambda_{max}(r) = 0.9$ for $1 \le r \le 8$. The stability region is the area under the curve.

$$\lambda_i = \lambda \cdot 0.9^{-1} \tag{37}$$

$$\lambda_{r+1} = \lambda \cdot 0.9^{-1} \tag{38}$$

Since (a) the entire network is stable when all queues are stable, (b) every station is a M/M/1 system, and (c) such a system is considered to be stable when $\rho_k = \dfrac{\lambda_k}{\mu_k} < 1$, then, the network is stable if all of the following inequalities hold:

$$\lambda \cdot 0.9^{-1} < 1$$
$$\lambda \cdot 0.9^{-1} < 2$$

From these conditions we can see that the most restrictive condition is the first one, i.e. if the first one holds, then, inevitably, the second condition will be true. Figure 3 plots function $\lambda_{max}(r) = 0.9$ for $1 \le r \le 8$. The stability region is defined as the region where $\lambda < \lambda_{max}(r)$.

As $r \to \infty$, the stability region is not affected.

This is reasonable, since, as r increases, the number of relay stations increase, and each one of them carries a probability of erroneous transmission with it, but each erroneous packet doesn't have to be retransmitted from the base station, as in the previous two networks, but from the station where the error was made.

Figure 4 illustrates the maximum arrival rate per queueing network for $1 \le r \le 8$.

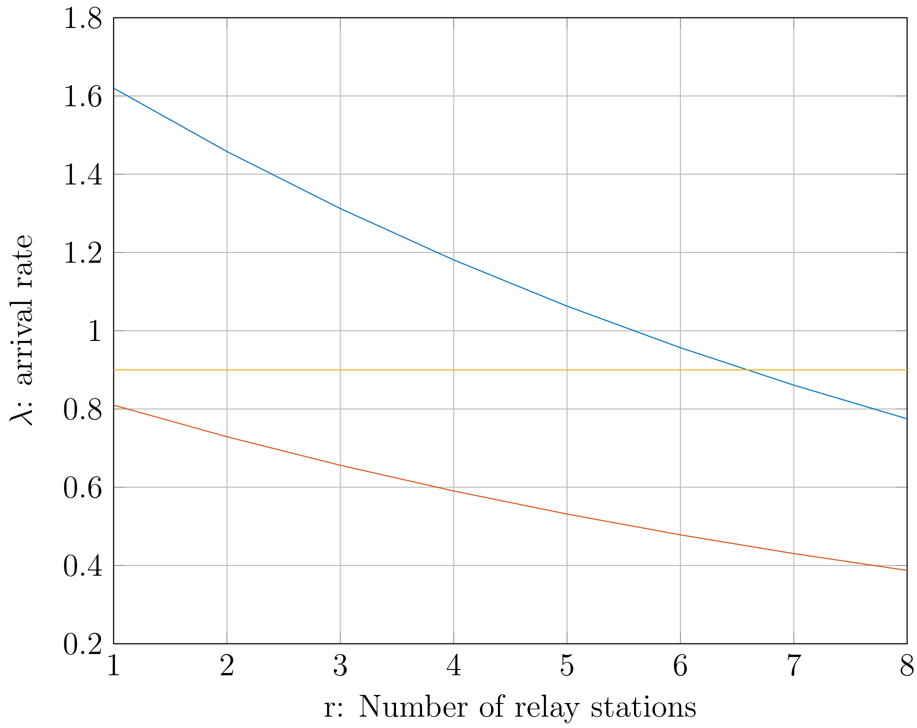

Figure 4: The maximum arrival rate for each of the three queueing networks considered here, for $1 \leq r \leq 8$. Colour `blue` denotes that of AF with end-to-end ARQ, `red` that of DF with end-to-end ARQ and `orange` that of DF with hop-by-hop ARQ.

Arrival rate-delay characteristics

4.4 AF relaying with end-to-end ARQ

Given that $p_{e,i} = 0.1$, $i = \{1, 2 \ldots r+1\}$, $r = 4$ and $\mu_{AF} = \mu_{MS} = 2$, the average end-to-end delay is

$$T = \frac{5}{2 \cdot 0.9^5 - \lambda} \tag{39}$$

In order for the network to be stable, $0 \leq \lambda \leq 2 \cdot 0.9^5 = 1.181$, for $r = 4$. Figure 5 illustrates how the end-to-end delay changes in relation to the arrival rate λ.

If $p_{e,i}$ increases, then λ_{max} decreases, as $\lambda_{max} = 2 \cdot (1 - p_{e,i})^5$, and the law that governs the average delay remains the same. Thus, as $p_{e,i}$ increases, the arrival rate-delay curve is shifted towards the left.

4.5 DF relaying with end-to-end ARQ

Given that $p_{e,i} = 0.1, i = \{1, 2 \ldots r+1\}$, $r = 4$ and $\mu_{MS} = 2\mu_{DF} = 2$, the average end-to-end delay is

$$T = \frac{1}{1.8 - \lambda} + \sum_{i=1}^{4} \frac{1}{0.9^{6-i} - \lambda} \tag{40}$$

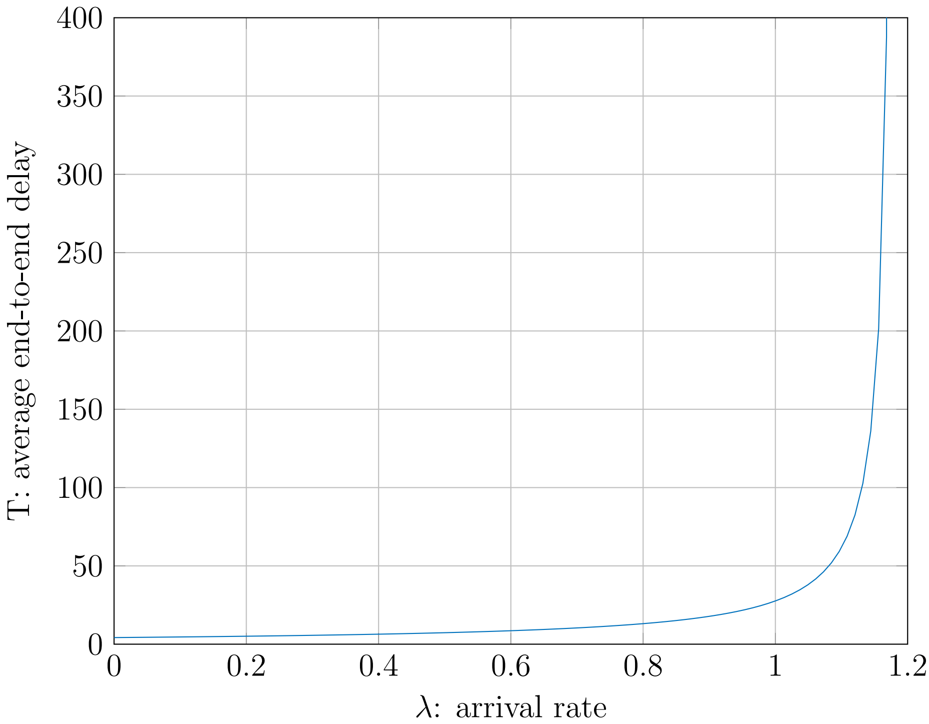

Figure 5: The relation between end-to-end delay and the arrival rate in a queueing network with AF relaying and end-to-end ARQ with $r = 4$.

In order for the network to be stable, $0 \leq \lambda \leq 0.9^5 = 0.5905$, for $r = 4$. Figure 6 illustrates how the end-to-end delay changes in relation to the arrival rate λ.

If $p_{e,i}$ increases, then λ_{max} decreases, as $\lambda_{max} = (1 - p_{e,i})^5$, and the law that governs the average delay remains the same. Thus, as $p_{e,i}$ increases, the arrival rate-delay curve is shifted towards the left.

4.6 DF relaying with hop-by-hop ARQ

Given that $p_{e,i} = 0.1, i = \{1, 2 \ldots r + 1\}$, $r = 4$ and $\mu_{MS} = 2\mu_{DF} = 2$, the average end-to-end delay is

$$\begin{aligned} T &= \frac{1 - p_{e,r+1}}{\mu_{MS}(1 - p_{e,r+1}) - \lambda} + \sum_{i=1}^{r} \frac{1 - p_{e,i}}{\mu_{DF}(1 - p_{e,i}) - \lambda} \\ &= \frac{0.9}{1.8 - \lambda} + \sum_{i=1}^{4} \frac{0.9}{0.9 - \lambda} \\ &= \frac{0.9}{1.8 - \lambda} + \frac{3.6}{0.9 - \lambda} \end{aligned}$$

In order for the network to be stable, $0 \leq \lambda \leq 0.9$, for $r = 4$. Figure 7 illustrates how the end-to-end delay changes in relation to the arrival rate λ.

If $p_{e,i}$ increases, then λ_{max} decreases, as $\lambda_{max} = 1 - p_{e,i}$, and the law that governs the average delay remains the same. Thus, as $p_{e,i}$ increases, the arrival rate-delay curve is shifted towards the left.

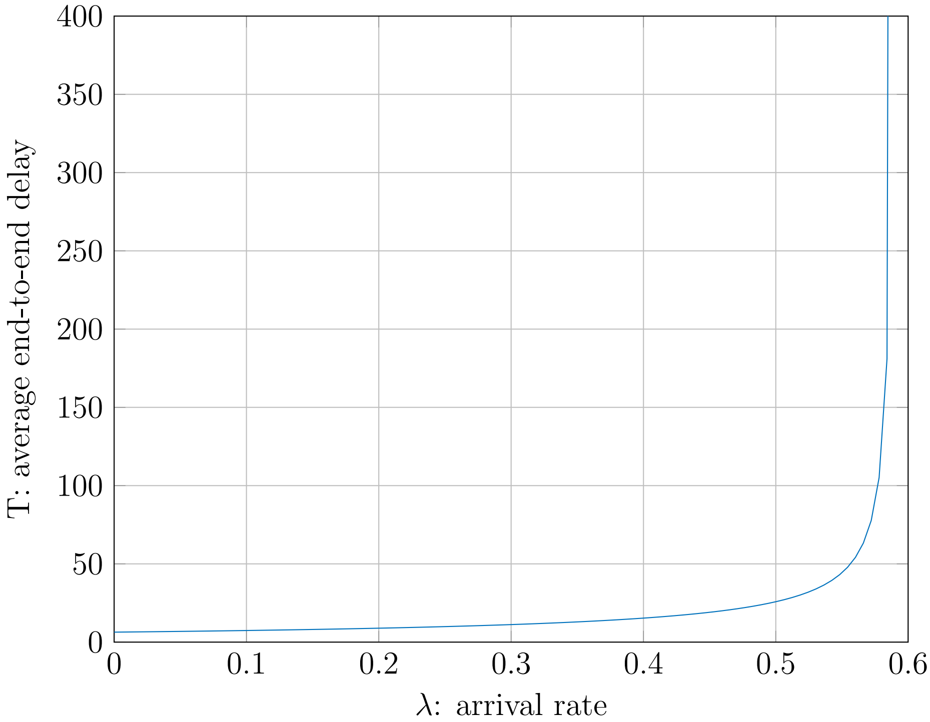

Figure 6: The relation between end-to-end delay and the arrival rate in a queueing network with DF relaying and end-to-end ARQ with $r = 4$.

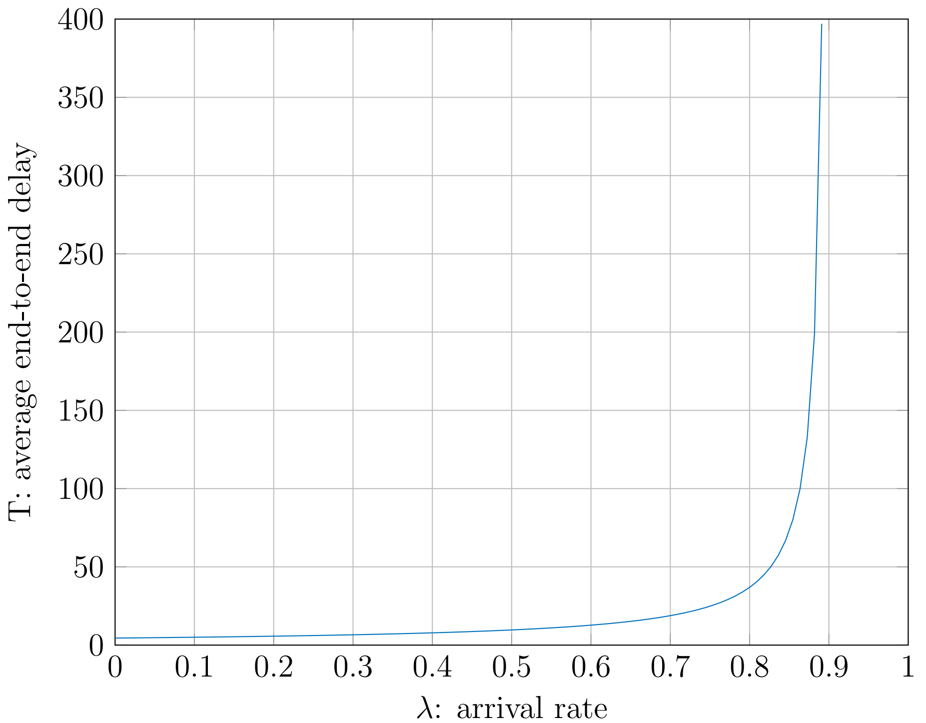

Figure 7: The relation between end-to-end delay and the arrival rate in a queueing network with DF relaying and hop-by-hop ARQ with $r = 4$.

Figure 8 shows the relation between the arrival rate and the end-to-end delay for all three queueing networks considered.

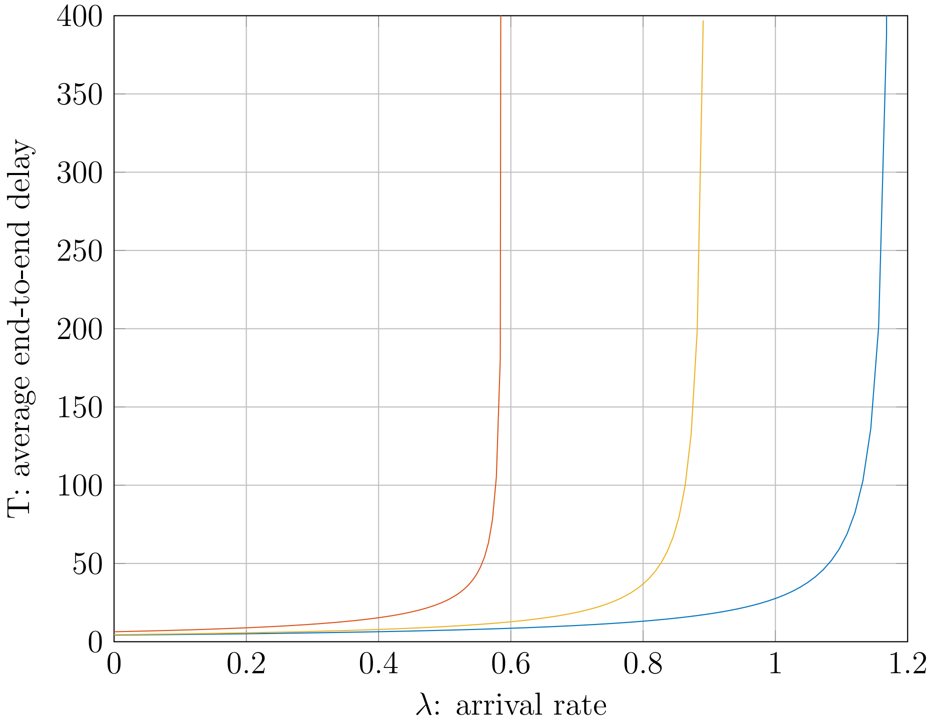

Figure 8: The relation between end-to-end delay and the arrival rate in a queueing network with AF relaying with end-to-end AQR (`blue`), DF relaying and end-to-end ARQ (`red`) and DF relaying and hop-by-hop ARQ (`orange`) for $r = 4$.

AF or DF

4.7 Part I

Since the arrival rate and the number of stations is fixed, and the probability of error varies, in order to select a scheme among the three queueing systems, it is reasonable to consider the metric of end-to-end delay as the decisive factor. The system that exhibits the least average delay is the one we shall choose as the most preferable.

Figure 9 shows the relation between the end-to-end delay and the probability of error $p_{e,i}$, $i = \{1, 2, \ldots r + 1\}$ for the three different queueing networks considered.

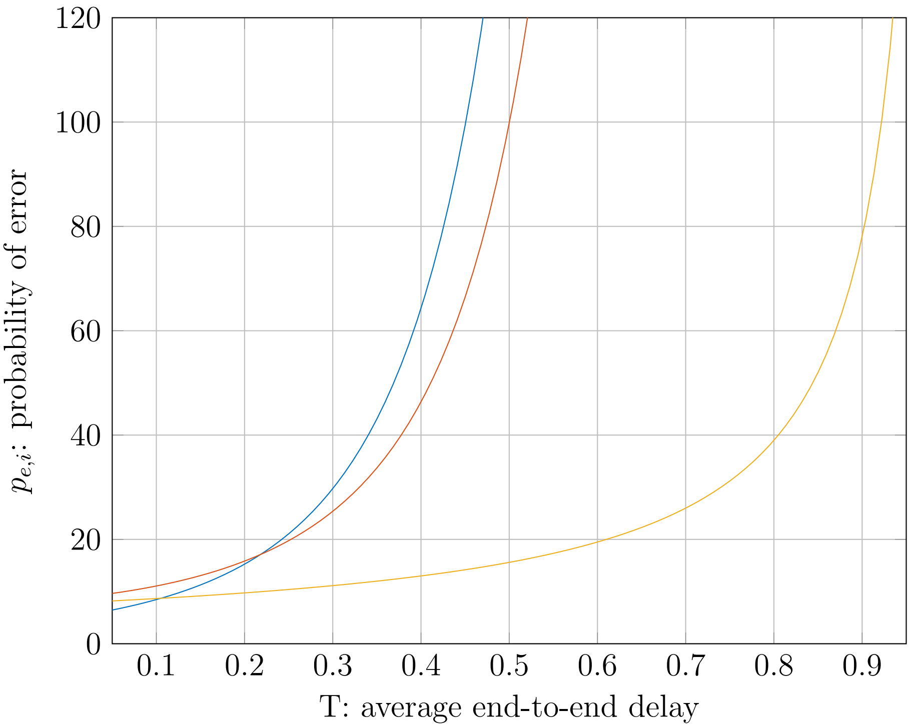

Figure 9: The relation between end-to-end delay and the probability of error in a queueing network with AF relaying with end-to-end ARQ (`blue`), DF relaying and end-to-end ARQ (`red`) and DF relaying and hop-by-hop ARQ (`orange`) for $r = 4$.

What is evident here is that as the probability of error increases, the average delay in the AF relaying with hop-by-hop ARQ system increases in a less rapid fashion compared to the other two queueing networks. This system can tolerate larger probability errors, and hence be more robust, while at the same time appear lower end-to-end delay times for the same values of the probability of error. However, if $p_{e,i} \leq 0.105$, the AF relaying with end-to-end ARQ system exhibits lower average end-to-end delay times.

4.8 Part II

Since the arrival rate, the number of stations, and the probability of error is fixed, in order to select a scheme among the three queueing systems, it is reasonable to consider the metric of end-to-end delay as the decisive factor. The system that exhibits the least average delay is the one we shall choose as the most preferable.

Figure 10 shows the relation between the end-to-end delay and the value of $k \in [1, 4]$ for the three different queueing networks considered.

Consulting figure 11, which shows a magnified portion of this relation, it is evident that for $k \in [1, 1.97]$, DF relaying with hop-by-hop ARQ exhibits lower average delay times than the other two queueing systems, while for $k \in [1.97, 4]$ the lowest average delay times are exhibited by the AF relaying with end-to-end ARQ system.

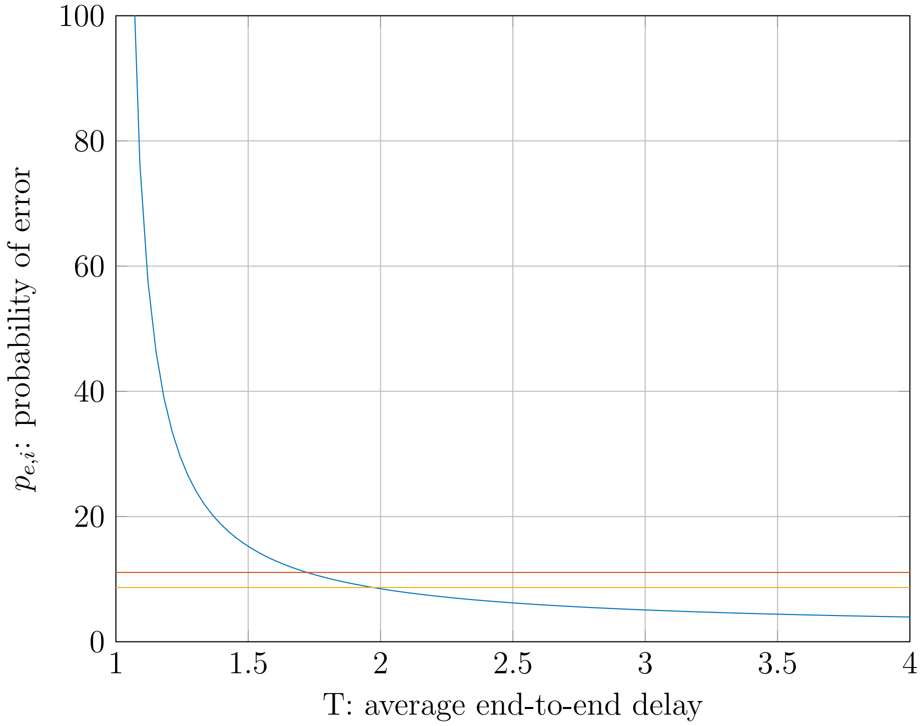

Figure 10: The relation between end-to-end delay and $k \in [1, 4]$ in a queueing network with AF relaying with end-to-end AQR (`blue`), DF relaying and end-to-end ARQ (`red`) and DF relaying and hop-by-hop ARQ (`orange`) for $r = 4$.

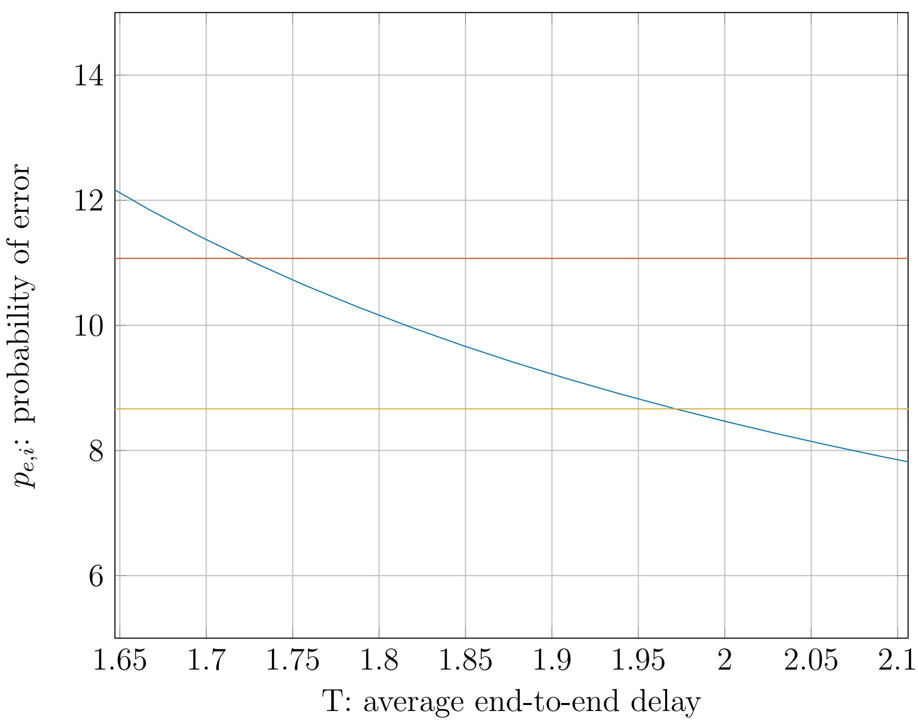

Figure 11: The relation between end-to-end delay and $k \in [1, 4]$ in a queueing network with AF relaying with end-to-end AQR (`blue`), DF relaying and end-to-end ARQ (`red`) and DF relaying and hop-by-hop ARQ (`orange`) for $r = 4$, magnified to display the fine details of when the system exhibiting the lowest average delay changes.

Finite number of retransmissions

4.9 Packet drop probability

The probability that a packet needs to be retransmitted is

$$p_{e,e2e} = 1 - \prod_{k=1}^{r+1}(1 - p_{e,k})$$

The probability that this exact packet will need to be retransmitted for the i^{th} time is independent of the $i-1$ number of times it was retransmitted in the past. Hence, if R is the number of maximum allowed transmission attempts, the packet drop probability is

$$p_{d,e2e} = p_{d,e2e}^{R} = (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^{R}$$

4.10 Packet loss ratio

From the structure of the network:

$$\begin{aligned} \lambda_1 &= \lambda + \lambda_1 \cdot p_{e,e2e} - \lambda_1 \cdot p_{d,e2e} \\ \lambda_1 &= \frac{\lambda}{1 - p_{e,e2e} + p_{d,e2e}} \end{aligned} \tag{41}$$

The number of error-free packets received at the mobile station (including it) is

$$\begin{aligned} \lambda_0 &= (1 - p_{e,e2e})\lambda_1 - p_{d,e2e}\lambda_1 \\ \frac{\lambda_0}{\lambda_1} &= 1 - p_{e,e2e} - p_{d,e2e} = \prod_{k=1}^{r+1}(1 - p_{e,k}) - (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^{R} \end{aligned}$$

and from equation 41

$$\frac{\lambda_0}{\lambda} = \frac{1 - p_{e,e2e} - p_{d,e2e}}{1 - p_{e,e2e} + p_{d,e2e}} = \frac{\prod_{k=1}^{r+1}(1 - p_{e,k}) - (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^{R}}{\prod_{k=1}^{r+1}(1 - p_{e,k}) + (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^{R}}$$

4.11 Stability region

In order for this system to be stable the following should hold, with $\lambda_i = \lambda_{r+1} = \lambda_1$:

$$\frac{\lambda_i}{\mu_{AF}} < 1$$
$$\frac{\lambda_{r+1}}{\mu_{MS}} < 1$$

Hence, due to $\mu_{MS} = \mu_{AF}$:

$$\begin{aligned}\lambda &< \mu_{MS}(1 - p_{e,e2e} + p_{d,e2e}) \\ &= \mu_{MS}\Big(\prod_{k=1}^{r+1}(1 - p_{e,k}) + (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^R\Big)\end{aligned}$$

This stability region is larger than when there is no retransmission limit, as was the case prior to this. This is reasonable, since, now, packets that need to be retransmitted more than this upper limit are not allowed to continue their life-cycle in the system.

4.12 Throughput

From the previous section:

$$\lambda_0 = \lambda \frac{1 - p_{e,e2e} - p_{d,e2e}}{1 - p_{e,e2e} + p_{d,e2e}} = \lambda \frac{\prod_{k=1}^{r+1}(1 - p_{e,k}) - (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^R}{\prod_{k=1}^{r+1}(1 - p_{e,k}) + (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^R}$$

4.13 Average end-to-end delay

With similar reasoning to the first part of the theoretic analysis, the average end-to-end delay is

$$\begin{aligned}T &= \frac{1}{\mu_{MS}(1 - p_{e,e2e} + p_{d,e2e}) - \lambda} + \sum_{i=1}^{r} \frac{1}{\mu_{AF}(1 - p_{e,e2e} + p_{d,e2e}) - \lambda} \\ &= \frac{1}{\mu_{MS}\Big(\prod_{k=1}^{r+1}(1 - p_{e,k}) + (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^R\Big) - \lambda} \\ &+ \sum_{i=1}^{r} \frac{1}{\mu_{AF}\Big(\prod_{k=1}^{r+1}(1 - p_{e,k}) + (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))^R\Big) - \lambda}\end{aligned}$$