VT16 – EP2200
Project I - Error Control in Relay Networks

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Theoretic Analysis

1 AF relaying with end-to-end ARQ

1.1 End-to-end error probability

For this queueing network, the different arrival rates for each station are:

$$\begin{aligned}
\lambda_2 &= \lambda_1(1 - p_{e,1}) \\
\lambda_3 &= \lambda_2(1 - p_{e,2}) = \lambda_1(1 - p_{e,1})(1 - p_{e,2}) \\
&\dots \\
\lambda_{r+1} &= \lambda_r(1 - p_{e,r}) = \lambda_1 \prod_{k=1}^{r}(1 - p_{e,k}), \text{ and, hence} \\
\lambda_0 &= \lambda_1 \prod_{k=1}^{r+1}(1 - p_{e,k})
\end{aligned}$$

The success rate, or end-to-end probability of successful receipt of packets at the mobile station is hence

$$p_{s,e2e} = \prod_{k=1}^{r+1}(1 - p_{e,k})$$

and the end-to-end error probability is then

$$p_{e,e2e} = 1 - p_{s,e2e} = 1 - \prod_{k=1}^{r+1}(1 - p_{e,k})$$

1.2 Packet arrival rate at the first relay station

Then, at the first relay station, the packet arrival rate is

$$\begin{aligned}
\lambda_1 &= \lambda + \lambda_0 = \lambda + \lambda_1 p_{e,e2e} \\
\lambda_1(1 - p_{e,e2e}) &= \lambda \\
\lambda_1 &= \frac{\lambda}{1 - p_{e,e2e}} \\
\lambda_1 &= \frac{\lambda}{1 - (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))} \\
\lambda_1 &= \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k})}
\end{aligned}$$

1.3 Average number of packets

Since there is no blocking of erroneous packets made in this queueing network, the arrival rate at every relay station and at the mobile station will be equal to $\lambda_i = \lambda_1$. The offered load at each relay station i and at the mobile station is then given by equations 1 and 2 respectively.

$$\rho_i^{AF} = \lambda_i \overline{x}_{AF} = \frac{\lambda_1}{\mu_{AF}} = \frac{\lambda}{\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{1}$$

$$\rho^{MS} = \lambda_{r+1} \overline{x}_{MS} = \frac{\lambda_1}{\mu_{MS}} = \frac{\lambda}{\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{2}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \frac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 3 and 4 respectively.

$$N_{i,q}^{AF} = \frac{\lambda^2}{\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k}) \Big(\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda\Big)} \tag{3}$$

$$N_q^{MS} = \frac{\lambda^2}{\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k}) \Big(\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda\Big)} \tag{4}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 1 and 2 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 5 and 6.

$$N_i^{AF} = \frac{\rho_i^{AF}}{1-\rho_i^{AF}} = \frac{\lambda}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{5}$$

$$N^{MS} = \frac{\rho^{MS}}{1-\rho^{MS}} = \frac{\lambda}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{6}$$

1.4 Average queueing delay

From Little's theorem, the average queueing delay is $W = \frac{N_q}{\lambda}$. Equations 7 and 8 give the average queueing delay at a relay station i and at the mobile station respectively.

$$W_i^{AF} = \frac{\lambda}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})\Big(\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda\Big)} \tag{7}$$

$$W^{MS} = \frac{\lambda}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})\Big(\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda\Big)} \tag{8}$$

1.5 Average end-to-end delay

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$T_i^{AF} = W_i^{AF} + \frac{1}{\mu_{AF}} = \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{9}$$

$$T^{MS} = W^{MS} + \frac{1}{\mu_{MS}} = \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{10}$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$T = T^{MS} + \sum_{i=1}^{r} T_i^{AF} = \frac{N^{MS} + \sum_{i=1}^{r} N_i^{AF}}{\lambda}$$

2 DF relaying with end-to-end ARQ

2.1 Packet arrival rate at each DF and at the MS

For this queueing network, the different arrival rates for each station are:

$$\left.\begin{array}{ll}\lambda_2 & = \lambda_1(1-p_{e,1}) \\ \lambda_3 & = \lambda_2(1-p_{e,2}) = \lambda_1(1-p_{e,1})(1-p_{e,2}) \\ & \dots \\ \lambda_{r+1} & = \lambda_r(1-p_{e,r}) = \lambda_1 \prod_{k=1}^{r}(1-p_{e,k})\end{array}\right\} \tag{11}$$

From the structure of the queueing network:

$$\begin{aligned}\lambda_1 &= \lambda + \lambda_1 p_{e,1} + \lambda_2 p_{e,2} + \lambda_3 p_{e,3} + \cdots + \lambda_{r+1} p_{e,r+1} \\ &= \lambda + \lambda_1 p_{e,1} + \lambda_1(1-p_{e,1})p_{e,2} + \lambda_1(1-p_{e,1})(1-p_{e,2})p_{e,3} + \cdots + \lambda_1 \prod_{i=1}^{r}(1-p_{e,i})p_{e,r+1} \\ &= \lambda + \lambda_1 \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\end{aligned}$$

Hence

$$\lambda_1 = \frac{\lambda}{1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)}$$

and from the set of equations 11, we obtain the packet arrival rates λ_i and λ_{r+1} at every relay station i and at the mobile station:

$$\lambda_i = \frac{\lambda \prod_{k=1}^{i-1}(1-p_{e,k})}{1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)} \tag{12}$$

$$\lambda_{r+1} = \frac{\lambda \prod_{k=1}^{r}(1-p_{e,k})}{1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)} \tag{13}$$

2.2 Average number of packets

The offered load at each relay station i and at the mobile station is then given by equations 14 and 15.

$$\rho_i^{DF} = \lambda_i \overline{x}_{DF} = \frac{\lambda_1}{\mu_{DF}} = \frac{\lambda \prod_{k=1}^{i-1} (1 - p_{e,k})}{\mu_{DF}\left(1 - \sum_{k=0}^{r} \left(p_{e,k+1} \prod_{i=1}^{k} (1 - p_{e,i})\right)\right)} \tag{14}$$

$$\rho^{MS} = \lambda_{r+1} \overline{x}_{MS} = \frac{\lambda_1}{\mu_{MS}} = \frac{\lambda \prod_{k=1}^{r} (1 - p_{e,k})}{\mu_{DF}\left(1 - \sum_{k=0}^{r} \left(p_{e,k+1} \prod_{i=1}^{k} (1 - p_{e,i})\right)\right)} \tag{15}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \dfrac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 16 and 17 respectively.

$$N_{i,q}^{DF} = \frac{(\rho_i^{DF})^2}{1 - \rho_i^{DF}} \tag{16}$$

$$N_q^{MS} = \frac{(\rho^{MS})^2}{1 - \rho^{MS}} \tag{17}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 14 and 15 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 18 and 19.

$$N_i^{DF} = \frac{\rho_i^{DF}}{1 - \rho_i^{DF}} \tag{18}$$

$$N^{MS} = \frac{\rho^{MS}}{1 - \rho^{MS}} \tag{19}$$

2.3 Average queueing delay

From Little's theorem, the average queueing delay is $W = \dfrac{N_q}{\lambda}$. Equations 20 and 21 give the average queueing delay at a relay station i and at the mobile station respectively.

$$W_i^{DF} = \frac{N_{i,q}^{DF}}{\lambda_i} \tag{20}$$

$$W^{MS} = \frac{N_q^{MS}}{\lambda_{r+1}} \tag{21}$$

2.4 Average end-to-end delay

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$T_i^{DF} = W_i^{DF} + \frac{1}{\mu_{DF}} \tag{22}$$

$$T^{MS} = W^{MS} + \frac{1}{\mu_{MS}} \tag{23}$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$T = T^{MS} + \sum_{i=1}^{r} T_i^{DF} = \frac{N^{MS} + \sum_{i=1}^{r} N_i^{DF}}{\lambda}$$

3 DF relaying with hop-by-hop ARQ

3.1 Packet arrival rate at each DF and at the MS

For this queueing network, the arrival rate at the first relay station is

$$\lambda_1 = \lambda + \lambda_1 p_{e,1} \Leftrightarrow \lambda_1 = \frac{\lambda}{1 - p_{e,1}}$$

From the structure of the queueing network, the arrival at the subsequent relay stations rate will be the sum of the error-free arrivals from its previous station and the erroneous packets that were processed at that specific station

$$\left.\begin{array}{rl} \lambda_2 & = \lambda_2 p_{e,2} + (1 - p_{e,1})\lambda_1 \Leftrightarrow \lambda_2(1 - p_{e,2}) = (1 - p_{e,1})\lambda_1 \\ \lambda_3 & = \lambda_3 p_{e,3} + (1 - p_{e,2})\lambda_2 \Leftrightarrow \lambda_3(1 - p_{e,3}) = (1 - p_{e,2})\lambda_2 \\ & \dots \\ \lambda_{r+1} & = \lambda_r p_{e,r} + (1 - p_{e,r})\lambda_r \Leftrightarrow \lambda_{r+1}(1 - p_{e,r+1}) = (1 - p_{e,r})\lambda_r \end{array}\right\} \tag{24}$$

From this set of equations we observe that

$$\lambda_1(1 - p_{e,1}) = \lambda_2(1 - p_{e,2}) = \lambda_3(1 - p_{e,3}) = \cdots = \lambda_{r+1}(1 - p_{e,r+1})$$

Hence the arrival rates at each relay station i and at the mobile station are

$$\lambda_i = \frac{\lambda}{1 - p_{e,i}}$$

$$\lambda_{r+1} = \frac{\lambda}{1 - p_{e,r+1}}$$

3.2 Average number of packets

The offered load at each relay station i and at the mobile station is then given by equations 25 and 26.

$$\rho_i^{DF} = \lambda_i \overline{x}_i = \frac{\lambda}{\mu_{DF}(1 - p_{e,i})} \tag{25}$$

$$\rho^{MS} = \lambda_{r+1} \overline{x}_{r+1} = \frac{\lambda}{\mu_{MS}(1 - p_{e,r+1})} \tag{26}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \dfrac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 27 and 28 respectively.

$$N_{i,q}^{DF} = \frac{(\rho_i^{DF})^2}{1 - \rho_i^{DF}} = \frac{\lambda^2}{\mu_{DF}(1 - p_{e,i})\Big(\mu_{DF}(1 - p_{e,i}) - \lambda\Big)} \tag{27}$$

$$N_q^{MS} = \frac{(\rho^{MS})^2}{1 - \rho^{MS}} = \frac{\lambda^2}{\mu_{MS}(1 - p_{e,r+1})\Big(\mu_{MS}(1 - p_{e,r+1}) - \lambda\Big)} \tag{28}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 25 and 26 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 18 and 19.

$$N_i^{DF} = \frac{\rho_i^{DF}}{1 - \rho_i^{DF}} = \frac{\lambda}{\mu_{DF}(1 - p_{e,i}) - \lambda} \tag{29}$$

$$N^{MS} = \frac{\rho^{MS}}{1 - \rho^{MS}} = \frac{\lambda}{\mu_{MS}(1 - p_{e,r+1}) - \lambda} \tag{30}$$

3.3 Average queueing delay

From Little's theorem, the average queueing delay is $W = \dfrac{N_q}{\lambda}$. Equations 31 and 32 give the average queueing delay at a relay station i and at the mobile station respectively.

$$\begin{aligned} W_i^{DF} = \frac{N_{i,q}^{DF}}{\lambda_i} &= \frac{\dfrac{\lambda^2}{\mu_{DF}(1 - p_{e,i})\Big(\mu_{DF}(1 - p_{e,i}) - \lambda\Big)}}{\dfrac{\lambda}{1 - p_{e,i}}} \\ &= \frac{\lambda}{\mu_{DF}\Big(\mu_{DF}(1 - p_{e,i}) - \lambda\Big)} \end{aligned} \tag{31}$$

$$
W^{MS} = \frac{N_q^{MS}}{\lambda_{r+1}} = \frac{\dfrac{\lambda^2}{\mu_{MS}(1-p_{e,r+1})\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)}}{\dfrac{\lambda}{1-p_{e,r+1}}}
$$

$$
= \frac{\lambda}{\mu_{MS}\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)} \tag{32}
$$

3.4 Average end-to-end delay

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$
\begin{aligned}
T_i^{DF} &= W_i^{DF} + \frac{1}{\mu_{DF}} \\
&= \frac{\lambda}{\mu_{DF}\Big(\mu_{DF}(1-p_{e,i})-\lambda\Big)} + \frac{1}{\mu_{DF}} \\
&= \frac{1-p_{e,i}}{\mu_{DF}(1-p_{e,i})-\lambda}
\end{aligned}
$$

$$
\begin{aligned}
T^{MS} &= W^{MS} + \frac{1}{\mu_{MS}} \\
&= \frac{\lambda}{\mu_{MS}\Big(\mu_{MS}(1-p_{e,r+1})-\lambda\Big)} + \frac{1}{\mu_{MS}} \\
&= \frac{1-p_{e,r+1}}{\mu_{MS}(1-p_{e,r+1})-\lambda}
\end{aligned}
$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$
\begin{aligned}
T &= T^{MS} + \sum_{i=1}^{r} T_i^{DF} \\
&= \frac{1-p_{e,r+1}}{\mu_{MS}(1-p_{e,r+1})-\lambda} + \sum_{i=1}^{r} \frac{1-p_{e,i}}{\mu_{DF}(1-p_{e,i})-\lambda}
\end{aligned}
$$

Numerical Evaluation

4 Stability region

4.1 AF relaying with end-to-end ARQ

Given that $p_{e,i} = 0.1, i = \{1, 2 \dots r+1\}$ and $\mu_{AF} = \mu_{MS} = 2$, the arrival rates at each relay station and at the mobile station are:

$$\begin{aligned}
\lambda_1 &= \frac{\lambda}{(1-0.1)^{r+1}} = \lambda\ 0.9^{-r-1} \\
\lambda_2 &= \lambda_1(1-0.1) = \lambda\ 0.9^{-r} \\
\lambda_3 &= \lambda_2(1-0.1) = \lambda\ 0.9^{-r+1} \\
&\dots \\
\lambda_{r+1} &= \lambda\ 0.9^{-1}
\end{aligned}$$

Since (a) the entire network is stable when all queues are stable, (b) every station is a M/M/1 system, and (c) such a system is considered to be stable when $\rho = \dfrac{\lambda}{\mu} < 1$, then the network is stable if all of the following inequalities hold:

$$\begin{aligned}
&\lambda \cdot 0.9^{-r-1} < 2 \\
&\lambda \cdot 0.9^{-r} < 2 \\
&\lambda \cdot 0.9^{-r+1} < 2 \\
&\dots \\
&\lambda \cdot 0.9^{-1} < 2
\end{aligned}$$

or

$$\begin{aligned}
&\lambda < 2 \cdot 0.9^{r+1} \\
&\lambda < 2 \cdot 0.9^{r} \\
&\lambda < 2 \cdot 0.9^{r-1} \\
&\dots \\
&\lambda < 2 \cdot 0.9^{1}
\end{aligned}$$

From these conditions, since $r \geq 1$ and $0.9 < 1$, we can see that the most restrictive condition is the first one, i.e. if the first one holds, then, inevitably, all the following conditions will be true. Figure 1 plots function $\lambda_{max}(r) = 2 \cdot 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is defined as the region where $\lambda < \lambda_{max}(r)$.

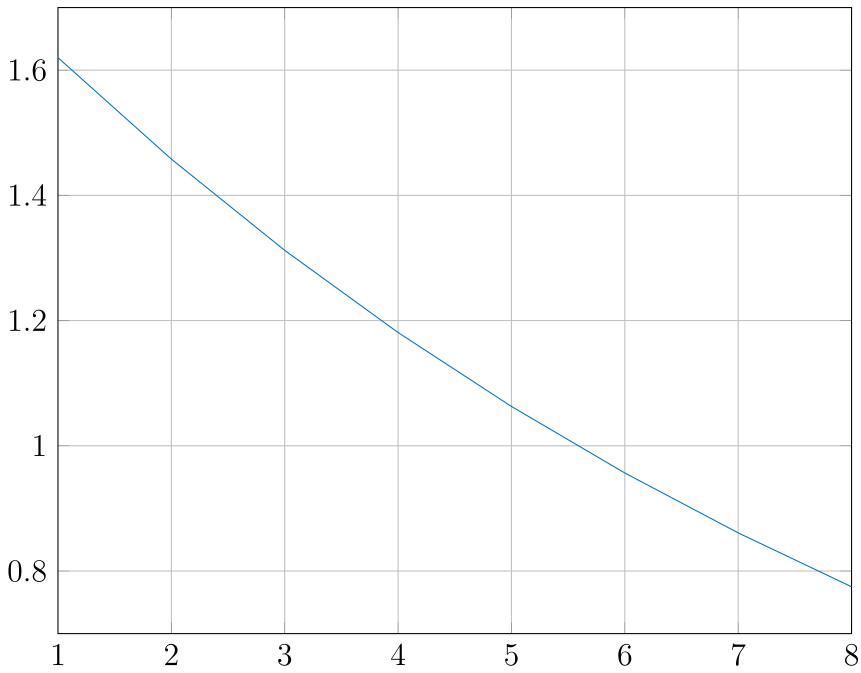

Figure 1: Function $\lambda_{max}(r) = 2 \cdot 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is the area under the curve.

As $r \to \infty$, the stability region gets smaller and smaller in size, until it is obliterated, since $\lambda_{max}(r) \to 0$ as $r \to \infty$.

This is reasonable, since, as r increases, the number of relay stations increase, and each one of them carries a probability of erroneous transmission with it. Each erroneous packet has to be retransmitted from the base station, and the end-to-end error probability goes to 1 as $r \to \infty$:

$$\lim_{r\to\infty} p_{e,e2e} = \lim_{r\to\infty} (1 - \prod_{k=1}^{r+1} 0.9) = 1 - 0 = 1$$

4.2 DF relaying with end-to-end ARQ

With $p_{e,i} = 0.1, i = \{1, 2 \dots r+1\}$, the arrival rates at each relay station and at the mobile station, given by equations 12 and 13, become

$$\lambda_i = \frac{\lambda}{1 - \sum_{k=0}^{r} \left(0.1 \prod_{i=1}^{k} (1-0.1)\right)} \prod_{k=1}^{i-1} (1-0.1) \tag{33}$$

$$\lambda_{r+1} = \frac{\lambda}{1 - \sum_{k=0}^{r} \left(0.1 \prod_{i=1}^{k} (1-0.1)\right)} \prod_{k=1}^{r} (1-0.1) \tag{34}$$

The first part of the right-hand side of the above 2 equations does not change over each station, and is equal to

$$\frac{\lambda}{1-\sum_{k=0}^{r}\left(0.1\prod_{i=1}^{k}(1-0.1)\right)}=\frac{\lambda}{1-0.1\sum_{k=0}^{r}0.9^k}=\frac{\lambda}{1-0.1\frac{1-0.9^{r+1}}{1-0.9}}=\lambda\cdot 0.9^{-r-1}$$

Hence, equations 33 and 34 become

$$\lambda_i=\lambda\cdot 0.9^{-r-1}\prod_{k=1}^{i-1}(1-0.1)=\lambda\cdot 0.9^{-r-1}\cdot 0.9^{i-1}=\lambda\cdot 0.9^{i-r-2}$$

$$\lambda_{r+1}=\lambda\cdot 0.9^{-r-1}\prod_{k=1}^{r}(1-0.1)=\lambda\cdot 0.9^{-r-1}\cdot 0.9^{r}=\lambda\cdot 0.9^{-1}$$

Since (a) the entire network is stable when all queues are stable, (b) every station is a M/M/1 system, and (c) such a system is considered to be stable when $\rho_k=\frac{\lambda_k}{\mu_k}<1$, then, the network is stable if all of the following inequalities hold:

$$\begin{aligned}\lambda_i &< \mu_{DF}\\ \lambda_{r+1} &< \mu_{MS}\end{aligned}$$

or

$$\begin{aligned}\lambda &< 0.9^{r+1}\\ \lambda &< 0.9^{r}\\ \lambda &< 0.9^{+r-1}\\ &\dots\\ \lambda &< 0.9^{2}\\ \text{and}&\\ \lambda &< 2\cdot 0.9\end{aligned}$$

From these conditions, since $r\geq 1$, we can see that the most restrictive condition is the first one, i.e. if the first one holds, then, inevitably, all the following conditions will be true. Figure 2 plots function $\lambda_{max}(r)=0.9^{r+1}$ for $1\leq r\leq 8$. The stability region is defined as the region where $\lambda<\lambda_{max}(r)$.

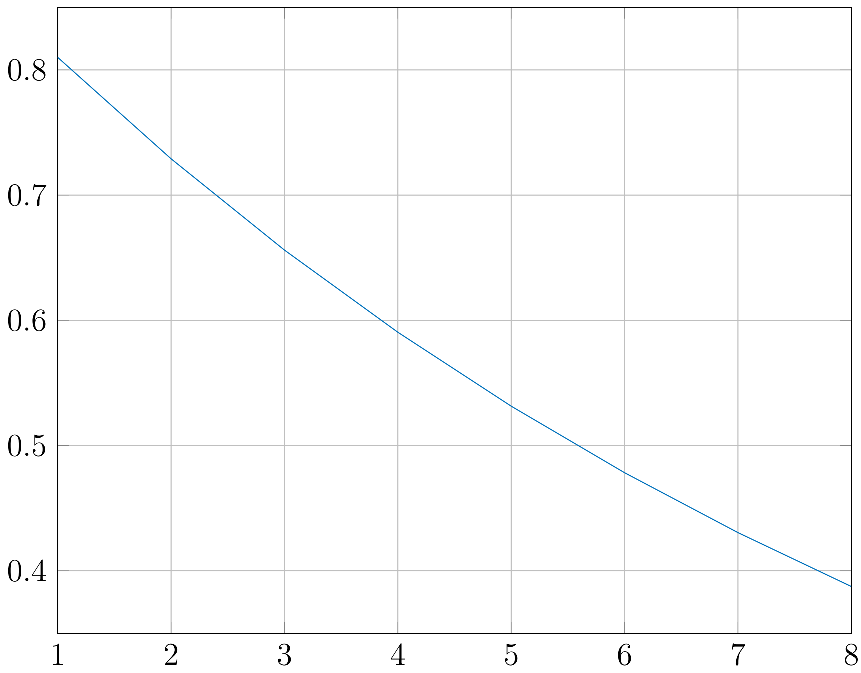

Figure 2: Function $\lambda_{max}(r) = 0.9^{r+1}$ for $1 \leq r \leq 8$. The stability region is the area under the curve.