VT16 – EP2200
Project I - Error Control in Relay Networks

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Theoretic Analysis

1 AF relaying with end-to-end ARQ

1.1 End-to-end error probability

For this queueing network, the different arrival rates for each station are:

$$\begin{aligned}
\lambda_2 &= \lambda_1(1 - p_{e,1}) \\
\lambda_3 &= \lambda_2(1 - p_{e,2}) = \lambda_1(1 - p_{e,1})(1 - p_{e,2}) \\
&\dots \\
\lambda_{r+1} &= \lambda_r(1 - p_{e,r}) = \lambda_1 \prod_{k=1}^{r}(1 - p_{e,k}), \text{ and, hence} \\
\lambda_0 &= \lambda_1 \prod_{k=1}^{r+1}(1 - p_{e,k})
\end{aligned}$$

The success rate, or end-to-end probability of successful receipt of packets at the mobile station is hence

$$p_{s,e2e} = \prod_{k=1}^{r+1}(1 - p_{e,k})$$

and the end-to-end error probability is then

$$p_{e,e2e} = 1 - p_{s,e2e} = 1 - \prod_{k=1}^{r+1}(1 - p_{e,k})$$

1.2 Packet arrival rate at the first relay station

Then, at the first relay station, the packet arrival rate is

$$\begin{aligned}
\lambda_1 &= \lambda + \lambda_0 = \lambda + \lambda_1 p_{e,e2e} \\
\lambda_1(1 - p_{e,e2e}) &= \lambda \\
\lambda_1 &= \frac{\lambda}{1 - p_{e,e2e}} \\
\lambda_1 &= \frac{\lambda}{1 - (1 - \prod_{k=1}^{r+1}(1 - p_{e,k}))} \\
\lambda_1 &= \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{e,k})}
\end{aligned}$$

1.3 Average number of packets

Since there is no blocking of erroneous packets made in this queueing network, the arrival rate at every relay station and at the mobile station will be equal to $\lambda_i = \lambda_1$. The offered load at each relay station i and at the mobile station is then given by equations 1 and 2 respectively.

$$\rho_i^{AF} = \lambda_i \overline{x}_{AF} = \frac{\lambda_1}{\mu_{AF}} = \frac{\lambda}{\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{1}$$

$$\rho^{MS} = \lambda_{r+1} \overline{x}_{MS} = \frac{\lambda_1}{\mu_{MS}} = \frac{\lambda}{\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k})} \tag{2}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \frac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 3 and 4 respectively.

$$N_{i,q}^{AF} = \frac{\lambda^2}{\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k}) \Big(\mu_{AF} \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda\Big)} \tag{3}$$

$$N_q^{MS} = \frac{\lambda^2}{\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k}) \Big(\mu_{MS} \prod_{k=1}^{r+1}(1 - p_{e,k}) - \lambda\Big)} \tag{4}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 1 and 2 respectively.

Hence, the average number of packets at every relay station i and at the mobile station are given by equations 5 and 6.

$$N_i^{AF} = \frac{\rho_i^{AF}}{1-\rho_i^{AF}} = \frac{\lambda}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{5}$$

$$N^{MS} = \frac{\rho^{MS}}{1-\rho^{MS}} = \frac{\lambda}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{6}$$

1.4 Average queueing delay

From Little's theorem, the average queueing delay is $W = \frac{N_q}{\lambda}$. Equations 7 and 8 give the average queueing delay at a relay station i and at the mobile station respectively.

$$W_i^{AF} = \frac{\lambda}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})\left(\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda\right)} \tag{7}$$

$$W^{MS} = \frac{\lambda}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})\left(\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda\right)} \tag{8}$$

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$T_i^{AF} = W_i^{AF} + \frac{1}{\mu_{AF}} = \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{9}$$

$$T^{MS} = W^{MS} + \frac{1}{\mu_{MS}} = \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \tag{10}$$

1.5 Average end-to-end delay

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$\begin{aligned} T &= \frac{r\cdot N_i^{AF} + N^{MS}}{\lambda} = r\cdot T_i^{AF} + T^{MS} \\ &= r\cdot\frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{AF}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} + \frac{\prod_{k=1}^{r+1}(1-p_{e,k})}{\mu_{MS}\prod_{k=1}^{r+1}(1-p_{e,k})-\lambda} \end{aligned}$$

2 DF relaying with end-to-end ARQ

For this queueing network, the different arrival rates for each station are:

$$\left.\begin{array}{ll} \lambda_2 & = \lambda_1(1-p_{e,1}) \\ \lambda_3 & = \lambda_2(1-p_{e,2}) = \lambda_1(1-p_{e,1})(1-p_{e,2}) \\ & \dots \\ \lambda_{r+1} & = \lambda_r(1-p_{e,r}) = \lambda_1 \prod_{k=1}^{r}(1-p_{e,k}) \end{array}\right\} \tag{11}$$

From the structure of the queueing network:

$$\begin{aligned} \lambda_1 &= \lambda + \lambda_1 p_{e,1} + \lambda_2 p_{e,2} + \lambda_3 p_{e,3} + \cdots + \lambda_{r+1} p_{e,r+1} \\ &= \lambda + \lambda_1 p_{e,1} + \lambda_1(1-p_{e,1})p_{e,2} + \lambda_1(1-p_{e,1})(1-p_{e,2})p_{e,3} + \cdots + \lambda_1 \prod_{i=1}^{r}(1-p_{e,i})p_{e,r+1} \\ &= \lambda + \lambda_1 \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right) \end{aligned}$$

Hence

$$\lambda_1 = \frac{\lambda}{1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)}$$

And from the set of equations 11, we obtain the packet arrival rates λ_i and λ_{r+1} at every relay station i and at the mobile station:

$$\lambda_i = \frac{\lambda \prod_{k=1}^{i-1}(1-p_{e,k})}{1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)}$$

$$\lambda_{r+1} = \frac{\lambda \prod_{k=1}^{r}(1-p_{e,k})}{1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)}$$

The offered load at each relay station i and at the mobile station is then given by equations 12 and 13.

$$\rho_i^{DF} = \lambda_i \overline{x}_{DF} = \frac{\lambda_1}{\mu_{DF}} = \frac{\lambda \prod_{k=1}^{i-1}(1-p_{e,k})}{\mu_{DF}\left(1-\sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right)} \tag{12}$$

$$\rho^{MS} = \lambda_{r+1}\overline{x}_{MS} = \frac{\lambda_1}{\mu_{MS}} = \frac{\lambda \prod_{k=1}^{r}(1-p_{e,k})}{\mu_{DF}\left(1 - \sum_{k=0}^{r}\left(p_{e,k+1}\prod_{i=1}^{k}(1-p_{e,i})\right)\right)} \tag{13}$$

With these pieces of information, we can derive the average number of packets in each queue, the average number of packets being serviced, and then, from Little's theorem, the average queueing delay.

The average number of packets in a queue of a M/M/1 system is $N_q = \frac{\rho^2}{1-\rho}$, hence the average number of packets in the queues of a relay station i and in the mobile station is given by equations 14 and 15 respectively.

$$N_{i,q}^{DF} = \frac{(\rho_i^{DF})^2}{1-\rho_i^{DF}} \tag{14}$$

$$N_q^{MS} = \frac{(\rho^{MS})^2}{1-\rho^{MS}} \tag{15}$$

The average number of packets under service at a relay station i and at the mobile station is given by equations 12 and 13 respectively.

From Little's theorem, the average queueing delay is $W = \frac{N_q}{\lambda}$. Equations 16 and 17 give the average queueing delay at a relay station i and at the mobile station respectively.

$$W_i^{AF} = \tag{16}$$

$$W^{MS} = \tag{17}$$

The average delay at a relay station i and at the mobile station is then the sum of the waiting time and the service time at that station:

$$T_i^{AF} = W_i^{AF} + \frac{1}{\mu_{AF}} = \tag{18}$$

$$T^{MS} = W^{MS} + \frac{1}{\mu_{MS}} = \tag{19}$$

The average end-to-end delay T is then the sum of all waiting and serving times, over r relay stations and at the mobile station.

$$T = r \cdot T_i^{AF} + T^{MS} = \tag{20}$$

3 DF relaying with hop-by-hop ARQ

Numerical Evaluation