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# Euler Graph (G)

closed walk All the ~~vertices~~ edges are included and vertices are even.

Case-I - Suppose Euler graph

PT V→ Even.

• always connected without isosceles.

• we can enter through one edge exit through another edge.

even degree

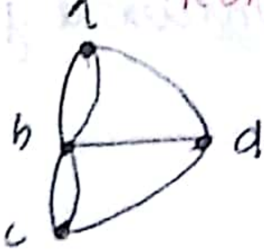
• Here we need to use one edge more than once

odd degree

case-II suppose all V→ even & PT Euler graph

consider a graph G starting from one vertex and reaching at the same vertex.
Since all the

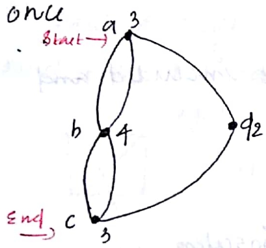
## Konigsberg Bridge Problem




odd degree
It is not Euler graph.

OPEN EULER LINE / UNICURSAL LINE

Open ~~closed~~ walk all edges are ~~not~~ included exactly once




2 Exactly two odd degree.

* Open euler line into Euler graph when start vertex and end vertex are connected

Theorems:-

If Graph G with 2K odd vertices there K edge disjoint subgraph or K unicursal graph.

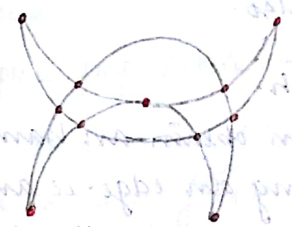
PROOF

G - 2K odd vertices $(v_1, v_2, \dots v_K)$, $(w_1, w_2, \dots w_K)$ } K edges to connect.

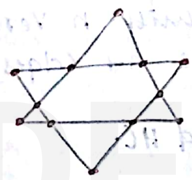
$(v_1 w_1)(v_2 w_2) \dots (v_K w_K)$

Removal of each edge results in each unicursal graph. If K edges are added then by removing K edges it forms K unicursal graph

Mohammed's Scimitars.



Star of David.





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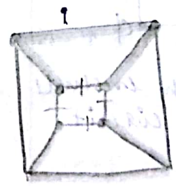
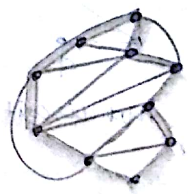
## Hamiltonian Path and circuits.

It is a closed walk that travels every vertex exactly once excepting the starting vertex. It is called a Hamiltonian circuit.

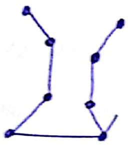




* In a hamiltonian ckt with n vertices will have n edges also.

Hamiltonian path.

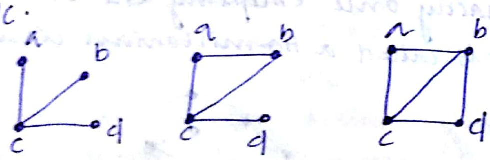
we can obtain an hamiltonian Path by removing an edge. ie an open walk that travels through every ~~ver~~ vertex exactly once



An hamiltonian path ~~ha~~ with n vertex will have n-1 edges.

propeaties of HP and HC.

- HP is a subset of HC. and HC is a subset of G
- All HC includes HP but HP doesn't includes HC.



no path    not circuit    • Path includ • no circuit

* In HP and HC there is no self loop and parallel edges.

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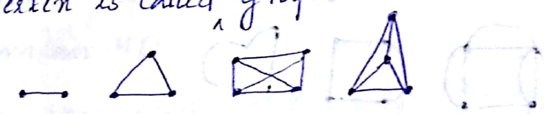
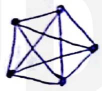
Complete graph/universal graph/ clique.

There is an edge b/w each pair of vertex is called complete graph.

eg:



~~Grag~~ complete graph with n vertices will have degree of n-1, no: of edges in a complete graph with n vertices is $\frac{n(n-1)}{2}$



• complete graph with 5 vertices

* There is a hamiltonian circuit and path in complete graph.

Theorem:

In a complete graph with n vertices there are $\frac{n-1}{2}$ edge disjoint hamiltonian circuits if n is odd and $n \geq 3$

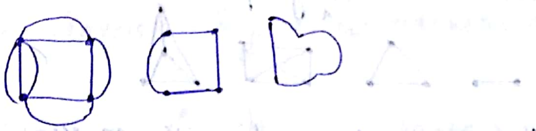
suppose this is our graph

$n=5$ - odd $\geq 3$

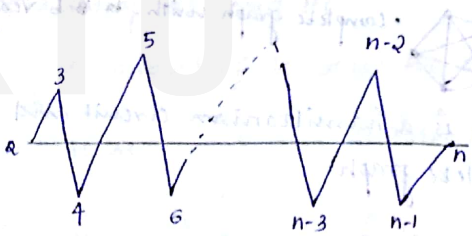
$\left(\frac{n-1}{2}\right) \Rightarrow \frac{4}{2} = 2$

2 edge disjoint Hamiltonian circuit

eg for edge disjoint



eg:




$\frac{360}{n-1}, \; 2 \cdot \frac{360}{n-1}$

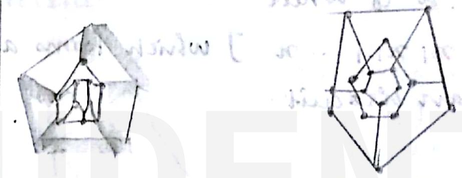
Q Observe that there can be no path longer than a hamiltonian path is it exist in a graph.

Q Draw a graph that have hamiltonian path but doesn't have HC.

Q Draw a graph in which an Euler line is also a HC.

Q Draw a graph which is an euler line but don't form an HC.

Q Show that neither of the graphs have a HP



Diracs theorem for Hamiltonicity

Theorem:

Derive theorem for hamitonicity

If $G$ is a graph with $n$- vertices where $n \geq 3$ and degree of $d \geq \frac{n}{2}$ for every vertex $v \in G$) then $G$ is hamiltonion.

PROOF:

Let $P = x_0, x_1, x_2 \dots x_k$ be a longest path in $G$ since $P$ is the longest path all neighbours of $x_0$ & all neighbours of $x_k$ lies on P. Hence atleast $n/2$ of the vertices $x_0, x_1, \ldots x_{k-1}$ are adjuscent to $x_k$ and atleast $n/2$ vertices of $x_1, x_2 \ldots x_k$ are adjuscent to $x_0$ hence atleast $\frac{n}{2}$ vertices of $x_i \in \{x_0, x_1 \ldots x_{k-1}\}$ some $x_i$

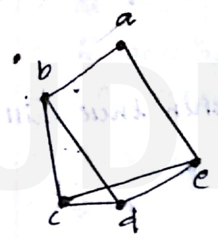
$x_i x_k \in E$

$\therefore$ It forms a C where $C = x_0, x_{i+1}, x_{i+2} \ldots x_{k-1}, x_k, x_i x_{i-1} \ldots x$ ] which forms a hamiltonian circuit

~~Dirac's theorem for Hamiltonicity~~ :-

## Ore's theorem :-

Ore's theorem the connected graph G has an hamiltonian ckt if for any 2 vertices u and v which are not adjuscent, $deg(u) + deg(v) \geq n$ where n is the no: of vertices




non-adjuscent vertices

(a, c)
(a, d)
(b, e)

$d(a) = 2$  $d(c) = 3$  $d(d) = 3$

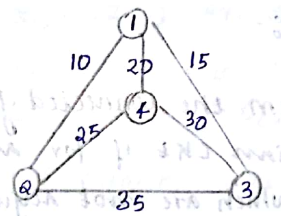
$d(b) = 3$  $d(e) = 3$

$d(a,c) = 5 \geq \frac{n}{2}$  $\frac{5}{2} = 2.5$

$d(a,d) = 5$

$d(b,e) = 6$

13/9/17 Travelling Sales Man Problem: (TSP)




Vertex - cities
Edge - Path.

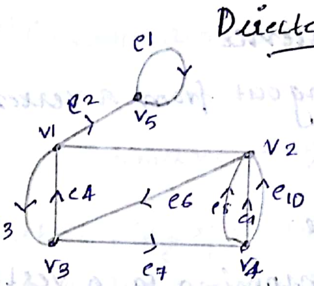
distance b/w the cities is represented by a weight on the edge.

- If there are n vertices then there will be $\frac{(n-1)!}{2}$ paths.

$\frac{(4-1)!}{2} = \frac{3!}{2} = \frac{3\times 2}{2} = \frac{6}{2} = 3$ //.

① 1 - 2 - 4 - 3 - 1 = 80
1 - 4 - 2 - 3 - 1 = 95
1 - 3 - 2 - 4 - 1 = 80.

## Directed Graphs




$V = \{v_1, v_2, \ldots v_n\}$

$E = \{e_1, e_2, \ldots e_k\}$

A mapping sign $\psi$ such that mapping each edges of it mapping to an ordered pair of vertices.

$e_3 \rightarrow (v_1, v_3)$

$e_4 \rightarrow (v_3, v_1)$

(v) ⟶ e initial vertex

(v) ⟵ e terminal vertex

A Directed graph is a graph which initial vertex and terminal vertex are as same.

$e_3$ and $e_4$ are not parallel edges.

Outdegree / out valence

The no: of edges going out from a vertex.

$d^+(v)$

Indegree / invalence.

The no: of edges incoming to a vertex.

$d^-(v)$

- $d^+(v_1) = 3$

$d^-(v_1) = 1$

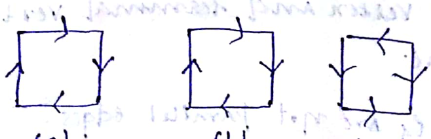
$d^+(v_1) + d^-(v_1) = d(v_1) = 4$

DIGRAPHS

Types of digraphs:

1) Isomorphic diagram:

Graph with same no of vertices, edges and same degree, direction will also same.




a is isomorphic to

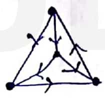
2) Simple digraph:

Symmetric    Asymmetric

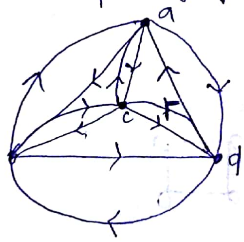
Symmetric diagraph: when a is related to b, then b is also related to a.


3) Complete digraph:

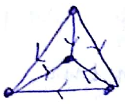
A digraph in which all vertices are connected to each edge.



Symmetric complete graph



Assymmetric complete graph :



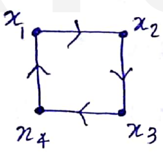
4 Balanced digraph
- " " is for each vertex indegree = outdegree

Digraph and binary relation :

$X = \{x_1, x_2, x_3, \dots\}$

$x_i \, R \, x_j$

The elements in a digraph is the vertex, points and directions include in edges .

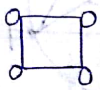



PROPERTIES :-

1) Reflexive

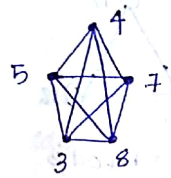
$x_i R x_i$

* Selfloop

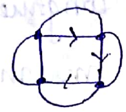


eg R: "is greater than"

$\{3, 4, 5, 7, 8\}$




2) Symmetry



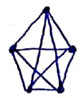
$x_i R x_j$ then $x_j R x_i$

eg: "is equal to" – symmetric and Reflexive .

"is spouse to" – Symmetric and irreflexive .

3) Transitive

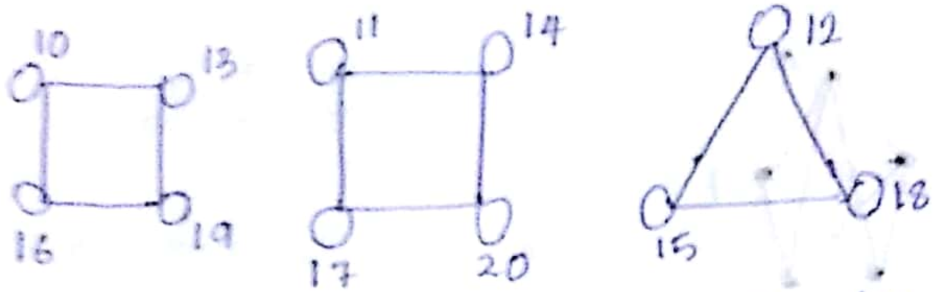
aRb, bRc then aRc



4) Equivalance

A relation which reflexive, symmetric and transitive are called equivalance relation eg: consider some digraphs which have 10-20

20 elements




= 1 mod 3     = 2 mod 3     = mod 3

R : is congruent to mod m

a = b mod m when a mod m = b mod m

Relation matrix

$A = \{3, 4, 5, 7, 8\}$

$$\begin{array}{c|c} & \begin{matrix} 3 & 4 & 5 & 7 & 8 \end{matrix} \\ \begin{matrix} 3 \\ 4 \\ 5 \\ 7 \\ 8 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$= 1 \quad x_i R x_j$

$= 0 \quad x_i \not R x_j$