22/08

MODULE 2

Regular Grammar

* Set of rules for language.
* A grammar is represented by 4 tuple representation

$G = (V, T, S, P)$

V is set of objects called variables.

T terminal symbols.

$S \in V$ called start symbol

P is called productions.

Production format is $x \to y$, x can be replaced with y

ab

eg:- $G = (\{S\}, \{a,b\}, S, P)$

where P is given by $S \to aSb$

$S \to \varepsilon$.

? Write down the language generate by the grammar.

$L = \{\varepsilon, ab, aabb, aaabbb, \ldots\}$

? • $G = \{\{S, A\}, \{a, b\}, S, P\}$ $\left\{\begin{array}{l} Ab\ aAb, \\ bab. \end{array}\right\}$

$S \Rightarrow Ab$

$A \Rightarrow aAb$

$A \Rightarrow \varepsilon.$

$L = \{b, abb, aabbb \ldots\ldots\}$

$a^n b^{n+1}$ where $n \geq 0$

? $\Sigma = a, b$

$G = (\{S\}, \{a, b\}, S, P)$

P

$S \to SS$

$S \to \varepsilon$

$S \to aSb$

$S \to bSb$

bsb

$b\,s\,b \downarrow basbb.$

$L = \{\varepsilon ab, bb, babb, abbb, ababab, \ldots\ldots\}$

? $G = (\{A, S\}, \{a, b\}, S, P)$

$S \to aAb \mid \varepsilon$

$A \to aAb \mid \varepsilon.$

$L = \{\varepsilon, ab, aabb, aaabbb, \ldots\ldots\}.$

$a^n b^n$ where $n \geq 0.$

Q) construct a grammar to generate palindrom string over the alphabet a,b.

$L = \{ aba,\ abba, baab, bab, \ldots \}$

$S \to \varepsilon$ $S \to a$

$S \to aSa$ $S \to b.$

$S \to bSb.$

$G = (\{S\}, \{a,b\}, S, P)$

aSa
$\downarrow$
aa

aSa
$\downarrow$
$absba.$

Q? Generate the regular grammar over the alphabet a,b. Strings of language contain even no. of a's and even no. of b's

$L = \{ \varepsilon, aabb, abab, bbaa, baba, aaaabbbb, \ldots \}$

~~S→ε~~

$S \to aAb \mid \varepsilon$

$A \to aSb.$

~~S→aAa~~

$S \to \varepsilon \mid aAa$

$A \to bSb$

aSb

ε

ab

$S \to \varepsilon \mid aAb$

$A \to bSa.$

$S \to \varepsilon \mid bAb$

$A \to aSa.$

$S \to \varepsilon \mid bAa.$

$A \to aSb$

$S \to aAa \mid bAb$

$A \to \varepsilon$

H.W → Find the ~~alphabet~~ grammar over the alphabet (a,b) having the language which contains exactly one 'a'. $L = \{abb, bab, a, abbb, bba \ldots\}$

→ All strings have atleast one a over the alphabet a,b.

Answer

1) $G = (\{S, \} \{a,b\}, S, P)$

$L = \{abb, bba, babb, abbb., a, \ldots\}$

~~$S \to a \mid bAb$~~ ~~$S \to bAb$~~

~~$A \to b$~~ ~~$A \to a$~~

$A \to aAb \mid \varepsilon$

$S \to aAb \mid bAa$

$A \to bAb \mid \varepsilon$

29/08 Regular Expression.

A production rule is the form.

$$A \rightarrow Bx \quad | \quad A \rightarrow xB.$$
$$A \rightarrow x$$

where x is terminal.

eg: $S \rightarrow aA$
$S \rightarrow b$

$\varepsilon \rightarrow \{\varepsilon\}$ $\quad aa \rightarrow \{a\}$

$a \rightarrow \{a\}$ $\quad a+b \rightarrow \{a,b\}$

$a+b+c \rightarrow \{a,b,c\}$ $\quad a^* \rightarrow \{\varepsilon, a, aa, \ldots\}$

$aa^* \rightarrow \{a, aa, aaa, \ldots\}$ $\quad a^+ \rightarrow \{a, aa, aaa, \ldots\}$

? write regular expression of the language
$L = \{\varepsilon, ab, abab, ababab, \ldots\}$

regular expression $= (ab)^*$

? write R.E for language which contain even no. of ones over

$R = \{\varepsilon, 11, 1111, 111111\}$

$RE = (11)^*$

? write R.E with an even no.of a's followed by odd no.of b's

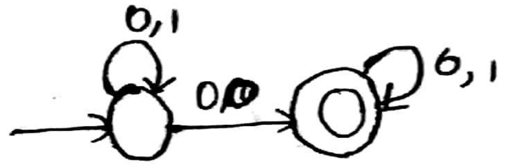
$L = \{aab, aaaab, \ldots\}$

$RE = (aa)^* (bb)^* b$

? write the R.E on language over (0,1) which contains at least one pair of consecutive 0's.

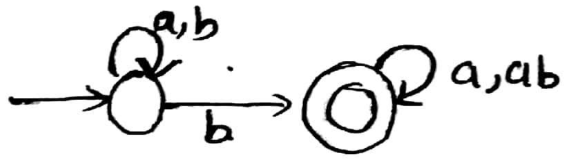
$L = \{00, 0011, 1100, 100, 000, 0001, \ldots\}$

$(0+1)^* 00 (0+1)^*$



? find all strings on $((a+b)^* b (a+ab)^*)$

$L = \{aba, b, abab, bba, ababab, \ldots\}$



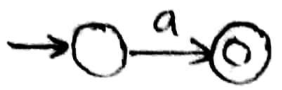
? write RE of language which contains all the alphabets (0,1) which ends with (01)

$L = \{101, 1101, 0001, 01, \ldots\}$

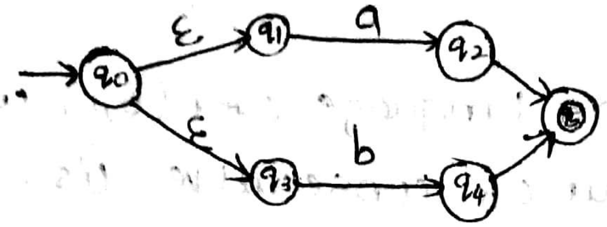
$R.E = (0+1)^* 01$

24/08 Conversion of regular expression to NFA with epsilon transition.

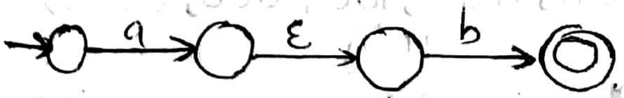
a



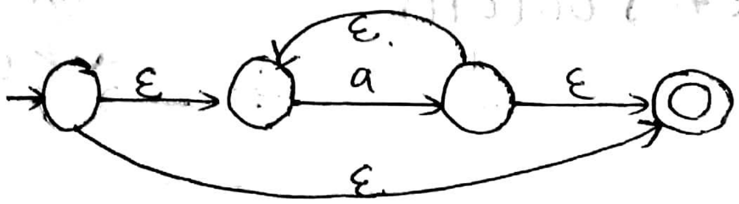
a+b



ab

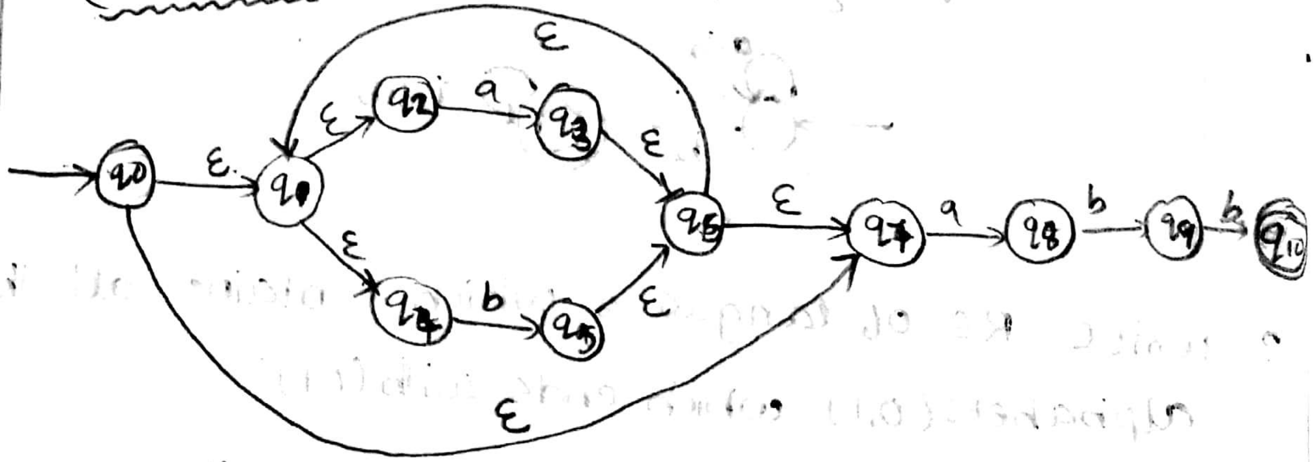


a*



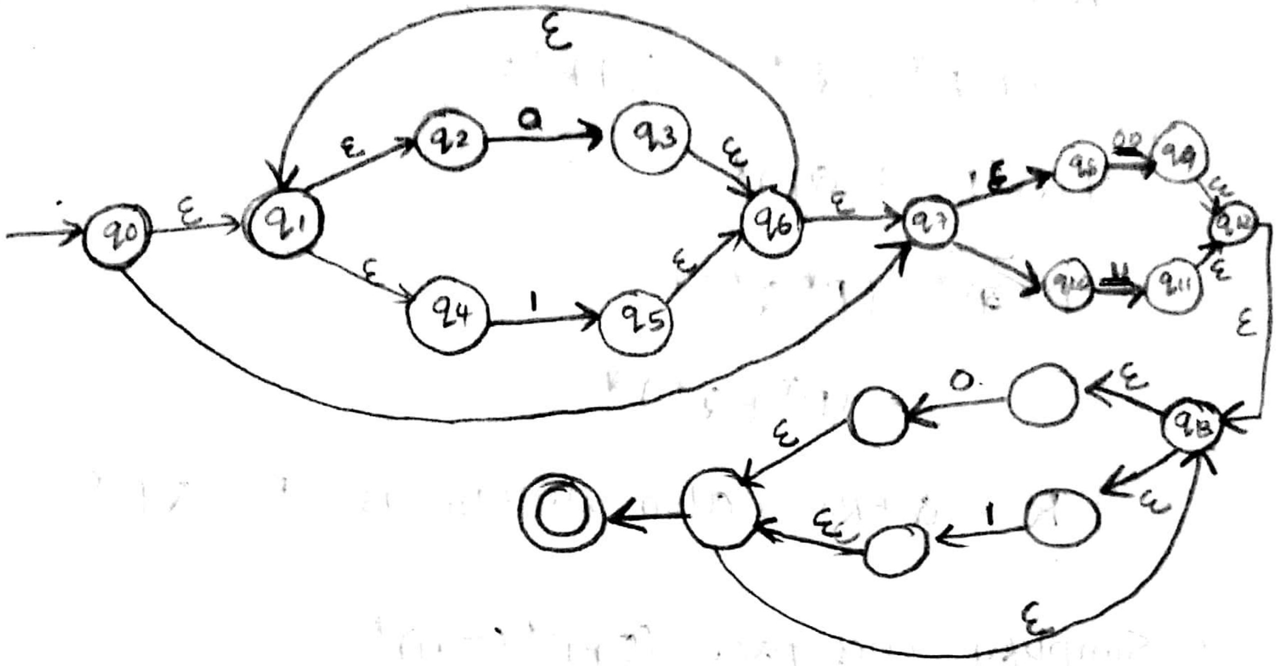
ε, a, aa

(a+b)*abb

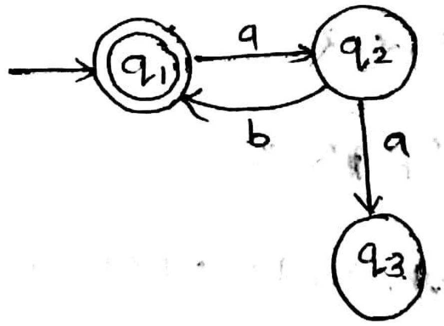


? Regular Expression to NFA, regular expression is.

$(0+1)^*(00+11)(0+1)^*$



? Conversion of DFA to regular Expression.



$\phi + R = R$

$\phi R = R\phi = \phi$

$\varepsilon R = R\varepsilon = R$

$\varepsilon^* R = \varepsilon$

$\phi^* = \varepsilon$

$R + R = R.$

$R^* R^* = R^*$

$$RR^* = R^*R$$

$$(R^*)^* = R$$

$$\varepsilon + RR^* = R^* = \varepsilon + R^*R$$

$$(PQ)^*P = P(QP)^*$$

$$(P+Q)^* = (P^*Q^*)^*$$

$$= (P^* + Q^*)^*$$

$R = Q + RP$ also written as $R = QP^*$

? Simplify and prove $(0+1)^*(0+1)^*$

$$(0+1)^*(0+1)^* = (0+1)^*$$

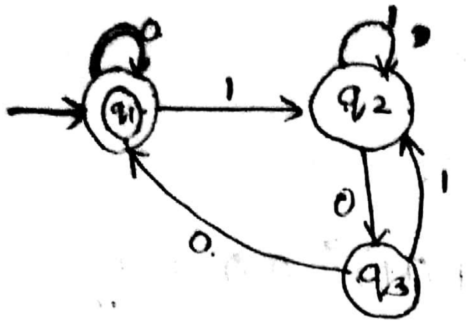
$$= (0^* 1^*)^*$$

$$= (0^* + 1^*)^*$$

$L\ (0+1)^*(0+1)^* = \{\varepsilon, 0, 1, 01, 11, 0101, 010101 \ldots\}$

$L\ (0+1)^* = \{\varepsilon, 0, 1, 00, 11, 01, 0101, \ldots\}$

?



convert to regular expression.

A. Follow the procedure.

$q_1 = q_1 0$ (inflow to q_1) $+ q_3 0$

- Inflow of each state, if it is start state also ε.

$$q_1 = q_1 0 + q_3 0 + \varepsilon$$

$$q_2 = q_1 1 + q_2 1 + q_3 1$$

$$q_3 = q_2 0$$

- Simplyfy the following.

On q_2,

$$q_2 = q_2 1 + q_3 1 + q_1 1$$

$$= q_2 1 + q_2 0 1 + q_1 1$$

$$\overset{R}{q_2} = \overset{Q}{q_1 1} \overset{+}{+} \overset{R}{q_2} \overset{P}{(1+01)}$$

$$q_2 = q_1 1 (1+01)^* \quad (R = Q + RP \Rightarrow QP^*$$

$$= q_1 1 (1+01)^*$$

$$q_1 = q_1 0 + q_3 0 + \varepsilon$$

$$= q_1 0 + q_2 00 + \varepsilon$$

$$= q_1 0 + q_1 1(1+01)^* 00 + \varepsilon$$

$$\underset{R}{q_1} = \underset{R}{q_1}\big(\underbrace{0+1(1+01)^*00}_{P}\big) + \underset{Q}{\varepsilon}$$

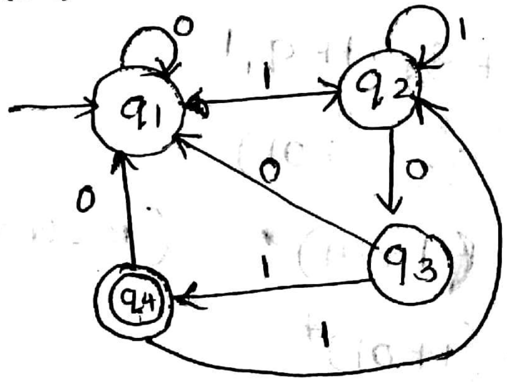
$$q_1 = \varepsilon(0+1(1+01)^*00)^* \qquad (\varepsilon R = R)$$

$$= (0+1(1+01)^*00)^*$$

q_1 is the final state you have to find regular expression corresponding to q_1.

Ans: $(0+1(1+01)^*00)^*$

? Reduce the following finite state automata to regular expression.



$$q_1 = q_1 0 + q_4 0 + q_3 0 + \varepsilon.$$

$$q_2 = q_1 1 + q_2 1 + q_4 1$$

$$q_3 = q_2 0$$

$$q_4 = q_3 1$$

$$\cancel{q_2 = q_1 1 + q_2 1 + q_4 1}$$

$$\cancel{= q_1 1 + q}$$

$$q_1 = q_1 0 + q_4 0 + q_3 0 + \varepsilon.$$

$$= q_1 0 + q_3 10 + q_3 0 + \varepsilon$$

$$= q_1 0 + q_3 (10 + 0) + \varepsilon.$$

$$q_2 = q_1 1 + q_2 1 + q_4 1$$

$$q_2 = q_1 1 + q_2 1 + q_3 11$$

$$q_2 = q_1 1 + q_2 1 + q_2 011$$

$$\underset{R}{q_2} = \underset{Q}{q_1 1} + \underset{R}{q_2} \underset{P}{(1 + 011)}$$

$$q_2 = q_1 1 (1 + 011)^*$$

$$\cancel{q_3 = q_2 0}$$

$$\cancel{= q_1 1 (1 + 011)^* 0}$$

$$q_1 = q_1 0 + q_4 0 + q_3 0 + \varepsilon$$

$$= q_1 0 + q_3 10 + q_2 00 + \varepsilon$$

$$= q_1 0 + q_2 010 + q_2 00 + \varepsilon$$

$$q_1 = q_1 0 + q_2 (010 + 00) + \varepsilon$$

$$= q_1 0 + q_1 1 (1 + 011)^* (010 + 00) + \varepsilon$$

$$q_1 = q_1 (0 + 1 (1 + 011)^* (010 + 00)) + \varepsilon.$$

$$R = R \; P.$$

$$q_3 = q_2 0.$$

$$q_4 = q_2 01$$

$$q_4 = q_1 1 (1 + 011)^* 01$$

$$\underset{R}{q_1} = \underset{R}{q_1} \underset{P}{(0 + 1 (1 + 011)^* (010 + 00))} \underset{+\,Q}{+\, \varepsilon}$$

$$= \varepsilon (0 + 1 (1 + 011)^* (010 + 00))^*$$

$$q_1 = (0 + 1 (1 + 011)^* (010 + 00))^*$$

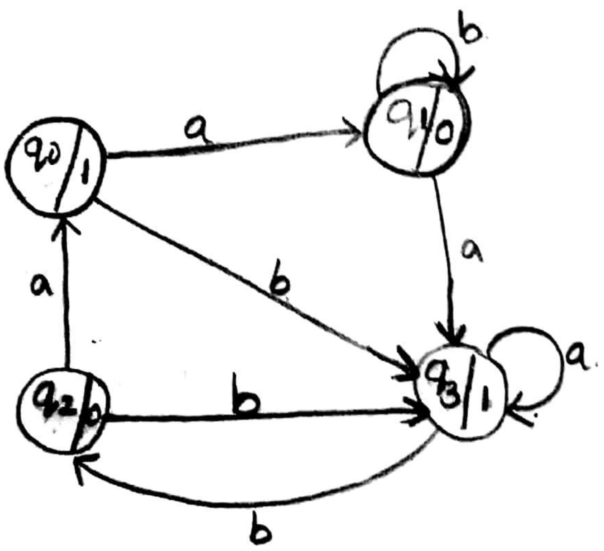
$$q_4 = (0 + 1 (1 + 011)^* (010 + 00))^* 1 (1 + 011)^* 01$$

Finite State Machines with Output

Mealy and Moore machine

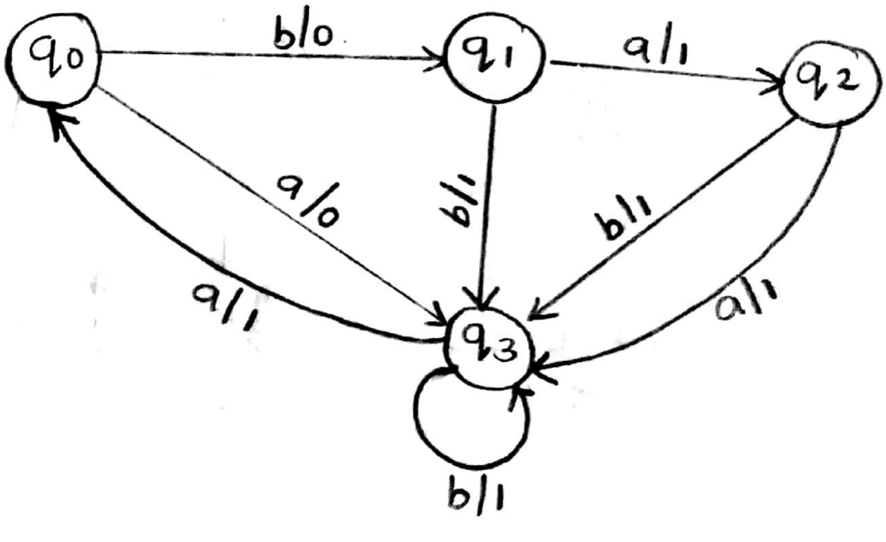
Present State.	Next state		Output
	After input a	After input b	
q_0	q_1	q_3	1
q_1	q_3	q_1	0
q_2	q_0	q_3	0
q_3	q_3	q_2	1

moore machine.



meanly machine.

Present State	Next State			
	input a		input b.	
	State	output	state	output
q_0	q_3	0	q_1	0.
q_1	q_2	1	q_3	1
q_2	q_3	1	q_3	0.
q_3	q_0	1	q_3	1



If the output function depends only on the present State, the automata is called moore machine.

If output fn. depends both present state and input the automata is called meanly machine.