MODULE-1

General graph includes

i) Self loop

ii) Parallelism

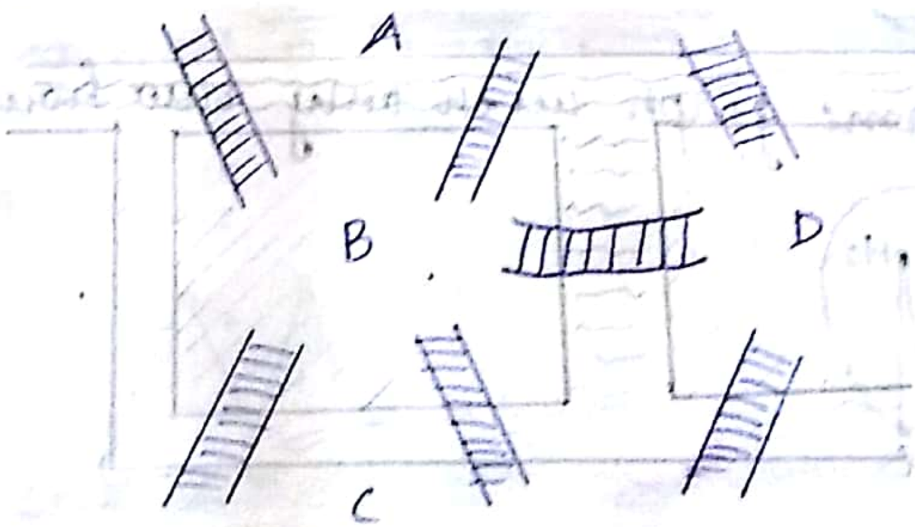
But in the case of Simple graph that two properties will not be included

? Draw all simple graphs of 1, 2, 3, 4 Vertices

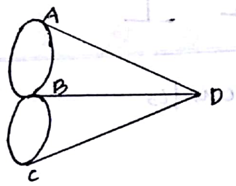
→ Applications of Graph

① Konigsberg Bridge Problem:-

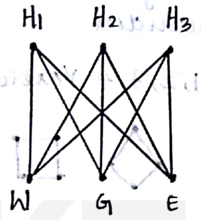
Leonhard Euler developed Konigsberg Bridge problem in 1735 at Russia. This problem doesn't exist solution



Graphical representation of above problem.

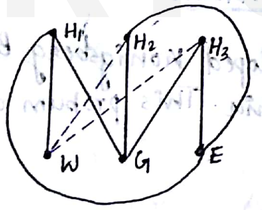


② Utility problem

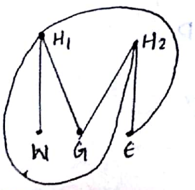


Three houses H_1 H_2 H_3 and Three utility

without crossing convert into graph.

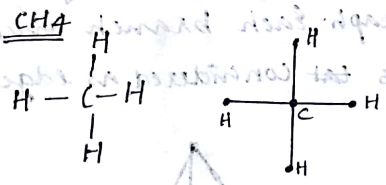


Draw the same graph with only two houses.

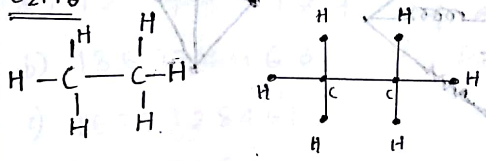


Draw graph for chemical compounds for CH_4, C_2H_6, C_6H_6, N_2O_3:

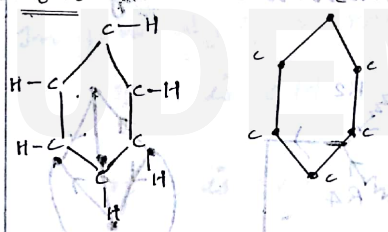
CH_4



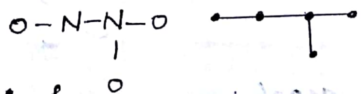
C_2H_6



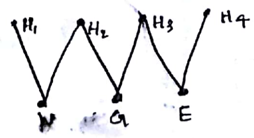
C_6H_6



N_2O_3:-



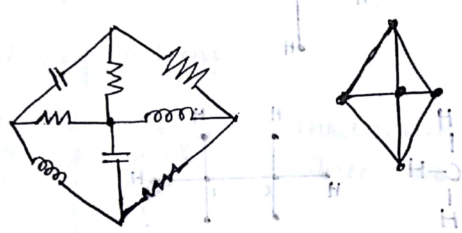
4 houses.



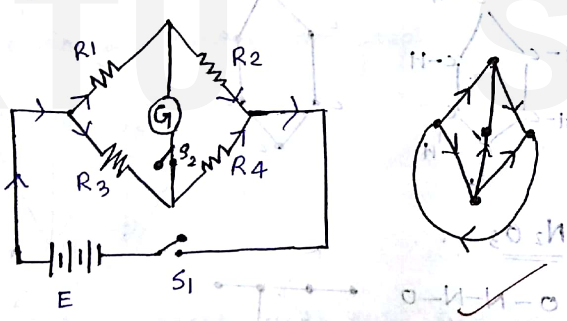
3/8/17

Electrical Network Problem

Consider an electrical network. Each junct^n is a vertex of the graph. Each branch including electrical element is ~~tat~~ considered as edge.

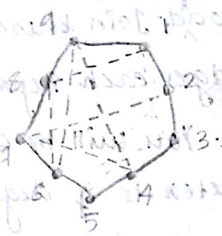


Q Draw a graph of wheatstone bridge circuit.



This is a directed graph.

Seating problem:



9 members need to change their neighbours each day. How many ~~such~~ arrangements possible?

a) 1 2 3 4 5 6 7 8 9

b) 1 3 5 2 7 4 9 6 8 1

c) 1 5 7 3 9 2 8 4 6 1

d 1 7 9 5 8 3 6 2 4 1

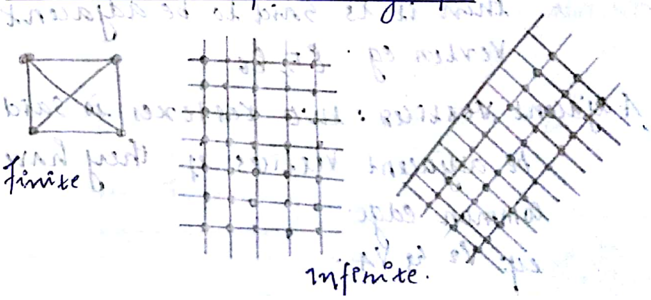
4 arrangements possible.

In general, if n members

If n is odd $= \frac{n-1}{2}$

If n is even $= \frac{n-2}{2}$

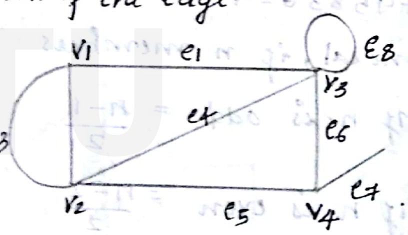
Finite and Infinite graph:



Q Draw a graph with 64 vertices representing squares of a chessboard. Join these vertices appropriated by edges each representing a move of the knight. You will be see that in this graph every vertex is of degree 2, 3, 4, 6/8. How many vertices are of each type.

Incidence and degree:

Incidence: is a vertex if the vertex of is an end vertex of the edge.



Adjacent: if they have a common end vertex & non-parallel end vertex. then it is said to be adjacent vertex eg: e_5, e_6

Adjacent vertices: two vertexes is said to be adjacent vertices if they have a common edge.
eg: V_2 & V_4.

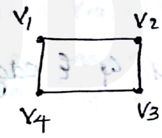
- Degree of vertex: no: of edges incident on the vertex.
  eg: $d(V_4) = 3$, $d(V_1) = 3$, $d(V_2) = 4$.
  $d(V_3) = \cancel{3}5$, ~~d(V~~ self loop counted twice.

- Order of graph: no: of vertices in a graph.
  eg: $|V| = 4$

- Size of graph: no: of edges in a graph
  eg $|E| = 8$.

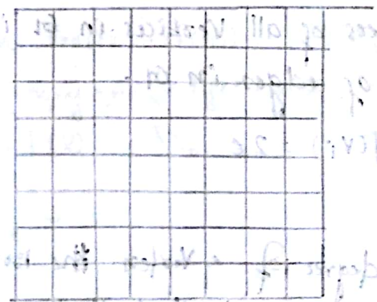
- Regular graph: is said if every vertex has the same degree.

eg:



$d(V_1) = 2$ $\quad$ $d(V_3) = 2$

$d(V_2) = 2$ $\quad$ $d(V_4) = 2$



ans?

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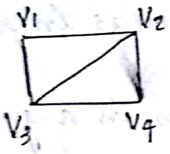
$G = (V, E)$

n vertices $V = \{v_1, v_2, v_3, \dots v_n\}$

$$\sum_{i=1}^{n} d(v_i) = 2e$$



$d(v_1) = 2$
$d(v_2) = 3$
$d(v_3) = 3$
$d(v_4) = \frac{2}{10}$.

no: of edges = 5

$d(v_1) + d(v_2) + d(v_3) + d(v_4)$.
$= 10 = 2 \times e = 2 \times 5$

A graph G with V - Vertices & E edges,
$V = \{v_1, v_2, v_3, \dots v_n\}$

Sum of degrees of all vertices in G is twice the number of edges in G.

$$\sum_{i=1}^{n} d(v_i) = 2e$$

Proof

Since the degree of a vertex the no: of edges incident with that vertex, the sum of degree counts the total no: of times and is edge incident with 'v'. Since every edges " " exactly with two vertices each edge gets counted twice. once at each end.

Hence the theorem.

The no: of odd degree vertex is always is a even.

$$\sum_{i=1}^{n} d(v_i) = \sum_{even} d(v_j) + \sum_{odd} d(v_k)$$

Isolated vertex: where no edges incident on it ~~or the number of deg edges incid~~

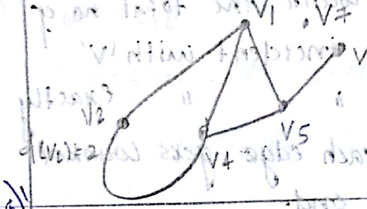
$d(v_i) = 0 \rightarrow v_3, v_7$

Pendant vertex - $d(v_i) = 1 \rightarrow V_6$

~~No~~ only one edges incident on a vertex

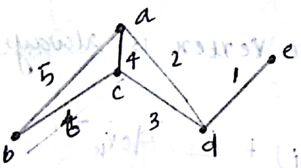
$d(v) = 1 \rightarrow V_6$.

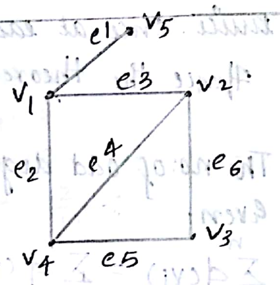
Null graph.



Isomorphism:-

eg:1)





5 vertices.

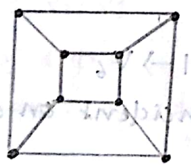
a b c d e

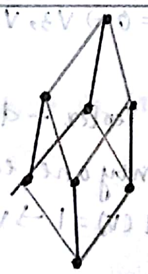
1 2 3 4 5 6

v_1 v_2 v_3 v_4 v_5

e_1 e_2 e_3 e_4 e_5 e_6

eg:2)



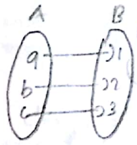


V -> 8 vertices
E -> 12 edges
d(V) -> 3

V -> 8 vertices
E -> 12 edges
d(V) -> 3.

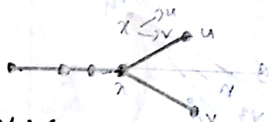
2 graphs are said to isomorphic if they have same no: edges in 1:1 correspondance & Same no: ~~ed~~ vertices in 1:1 correspondance. Degree of vertices should be same.

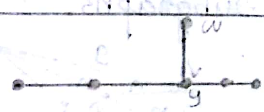
Two



1:1 correspondance

eg:3



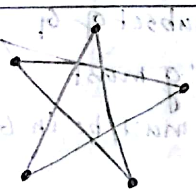


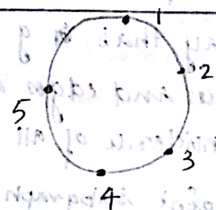
V -> 6, E -> 5
d -> 1 -> 3
d -> 2 -> 2
d -> 3 -> 1

V -> 6, E -> 5
d -> 1 -> 3
d -> 2 -> 2
d -> 3 -> 1

- It is not isomorphic bcz not satisfy one to one correspondance

eg4





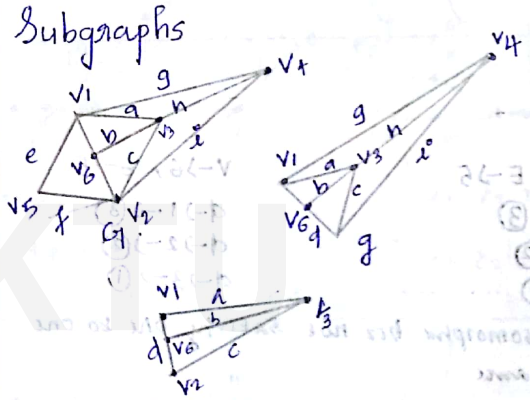
V -> 5
e -> 5
d -> 2

V -> 5
e -> 5
d -> 2

To check one to one correspondance.

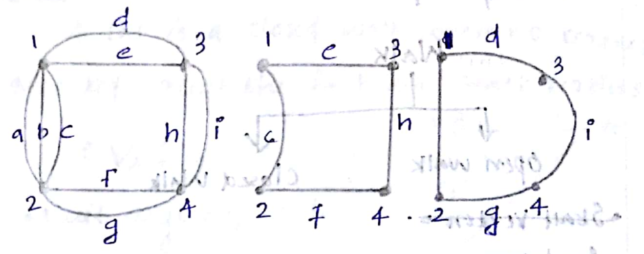
		I	II
a -> 1		(ad)	(1,5)
b -> 4		(ac)	(1,2)
c -> 2		(bd)	(4,5)
d -> 5		(be)	(4,3)
e -> 3			

-> Subgraphs

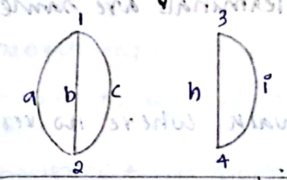


we can say that ~~a~~ g is a subset of G if all vertices and edges ~~are~~ in g must be in G. Incidence of all edges must be in G.

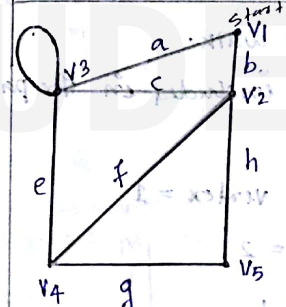
- Edge disjoint subgraph:
- Vertex disjoint No: 2 vertex in the subgraph ~~is~~ are



Vertex disjoint subgraph



Walks, Path & Circuit



Walk:
Finite alternative sequence of vertices and edges. Starting with a vertex & ending with another vertexes each vertex before that & there is a vertex and after that vertex.

- In a walk none of the edges repeated but, vertices are repeated
- $V_1 b V_2 c V_3 d V_3 e V_4 f V_2 h V_5$.

a self loop can be included in a walk.

Walk

Open walk

Closed walk

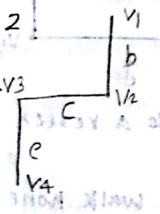
~~Start vertex = End ver~~

- Terminal vertices are different
- Terminals are same.

PATH : is an open walk where no vertices are repeated.

$v_1 b v_2 c v_3 e v_4$ — open walk.

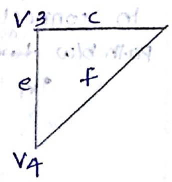
- length of path: no: of edges included in the path.
  in the above eg: length = 3
- degree of start and end vertex = 1
- degree of other vertices = 2
- Self loops are not included in path.



CIRCUIT

A ckt is a closed walk where no vertices are repeated except the last and ~~start~~ First vertices.

$V_2 c V_3 e V_4 f V_2$

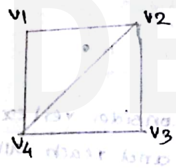


In a circuit degree of each vertex = 2

Theorem

Maximum degree of simple graph with n vertices is n-1

consider a simple graph with v_1, v_2, v_3, v_4.



4 vertices.

$d(v_1) = 2$

$d(v_2) = 3$

$d(v_3) = 2$

$d(v_4) = 3$

$G \rightarrow n$ vertices

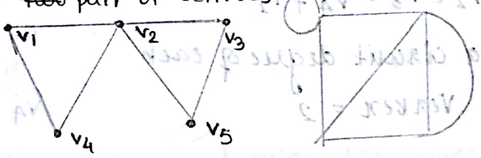
$d(v) \rightarrow n-1$ max

Proof:

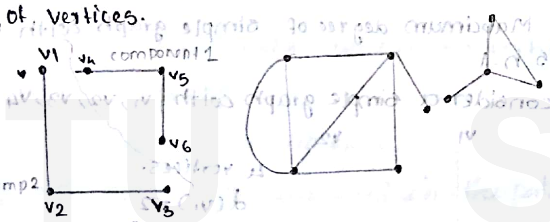
let G be a simple graph with n vertices. consider a vertex V(G), since the graph is simple (with-out selfloop and parallel edges) the vertex be can be adjacent to almost remaining n-1 vertices. Hence the max degree of any vertex in a simple graph with n vertices is n-1.

Connected graph & disconnected graphs.

In connected graph there will be atleast one path b/w ~~two~~ each pair of vertices.

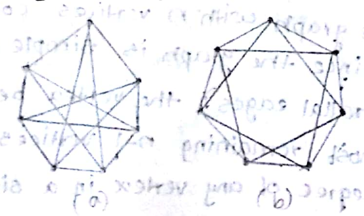


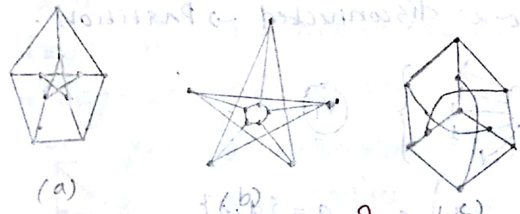
In disconnected graph there is no path b/w every pair of vertices.



A componenent is when you consider vertex v_1 when you start from one vertex and reach all vertex a disconnected graph is composed of components.

? verify these 2 graph are isomorphic.

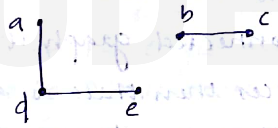




16/8/17. Theorem ans??

In a disconnected graph the whole vertex set can be divided into two subsets ~~vertex~~ V_1 and V_2 such that V_1 form subset of one component and V_2 form subset of another component.

eg:



$V = \{a, b, c, d, e\}$

$V_1 = \{a, d, e\}$ $V_2 = \{b, c\}$

~~case-1~~

- In a disconnected graph there will be partition.

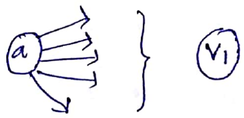
case-1: Partition ⇒ disconnected.

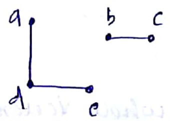
V_1 V_2

$a \in V_1$ & $b \in V_2$

There is no path b/w a and b.

case-2: disconnected → Partition.





$a = \{d, e\}$

$V_2 = \{b, c\}$

Theorem-2

Whether a connected or disconnected graph if there exists ~~atleast~~ exactly two odd vertices then there is a path b/w these two odd vertices.

If it is a disconnected graph, it includes two odd vertices then there is a connected path b/w those two vertices. Then this two vertices should be in same component.

Theorem

The maximum no:of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

PROOF

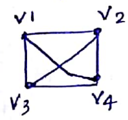
Graph with n vertices

1 vertex → (n-1) degree → (n-1) edges.

n vertices → n(n-1) → n(n-1) edges.

each edges is counted twice.

eg:



V_1 - 3 vertices - 3 edges

V_2 - 3 edges

V_3 - 3 edges

V_4 - 3 edges

$4 \times 3 = 12$

$\frac{12}{2} = 6$ edges.

PROOF

Let G be a simple graph with n vertices. Since the graph is simple (without self loop and parallel edge), each vertex ~~be~~ v is connected to all other n-1 vertices. So, that it form n-1 edges. So, all n vertices are connected to n(n-1) other vertices which forms n(n-1) edges but by this method each edge is counted twice and therefore the result is divided by two. Therefore the max. no:of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

→ THEOREM:

A simple graph with n vertices and K components have atmost $\frac{(n-K)(n-K+1)}{2}$ edges.

PROOF:- of inequality:

K components, n vertices

$$\underset{n_1}{\overset{1}{\downarrow}}, \underset{n_2}{\overset{2}{\downarrow}}, \underset{n_3}{\overset{3}{\downarrow}} \dots \underset{n_K}{\overset{K}{\downarrow}}.$$

$$n_1+n_2+n_3+\dots+n_K = n$$

$$\boxed{\sum_{i=1}^{K} n_i = n} \quad \text{— ①}$$

$$\sum_{i=1}^{K} n_i - 1 = n-K$$

squaring on both sides.

$$\left[\sum_{i=1}^{K} n_i - 1\right]^2 = (n-K)^2$$

$$\sum_{i=1}^{K} ($$

proof for above theorem.

i^{th} term, $\frac{n_i(n_i-1)}{2}$

$G \to$

$$\frac{1}{2}\sum_{i=1}^{K} n_i(n_i-1) = \frac{1}{2}\sum_{i=1}^{K}(n_i)^2 - \frac{n}{2}$$

$$= \frac{1}{2}\left[n^2 - (K-1)(2n-K) - n\right]$$

$$= \frac{1}{2}\left[n^2 - (2nK - K^2 - 2n + K) - n\right]$$

$$= \frac{1}{2}\left[n^2 - 2nK + K^2 + 2n - K - n\right]$$

$$= \frac{1}{2}\left[(n-K)^2 - K + n\right]$$

$$= \frac{1}{2}\left[(n-K)^2 + (n-K)\right]$$

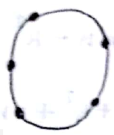
$$= \frac{n-K}{2}\left[n-K+1\right]$$

Excercise

Q) prove that any two simple connected graphs with n vertices all of degree two are isomorphic

A) A simple connected graphs are isomorphics if they satisfy same no: of vertices and edges with same degree two.

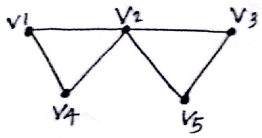
eg:

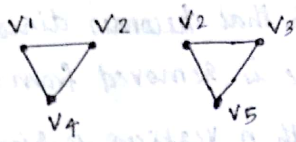


V -> 5
e -> 5
degree -> 2

V -> 5
e -> 5
degree -> 2

Q) PT if a connected graph G decomposed into two subgraph g_1 and g_2, there must be atleast one vertex ~~..~~ common b/w g_1 & g_2.





A graph G is decomposed into g_1 & g_2

$V(g_1) \cup V(g_2) = V(G)$

$E(g_1) \cup E(g_2) = E(G)$

$E(g_1) \cap E(g_2) = \emptyset$

By contradiction suppose that

$V(g_1) \cap V(g_2) = \emptyset$

~~$\exists x,y \; x \in V(g_1)$ & $y \in V(g_2)$~~ (no common vertex)

∴ There doesn't exit (u,v) such that $u \in V(G_1)$ and $v \in U(G_1)$ $(u,v) \in$

$\exists x,y \; x \in V(g_1)$ & $y \in V(g_2)$ There doesn't exist a path b/w x and y.

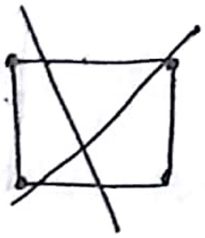
∴ G can't be connected. This is a contradiction

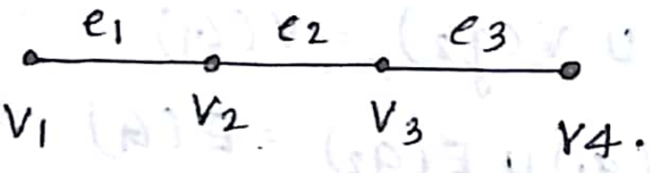
$V(g_1) \cap V(g_2) \neq \emptyset$

Hence the proof.

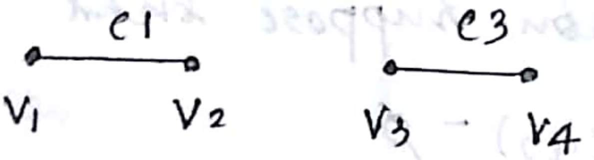
Q Draw a connected graph that becomes disconnected graph when any edge is removed from that

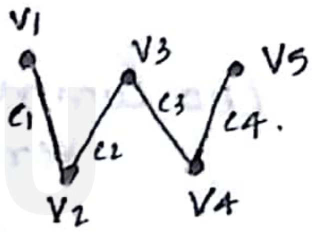
a) PT such a graph with n vertices is simple and have n-1 edges.

A) 

(eg:1) 

Remove edge e_2 then.

(eg:1) 



Remove e_2.