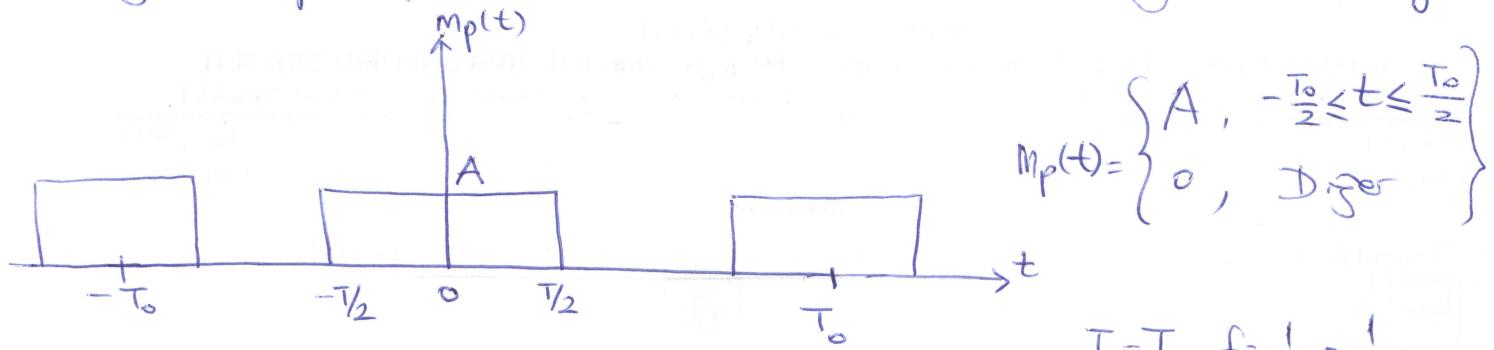


Asagidakı gibi işaretin Üstel Fourier Seri açılımı gereklidir.



$$m_p(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} m_p(t) \cdot e^{-jn\omega_0 t} dt$$

$$T_0 = T_0 \quad f = \frac{1}{T} = \frac{1}{T_0}$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A \cdot e^{-j\frac{2\pi n t}{T_0}} dt = -\frac{1}{T_0} \cdot \frac{A}{j\frac{2\pi n}{T_0}} \left[e^{\frac{-j2\pi n t}{T_0}} \right]_{-T_0/2}^{T_0/2}$$

$$= -\frac{A}{j2\pi n} \left[e^{\frac{-j2\pi n T}{T_0}} - e^{\frac{j2\pi n T}{T_0}} \right] = \frac{A}{\pi n} \cdot -\frac{1}{2j} \left[e^{\frac{-j\pi n T}{T_0}} - e^{\frac{j\pi n T}{T_0}} \right]$$

$$= \frac{A}{\pi n} \cdot \frac{1}{2j} \left[e^{\frac{j\pi n T}{T_0}} - e^{\frac{-j\pi n T}{T_0}} \right] = \frac{A}{\pi n} \cdot \sin\left(\frac{\pi n T}{T_0}\right)$$

$$\sin\left(\frac{\pi n T}{T_0}\right)$$

$$\boxed{Sinc(\lambda) = \frac{\sin(\pi\lambda)}{\pi\lambda}} = \frac{A}{\pi n} \cdot \sin\left(\frac{\pi n T}{T_0}\right)$$

$$= \frac{A \cdot \sin(\pi\lambda)}{\pi n \cdot \lambda} \rightarrow \text{sinc}(\lambda)$$

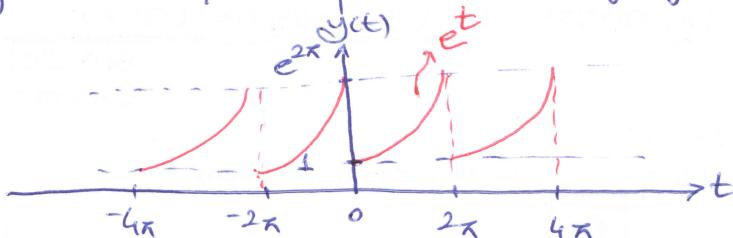
$$= \frac{A\lambda}{n} \cdot \text{sinc}(\lambda)$$

$$= \frac{A \cdot n T}{n \cdot T_0} \cdot \text{sinc}\left(\frac{n T}{T_0}\right)$$

$$\boxed{D_n = \frac{AT}{T_0} \cdot \text{sinc}\left(\frac{n T}{T_0}\right)}$$

$$m_p(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{AT}{T_0} \cdot \text{sinc}\left(\frac{n T}{T_0}\right) \cdot e^{j\frac{2\pi n t}{T_0}}$$

Aşağıdaki $y(t)$ işaretinin Üstel Fourier Seri açılımını 5. harmonik kader (5 dahlı) açanız ve Enerji/faz spektrumlarını çiziniz.



$$T = 2\pi \quad \omega = 2\pi f = 1 \text{ Hz}$$

$$f = \frac{1}{2\pi}$$

$$y(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega t} \quad D_n = \frac{1}{T} \int_{-T/2}^{T/2} y(t) e^{-jn\omega t} dt$$

$$D_n = \frac{1}{2\pi} \int_0^{2\pi} e^t e^{-jn\omega t} dt = \frac{1}{2\pi} \int_0^{2\pi} e^{(1-jn)t} dt$$

$$= \frac{1}{2\pi(1-jn)} \cdot e^{(1-jn)t} \Big|_0^{2\pi} = \frac{1}{2\pi(1-jn)} \left(e^{(1-jn)2\pi} - 1 \right) = \frac{e^{2\pi} - 1}{2\pi(1-jn)}$$

$$D_n = \frac{e^{2\pi} - 1}{2\pi(1-jn)}$$

$$|D_n| = \frac{e^{2\pi} - 1}{2\pi\sqrt{1+n^2}}$$

$$|D_0| = \frac{e^{2\pi} - 1}{2\pi} = 85.06 \quad \angle \theta_0 = 0^\circ$$

$$|D_1| = \frac{e^{2\pi} - 1}{2\sqrt{2}\pi} = 60.15 \quad \angle \theta_1 = \tan^{-1}\left(\frac{-b}{a}\right) = \tan\left(\frac{-(-1)}{1}\right) = \tan(1) = 45^\circ$$

$$|D_2| = \frac{e^{2\pi} - 1}{2\pi\sqrt{5}} = 38.04 \quad \angle \theta_2 = 63.43^\circ$$

$$|D_3| = \frac{e^{2\pi} - 1}{2\sqrt{10}\pi} = 26.90 \quad \angle \theta_3 = 71.56^\circ$$

$$|D_4| = \frac{e^{2\pi} - 1}{2\sqrt{17}\pi} = 20.63 \quad \angle \theta_4 = 75.96^\circ$$

$$|D_5| = \frac{e^{2\pi} - 1}{2\sqrt{26}\pi} = 16.68 \quad \angle \theta_5 = 78.69^\circ$$

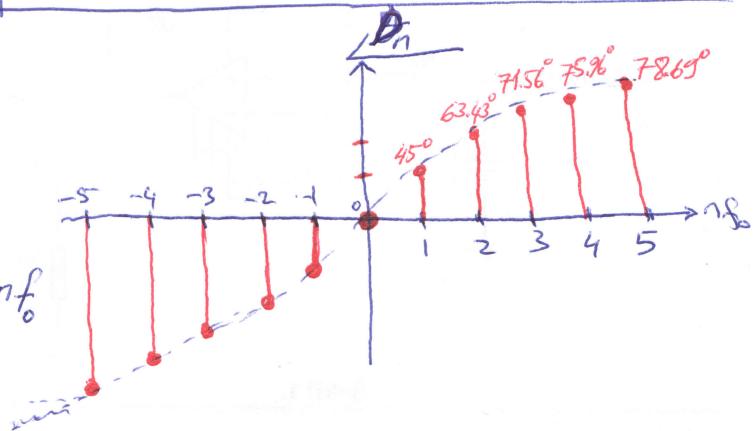
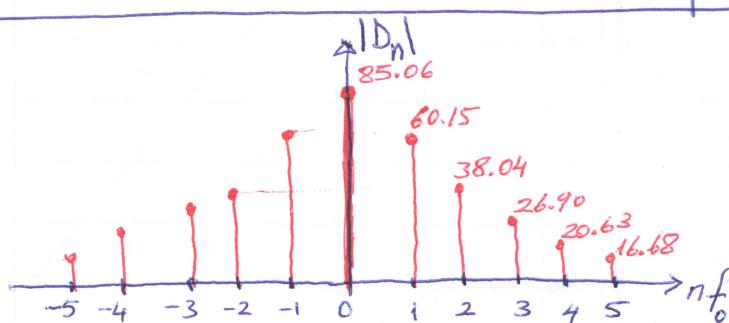
$$|D_{-1}| = 60.15 \quad \angle \theta_{-1} = -45^\circ$$

$$|D_{-2}| = 38.04 \quad \angle \theta_{-2} = -63.43^\circ$$

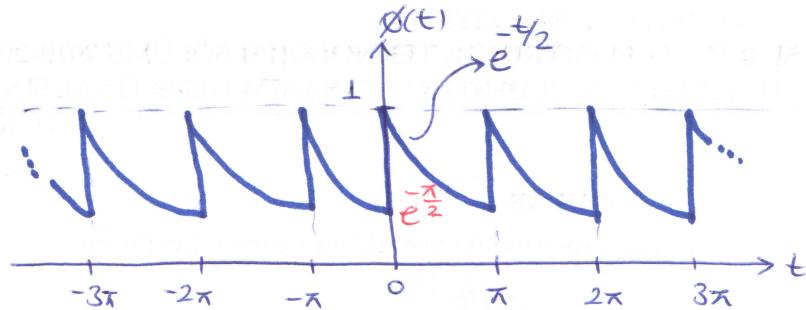
$$|D_{-3}| = 26.90 \quad \angle \theta_{-3} = -71.56^\circ$$

$$|D_{-4}| = 20.63 \quad \angle \theta_{-4} = -75.96^\circ$$

$$|D_{-5}| = 16.68 \quad \angle \theta_{-5} = -78.69^\circ$$



dr: Aşağıdaki $\phi(t)$ işaretinin Üstel Fourier Seri açılımını gerçekleştirdiğin, frekans spektrumlarını çiziniz.



ölen

$$T_0 = \pi, \omega = 2 \quad \phi(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt}$$

$$D_n = \frac{1}{\pi} \int_0^{\pi} e^{-j2t} \cdot e^{j2nt} dt = \frac{1}{\pi} \int_0^{\pi} e^{-(\frac{1}{2}+j2n)t} dt = \left[\frac{-1}{(\frac{1}{2}+j2n)} e^{-(\frac{1}{2}+j2n)t} \right]_0^{\pi} = \frac{0.504}{1+j4n}$$

$$\phi(t) = 0.504 \sum_{n=-\infty}^{\infty} \frac{1}{1+j4n} e^{j2nt}$$

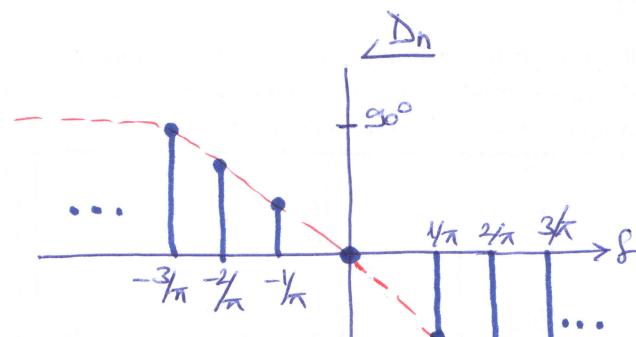
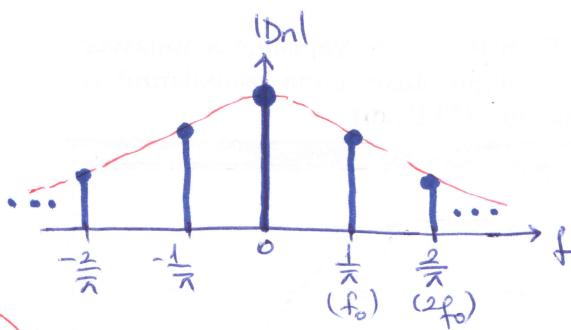
$$= 0.504 \left[1 + \underbrace{\frac{1}{1+j4} e^{j2t}}_{D_n} + \underbrace{\frac{1}{1+j8} e^{j4t}}_{D_{-n}} + \underbrace{\frac{1}{1+j12} e^{j6t}}_{D_n} + \dots + \underbrace{\frac{1}{1-j4} e^{-j2t}}_{D_n} + \underbrace{\frac{1}{1-j8} e^{-j4t}}_{D_{-n}} + \underbrace{\frac{1}{1-j12} e^{-j6t}}_{D_n} + \dots \right]$$

$$D_0 = 0.504$$

$$D_1 = \frac{0.504}{1+j4} = 0.122 \cdot e^{j-75.96^\circ} \quad D_2 = \frac{0.504}{1+j8} = 0.0625 \cdot e^{j-82.87^\circ}$$

$$D_{-1} = \frac{0.504}{1-j4} = 0.122 \cdot e^{j75.96^\circ} \quad D_{-2} = \frac{0.504}{1-j8} = 0.0625 \cdot e^{j82.87^\circ}$$

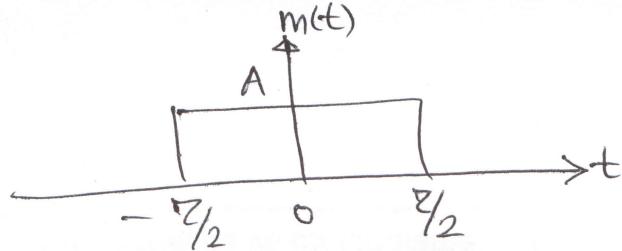
$$D_n = \sqrt{a^2 + b^2} \quad \theta_n = \arctan\left(\frac{b}{a}\right)$$



$$\begin{aligned} & \frac{-2}{(1+j4n)\pi} \left(e^{-j(\frac{1}{2}+j2n)t} \Big|_0^\pi \right) = \frac{-2}{(1+j4n)\pi} \left(e^{-j\frac{1}{2}\pi} \cdot e^{-j2\pi n} - 1 \right) \\ & = -\frac{2}{(1+j4n)\pi} \cdot \left(e^{-j\frac{\pi}{2}} - 1 \right) \\ & = -\frac{2}{(1+j4n)\pi} \cdot (-0.7921) \end{aligned}$$

$$= \frac{1.5842}{(1+j4n)\pi} = \frac{0.504}{1+j4n}$$

Aşağıdakii $m(t)$ işaretinin Fourier dönüşümünü bulunuz.



$$m(t) = \begin{cases} A, & -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \\ 0, & \text{diğer} \end{cases}$$

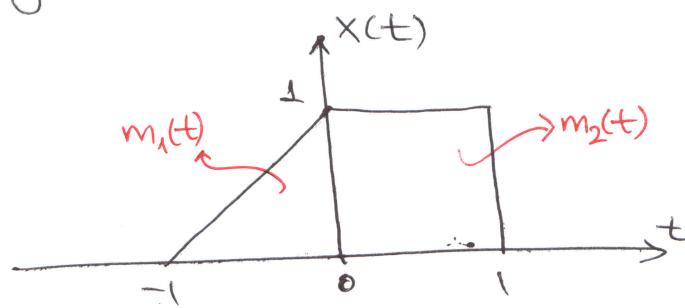
$$\begin{aligned} M(f) &= \int_{-\pi/2}^{\pi/2} A \cdot e^{-j2\pi ft} dt = A \cdot \left[\frac{-e^{-j2\pi ft}}{j2\pi f} \right]_{-\pi/2}^{\pi/2} = \frac{-A}{j2\pi f} \cdot \left(e^{-j\pi f\pi/2} - e^{j\pi f\pi/2} \right) \\ &= \frac{A}{\pi f} \cdot \left(e^{j\pi f\pi/2} - e^{-j\pi f\pi/2} \right) = \frac{A}{\pi f} \cdot \sin(\pi f\pi/2) = \frac{A \cdot \sin(\pi f\pi/2)}{\pi f \cdot \pi/2} \end{aligned}$$

$$\text{Sinc}(\lambda) = \frac{\sin(\pi\lambda)}{\pi\lambda} = \boxed{A\pi \cdot \text{Sinc}(f\pi)}$$

$$A \cdot \text{Dikd}\left(\frac{t}{\pi}\right) \longleftrightarrow A\pi \cdot \text{Sinc}(f\pi)$$

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Örnek: Aşağıdaki $x(t)$ işaretinin Fourier dönüştürünü bulunuz.



$$x(t) = m_1(t) + m_2(t)$$

$$\mathcal{F}\{x(t)\} = \mathcal{F}\{m_1(t)\} + \mathcal{F}\{m_2(t)\}$$

$$m_1(t) = \begin{cases} t+1, & -1 \leq t \leq 0 \\ 0, & \text{diğer} \end{cases}$$

$$\mathcal{F}\{m_1(t)\} = \int_{-1}^0 (t+1) e^{-j2\pi f t} dt = \int_{-1}^0 t e^{-j2\pi f t} dt + \int_{-1}^0 e^{-j2\pi f t} dt$$

$$\boxed{\int t e^{at} dt = \frac{e^{at}}{a^2} (at-1)}$$
$$= \left[\frac{e^{-j2\pi f t}}{(-j2\pi f)^2} \cdot (-j2\pi f t - 1) \right]_{-1}^0 + \left(\frac{-1}{j2\pi f} \cdot e^{-j2\pi f t} \right)_{-1}^0$$

$$= \left[\frac{1}{4\pi^2 f^2} - \left(\frac{e^{j2\pi f} \cdot (1-j2\pi f)}{4\pi^2 f^2} \right) \right] + \left[-\frac{1}{j2\pi f} - \left(\frac{-e^{j2\pi f}}{j2\pi f} \right) \right]$$

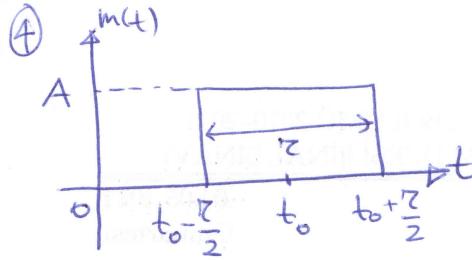
$$= \left[\frac{1}{4\pi^2 f^2} - \frac{e^{j2\pi f}}{4\pi^2 f^2} + \frac{j2\pi f \cdot e^{j2\pi f}}{4\pi^2 f^2} \right] + \left[-\frac{1}{j2\pi f} + \frac{e^{j2\pi f}}{j2\pi f} \right]$$

$$e^{j2\pi f} = \cos 2\pi f - j \sin 2\pi f$$
$$\equiv 1$$

$$= -\frac{j2\pi f}{4\pi^2 f^2} = \frac{j2\pi}{4\pi^2 f} = \frac{j}{2\pi f}$$

$$\boxed{\mathcal{F}\{m_2(t)\} = \text{Sinc}(f) \cdot e^{-j\pi f}}$$

$$\boxed{\mathcal{F}\{x(t)\} = X(f) = \frac{j2\pi}{2\pi f} + \text{Sinc}(f) \cdot e^{-j\pi f}}$$



Yandaki işaretin Fourier dönüşümü;
 a) Jeman ötelelesi ögelleşti kullanarak,
 b) Direk olarak (ögelleşti kullanmadan) elde edin.

②

⑤ $m(t) = A \cdot \text{Dihd}\left(\frac{t-t_0}{2}\right) \Rightarrow M(f) = A \cdot \text{Sinc}(f\pi) \cdot e^{-j2\pi f t_0}$

b) $\text{A} \cdot \text{Dihd}\left(\frac{t-t_0}{2}\right) = \begin{cases} A & ; t_0 - \frac{\pi}{2} \leq t \leq t_0 + \frac{\pi}{2} \\ 0 & ; \text{diğer aralıklar} \end{cases}$

$$M(f) = \int_{t_0 - \frac{\pi}{2}}^{t_0 + \frac{\pi}{2}} A \cdot e^{-j2\pi f t} dt = -\frac{A}{j2\pi f} \cdot e^{-j2\pi f t} \Big|_{t_0 - \frac{\pi}{2}}^{t_0 + \frac{\pi}{2}} = -\frac{A}{j2\pi f} \left(e^{-j2\pi f (t_0 + \frac{\pi}{2})} - e^{-j2\pi f (t_0 - \frac{\pi}{2})} \right)$$

$$= -\frac{A}{j2\pi f} \left(e^{-j\pi f (2t_0 + \pi)} - e^{-j\pi f (2t_0 - \pi)} \right)$$

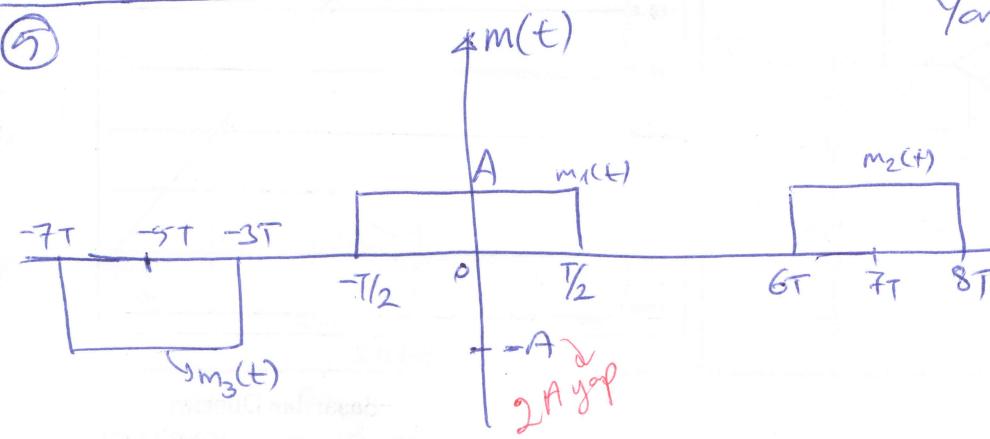
$$= -\frac{A}{j2\pi f} \left(\left(e^{j2\pi f t_0} \cdot e^{-j\pi f \pi} \right) - \left(e^{-j2\pi f t_0} \cdot e^{j\pi f \pi} \right) \right)$$

$$= \frac{A}{j2\pi f} \cdot e^{-j2\pi f t_0} \left(e^{j\pi f \pi} - e^{-j\pi f \pi} \right) = \frac{A}{\pi f} \cdot e^{-j2\pi f t_0} \cdot \text{Sin}(\pi f \pi)$$

$$= A \cdot e^{-j2\pi f t_0} \cdot \frac{\text{Sin}(\pi f \pi)}{\pi f \cdot \frac{\pi}{2}} = A \cdot \text{Sinc}(\pi f \pi) \cdot e^{-j2\pi f t_0}$$

$$= A \cdot \text{Sinc}(f\pi) \cdot e^{-j2\pi f t_0}$$

⑥



Yandaki işaretin Fourier dönüşümü

a) ögelleşti kullanarak

b) Değradan elde edin

$$a) m_1(t) = A \text{Dihd} \left(\frac{t}{T} \right) \Rightarrow M_1(f) = AT \text{Sinc}(fT)$$

$$m_2(t) = A \cdot \text{Dihd} \left(\frac{t-7T}{2T} \right) \Rightarrow M_2(f) = 2AT \text{Sinc}(2fT) \cdot e^{-j14\pi fT}$$

$$m_3(t) = -A \cdot \text{Dihd} \left(\frac{t+5T}{4T} \right) \Rightarrow M_3(f) = -4AT \text{Sinc}(4fT) \cdot e^{j10\pi fT}$$

$$M(f) = AT \text{Sinc}(fT) + 2AT \text{Sinc}(2fT) e^{-j14\pi fT} - 4AT \text{Sinc}(4fT) e^{j10\pi fT}$$

b) $\text{Dihd} \left(\frac{t}{T} \right) = \begin{cases} A; & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0; & \text{Diger Andelber} \end{cases}$ $\text{Dihd} \left(\frac{t-7T}{2T} \right) = \begin{cases} A; & 6T \leq t \leq 8T \\ 0; & \text{diger} \end{cases}$

$$-A \cdot \text{Dihd} \left(\frac{t+5T}{4T} \right) = \begin{cases} -A; & -7T \leq t \leq -3T \\ 0; & \text{diger} \end{cases}$$

$$\boxed{M_1(f) = \int_{-T/2}^{T/2} A \cdot e^{-j2\pi ft} dt = \frac{-A}{j2\pi f} \cdot e^{-j2\pi ft} \Big|_{-T/2}^{T/2} = \frac{-A}{j2\pi f} \left(e^{-j\pi fT} - e^{j\pi fT} \right) = \frac{A}{j2\pi f} \left(e^{j\pi fT} - e^{-j\pi fT} \right)}$$

$$= \frac{A}{\pi f} \sin(\pi fT) = AT \cdot \text{Sinc}(fT)$$

$$\boxed{M_2(f) = \int_{6T}^{8T} A \cdot e^{-j2\pi ft} dt = \frac{-A}{j2\pi f} \cdot e^{-j2\pi ft} \Big|_{6T}^{8T} = \frac{-A}{j2\pi f} \left(e^{-j16\pi fT} - e^{-j12\pi fT} \right)}$$

$$= \frac{-A}{j2\pi f} \cdot e^{-j14\pi fT} \left(e^{-j2\pi fT} - e^{-j2\pi fT} \right) = \frac{A}{\pi f} \cdot e^{-j14\pi fT} \cdot \sin(2\pi fT)$$

$$= 2AT \cdot \frac{\sin(2\pi fT)}{\pi f 2T} \cdot e^{-j14\pi fT} = 2AT \cdot \text{Sinc}(2fT) \cdot e^{-j14\pi fT}$$

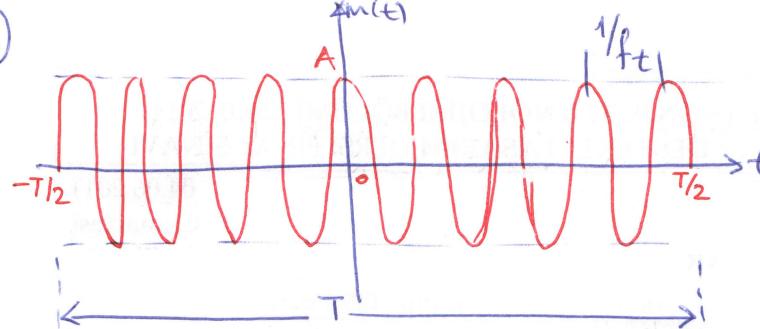
$$\boxed{M_3(f) = \int_{-7T}^{-3T} -A \cdot e^{-j2\pi ft} dt = \frac{A}{j2\pi f} \cdot e^{-j2\pi ft} \Big|_{-7T}^{-3T} = \frac{A}{j2\pi f} \left(e^{j6\pi fT} - e^{j14\pi fT} \right)}$$

$$= \frac{A}{j2\pi f} \cdot e^{j10\pi fT} \left(e^{-j4\pi fT} - e^{-j4\pi fT} \right) = -\frac{A}{\pi f} \cdot e^{j10\pi fT} \cdot \sin(4\pi fT)$$

$$= -4AT \cdot \text{Sinc}(4fT) e^{j10\pi fT}$$

$$\boxed{M(f) = M_1(f) + M_2(f) + M_3(f) = AT \text{Sinc}(fT) + 2AT \text{Sinc}(2fT) e^{-j14\pi fT} - 4AT \text{Sinc}(4fT) e^{j10\pi fT}}$$

(b)



Sekilde bir RF (Radyo Frekansı) dörtlüsü verilmelidir.
Bu dörtlünün Fourier dönüşümü elde edin.
ve spektral formunu algılayınız.

$$m(t) = A \cdot D \operatorname{hd}\left(\frac{t}{T}\right) \cdot \cos(2\pi f_t t)$$

$$\cos(2\pi f_t t) = \frac{1}{2} \left(e^{j2\pi f_t t} + e^{-j2\pi f_t t} \right) \text{ olduğundan}$$

$$m(t) = \frac{A}{2} \cdot D \operatorname{hd}\left(\frac{t}{T}\right) \cdot e^{j2\pi f_t t} + \frac{A}{2} \cdot D \operatorname{hd}\left(\frac{t}{T}\right) \cdot e^{-j2\pi f_t t}$$

Frekansa öteleme özelliği

$$m(t) \cdot e^{j2\pi f_t t} \leftrightarrow M(f - f_t) \text{ olduğu hatırlanır;}$$

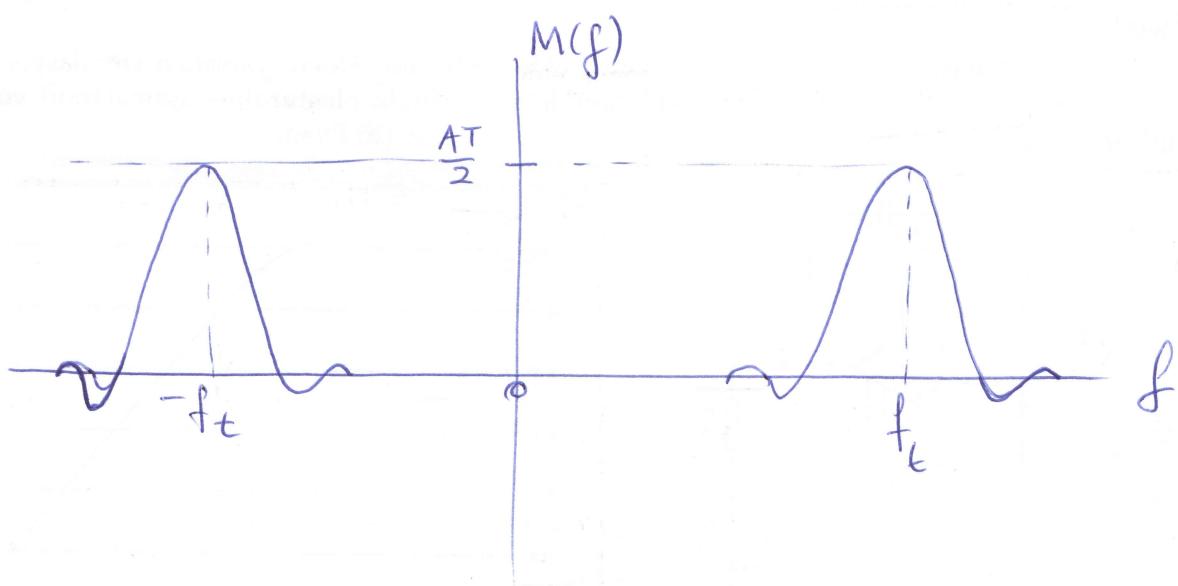
$$\frac{A}{2} D \operatorname{hd}\left(\frac{t}{T}\right) \leftrightarrow \frac{AT}{2} \operatorname{Sinc}(fT)$$

$$\frac{A}{2} D \operatorname{hd}\left(\frac{t}{T}\right) \cdot e^{j2\pi f_t t} \leftrightarrow \frac{AT}{2} \cdot \operatorname{Sinc}[(f - f_t)T]$$

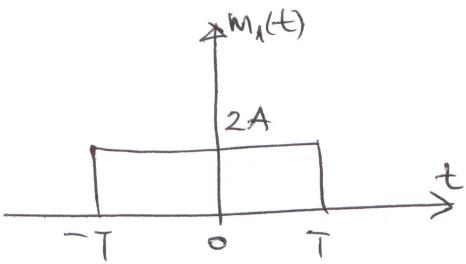
$$\frac{A}{2} D \operatorname{hd}\left(\frac{t}{T}\right) \cdot e^{-j2\pi f_t t} \leftrightarrow \frac{AT}{2} \operatorname{Sinc}[(f + f_t)T]$$

$$M(f) = \begin{cases} \frac{AT}{2} \operatorname{Sinc}[T(f - f_t)] & ; f > 0 \\ \frac{AT}{2} \operatorname{Sinc}[T(f + f_t)] & ; f < 0 \end{cases}$$

Sekilde elde edilir

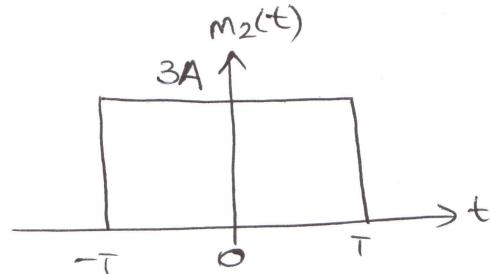


Örnek: Sekilde verilen $m_1(t)$ ve $m_2(t)$ işaretlerinin konvolüsyonunu grafik yoluyla bulunuz.

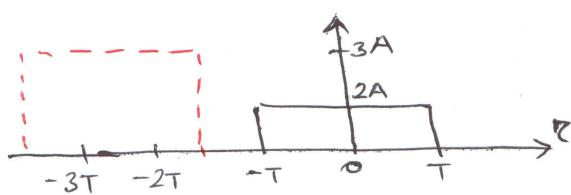


$$m_1(t) = 2A$$

$$m_2(t) = 3A$$



1. Adım $t < -T$ ise



$$m_1 \otimes m_2 = 0$$

$$t < -T$$

2. Adım: $-T < t < T$ ise

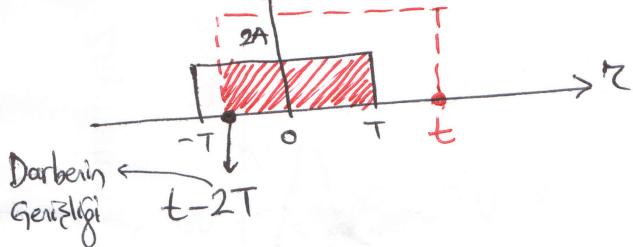
$$m_1 \otimes m_2 = \int_{-T}^t 6A^2 dz = 6A^2 z \Big|_{-T}^t = 6A^2(t+T)$$

$$-T < t < T$$

$$m_1 \otimes m_2 = \int_{t-2T}^T 6A^2 dz = 6A^2(3T-t)$$

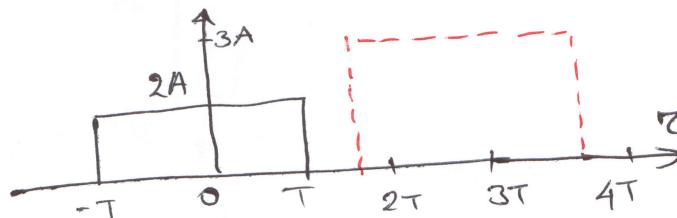
$$t < t < 3T$$

3. Adım: $T < t < 3T$ ise



Darbein
Genişliği

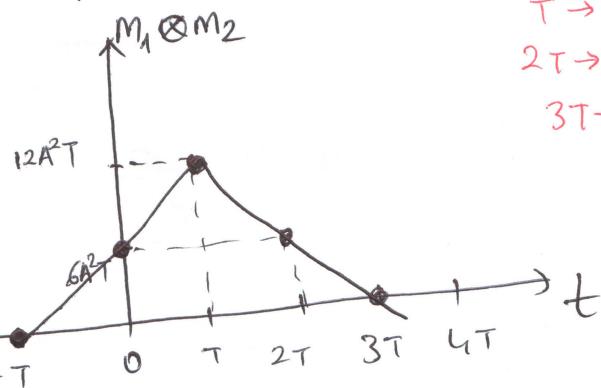
4. Adım $t > 3T$ ise



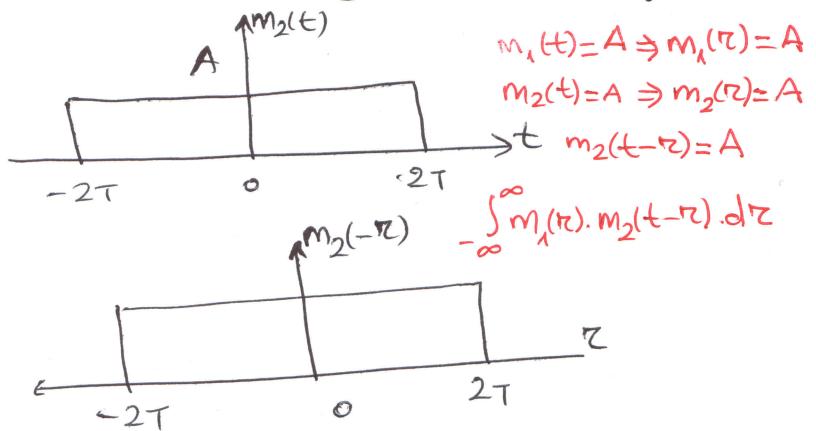
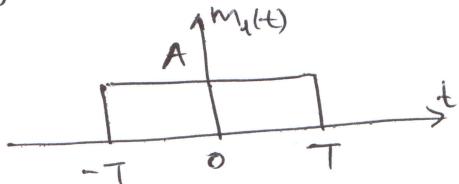
$$m_1 \otimes m_2 = 0$$

$$t > 3T$$

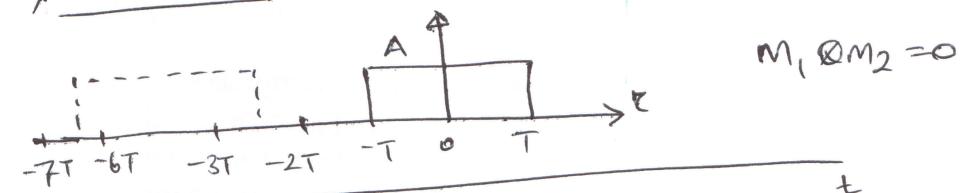
$$\begin{aligned} -T &\rightarrow 0 \\ 0 &\rightarrow 6A^2 T \\ T &\rightarrow 12A^2 T \\ 2T &\rightarrow 6A^2 T \\ 3T &\rightarrow 0 \end{aligned}$$



Aşağıda verilen $m_1(t)$ ve $m_2(t)$ işaretlerinin konvolüsyonunu bulunuz



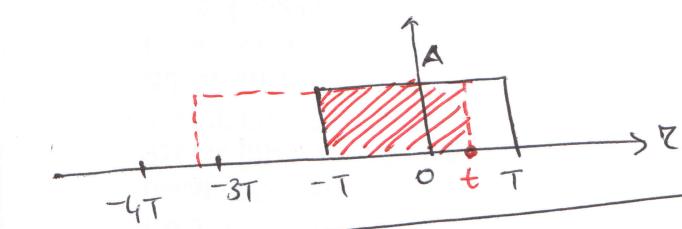
1) $t < -T$ ise



2) $-T < t < 3T$ ise

$$m_1 \otimes m_2 = \int_{-T}^t A^2 d\tau = A^2(t+T)$$

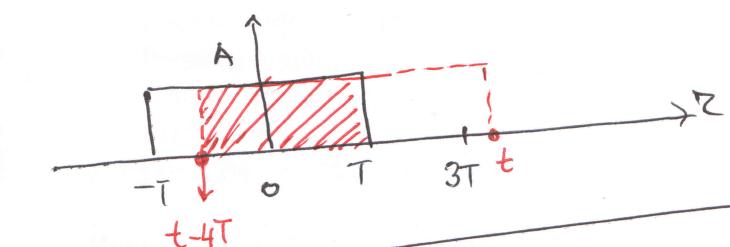
$\overline{-T < t < 3T}$



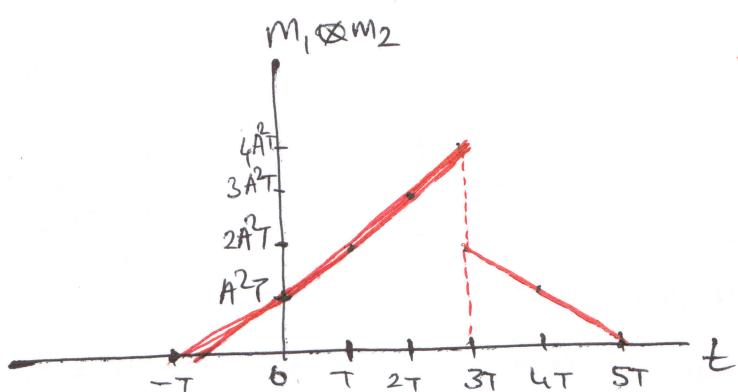
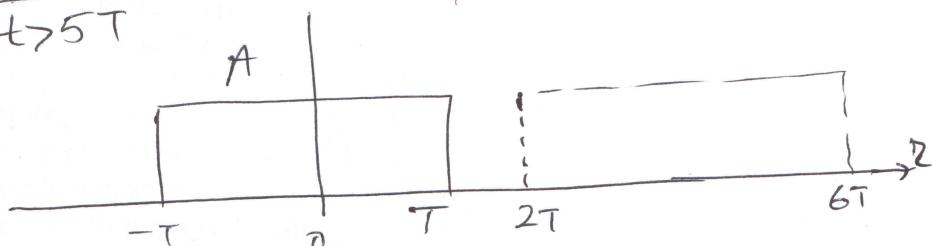
3) $3T < t < 5T$ ise

$$m_1 \otimes m_2 = \int_{t-4T}^T A^2 d\tau = A^2(5T-t)$$

$\overline{3T < t < 5T}$



4) $t > 5T$



$$\begin{aligned} -T &\rightarrow 0 \\ 0 &\rightarrow A^2T \\ T &\rightarrow 2A^2T \\ 3T &\rightarrow 4A^2T \\ 3T &\rightarrow 2A^2T \\ 4T &\rightarrow A^2T \\ 5T &\rightarrow 0 \end{aligned}$$