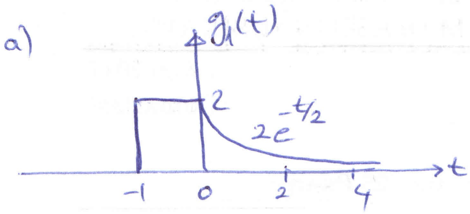
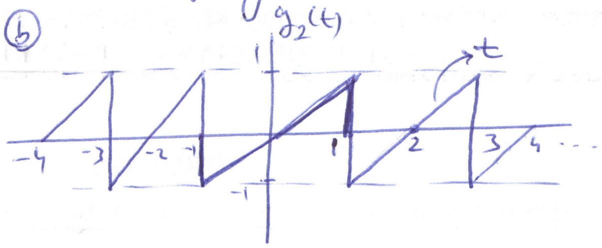
1) Aşağıdaki işaretler için uygun ölçümleri belirleyiniz (Enerji mi, Güç mü?)

a)



b)



Enerji işaretidir çünkü $t \to \infty$ genlik $\to 0$

$$E_{g_1} = \int_{-\infty}^{\infty} g_1^2(t)\,dt = \int_{-1}^{0} 2^2\,dt + \int_{0}^{\infty} \left(2e^{-t/2}\right)^2 dt$$

$$= 4\underbrace{\int_{-1}^{0} dt}_{1} + \underbrace{\int_{0}^{\infty} 4\,e^{-t}\,dt}_{4}$$

$$= 4 + 4 = 8$$

$$\boxed{\int_{0}^{\infty} e^{-at}\,dt = \frac{1}{a}}$$

Periyodik bir işarettir, dolayısıyla güç işaretidir. Aynı zamanda $t \to \infty$ ~~genlik $\to 0$~~

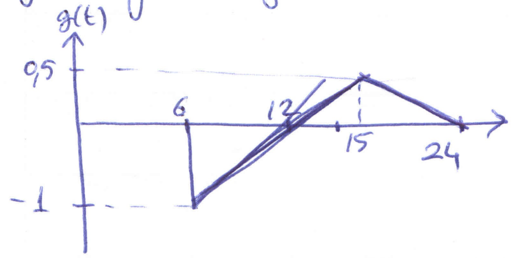
$T = 2$

$$P_{g_2} = \frac{1}{T}\int_{-T/2}^{T/2} (g(t))^2\,dt = \frac{1}{2}\int_{-1}^{1} t^2\,dt$$

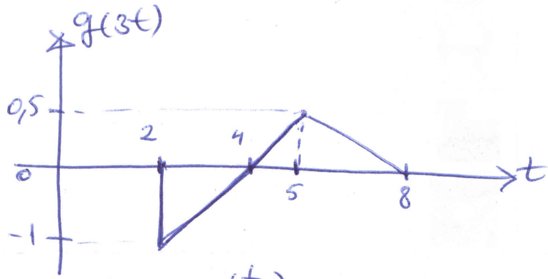
$$= \frac{1}{2}\,\frac{t^3}{3}\Big|_{-1}^{1} = \frac{1}{6}\,t^3\Big|_{-1}^{1} = \frac{1}{3}$$

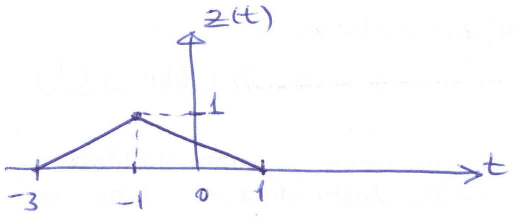
$$P_{rms} = \sqrt{P} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

2) Aşağıdaki $g(t)$ ve $z(t)$ işaretleri için $g(3t)$ ve $z(t/2)$ işaretlerini belirleyiniz.

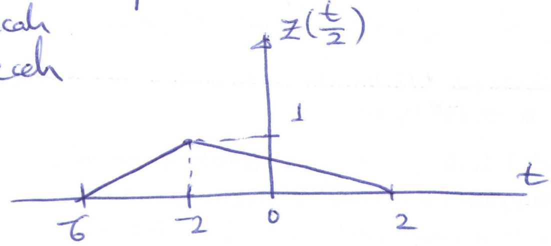


3 kat hızlanacak yani sıkıştırılacak,

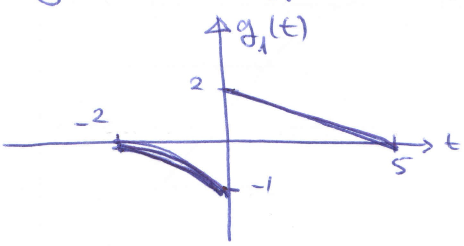


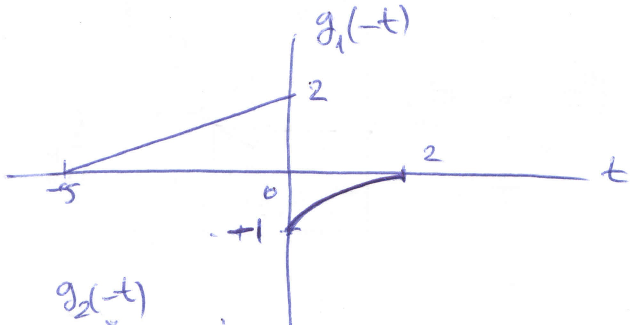


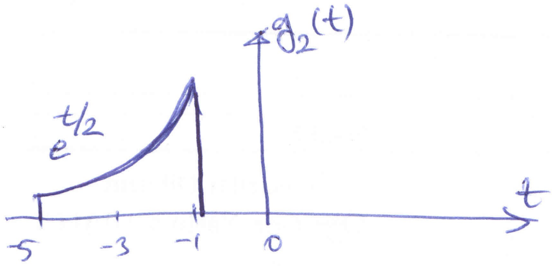
2 kat yavaşlatılacak yani genişletilecek

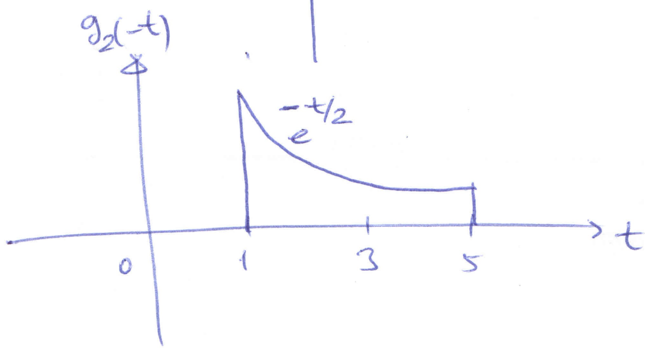


3) Aşağıda verilen $g_1(t)$ ve $g_2(t)$ işaretleri için $g_1(-t)$ ve $g_2(-t)$ işaretlerini belirleyiniz.

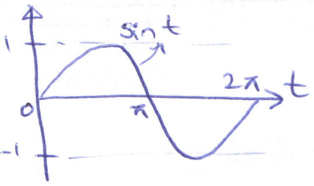


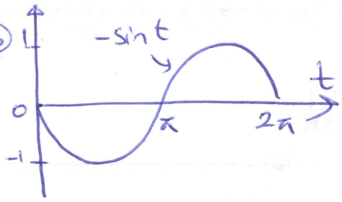


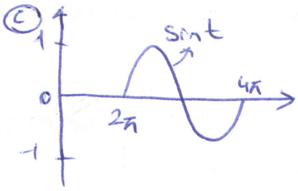


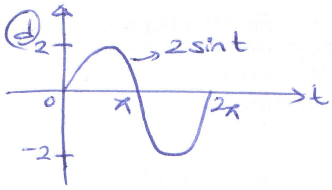


④ Aşağıda gösterilen işaretlerin enerjilerini bulunuz. İşaret değişiminin (terslendirmesinin), zaman ötelemesinin veya işaret genliğinin değişiminin enerji üzerindeki etkisini yorumlayınız.

ⓐ 

ⓑ 

ⓒ 

ⓓ 

a) $E_a = \int_0^{2\pi} (\sin t)^2 . dt = \int_0^{2\pi} \sin^2 t . dt = \frac{1}{2}\int_0^{2\pi} (1-\cos 2t) . dt = \frac{1}{2} t \Big|_0^{2\pi} - \frac{1}{2}.\frac{1}{2}.\sin 2t \Big|_0^{2\pi}$

$= \frac{1}{2}(2\pi - 0) - \frac{1}{4}\underbrace{(\underbrace{\sin 4\pi}_{0} - \underbrace{\sin 0}_{0})}_{0} = \frac{1}{\cancel{2}}.\cancel{2}\pi = \pi$

ⓑ $E_b = \int_0^{2\pi} (-\sin t)^2 . dt = \int_0^{2\pi} \sin^2 t . dt = \pi$

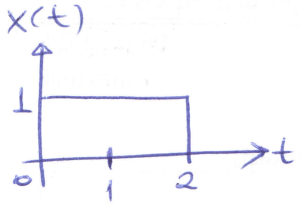
ⓒ $E_c = \int_{2\pi}^{4\pi} (\sin t)^2 . dt = \frac{1}{2}\int_{2\pi}^{4\pi} (1-\cos 2t) . dt = \frac{1}{2} t \Big|_{2\pi}^{4\pi} - \frac{1}{4}\sin 2t \Big|_{2\pi}^{4\pi} = \pi$

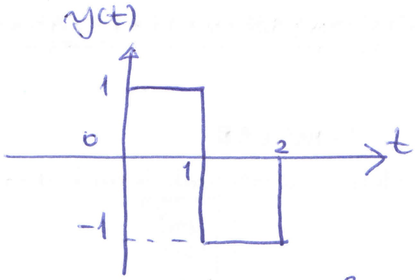
ⓓ $E_d = \int_0^{2\pi} (2\sin t)^2 . dt = \int_0^{2\pi} 4\sin^2 t . dt = 4.\frac{1}{2}\int_0^{2\pi} (1-\cos 2t) . dt = 2\left[\int_0^{2\pi} dt - \int_0^{2\pi} \cos 2t . dt\right]$

$= 2.t\Big|_0^{2\pi} - \sin 2t \Big|_0^{2\pi} = 4\pi - 0 = 4\pi$

Bir işaretin zamanda ötelemesi veya terslendirilmesi işaretin enerjisi üzerinde herhangi bir değişikliğe sebep olmamaktadır ancak işaretin genliğinin "k" ile çarpılması durumunda enerjisi "k^2" ile artış göstermektedir.

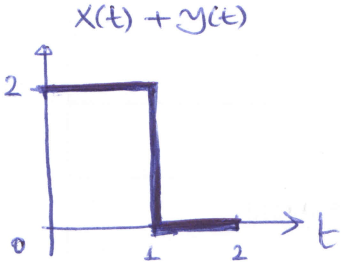
⑤ Aşağıda verilen x(t) ve y(t) işaretlerinin enerjilerini bulunuz. x(t)+y(t) ve x(t)−y(t) işaretlerini çizerek toplam ve fark işaretlerinin enerjilerinin E_x+E_y olduğunu gösteriniz.



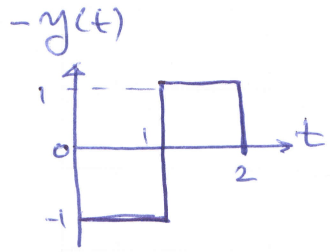


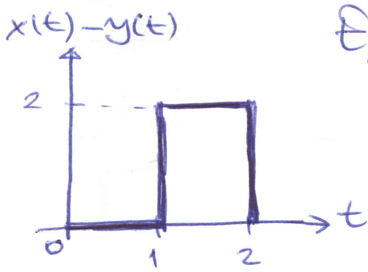
$$E_x=\int_0^2 (1)^2\,dt = t\Big|_0^2 = 2$$

$$E_y=\int_0^1 (1)^2\,dt+\int_1^2 (-1)^2\,dt = t\Big|_0^1 + t\Big|_1^2 = 2$$



$$E_{x+y}=\int_0^1 (2)^2\,dt = 4t\Big|_0^1 = 4$$

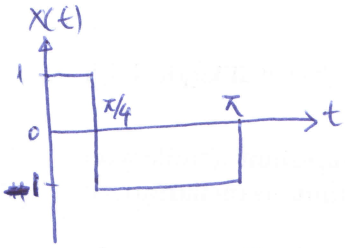


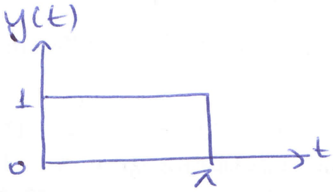


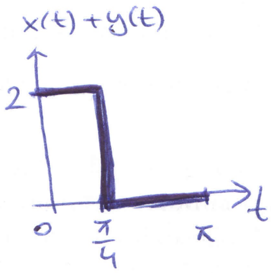
$$E_{x-y}=\int_1^2 2^2\,dt = 4t\Big|_1^2 = 4$$

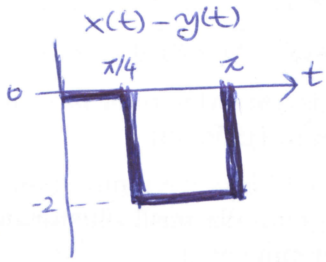
$$\boxed{E_x+E_y=E_{x+y}=E_{x-y}}$$

Ödev: Aynı işlemi aşağıdaki işaretler için gerçekleştirin ve sonuçları karşılaştırın.









$$E_x=\int_0^{\pi/4} 1^2\,dt+\int_{\pi/4}^{\pi} (-1)^2\,dt = t\Big|_0^{\pi/4} + t\Big|_{\pi/4}^{\pi} = \pi$$

$$E_y=\int_0^{\pi} (1)^2\,dt = t\Big|_0^{\pi} = \pi$$

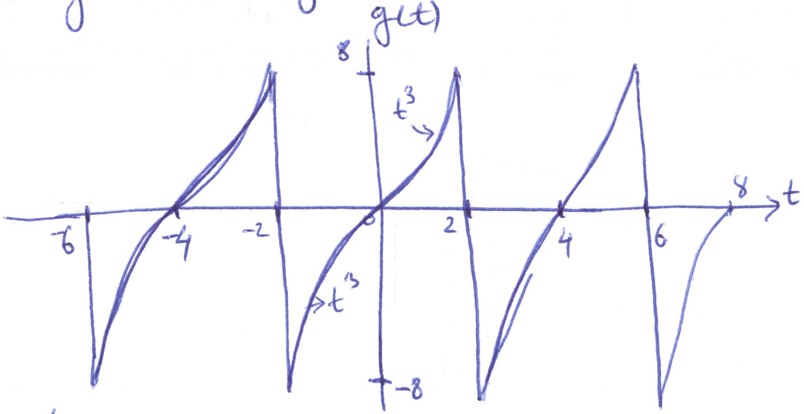
$$E_{x+y}=\int_0^{\pi/4} (2)^2\,dt = 4t\Big|_0^{\pi/4} = \pi$$

$$E_{x-y}=\int_{\pi/4}^{\pi} (-2)^2\,dt = 4t\Big|_{\pi/4}^{\pi} = 4\cdot\left(\pi-\frac{\pi}{4}\right) = 3\pi$$

Elde edilen sonuçtan görülmektedir ki, Genel olarak;

$$\boxed{E_{x\pm y}\neq E_x+E_y}$$

⑥ Bir peryodik $g(t)$ işareti aşağıda görülmektedir. Buna göre $g(t)$, $2g(t)$ ve $c.g(t)$ işaretlerinin gücünü ve rms değerlerini bulunuz.



$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g^2(t).dt \qquad RMS = \sqrt{P_g} \qquad T=4$$

$$P_g = \frac{1}{4}\int_{-2}^{2}(t^3)^2.dt = \frac{1}{4}\int_{-2}^{2} t^6.dt = \frac{1}{4}\cdot\frac{t^7}{7}\Big|_{-2}^{2} = \frac{1}{28}\underbrace{\left(2^7-(-2)^7\right)}_{2^8} = \frac{1}{28}\cdot 2^8 = \boxed{\frac{64}{7}}$$

$$RMS = \sqrt{P_g} = \sqrt{\frac{64}{7}} = \frac{8}{\sqrt{7}} \, //$$

$$P_{2g} = \frac{1}{T}\int_{-T/2}^{T/2}(2g(t))^2.dt = \frac{1}{4}\int_{-2}^{2}(2t^3)^2.dt = \frac{1}{4}\int_{-2}^{2}4t^6 dt = \frac{t^7}{7}\Big|_{-2}^{2} = \frac{1}{7}\left[2^7+2^7\right] = \frac{256}{7}$$

$$RMS = \sqrt{P_{2g}} = \sqrt{\frac{256}{7}} = \frac{16}{\sqrt{7}} \, //$$

$$P_{cg} = \frac{1}{T}\int_{-T/2}^{T/2}(c.g(t))^2 dt = \frac{1}{4}\int_{-2}^{2}c^2.t^6.dt = \frac{1}{4}\cdot c^2\cdot\frac{t^7}{7}\Big|_{-2}^{2} = \frac{64c^2}{7}$$

$$RMS = \sqrt{\frac{64c^2}{7}} = \frac{8c}{\sqrt{7}} \, //$$

---

⑦ Aşağıdaki ifadeleri basitleştiriniz.

a) $\left(\dfrac{\sin t}{t^2+2}\right).\delta(t)$ b) $\left[\dfrac{\sin\frac{\pi}{2}(t-2)}{t^2+4}\right].\delta(t-1)$ ⓒ $\left(\dfrac{1}{j\omega+2}\right).\delta(\omega+3)$

$\boxed{\phi(t).\delta(t)=\phi(0).\delta(t) \text{ özelliği kullanılırsa;}}$

a) $\left(\dfrac{\sin t}{t^2+2}\right).\delta(t)=\left(\dfrac{\sin 0}{0+2}\right).\delta(t)=0$

ⓑ $\left[\dfrac{\sin\frac{\pi}{2}(t-2)}{t^2+4}\right].\delta(t-1)$ $\quad t-1=0 \Rightarrow t=1$

$$\dfrac{\sin\frac{\pi}{2}(1-2)}{1^2+4}.\delta(t-1)=\dfrac{-\sin\frac{\pi}{2}}{5}.\delta(t-1)=\boxed{-\dfrac{1}{5}\delta(t-1)}$$

ⓒ $\left(\dfrac{1}{j\omega+2}\right).\delta(\omega+3)$ $\quad \omega+3=0 \Rightarrow \omega=-3$

$$\dfrac{1}{-j3+2}.\delta(\omega+3)=\boxed{\dfrac{1}{2-j3}.\delta(\omega+3)}$$

---

⑧ Aşağıda verilen integralleri değerlendiriniz.

a) $\int_{-\infty}^{\infty} g(\tau).\delta(t-\tau).d\tau$ ⓑ $\int_{-\infty}^{\infty}\delta(t).e^{-j\omega t}.dt$ ⓒ $\int_{-\infty}^{\infty}\delta(t-2).\sin\pi t.dt$

d) $\int_{-\infty}^{\infty}\delta(t+3).e^{-t}.dt$

$\boxed{\int_{-\infty}^{\infty}\phi(t).\delta(t-T).dt=\phi(T) \text{ özelliği kullanılırsa;}}$

ⓐ $\int_{-\infty}^{\infty} g(\tau).\delta(t-\tau).d\tau=\int_{-\infty}^{\infty} g(\tau).\delta(-(\tau-t)).d\tau=\int_{-\infty}^{\infty} g(\tau).\delta(\tau-t).d\tau=g(t)$

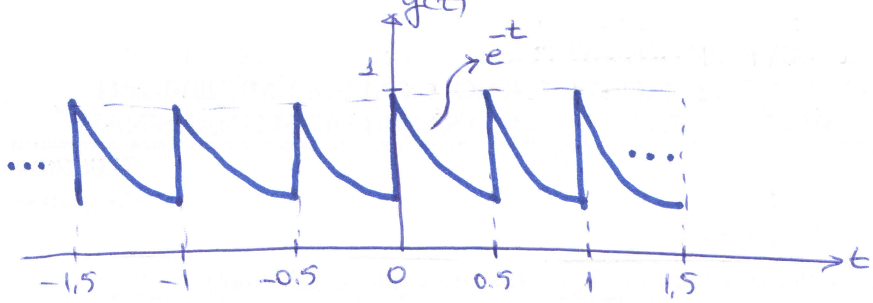
$\delta(t)=\delta(-t)$

ⓑ $\int_{-\infty}^{\infty}\delta(t).e^{-j\omega t}.dt \longrightarrow t=0$ için $\int_{-\infty}^{\infty}\delta(t)e^{-j\omega t}.dt=1$

ⓒ $\int_{-\infty}^{\infty}\delta(t-2).\sin\pi t.dt \Rightarrow t-2=0 \Rightarrow t=2$ ve $\sin\pi t=\sin 2\pi=0$

ⓓ $\int_{-\infty}^{\infty}\delta(t+3).e^{-t}.dt \Rightarrow t+3=0 \Rightarrow t=-3$ ve $\int_{-\infty}^{\infty}\delta(t+3).e^{-t}.dt=e^{3}$

Örnek: Aşağıdaki $g(t)$ işaretinin Trigonometrik Fourier Seri açılımını hesaplayınız.



$T_0 = 0.5 \Rightarrow f = 2\,\text{Hz} \qquad \omega_0 = 2\pi f = 4\pi$

$$g(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right)$$
$$= a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(4\pi n t) + b_n \sin(4\pi n t)\right)$$

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} g(t)\,dt = 2\int_0^{0.5} e^{-t}\,dt = -2e^{-t}\Big|_0^{0.5} = -2\left(e^{-0.5} - 1\right) = \boxed{0.79}$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\cos\left(\frac{2\pi n t}{T_0}\right)dt = 4\int_0^{0.5} e^{-t}\cos(4\pi n t)\,dt = 4\left[\frac{e^{-t}}{1+16\pi^2 n^2}\left(-\cos(4\pi n t) + 4\pi n \sin(4\pi n t)\right)\Big|_0^{0.5}\right]$$

$$\boxed{\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right)}$$

$$= 4\left[\frac{e^{-0,5}}{1+16\pi^2 n^2}\left(\underbrace{-\underbrace{\cos(2\pi n)}_{1}}_{-1} + \underbrace{4\pi n \underbrace{\sin(2\pi n)}_{0}}_{0}\right) - \left(\frac{1}{1+16\pi^2 n^2}\left(\underbrace{-\cos(0)}_{-1} + \underbrace{4\pi n \sin(0)}_{0}\right)\right)\right]$$

$$= 4\left[-\frac{e^{-0,5}}{1+16\pi^2 n^2} + \frac{1}{1+16\pi^2 n^2}\right] = 4\left(\frac{1-e^{-0,5}}{1+16\pi^2 n^2}\right) = \boxed{\frac{1.58}{1+16\pi^2 n^2}}$$

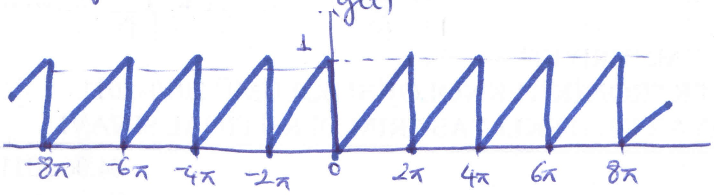
$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\sin\left(\frac{2\pi n t}{T_0}\right)dt = 4\int_0^{0.5} e^{-t}\sin(4\pi n t)\,dt = 4\left[\left(\frac{e^{-0,5}}{1+16\pi^2 n^2}\left(\underbrace{-\sin 2\pi n}_{0} - \underbrace{4\pi n \underbrace{\cos(2\pi n)}_{1}}_{-4\pi n}\right)\right) - \frac{e^{0}}{1+16\pi^2 n^2}\left(\underbrace{-\sin 0}_{0} - \underbrace{4\pi n \underbrace{\cos(0)}_{1}}_{-4\pi n}\right)\right]$$

$$\boxed{\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right)}$$

$$= 4\left[\frac{-4\pi n\, e^{-0,5}}{1+16\pi^2 n^2} - \frac{-4\pi n}{1+16\pi^2 n^2}\right] = \frac{16\pi n\left(1-e^{-0.5}\right)}{1+16\pi^2 n^2} = \boxed{\frac{6.30\pi n}{1+16\pi^2 n^2}}$$

$$\boxed{g(t) = 0.79 + \sum_{n=1}^{\infty}\frac{1.58}{1+16\pi^2 n^2}\cos(4\pi n t) + \frac{6.30\pi n}{1+16\pi^2 n^2}\sin(4\pi n t)}$$

Örnek: Aşağıdaki g(t) işaretinin Trigonometrik Fourier Seri açılımını 5. harmoniğe kadar (5 dahil) açınız ve genlik/faz spektrumlarını çiziniz.



$$\frac{y_2-y_1}{x_2-x_1}=\frac{y-y_1}{x-x_1} \Rightarrow y=\frac{t}{2\pi} \qquad T=2\pi \quad \omega=1$$

$$g(t)=a_0+\sum_{n=1}^{\infty} a_n\cos(n\omega t)+b_n\sin(n\omega t)$$

$$g(t)=a_0+\sum_{n=1}^{\infty} a_n\cos(nt)+b_n\sin(nt)$$

$$a_0=\frac{1}{T}\int_{t_1}^{t_1+T} g(t).dt=\frac{1}{2\pi}\int_0^{2\pi}\frac{t}{2\pi}dt=\frac{1}{4\pi^2}\int_0^{2\pi}t.dt=\frac{1}{8\pi^2}t^2\Big|_0^{2\pi}=\frac{1}{2}=\boxed{0.5}$$

$$a_n=\frac{2}{T}\int_{t_1}^{t_1+T} g(t).\cos(nt).dt=\frac{\cancel{2}}{\cancel{2}\pi}\int_0^{2\pi}\frac{t}{2\pi}.\cos(nt).dt=\frac{1}{2\pi^2}\left[\frac{t}{n}\sin(nt)+\frac{1}{n^2}\cos(nt)\right]_0^{2\pi}$$

$$\boxed{\int t.\cos(at).dt=\frac{t}{a}\sin(at)+\frac{1}{a^2}\cos(at)}$$

$$=\frac{1}{2\pi^2}\left[\left(\frac{2\pi}{n}.\cancelto{0}{\sin(2\pi n)}+\frac{1}{n^2}\cos(2\pi n)\right)-\left(\cancel{\frac{0}{n}\sin 0}+\frac{1}{n^2}\cos 0\right)\right]=\frac{1}{2\pi^2}\left[\frac{1}{n^2}\underbrace{\cos 2\pi n}_{1}-\frac{1}{n^2}\underbrace{\cos 0}_{1}\right]=\boxed{0}$$

$$b_n=\frac{2}{T}\int_{t_1}^{t_1+T} g(t).\sin(nt).dt=\frac{\cancel{2}}{\cancel{2}\pi}\int_0^{2\pi}\frac{t}{2\pi}\sin(nt).dt=\frac{1}{2\pi^2}\int_0^{2\pi}t.\sin(nt).dt$$

$$\boxed{\int t.\sin(nt)dt=\frac{1}{a^2}\sin at-\frac{t}{a}\cos at}=\frac{1}{2\pi^2}\left[\frac{1}{n^2}.\sin(nt)-\frac{t}{n}\cos(nt)\right]_0^{2\pi}$$

$$=\frac{1}{2\pi^2}\left[\left(\frac{1}{n^2}.\cancelto{0}{\sin(2\pi n)}-\frac{2\pi}{n}\underbrace{\cos(2\pi n)}_{1}\right)-\left(\frac{1}{n^2}\cancelto{0}{\sin(0)}-\cancelto{0}{\frac{0}{n}\cos(0)}\right)\right]=\frac{1}{2\pi^2}.\frac{-2\pi}{n}=\boxed{-\frac{1}{\pi n}}$$

$$\boxed{g(t)=0.5+\sum_{n=1}^{\infty}\left(-\frac{1}{\pi n}\right)\sin(nt)}$$

5. harmoniğe kadar açılırsa,

$$g(t)=0.5+\left(\left(-\frac{1}{\pi}\sin t\right)+\left(-\frac{1}{2\pi}\sin 2t\right)+\left(-\frac{1}{3\pi}\sin 3t\right)+\left(-\frac{1}{4\pi}\sin 4t\right)+\left(-\frac{1}{5\pi}\sin 5t\right)\right)$$

$$\boxed{g(t)=0.5-\frac{1}{\pi}\left(\sin t+\frac{1}{2}\sin 2t+\frac{1}{3}\sin 3t+\frac{1}{4}\sin 4t+\frac{1}{5}\sin 5t\right)}$$

$$=0.5-\frac{1}{\pi}\left(\cos\left(t-\frac{\pi}{2}\right)+\frac{1}{2}\cos\left(2t-\frac{\pi}{2}\right)+\frac{1}{3}\cos\left(3t-\frac{\pi}{2}\right)+\frac{1}{4}\cos\left(4t-\frac{\pi}{2}\right)+\frac{1}{5}\cos\left(5t-\frac{\pi}{2}\right)\right)$$

$$g(t)=0.5+\frac{1}{\pi}\left[\cos\left(t+\frac{\pi}{2}\right)+\frac{1}{2}\cos\left(2t+\frac{\pi}{2}\right)+\frac{1}{3}\cos\left(3t+\frac{\pi}{2}\right)+\frac{1}{4}\cos\left(4t+\frac{\pi}{2}\right)+\frac{1}{5}\cos\left(5t+\frac{\pi}{2}\right)\right)$$

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$$g(t) = 0.5 + \frac{1}{\pi}\left[\cos\left(t+\frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2t+\frac{\pi}{2}\right) + \frac{1}{3}\cos\left(3t+\frac{\pi}{2}\right) + \frac{1}{4}\cos\left(4t+\frac{\pi}{2}\right) + \frac{1}{5}\cos\left(5t+\frac{\pi}{2}\right)\right]$$