Signal Analysis with MATLAB

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Matlab Programming Structures

- **Loops**

```
for i=1:100
    sum = sum+i;
end
```

- Goes round the for loop 100 times, starting at i=1 and finishing at i=100

```
i=1;
while i<=100
    sum = sum+i;
    i = i+1;
end
```

- Similar, but uses a while loop instead of a for loop

- **Decisions**

```
if i==5
    a = i*2;
else
    a = i*4;
end
```

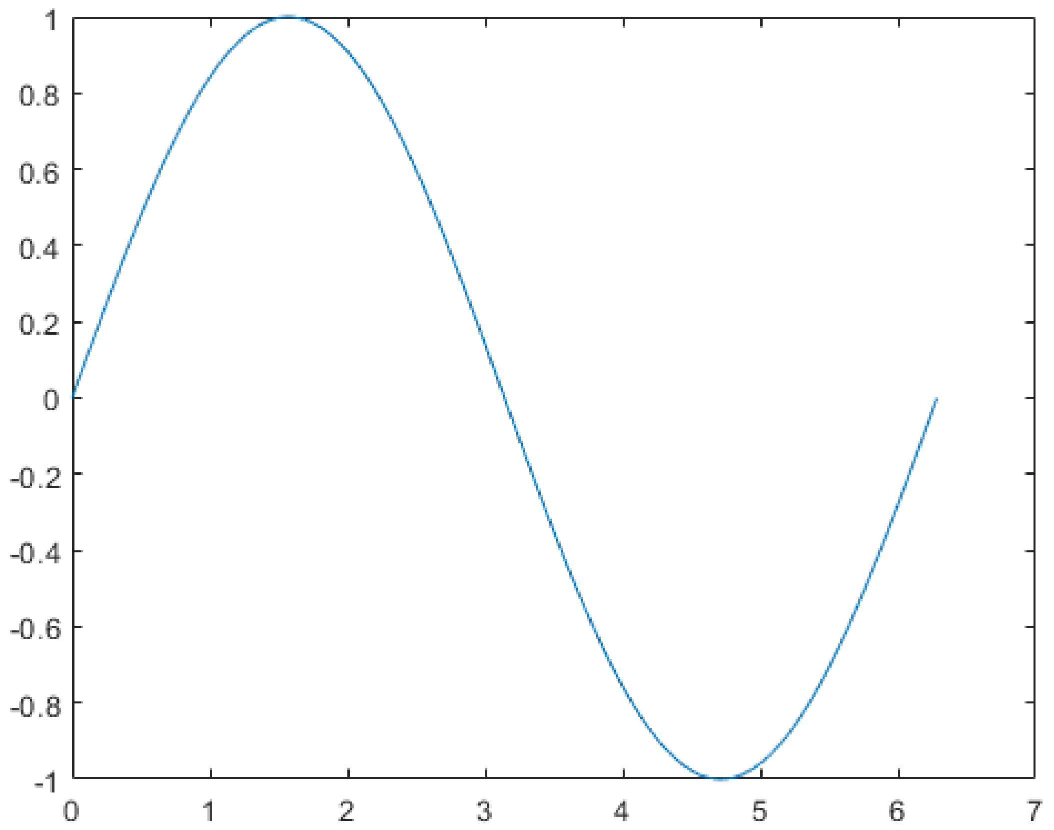
- Executes whichever branch is appropriate depending on test

```
switch i
case 5
    a = i*2;
otherwise
    a = i*4;
end
```

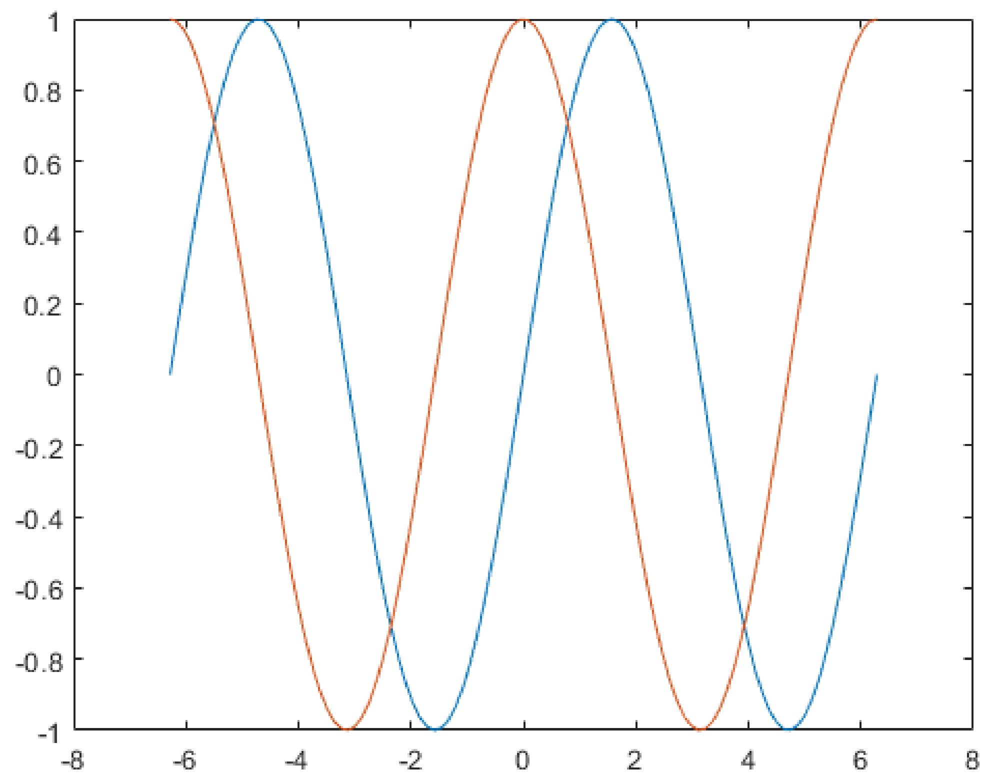
- Similar, but uses a switch

2-D Ploting

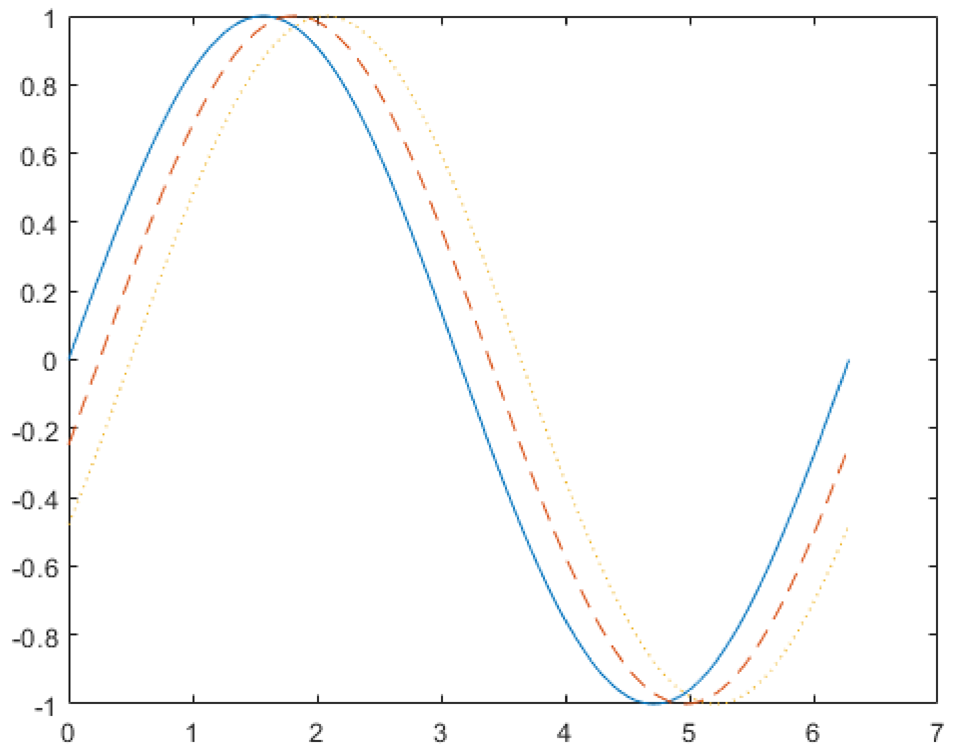
```
x = 0:pi/100:2*pi;
y = sin(x);
plot(x,y)
```



```
x = linspace(-2*pi,2*pi);
y1 = sin(x);
y2 = cos(x);

figure
plot(x,y1,x,y2)
```




```
x = 0:pi/100:2*pi;
y1 = sin(x);
y2 = sin(x-0.25);
y3 = sin(x-0.5);

figure
plot(x,y1,x,y2,'--',x,y3,':')
```

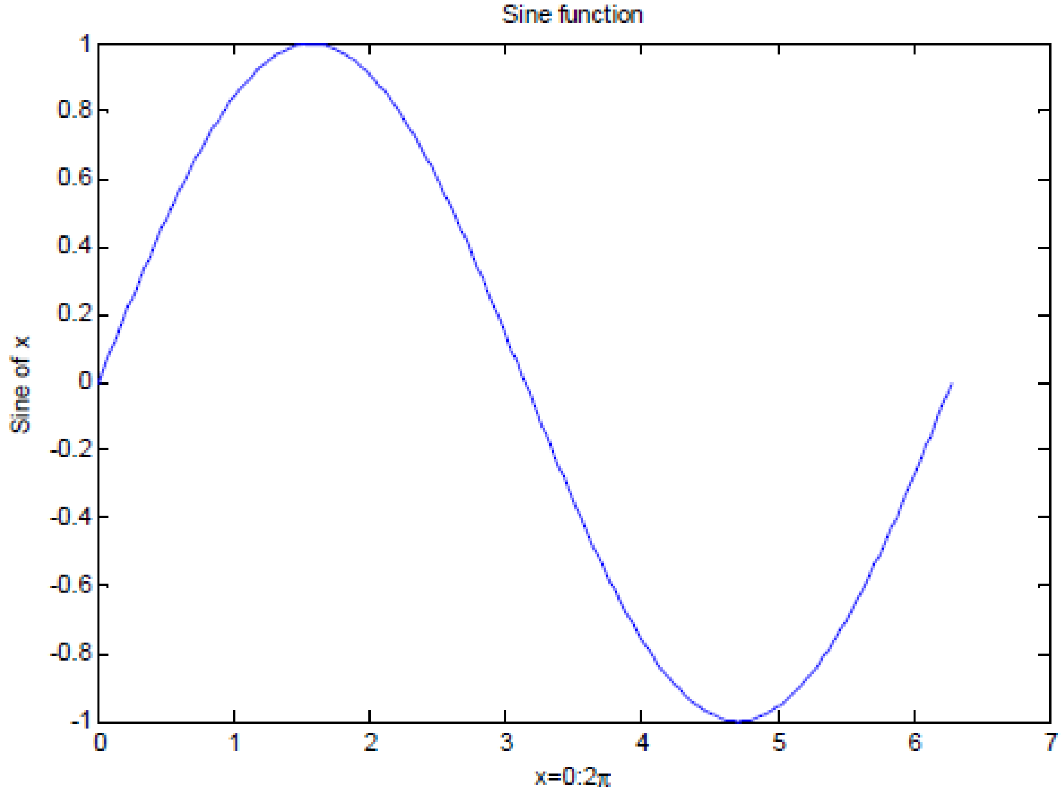


Basic plotting in MATLAB

MATLAB has an excellent set of graphic tools. Plotting a given data set or the results of computation is possible with very few commands

The MATLAB command to plot a graph is plot(x,y), e.g.

```
>> x = 0:pi/100:2*pi;
>> y = sin(x);
>> plot(x,y)
```



MATLAB enables you to add axis Labels and titles, e.g.

```
>> xlabel('x=0:2\pi');
>> ylabel('Sine of x');
>> tile('Sine function')
```

Nyquist Sampling Theorem

For lossless digitization, the sampling rate should be at least twice the maximum frequency response. Sayısal iletim ortamının performansını belirler.

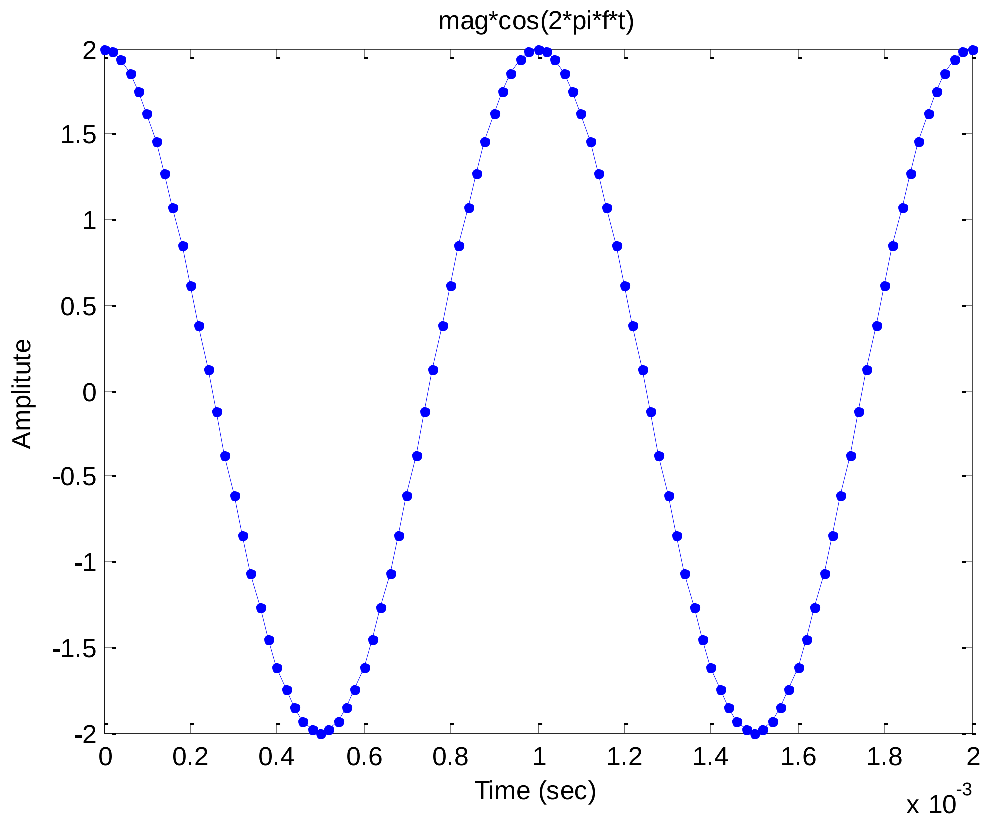
- In mathematical terms:

$$f_s \geq 2{*}f_m$$

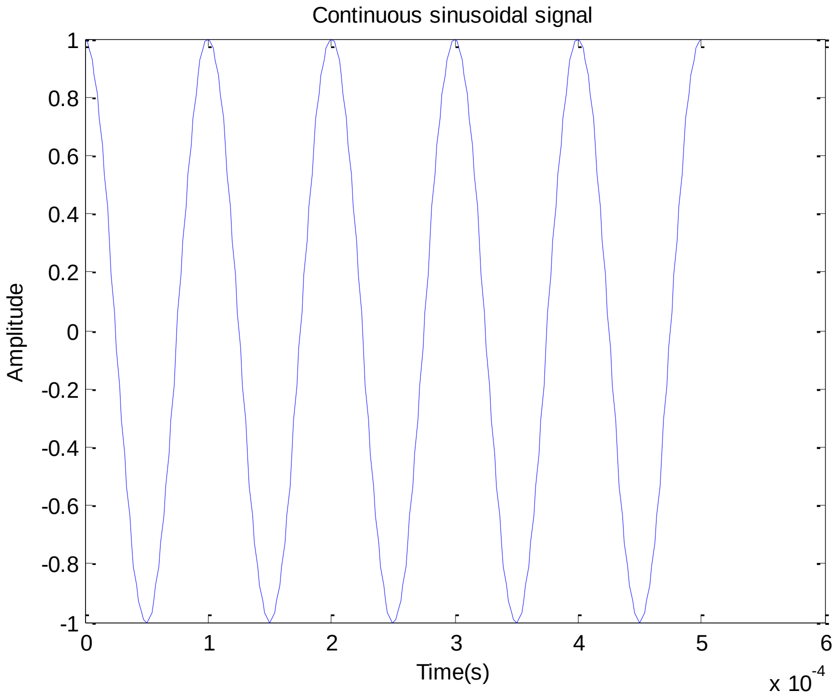
$$f_s \geq 2{*}B$$

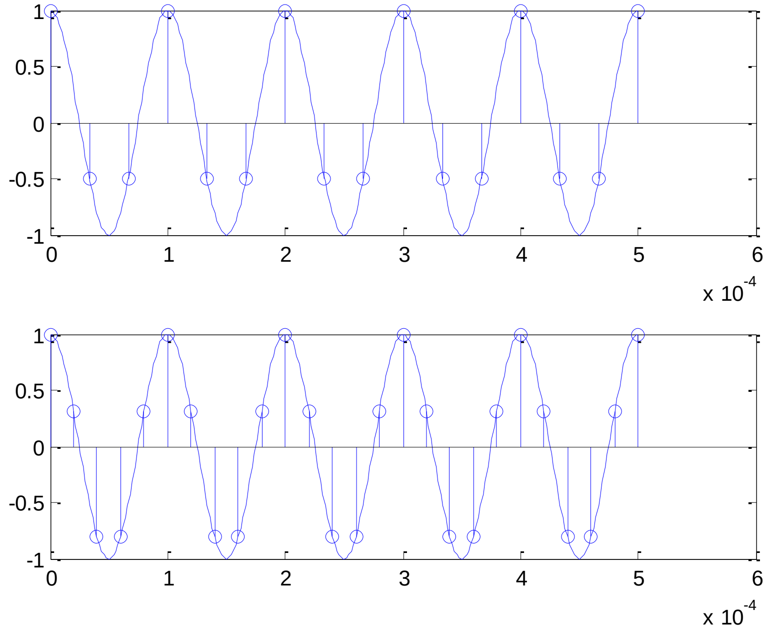
- where f_s is sampling frequency and f_m is the maximum frequency in the signal; B is the bandwidth. Birimleri Hz=1/sec. Frekans bir saniyedeki titreşim sayısıdır.

```
clear all
  close all
 mag  = 2;     % magnitude (arbitrary units)
 f = 1000;     % frequency in Hz
 Ps=50;        % number of sampling on a periot
 samp = f*Ps;   % sampling rate in Hz
 M=2;  %   Displaying number of period
 del=1/samp;


 t       = 0:del:M*Ps*del;  % time
 N       = length(t)
 x = mag*cos(2*pi*f*t);    % the signal equation
 figure
 plot(t,x,'.-');
 xlabel('Time (sec)');
 ylabel('Amplitute');
 title('mag*cos(2*pi*f*t)')
```

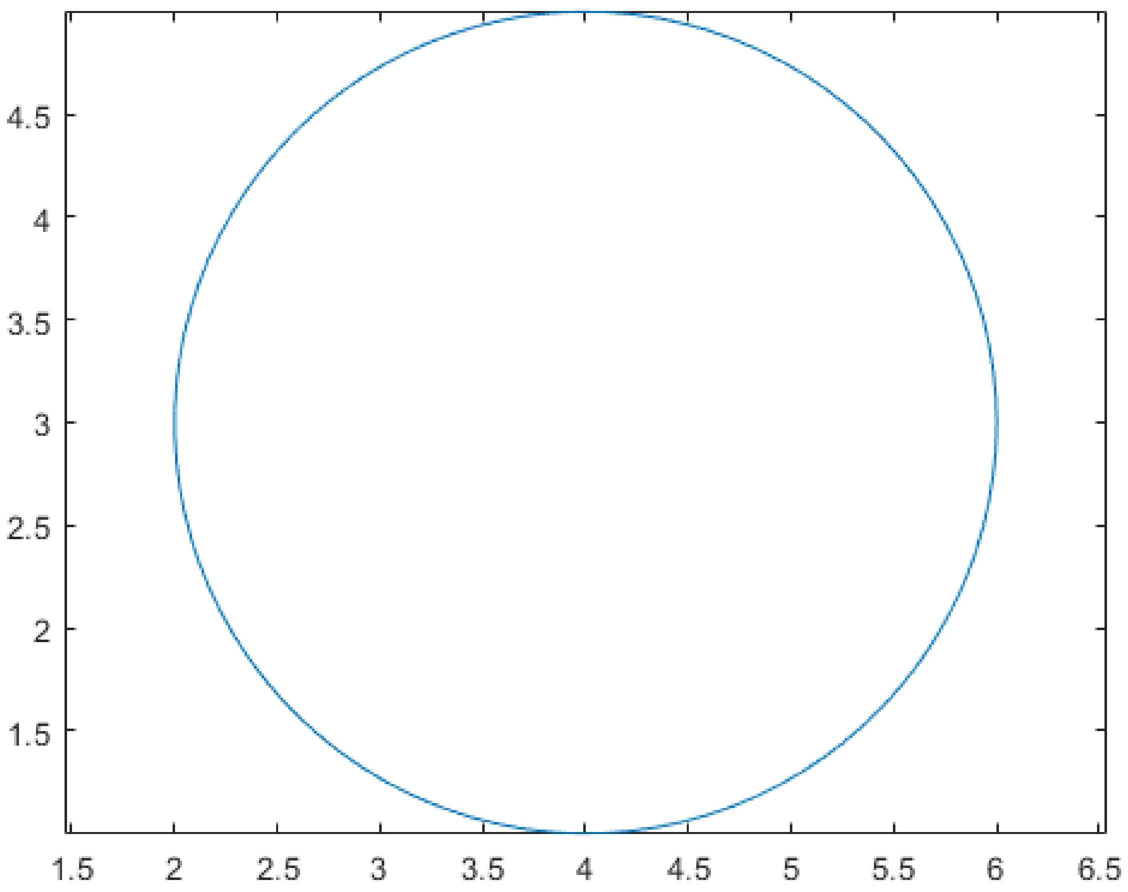


```
clear all
close all
 fs=500e3; %Very high sampling rate 500 kHz
f=10e3; %Frequency of sinusoid
nCyl=5; %generate five cycles of sinusoid
t=0:1/fs:nCyl*1/f; %time index
x=cos(2*pi*f*t);

figure, plot(t,x)
title('Continuous sinusoidal signal');
xlabel('Time(s)');
ylabel('Amplitude');


fs1=30e3; %30kHz sampling rate
t1=0:1/fs1:nCyl*1/f; %time index
x1=cos(2*pi*f*t1);

fs2=50e3; %50kHz sampling rate
t2=0:1/fs2:nCyl*1/f; %time index
x2=cos(2*pi*f*t2);

figure
subplot(2,1,1);
plot(t,x);
hold on;
stem(t1,x1);
subplot(2,1,2);
plot(t,x);
hold on;
stem(t2,x2);
```





```
r = 2;
xc = 4;
yc = 3;

theta = linspace(0,2*pi);
x = r*cos(theta) + xc;
y = r*sin(theta) + yc;
plot(x,y)
axis equal
```



Line Style	Description
-	Solid line (default)
--	Dashed line
:	Dotted line
-.	Dash-dot line

Color	Description
y	yellow
m	magenta
c	cyan
r	red
g	green
b	blue
w	white
k	black

Marker	Description
o	Circle
+	Plus sign
*	Asterisk
.	Point
x	Cross
s	Square
d	Diamond
^	Upward-pointing triangle
v	Downward-pointing triangle
>	Right-pointing triangle
<	Left-pointing triangle
p	Pentagram
h	Hexagram

Symbolic Expressions

Manipulating symbolic expressions

- numden()
- expand()
- factor()
- collect()
- simplify()
- simple()
- poly2sym()

Gradient of Function

Gradient vector of scalar function

The *gradient* of a function of two variables, $F(x, y)$, is defined as

$$\nabla F = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j}$$

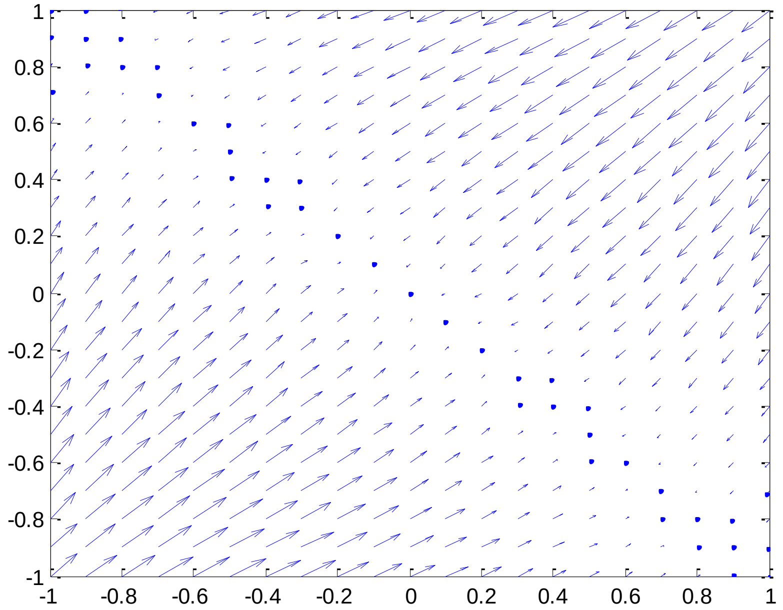
```
clear all
close all

syms x y z
f = 2*y*z*sin(x) + 3*x*sin(z)*cos(y);
gradient(f, [x, y, z])
```

Answer:
fx=2*y*z*cos(x) + 3*cos(y)*sin(z)
fy=2*z*sin(x) - 3*x*sin(y)*sin(z)
fz=2*y*sin(x) + 3*x*cos(y)*cos(z)

```
clear all
close all

syms x y
f = -(sin(x) + sin(y))^2;
g = gradient(f,[x,y])
```

Answaer
g =
 -2*cos(x)*(sin(x) + sin(y))
 -2*cos(y)*(sin(x) + sin(y))

```
[X, Y] = meshgrid(-1:.1:1,-1:.1:1);
G1 = subs(g(1),[x y],{X,Y});
G2 = subs(g(2),[x y],{X,Y});
quiver(X,Y,G1,G2)
```

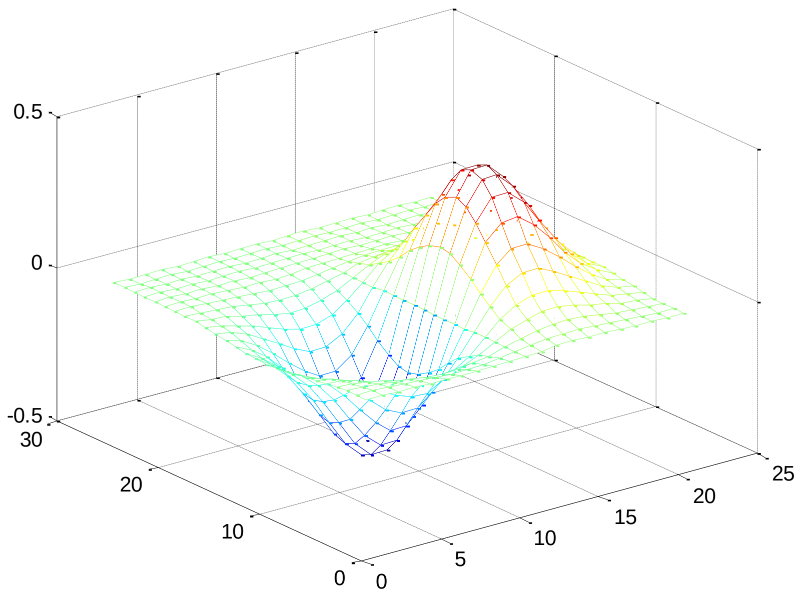


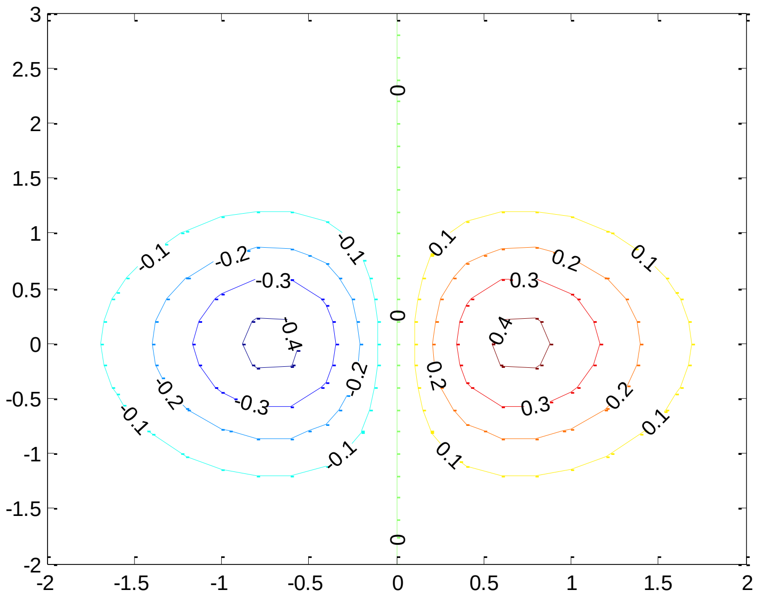
Not: subs(s,old,new) returns a copy of s, replacing all occurrences of old with new, and then evaluates s.

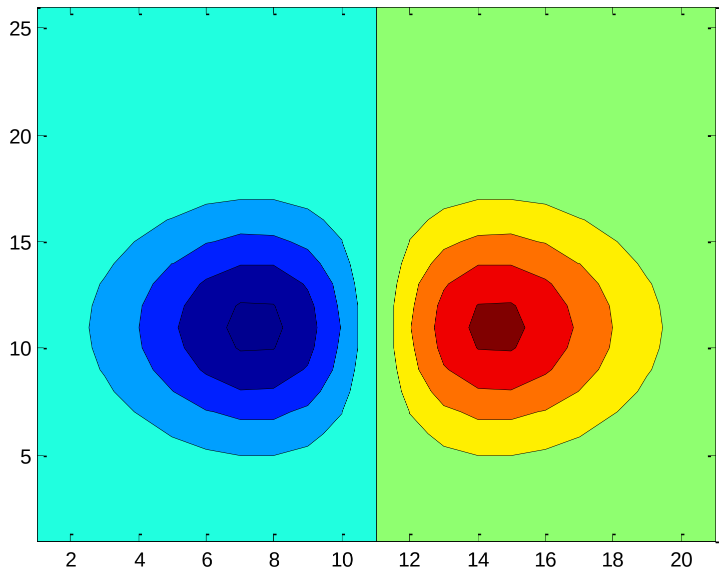
```
syms a b
subs(a + b, a, 4)
ans = b + 4
```

3-D Imaging

```
clear all
close all

x = -2:0.2:2;
y = -2:0.2:3;
[X,Y] = meshgrid(x,y);
Z = X.*exp(-X.^2-Y.^2);

figure, mesh(Z)
figure, contour(X,Y,Z)
figure, contourf(Z)
```

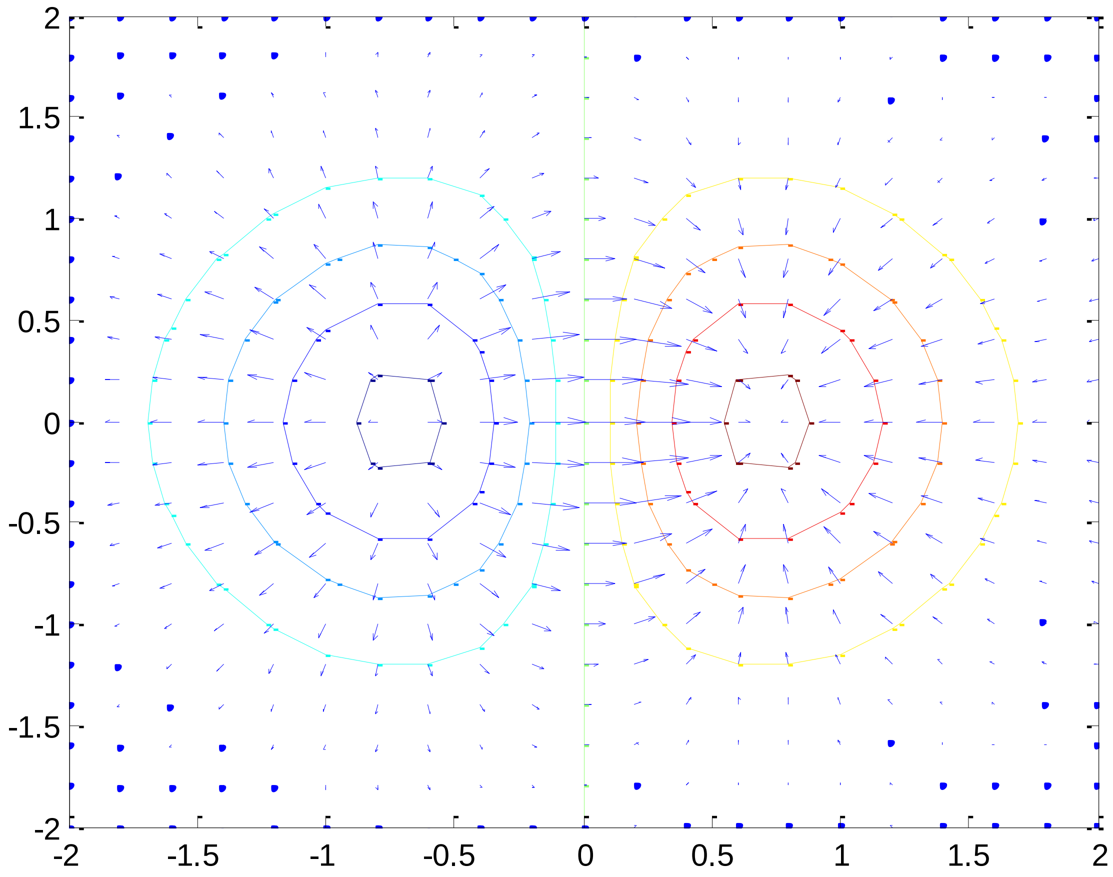






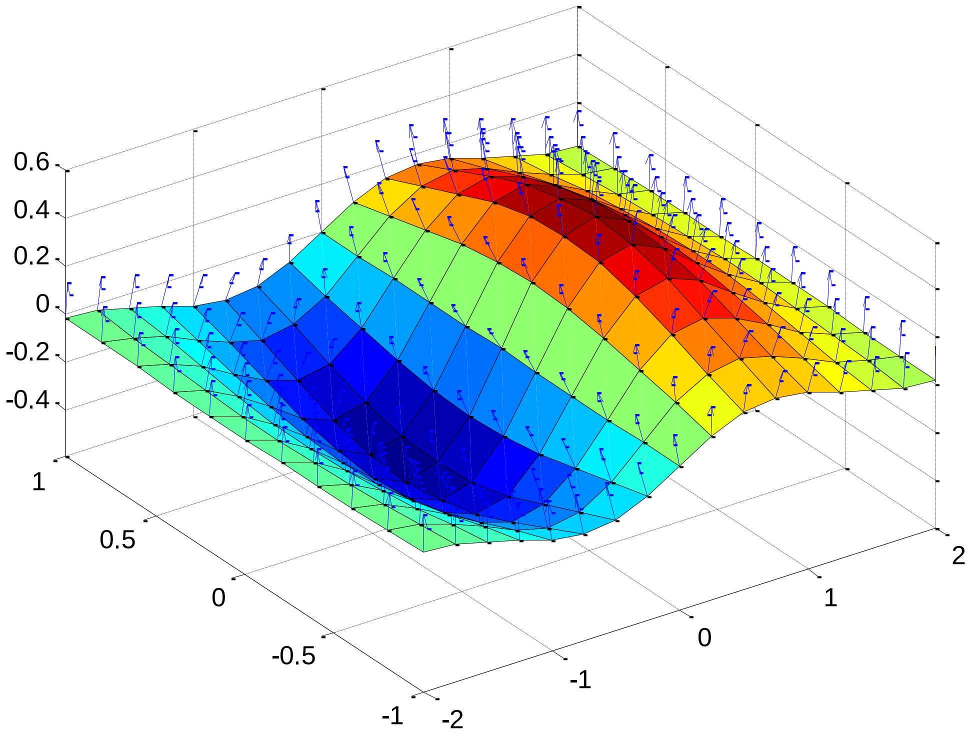
Plot the gradient of the function $z = xe^{-x2-y2}$

```
clear all
close all

[X,Y] = meshgrid(-2:.2:2);
Z = X.*exp(-X.^2 - Y.^2);
[DX,DY] = gradient(Z,.2,.2);

figure
contour(X,Y,Z)
hold on
quiver(X,Y,DX,DY)
hold off
```

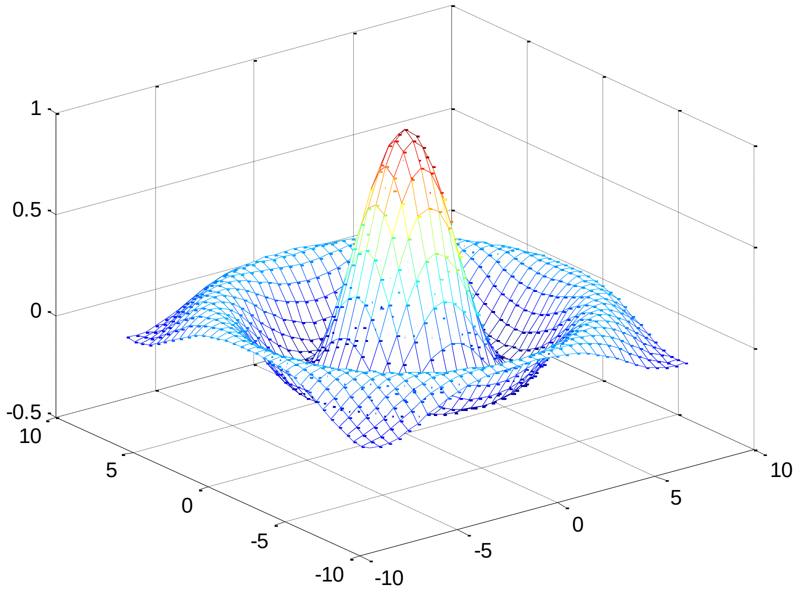


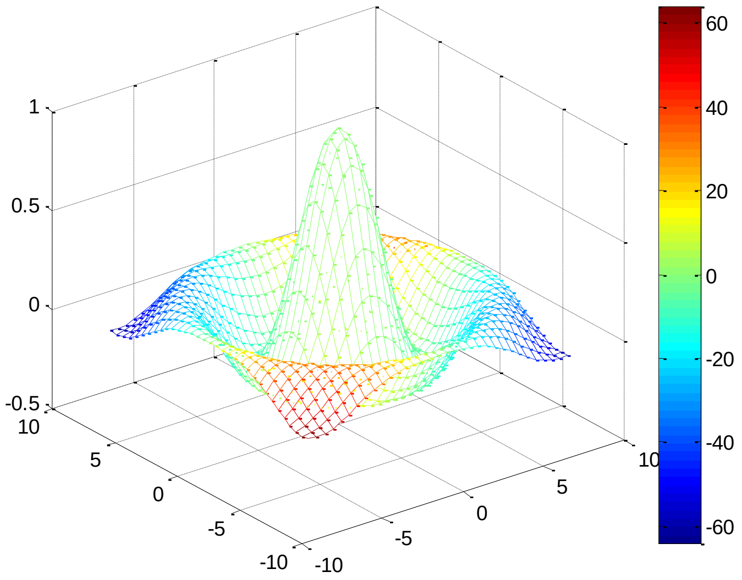
Plot the gradient of the function $z=xe^{-x^2-y^2}$

```
clear all
close all

[X,Y] = meshgrid(-2:0.25:2,-1:0.2:1);
Z = X.* exp(-X.^2 - Y.^2);
[U,V,W] = surfnorm(X,Y,Z);

figure
quiver3(X,Y,Z,U,V,W,0.5)

hold on
surf(X,Y,Z)
view(-35,45)
axis([-2 2 -1 1 -.6 .6])
hold off
```



```
clear all
close all

[X,Y] = meshgrid(-8:.5:8);
R = sqrt(X.^2 + Y.^2) + eps;
Z = sin(R)./R;
figure, mesh(X,Y,Z)


C = X.*Y;
figure, mesh(X,Y,Z,C), colorbar
```



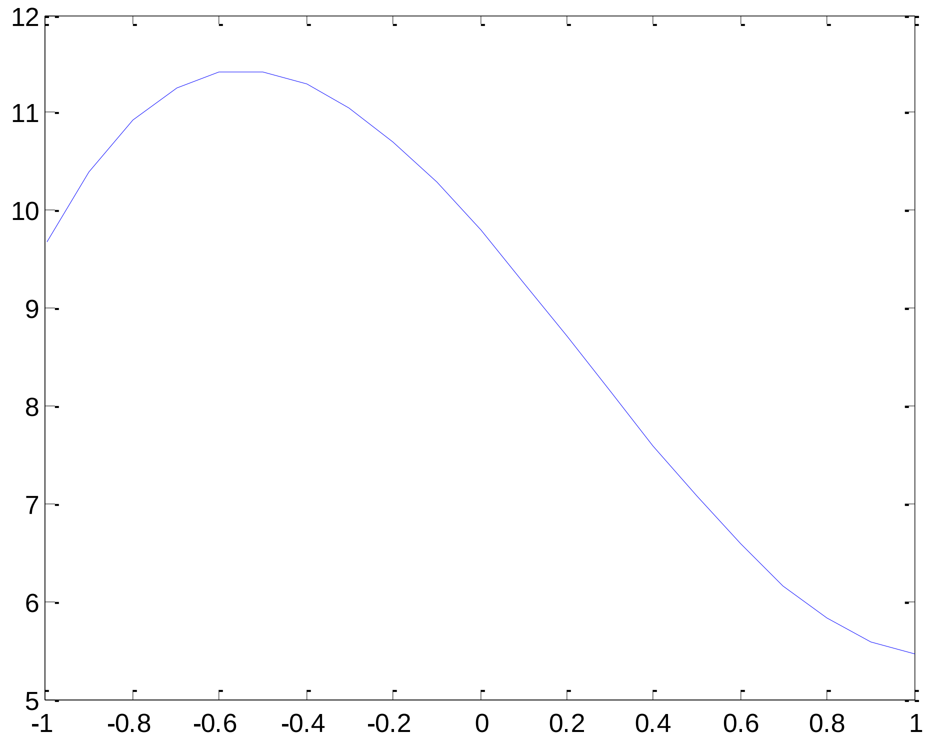


Polynomials and Roots

Polynomials and Roots

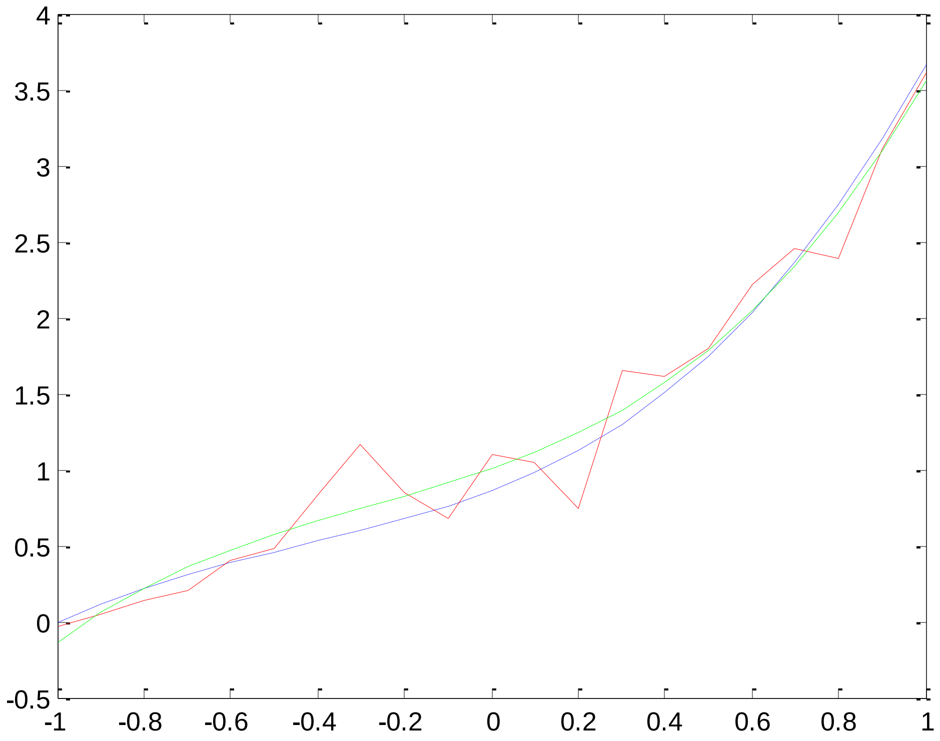
$$3x^3 - 2.23x^2 - 5.1x + 9.8$$

```
clear all
close all

c = [ 3 -2.23 -5.1 9.8 ]
 x = -1:0.1:1; % define range for plotting
 y = polyval(c,x); % compute samples
plot(x,y) % make the plot
r = roots(c)

cr = poly(r) % defines (x-r(1))*(x-r(2))*...
norm(c(1)*cr-c) % how far from original polynomial?

r =

  -1.5985 + 0.0000i
   1.1709 + 0.8200i
   1.1709 - 0.8200i

cr =    1.0000   -0.7433   -1.7000    3.2667
ans =   1.1374e-14
```



Polynomials and Roots

```
clear all
close all

 c3 = [0.74 0.97 1.1 0.86]; % data source
 x = -1:0.1:1; % sample range for x
 y = polyval(c3,x); % sample cubic at x
 noise = randn(1,size(y,2)); % random noise of same size as y
 noise = noise/norm(noise); % normalize the noise
ey = y + noise; % make noisy data
ec = polyfit(x,ey,3); % fit noisy data with cubic

 plot(x,y,'b--') % exact polynomial
 hold on
 plot(x,ey,'r') % noisy data
 hold on
 fy = polyval(ec,x) % sample fitted polynomial
 plot(x,fy,'g') % reconstructed data source
```



Integral – Derivative - Limit

Finding the integral of a function

Mathematical Operation	**MATLAB Command**
$\int x^n \, dx = \frac{x^{n+1}}{n+1}$	`int(x^n)` or `int(x^n,x)`
$\int_0^{\pi/2} \sin(2x) \, dx = 1$	`int(sin(2*x),0,pi/2)` or `int(sin(2*x),x,0,pi/2)`

$g = \cos(at+b)$
$\int g(t)dt = \frac{\sin(at+b)}{a}$
	`g = cos(a*t + b)`
	`int(g)` or
	`int(g,t)`

$\int J_1(z) \, dz = -J_0(z)$	`int(besselj(1,z))` or `int(besselj(1,z),z)`

$q = integral(fun,xmin,xmax),$ $$f(x)=e^{-x^2}(\ln x)^2$$

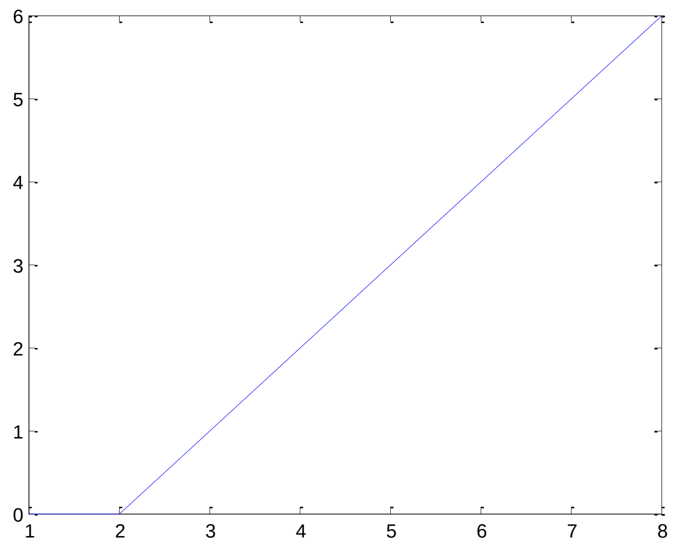
```
clear all
close all

fun = @(x) exp(-x.^2).*log(x).^2;

q = integral(fun,0,Inf)
```

q = 1.9475

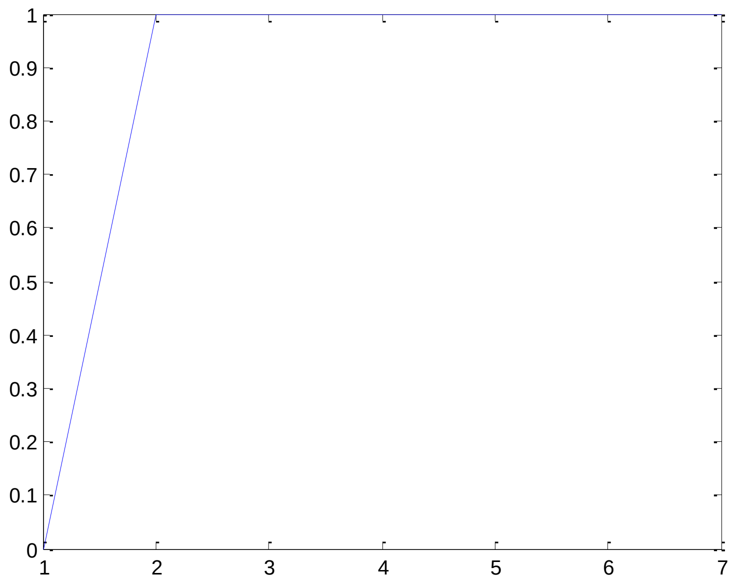
İntegral, derivative

```
clear all
close all

syms x
y1=int(4*x^2+3)
y2=int(4*x^2+3, x, -1, 3)

y1 = (4*x^3)/3 + 3*x
y2 = 26/3
```

```
clear all
close all

X = [0 0 1 2 3 4 5 6];
figure, plot(X)
Y = diff(X)
figure, plot(Y)
```




```
clear all
close all
syms x real
f = 1/x
y= diff(f)

y=-1/x^2
```



```
clear all
close all

syms x
f = cos(8*x)
g = sin(5*x)*exp(x)
h =(2*x^2+1)/(3*x)
a1=diff(f)
a2=diff(g)
a3=diff(h)
```

```
f =cos(8*x)
g = sin(5*x)*exp(x)
h =(2*x^2 + 1)/(3*x)

a1 = -8*sin(8*x)
a2 = 5*cos(5*x)*exp(x) + sin(5*x)*exp(x)
a3 = 4/3 - (2*x^2 + 1)/(3*x^2)
```

$$\frac{\partial f}{\partial x} = ?$$

```
clear all
close all

syms x y
f = sin(x*y)
diff(f,x)
```

```
Ans=y*cos(x*y)
```

```
clear all
close all

syms x a b c
S = simplify(sin(x)^2 + cos(x)^2)
S = simplify(exp(c*log(sqrt(a+b))))

S = 1
S =(a + b)^(c/2)
```

Finding the Limits of a function- f(x)

Mathematical Operation	MATLAB Command
$\lim_{x \to 0} f(x)$	`limit(f)`
$\lim_{x \to a} f(x)$	`limit(f,x,a)` or `limit(f,a)`
$\lim_{x \to a+} f(x)$	`limit(f,x,a,'right')`
$\lim_{x \to a-} f(x)$	`limit(f,x,a,'left')`

Evaluate: (a) $\lim_{x \to 0} \left(\frac{1}{x}\right)$ (b) $\lim_{x \to 0+} \left(\frac{1}{x}\right)$ (c) $\lim_{x \to 0-} \left(\frac{1}{x}\right)$

Solution of Differential Equations with MATLAB

$$d^2y/dx^2=\cos(2x)-y,\ y(0)=1, y'(0)=0.$$

```
clear all
close all

syms y(x)
Dy = diff(y);

ode = diff(y,x,2) == cos(2*x)-y;
cond1 = y(0) == 1;
cond2 = Dy(0) == 0;

conds = [cond1 cond2];
ySol(x) = dsolve(ode,conds);
ySol = simplify(ySol)


ySol(x) = 1 - (8*sin(x/2)^4)/3
```

Differential Equation	MATLAB® Commands

$\frac{dy}{dt} + 4y(t) = e^{-t},$
$y(0) = 1.$
	`syms y(t)`
	`ode = diff(y)+4*y == exp(-t);`
	`cond = y(0) == 1;`
	`ySol(t) = dsolve(ode,cond)`
	
	`ySol(t) =`
	`exp(-t)/3 + (2*exp(-4*t))/3`

$2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 0.$	`syms y(x)`
	`ode = 2*x^2*diff(y,x,2)+3*x*diff(y,x)-y == 0;`
	`ySol(x) = dsolve(ode)`
	
	`ySol(x) =`
	`C2/(3*x) + C3*x^(1/2)`

The Airy equation.
$\frac{d^2y}{dx^2} = xy(x).$
	`syms y(x)`
	`ode = diff(y,x,2) == x*y;`
	`ySol(x) = dsolve(ode)`
	
	`ySol(x) =`
	`C1*airy(0,x) + C2*airy(2,x)`

Solve Differential Equations in Matrix Form

Solve differential equations in matrix form by using `dsolve`.

Consider this system of differential equations.

$$\frac{dx}{dt} = x + 2y + 1,$$
$$\frac{dy}{dt} = -x + y + t.$$

The matrix form of the system is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ t \end{bmatrix}.$$

Let

$$Y = \begin{bmatrix} x \\ y \end{bmatrix}, A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ t \end{bmatrix}.$$

The system is now $Y' = AY + B$.

```
syms x(t) y(t)
A = [1 2; -1 1];
B = [1; t];
Y = [x; y];
odes = diff(Y) == A*Y + B
```

```
odes(t) =
  diff(x(t), t) == x(t) + 2*y(t) + 1
   diff(y(t), t) == t - x(t) + y(t)
```

Solve the matrix equation using `dsolve`. Simplify the solution by using the `simplify` function.

```
[xSol(t), ySol(t)] = dsolve(odes);
xSol(t) = simplify(xSol(t))
ySol(t) = simplify(ySol(t))
```

Suppose we want to solve and plot the solution to the second order equation

$$y''(x) + 8y'(x) + 2y(x) = \cos(x); \qquad y(0) = 0,\ y'(0) = 1.$$

```
clear all
close all

eqn2 = 'D2y + 8*Dy + 2*y = cos(x)';
inits2 = 'y(0)=0, Dy(0)=12';
y=dsolve(eqn2,inits2,'x')
y1 = simplify(y)
```

```
y1 = cos(x)/65 + (8*sin(x))/65 - exp(x*(14^(1/2) - 4))/130 - exp(-
x*(14^(1/2) + 4))/130 +
(192*14^(1/2)*exp(x*(14^(1/2) - 4)))/455 - (192*14^(1/2)*exp(-
x*(14^(1/2) + 4)))/455
```

Suppose we want to solve and plot solutions to the system of three ordinary differential equations

$$
\begin{aligned}
x'(t) &= x(t) + 2y(t) - z(t) \\
y'(t) &= x(t) + z(t) \\
z'(t) &= 4x(t) - 4y(t) + 5z(t).
\end{aligned}
$$

```
clear all
close all

[x,y,z]=dsolve('Dx=x+2*y-z','Dy=x+z','Dz=4*x-4*y+5*z')


x = - (C3*exp(t))/2 - (C1*exp(2*t))/2 - (C2*exp(3*t))/4
y = (C3*exp(t))/2 + (C1*exp(2*t))/4 + (C2*exp(3*t))/4
 z = C3*exp(t) + C1*exp(2*t) + C2*exp(3*
```

Suppose we want to solve and plot solutions to the system of three ordinary differential equations

$$
\begin{aligned}
x'(t) &= x(t) + 2y(t) - z(t)\\
y'(t) &= x(t) + z(t)\\
z'(t) &= 4x(t) - 4y(t) + 5z(t).
\end{aligned}
$$

```
clear all
close all
inits='x(0)=1,y(0)=2,z(0)=3';
[x,y,z]=dsolve('Dx=x+2*y-z','Dy=x+z','Dz=4*x-4*y+5*z',inits)

t=linspace(0,.5,25);
xx=eval(vectorize(x));
yy=eval(vectorize(y));
zz=eval(vectorize(z));
plot(t, xx, t, yy, t, zz)
```

Symbolic Differential Equation Terms

y	•y
$\dfrac{dy}{dt}$	•Dy
$\dfrac{d^2 y}{dt^2}$	•D2y
$\dfrac{d^n y}{dt^n}$	•Dny

$$b_2 \frac{d^2 y}{dt^2} + b_1 \frac{dy}{dt} + b_0 y = A \sin at$$

$$y(0) = C_1 \qquad \text{and } y'(0) = C_2$$

```
y = dsolve(‘b2*D2y+b1*D1y+b0*y=A*sin(a*t)’,
 ‘y(0)=C1’, ‘Dy(0)=C2’)

ezplot(y, [t1 t2])
```

$$\frac{dy}{dt} + 2y = 12 \qquad\qquad y(0) = 10$$

```
y = dsolve('Dy + 2*y = 12', 'y(0)=10')


>> y =6+4*exp(-2*t)
>> ezplot(y, [0 3])
>> axis([0 3 0 10])
```

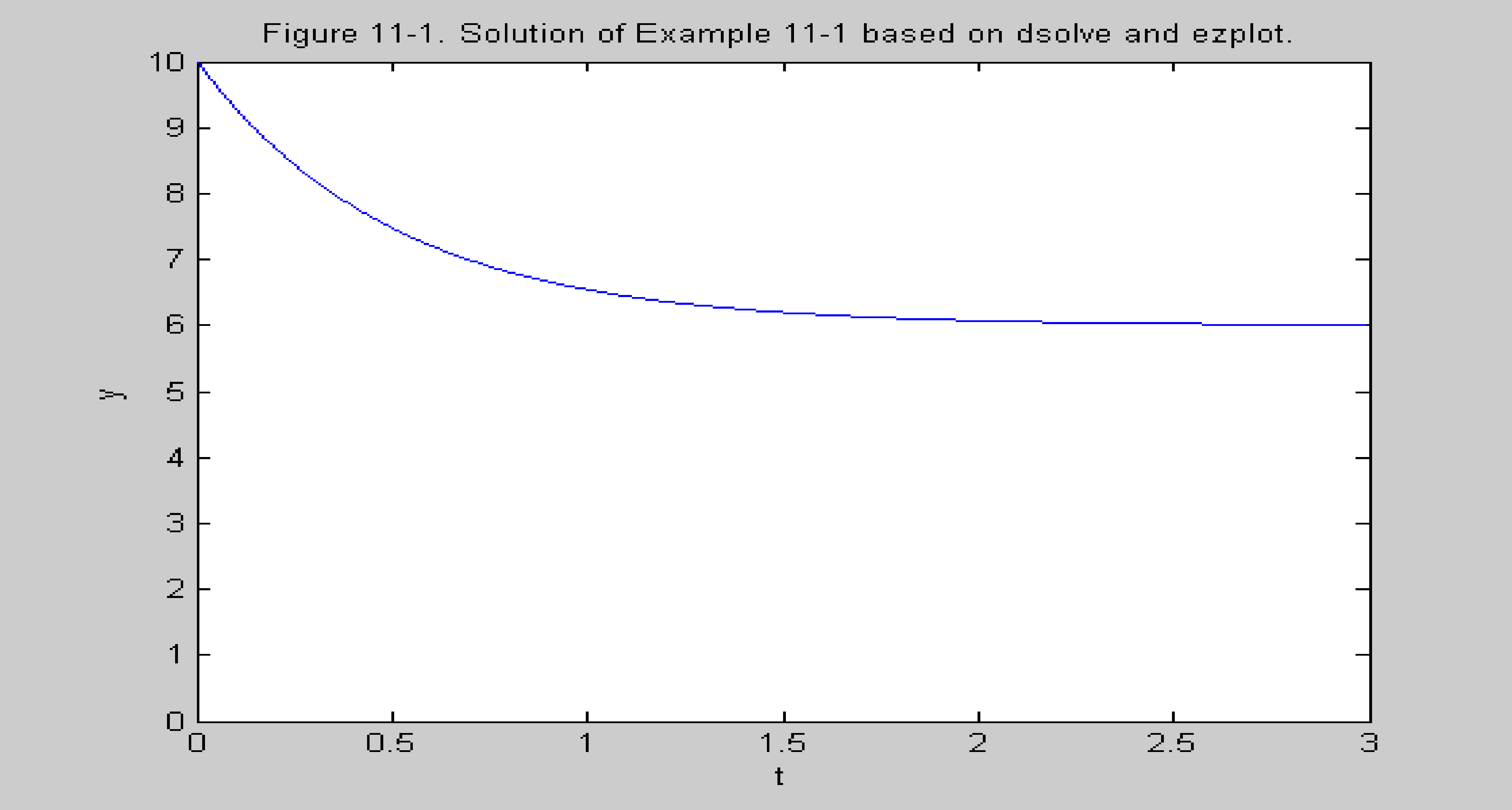


Figure 11-1. Solution of Example 11-1 based on dsolve and ezplot.
10
9
8
7
6
5
4
3
2
1
0
y
0
0.5
1
1.5
2
2.5
3
t

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=24$$

$$y(0)=10 \quad y'(0)=0$$

- >> y = dsolve('D2y + 3*Dy + 2*y = 24', 'y(0)=10', 'Dy(0)=0')

- y =
- 12+2*exp(-2*t)-4*exp(-t)

- >> ezplot(y, [0 6])

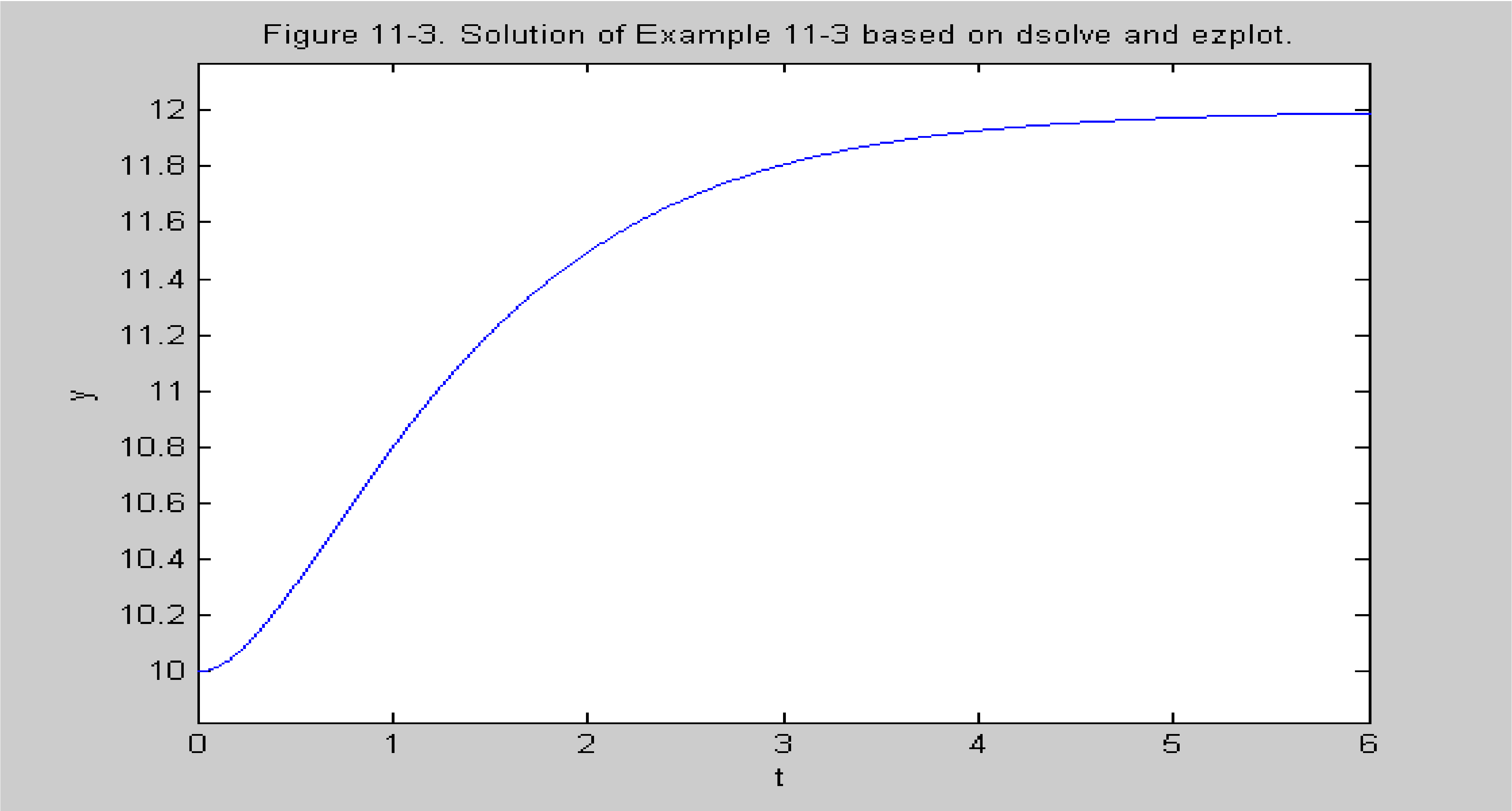
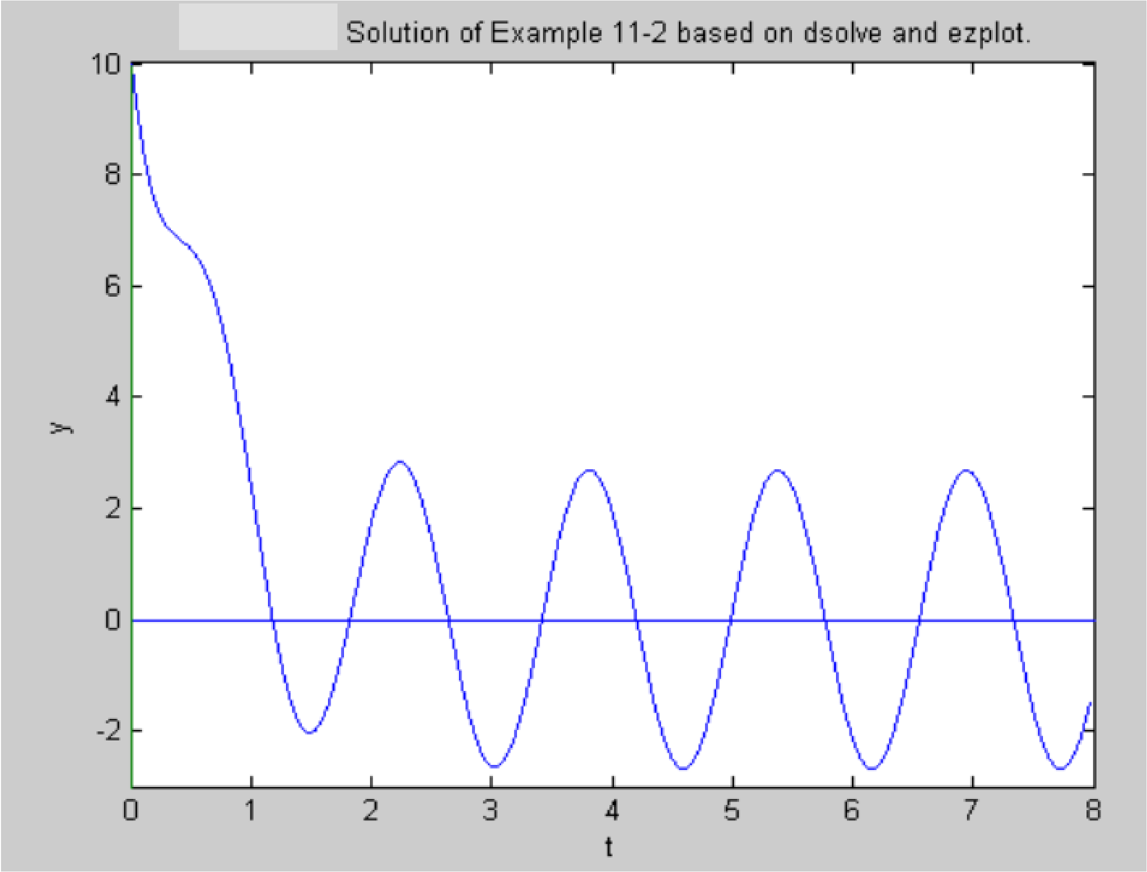


Figure 11-3. Solution of Example 11-3 based on dsolve and ezplot.
12
11.8
11.6
11.4
11.2
11
10.8
10.6
10.4
10.2
10
y
0
1
2
3
4
5
6
t

$$\frac{dy}{dt} + 2y = 12\sin 4t \qquad y(0) = 10$$

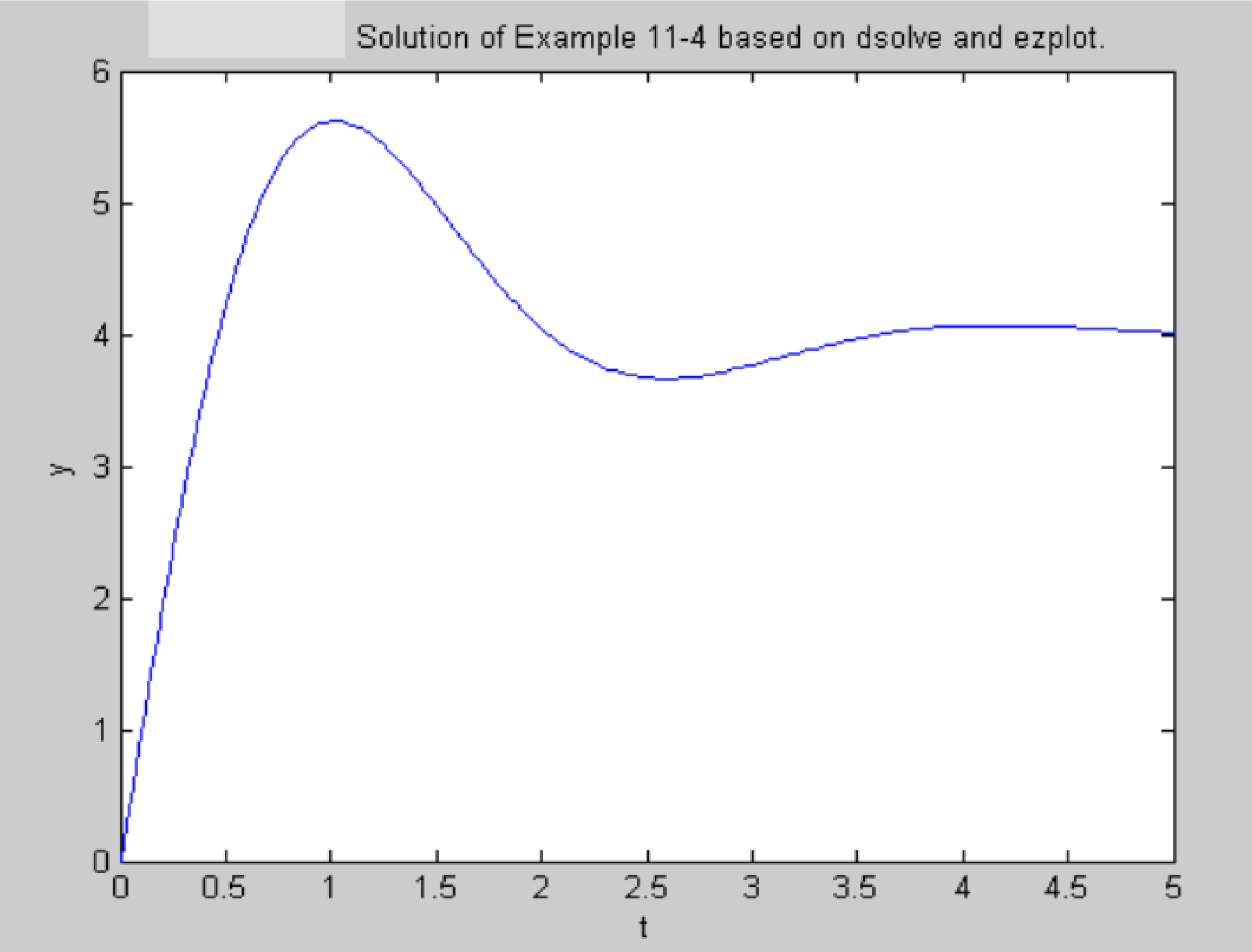
```
y = dsolve('Dy + 2*y = 12*sin(4*t)', 'y(0)=10')

ezplot(y, [0 8])
axis([0 8 -3 10])

y =  -12/5*cos(4*t)+6/5*sin(4*t)+62/5*exp(-2*t)
```



$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 20$$

$$y(0) = 0 \quad y'(0) = 10$$

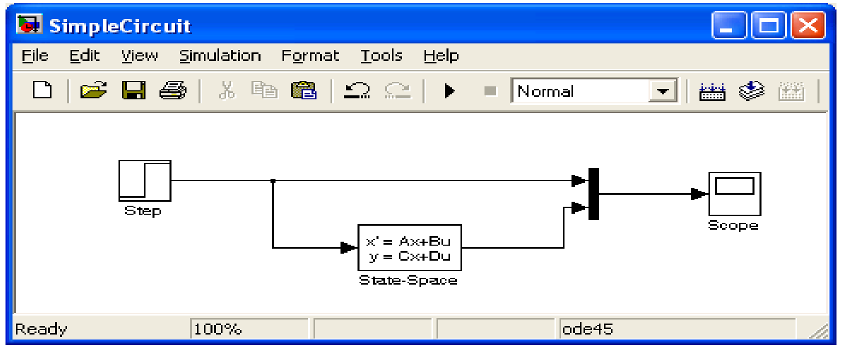
```
y = dsolve('D2y + 2*Dy + 5*y = 20',
    'y(0) = 0',  'Dy(0) = 10')
ezplot(y, [0 5])
y =   4+3*exp(-t)*sin(2*t)-4*exp(-t)*cos(2*t)
```

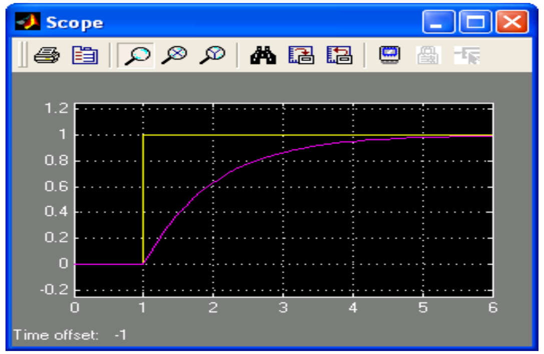


Simulink

Introduction to Simulink

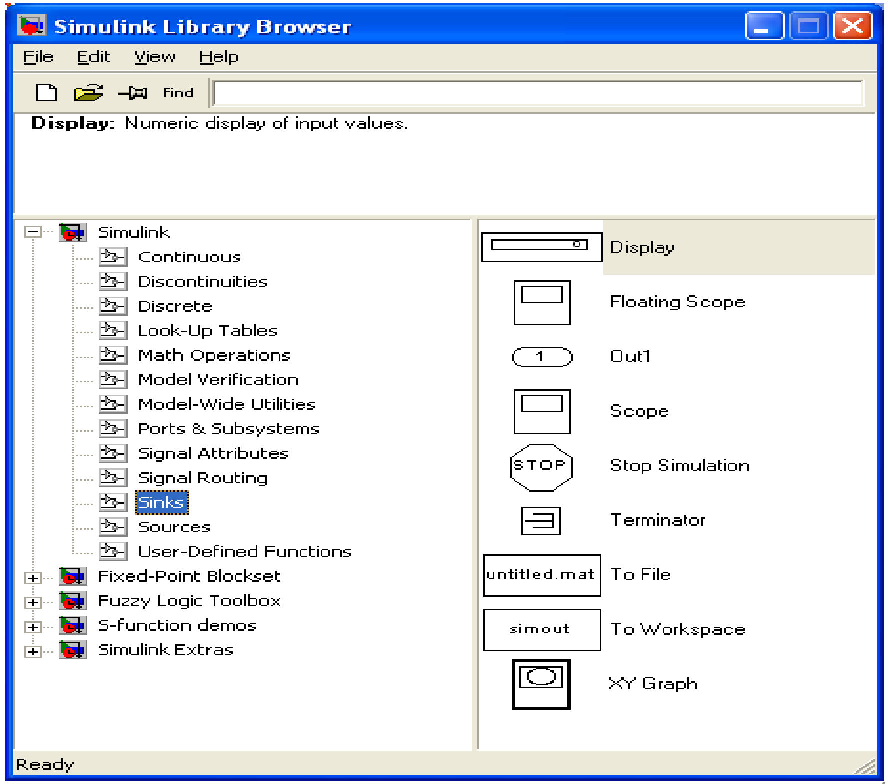
- Simulink is a graphical, “drag and drop” environment for building simple and complex signal and system dynamic simulations.
- It allows users to concentrate on the structure of the problem, rather than having to worry (too much) about a programming language.
- The parameters of each signal and system block is configured by the user (right click on block)
- Signals and systems are simulated over a particular time.

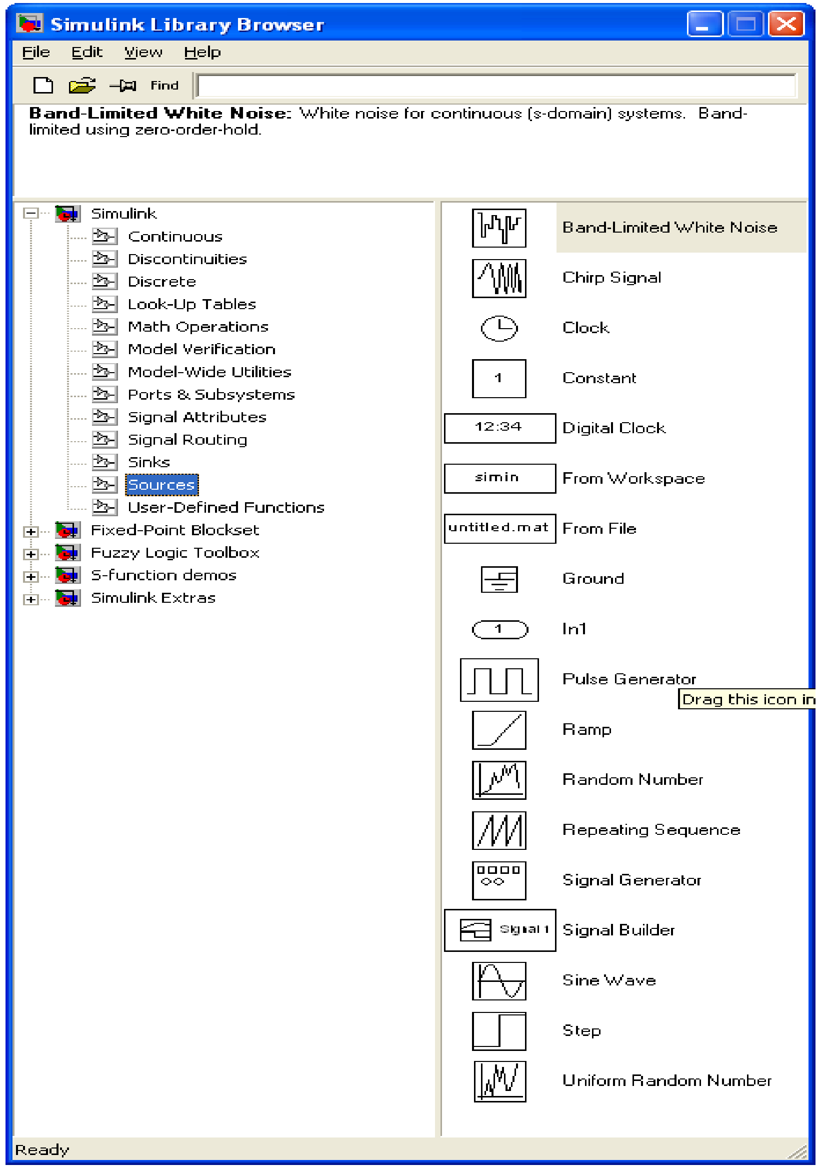




Signals in Simulink

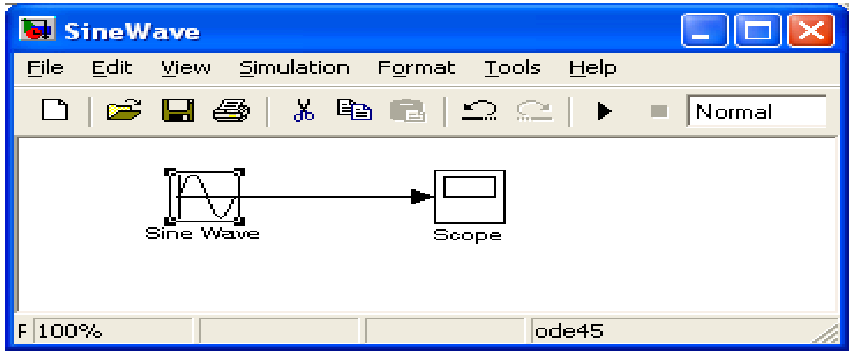
- Two main libraries for manipulating signals in Simulink:
- **Sources**: generate a signal
- **Sink**: display, read or store a signal

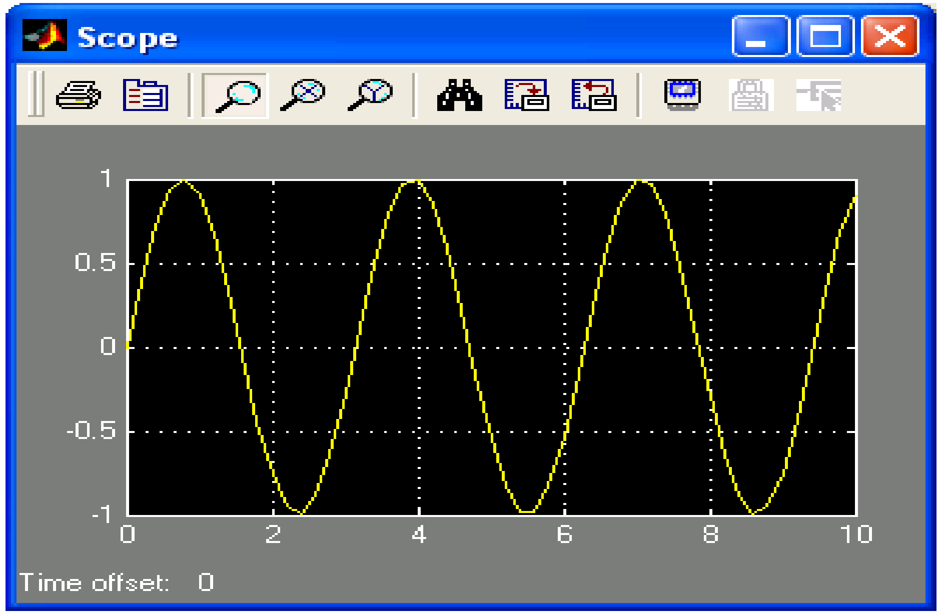




Example: Generate and View a Signal

- Copy “sine wave” source and “scope” sink onto a new Simulink work space and connect.
- Set sine wave parameters modify to 2 rad/sec
- Run the simulation:
- Simulation - Start
- Open the scope and leave open while you change parameters (sin or simulation parameters) and re-run





Summary

- This lecture has looked at signals:
- Power and energy
- Signal transformations
  - Time shift
  - Periodic
  - Even and odd signals
- Exponential and sinusoidal signals
- Unit impulse and step functions

- Matlab and Simulink are complementary environments for producing and analysing continuous and discrete signals.
- This will require some effort to learn the programming syntax and style!

Usage Notes

- These slides were gathered from the presentations published on the internet. I would like to thank who prepared slides and documents.
- Also, these slides are made publicly available on the web for anyone to use
- If you choose to use them, I ask that you alert me of any mistakes which were made and allow me the option of incorporating such changes (with an acknowledgment) in my set of slides.

Sincerely,

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