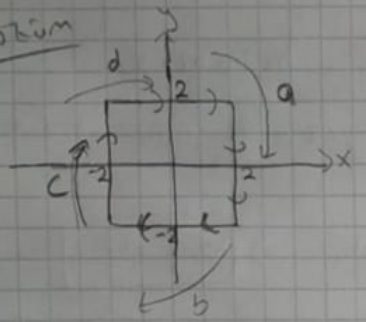
1-Soru $F = a_x xy - a_y 2x$ vektör alanının orjinde ortalanmış ve kenarları 4 birim olan kare yol ($-2 \le x \le 2$ ve $-2 \le y \le 2$) etrafındaki saat yönündeki dolaşımı bulunuz.

Çözüm




$F = a_x xy - a_y 2x$

$F \cdot dl$

dl (a) için: $x = 2$ $y = 2 \to -2$ $dl_y = a_y\,dy$

$-\int_2^{-2} 2x\,dy \Rightarrow -2x \int_2^{-2} dy = -2x\left(y \Big|_2^{-2}\right) \Rightarrow -2x(-2-(2))$

$= \underbrace{-2x}_{2} \cdot (-4) \Rightarrow -4 \cdot -4 = \boxed{16}$

dl (b) için: $x = 2 \to -2$ $y = -2$ $dl_x = a_x\,dx$

$\int_2^{-2} xy\,dx \Rightarrow y \int_2^{-2} x\,dx \Rightarrow y\left(\frac{x^2}{2}\Big|_2^{-2}\right) \Rightarrow y\,(2-(2))$

$\Rightarrow y \cdot (0) = \boxed{0}$

dl (c) için: $x = -2$ $y = -2 \to 2$ $dl_y = a_y\,dy$

$-\int_{-2}^{2} 2x\,dy \Rightarrow -2x \int_{-2}^{2} dy \Rightarrow -2x\left(y \Big|_{-2}^{2}\right) = -2x(2-(-2))$

$= \underbrace{-2x}_{-2}(4) = 4 \cdot 4 = \boxed{16}$

dl (d) için: $x = -2 \to 2$ $y = 2$ $dl_x = a_x\,dx$

$\int_{-2}^{2} xy\,dx = y \int_{-2}^{2} x\,dx = y\left(\frac{x^2}{2}\Big|_{-2}^{2}\right) = y\,(2-(2))$

$= y \cdot (0) = \boxed{0}$

cevap $= 16 + 0 + 16 + 0 = 32$

## 2. soru  $F = 2x^2y\,i + 3y^2z\,j - 4z^2x\,k$

**Çözüm**

a.) $\vec{\nabla}\cdot\vec{F} = ?$ $(x=1,\ y=2,\ z=1)$  $h_x h_y h_z = 1$

$$\Rightarrow \frac{1}{h_x h_y h_z}\left[\frac{\partial}{\partial x} h_y h_z\, 2x^2y + \frac{\partial}{\partial y} h_x h_z\, 3y2z + \frac{\partial}{\partial z} h_y h_x\, (-4z^2x)\right]$$

$\Rightarrow 4xy + 6yz - 8zx \Rightarrow$ değerler yerine yazılır $\Rightarrow \boxed{\vec{\nabla}\cdot\vec{F} = 12}$

b.) $\vec{\nabla}\cdot\vec{F} = ?$ $(r=2,\ \theta=\pi/3,\ \phi=\pi/4)$

$h_R = 1,\ h_\theta = R,\ h_\phi = R\sin\theta$

$x = R\sin\theta\cos\phi$

$y = R\sin\theta\sin\phi$

$z = R\cos\theta$

$F_1 = 2(R^2\sin^2\theta\cos\phi\, R\sin\theta\sin\phi)\,\partial_R$

$F_2 = (3R^2\sin^2\theta\sin^2\phi\, R\cos\theta)\,\partial_\theta$

$F_3 = (-4R^4\cos^2\theta\sin\theta\cos\phi)\,\partial_\phi$

$$F = 2(R^2\sin^2\theta\cos\phi\, R\sin\theta\sin\phi)\,\partial_R + (3R^2\sin^2\theta\sin^2\theta\, R\cos\theta)\,\partial_\theta + (-4R^4\cos^2\theta\sin\theta\cos\phi)\,\partial_\phi$$

1-) $(2R^3\sin^3\theta\cos\phi\sin\phi)(R^2\sin\theta) = F_1\,(R^2\sin\theta)$

$(2R^5\sin^4\theta\cos\phi\sin\phi)\,\frac{\partial}{\partial r} \Rightarrow \{10R^4\sin^4\theta\cos\phi\sin\phi$

2-) $F_2\,(R\sin\theta) \Rightarrow (3R^3\sin^2\theta\sin^2\phi\cos\theta)(R\sin\theta)$

$\Rightarrow (3R^4\sin^3\theta\sin^2\phi\cos\theta)\,\frac{\partial}{\partial\theta} \Rightarrow \{9R^4\sin^2\theta\sin^2\phi\cos\theta - 3R^4\sin^4\theta\sin^2\phi$

3-) $F_3\,(R) \Rightarrow (-4R^4\cos^2\theta\sin\theta\cos\phi)(R) = (-4R^5\cos^2\theta\sin\theta\cos\phi)\,\frac{\partial}{\partial\phi}$

$\Rightarrow \boxed{4R^5\cos^2\theta\sin\theta\sin\phi}$

$$\nabla F = \frac{1}{R^2\sin\theta}\Big[(10R^4\sin^4\phi\cos\phi\sin\phi) + (9R^4\sin^2\theta\sin^2\phi\cos\theta - 3R^4\sin^4\theta\sin^2\phi) + (4R^5\cos^2\theta\sin\theta\sin\phi)\Big]$$

$r=2,\ \theta=\pi/3,\ \phi=\pi/4$

değerleri denklemde yerine yazılırsa sonuç $\boxed{\vec{\nabla}\vec{F} = 22,54}$

__3 soru__ Dielektrik $\varepsilon_{r1} = 3$ ve $\varepsilon_{r2} = 2$ olan iki homojen yön bağımsız dielektrik ortamın xy düzlemi ile ayrıldığını kabul edelim. Bir ortak $E_1 = a_x - a_y 5 - a_z 4$'dir. $E_2$, $D_2$, $\alpha_1$, $\alpha_2$'yi bulunuz

__Çözüm__

$$\vec{E}_1 = \underbrace{a_x - a_y 5}_{\vec{E}_{1t}} \underbrace{- a_z 4}_{\vec{E}_{1n}} \Rightarrow \vec{E}_{1t} = a_x - a_y 5$$

$$\vec{E}_{1n} = -a_z 4$$

$$\vec{E}_{1t} = \vec{E}_{2t} \Rightarrow \vec{E}_{2t} = \vec{E}_{1t} = a_x - a_y 5$$

$$D_{1n} - D_{2n} = \rho_s \Rightarrow D_{1n} - D_{2n} = 0 \Rightarrow D_{1n} = D_{2n}$$

$$\vec{D}_{2n} = \vec{D}_{1n} = \varepsilon_1 \vec{E}_{1n} = \varepsilon_{r_1} \varepsilon_0 \vec{E}_{1n} = 3\varepsilon_0(-a_z 4) = -a_z 12 \varepsilon_0$$

$$\vec{E}_{2n} = \frac{\vec{D}_{2n}}{\varepsilon_2} = \frac{\vec{D}_{2n}}{\varepsilon_{r_2}\varepsilon_0} = \frac{\vec{D}_{2n}}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(-a_z 12 \varepsilon_0) = -a_z \frac{12\varepsilon_0}{2\varepsilon_0} = -a_z 6$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

$$\boxed{\vec{E}_2 = a_x - a_y 5 - a_z 6}$$

$$\vec{D}_2 = \varepsilon_2 \vec{E}_2 = \varepsilon_{r_2} \varepsilon_0 \vec{E}_2 = 2\varepsilon_0 \vec{E}_2$$

$$\boxed{\vec{D}_2 = \varepsilon_0 (2a_x - a_y 10 - a_z 12)}$$

$$\tan\alpha_2 = E_2$$

$$\tan\alpha_2 = a_x - a_y 5 - a_z 6$$

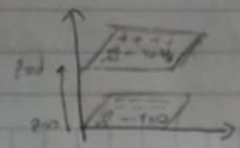
$$\boxed{\alpha_2 = \tan^{-1}(a_x - a_y 5 - a_z 6)}$$

$$\tan\alpha_1 = E_1$$

$$\tan\alpha_1 = a_x - a_y 5 - a_z 4$$

$$\boxed{\alpha_1 = \tan^{-1}(a_x - a_y 5 - a_z 4)}$$

4. soru



Çözüm

b.) elektrik alan

$$E = \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_z) \Rightarrow E = E_+ + E_- \Rightarrow \frac{2\rho_s}{2\varepsilon_0} \Rightarrow \boxed{E = \frac{\rho_s}{\varepsilon_0}(-\hat{a}_z)}$$

a.) Potansiyel dağılım

$$V = -\int_0^d -\hat{a}_z \frac{\rho_s}{\varepsilon_0}\hat{a}_z\, dz \qquad dl = dz \cdot \hat{a}_z$$

$$V = \int_{z=0}^{d} \frac{\rho_s}{\varepsilon_0} dz \Rightarrow \frac{\rho_s}{\varepsilon_0} z \Big|_0^d = \boxed{\frac{\rho_s \cdot d}{\varepsilon_0}}$$

c.) yük dağılım

+ plaka için

$$\boxed{Q_+ = \frac{\rho_s \cdot A}{2}}$$

− plaka için

$$\boxed{Q_- = \frac{\rho_s \cdot A}{2}}$$

d.) Kapasitörün kapasitansı

$$C = \frac{Q}{V} \Rightarrow \frac{\cancel{\rho_s} \cdot A}{\frac{\cancel{\rho_s} \cdot d}{\varepsilon_0}} \Rightarrow \boxed{C = \varepsilon_0 \cdot \frac{A}{d}}$$

e.) $\frac{-\rho v z}{d}$ için işlemleri tekrarla

$$E = \frac{\frac{-\rho v z}{d}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{-\rho v z}{d \cdot \varepsilon_0}(-\hat{a}_z)}$$

$$V = \int_{z=0}^{d} \frac{\frac{-\rho . v . z}{d}}{\varepsilon_0} \cdot dz \Rightarrow \frac{\frac{-\rho . v . z^2}{2d}}{\varepsilon_0}$$

$$= \frac{\frac{-\rho . v . d^2}{2d}}{\varepsilon_0} \Rightarrow \frac{\frac{-\rho v d}{2}}{\varepsilon_0}$$

$$\boxed{V = \frac{-\rho . v . d}{2\varepsilon_0}}$$

+ plakalar için

$$Q_+ = \frac{\frac{-\rho . v . z}{d}}{2} = \boxed{\frac{-\rho v . z}{2d}}$$

− plakalar için

$$\boxed{\frac{-\rho . v . z}{2d}}$$

$$C = \frac{Q}{V} = \frac{\frac{-\rho v z}{d} A}{\frac{-\rho v d}{2\varepsilon_0}} \Rightarrow \frac{-\rho . v . z}{d} A \frac{2\varepsilon_0}{-\rho v d}$$

$$\boxed{C = \varepsilon_0 \frac{2z}{d^2} A}$$

$$\Rightarrow C = \frac{z}{d} A \frac{2\varepsilon_0}{d} = \varepsilon_0 \frac{2z}{d^2} A$$

ABCDA

5. soru

$$F = 5r\sin\phi\, a_r + r^2\cos\phi\, a_\phi$$

$\frac{d}{2}$ $\boxed{2d}$ $\boxed{2d}$ $\boxed{V = \frac{-\rho V d}{2\varepsilon_0}}$

$$C = \frac{Q}{V} = \frac{\frac{-\rho v z}{d} A}{\frac{-\rho v d}{2\varepsilon_0}} \Rightarrow \frac{-\rho v z}{d} A \, \frac{2\varepsilon_0}{-\rho v d}$$

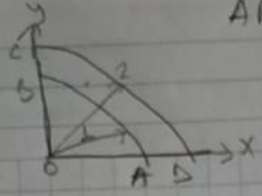
$$\boxed{C = \varepsilon_0 \frac{2z}{d^2} \cdot A}$$

$$\Rightarrow C = \frac{z}{d} \cdot A \, \frac{2\varepsilon_0}{d} = \varepsilon_0 \frac{2z}{d^2} A$$



ABCDA

5. soru




$$F = 5r \sin\phi \, a_r + r^2 \cos\phi \, a_\phi$$

$$\vec{\nabla} \times \vec{F} = \frac{1}{r} \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 5r\sin\phi & r^3\cos\phi & 0 \end{vmatrix}$$

$$= \frac{1}{r} \left( \hat{a}_r (0) + \hat{a}_\phi (0) + (3r^2 \cos\phi - 5r\cos\phi) \hat{a}_z \right)$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{F} = \hat{a}_z (3r\cos\phi - 5\cos\phi)}$$

$d\vec{s} = -\hat{a}_z \, r \, dr \, d\phi$ ⇒ h parmak kuralı ile bu şekilde bulunur

$$\iint \vec{\nabla} \times \vec{F} \cdot d\vec{s} = \iint -\hat{a}_z \, r \, dr \, d\phi \cdot (3r\cos\phi - 5\cos\phi) \hat{a}_z$$

$$= -\int_1^2 \int_0^{\pi/2} \cos\phi \, (3r^2 - 5r) \, dr \, d\phi \Rightarrow -\int_1^2 3r^2 - 5r \, dr \cdot \int_0^{\pi/2} \cos\phi \, d\phi$$

$$-\left( \frac{3r^3}{3} - \frac{5r^2}{2} \right) \Big|_1^2 \qquad \underbrace{\sin \pi/2}_{1} - \underbrace{\sin 0}_{0} = 1$$

$$\boxed{\iint \vec{\nabla} \times \vec{F} \cdot d\vec{s} = \frac{1}{2}}$$

$$\Rightarrow -\left[ \left( \frac{8-20}{2} \right) - \left( 1 - \frac{5}{2} \right) \right]$$

$$\frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2}$$

$$F = 5r \sin\phi \, \hat{a}_r + r^2 \cos\phi \, a_\phi$$

$$dl_\phi = \hat{a}_\phi \, r \, d\phi$$

$$dl_\phi \cdot F = r^3 \cos\phi \, d\phi$$

$$\int_{0\,AB}^{\pi/2} r^3 \cos\phi \, d\phi = r^3 \sin\phi \Big|_0^{\pi/2} = r^3 (\sin \pi/2 - \sin 0) = \underbrace{r^3}_{1} \cdot 1 = 1$$

$$\int_{\pi/2\,CD}^{0} r^3 \cos\phi = r^3 \sin\phi \Big|_{\pi/2}^{0} = r^3 (\sin 0 - \sin \pi/2) = \underbrace{r^3}_{8} \cdot -1 = -8$$

$$\int_{0\,AB}^{\pi/2} r^3 \cos\phi \, d\phi = r^3 \sin\phi \Big|_0^{\pi/2} = r^3 (\sin \pi/2 - \sin 0) = \underbrace{r^3}_{1} \cdot 1 = 1$$

$$\int_{\pi/2\,CD}^{0} r^3 \cos\phi = r^3 \sin\phi \Big|_{\pi/2}^{0} = r^3 (\sin 0 - \sin \pi/2) = \underbrace{r^3}_{8} \cdot -1 = -8$$

5. sorunun devamı

$dl_r = \hat{a}_r dr \qquad dl_r \cdot F = 5r \sin\phi \, dr$

$$\int_{1\,BC}^{2} 5r \sin\phi \, dr = \sin\phi \frac{5r^2}{2} \Big|_1^2 = \sin\phi \left(\frac{20}{2} - \frac{5}{2}\right) = \sin\phi \left(\frac{15}{2}\right)$$

$$= \underbrace{\sin \pi/2}_{1} \left(\frac{15}{2}\right) = \frac{15}{2}$$

$$\int_{2\,DA}^{1} 5r \sin\phi \, dr \Rightarrow \sin\phi \left(\frac{5r^2}{2}\right) \Big|_2^1 \Rightarrow \sin\phi \left(\frac{5}{2} - \frac{20}{2}\right) = \sin\phi \left(-\frac{15}{2}\right)$$

$$= \sin(0) \cdot \left(-\frac{15}{2}\right) \Rightarrow 0 \cdot \left(-\frac{15}{2}\right) = 0$$

$$1 + (-8) + \left(\frac{15}{2}\right) + (0) = \frac{1}{2}$$

$$\boxed{\int \vec{F} \, dl = \frac{1}{2}}$$

$$\underbrace{\underbrace{\int \vec{F} \, dl}_{1/2} = \underbrace{\iint \vec{\nabla} \times \vec{F} \, d\vec{s}}_{1/2}}$$

eşit oldukları için stokes teoremini doğrular.