FOURIER DÖNÜŞÜMÜ

Serkan Ataede 1,65 Özlem

- $F(w) = \int_{-\infty}^{\infty} f(t) \cdot e^{-jwt} \cdot dt$
- $F(f) = F(f(t)) = \int_{-\infty}^{\infty} f(t) \cdot e^{-jwt} \cdot dt$
- $f(t) = F^{-1}(F(w)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) \cdot e^{jwt} \cdot dw$
- $f(f) = F^{-1}(F(f)) = \int_{-\infty}^{\infty} F(f) \cdot e^{j2\pi f t} \cdot df$

* Bir işaretin fourier dönüşümünün alınabilmesi için;

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty$$ şartı sağlanmalıdır.

* Fourier dönüşümü ile fourier serisi katsayıları arasındaki ilişki;

$$F_k = \frac{1}{T_0} F(w) \Big|_{w = k w_0} \Rightarrow F_k = \frac{1}{T_0} F(k w_0)$$

* Periyodik olmayan işaretlerin genlik ve faz tayfı süreklidir.

* Periyodik işaretlerin fourier dönüşümü hesaplanamaz; fakat dolaylı olarak hesaplanabilir. Bu işaretlerin genlik ve faz tayfı çizgiden oluşur.

* $x(t) = e^{\pm j w_0 t}$

$X(w) = 2\pi \delta(w \mp w_0)$

$X(f) = \delta(f \mp f_0)$

* $x(t) = e^{\pm j k w_0 t}$

$X(w) = 2\pi \delta(w \mp k w_0)$

$X(f) = \delta(f \mp k f_0)$

* $x(t) = e^{\pm j(k w_0 t \pm \phi)}$

$X(w) = e^{\pm j\phi} 2\pi \delta(w \mp k w_0)$

$X(f) = e^{\pm j\phi} \delta(f \mp k f_0)$

* Örneğin; $x(t) = e^{j2\pi t}$

$\Rightarrow X(w) = 2\pi \delta(w - 2\pi)$

$X(f) = \delta(f - 1)$

* Örneğin; $x(t) = \cos w_0 t = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2}$

$\Rightarrow X(w) = \frac{1}{2} (2\pi \delta(w - w_0) + 2\pi \delta(w + w_0))$

$\Rightarrow X(w) = \pi (\delta(w - w_0) + \delta(w + w_0))$

$\Rightarrow X(f) = \frac{1}{2} (\delta(f - f_0) + \delta(f + f_0))$

* Örneğin; $x(t) = e^{-j3\pi t/2}$

$\Rightarrow X(w) = 2\pi \delta(w + 3\pi/2)$

$X(f) = \delta(f + 3/4)$

* Örneğin: $x(t) = \sin w_0 t = (e^{jw_0t} - e^{-jw_0t}) / 2j$

$\Rightarrow X(w) = \frac{1}{2j} (2\pi\delta(w-w_0) - 2\pi\delta(w+w_0))$

$\Rightarrow X(w) = \frac{\pi}{j} (\delta(w-w_0) - \delta(w+w_0))$

$\Rightarrow X(f) = \frac{1}{2j} (\delta(f-f_0) - \delta(f+f_0))$

* Örneğin: $x(t) = e^{-j(25\pi/2\, t + \pi/4)}$

$\Rightarrow X(w) = e^{-j\pi/4} \cdot 2\pi\,\delta(w + 25\pi/2)$

$\Rightarrow X(f) = e^{-j\pi/4}\,\delta(f + 25/4)$

* Örneğin: $x(t) = \cos(\frac{2\pi}{3}t - \pi/3) = \frac{e^{j(2\pi t/3 - \pi/3)} + e^{-j(\frac{2\pi t}{3} - \pi/3)}}{2}$

$\Rightarrow X(w) = \frac{1}{2} (e^{-j\pi/3} \cdot 2\pi\delta(w - 2\pi/3) + e^{+j\pi/3} 2\pi\delta(w + 2\pi/3))$

$\Rightarrow X(f) = \frac{1}{2} e^{-j\pi/3} \delta(f - 1/3) + \frac{1}{2} e^{j\pi/3} \delta(f + 1/3)$

* Fourier Dönüşümünün Özellikleri

1) Doğrusallık özelliği

$$\left.\begin{matrix} f(t) \longleftrightarrow F(w) \\ g(t) \longleftrightarrow G(w) \end{matrix}\right\} af(t) + bg(t) \leftrightarrow aF(w) + bG(w)$$

2) Dikgenlik Özelliği

$$\int_{-\infty}^{\infty} f(t) \cdot g^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) \cdot G^*(w) \cdot dw = \int_{-\infty}^{\infty} F(f) \cdot G^*(f) \cdot df$$

$$f(t) = g(t) \Rightarrow \underbrace{\int_{-\infty}^{\infty} |f(t)|^2 dt}_{\text{zaman domeninde hesaplanan enerji}} = \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} |F(w)|^2 dw = \int_{-\infty}^{\infty} |F(f)|^2 df}_{\text{Frekans domeninde hesaplanan enerji.}}$$

PARSEVAL TEOREMİ!

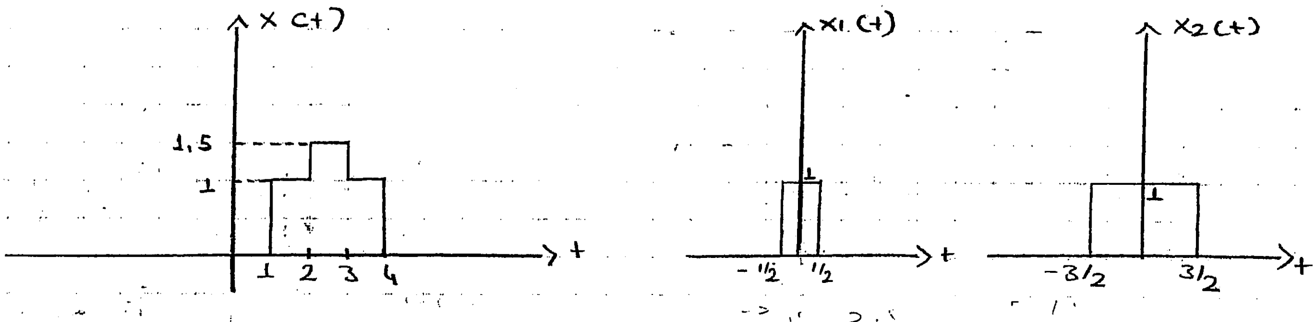
3) Kayma özellikleri

3-a) Zamanda kayma özelliği

$f(t) \leftrightarrow F(w)$

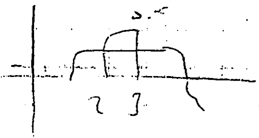
$f(t-t_0) \leftrightarrow e^{-jwt_0} F(w)$ $\quad$ t_0: Kayma miktarı !

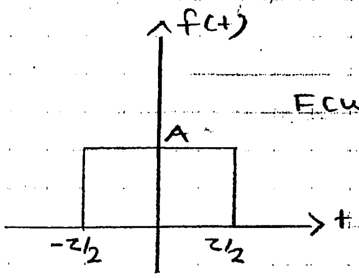
örnek:



x(t)'yi fourier dönüşümünü kolayca hesaplayabileceğimiz $x_1(t)$ ve $x_2(t)$ cinsinden ifade edebiliriz.

$$x(t) = \frac{1}{2} x_1(t-2{,}5) + x_2(t-2{,}5)$$





$F(w) = A\tau \operatorname{sinc}(f\tau) \Rightarrow$

$*$ $x_1(t)$ için $\left.\begin{matrix} A=1 \\ \tau=1 \end{matrix}\right\}$ $x_1(w) = \operatorname{sinc}(f)$

$*$ $x_2(t)$ için $\left.\begin{matrix} A=1 \\ \tau=3 \end{matrix}\right\}$ $x_2(w) = 3\sin(3f)$

$x_1(f) = \operatorname{sinc}(f) \Rightarrow x_1(w) = \sin(w/2)/w/2$

$x_2(f) = 3\operatorname{sinc}(3f) \Rightarrow x_2(w) = 3\sin(3w/2)/3w/2$

$$\Rightarrow x(w) = \frac{1}{2} e^{-j2{,}5w} \cdot \sin(w/2)/w/2 + e^{-j2{,}5w} \cdot 3 \cdot \sin(3w/2)/\tfrac{3w}{2}$$

$$\Rightarrow x(w) = e^{-j2{,}5w}\left(\frac{\sin(w/2)}{w} + \frac{2\sin(3w/2)}{w}\right)$$

$*$ Şimdide formülden yapalım;

$$x(w) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} dt \Rightarrow x(w) = \int_1^2 e^{-jwt} dt + \frac{3}{2}\int_2^3 e^{-jwt} dt$$

$$+ \int_3^4 e^{-jwt} dt$$

$$\Rightarrow X(\omega) = \frac{-1}{j\omega} e^{-j\omega t}\Big|_1^2 - \frac{3}{2j\omega} e^{-j\omega t}\Big|_2^3 - \frac{1}{j\omega} e^{-j\omega t}\Big|_3^4$$

$$\Rightarrow X(\omega) = \frac{-1}{j\omega}\left(e^{-j2\omega} - e^{-j\omega}\right) - \frac{3}{2j\omega}\left(e^{-j3\omega} - e^{-j2\omega}\right) - \frac{1}{j\omega}\left(e^{-j4\omega} - e^{-j3\omega}\right)$$

$$\Rightarrow X(\omega) = \frac{-1}{j\omega} e^{-j2\omega} + \frac{1}{j\omega} e^{-j\omega} - \frac{3}{2j\omega} e^{-j3\omega} + \frac{3}{2j\omega} e^{-j2\omega} - \frac{1}{j\omega} e^{-j4\omega} + \frac{1}{j\omega} e^{-j3\omega}$$

$$\Rightarrow X(\omega) = \frac{1}{j2\omega} e^{-j2\omega} + \frac{1}{j\omega} e^{-j\omega} - \frac{1}{2j\omega} e^{-j3\omega} - \frac{1}{j\omega} e^{-j4\omega}$$

$$X(\omega) = \frac{e^{-j2,5\omega}}{j2\omega}\left(e^{j\omega/2} - e^{-j\omega/2}\right) + \frac{1}{j\omega} e^{-j2,5\omega}\left(e^{j3\omega/2} - e^{-j3\omega/2}\right)$$

$$\Rightarrow X(\omega) = e^{-j2,5\omega} \times \sin(\omega/2)/\omega + 2e^{-j2,5\omega} \times \sin(3\omega/2)/\omega$$

$$\Rightarrow X(\omega) = e^{-j2,5\omega}\underbrace{\left(\sin(\omega/2)/\omega + 2\sin(3\omega/2)/\omega\right)}_{\text{Aynı sonucu verdi!}}$$

3-b) Zamanda katlama özelliği

Sözlü ifadesi: Bir işaret herhangi iki işaretin katlanmasından meydana geliyorsa bu işaretin fourier dönüşümü, diğer iki işaretin fourier dönüşümünün çarpımıdır.

$$x(t) = f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) . g(t-\tau) d\tau$$

$$\Rightarrow X(\omega) = F(\omega) . G(\omega)$$

Example: $h(t) = \delta(t - t_0)$, $x(t)$ sürekli bir sinyal ve fourier dönüşümü $F\left\{\int_{-\infty}^{\infty} x(t) dt\right\} = X(\omega)$ dır.

$$y(t) = x(t) * h(t) = \underset{\smile}{x(t-t_0)}$$

çarpımı $x(t_0)$, katlaması $x(t-t_0)$

$\Rightarrow Y(w) = X(w) \cdot H(w) = X(w) \cdot \underbrace{e^{-jwt_0}}_{\delta(t-t_0)\text{'ın fourier dönüşümü!}}$

$* \; |e^{-jwt_0}| = 1$, $-wt_0$: Faz özelliği!

3-c) Modulasyon özelliği

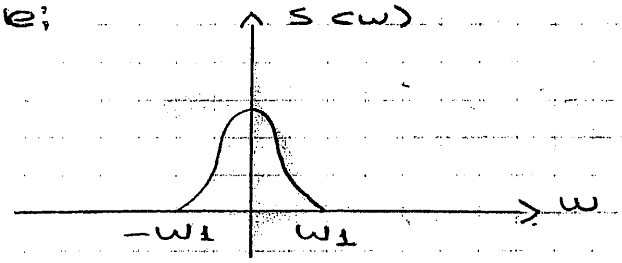
$$e^{jw_0t} \cdot f(t) \Leftrightarrow F(w - w_0)$$

3-d) Zamanda Çarpma özelliği

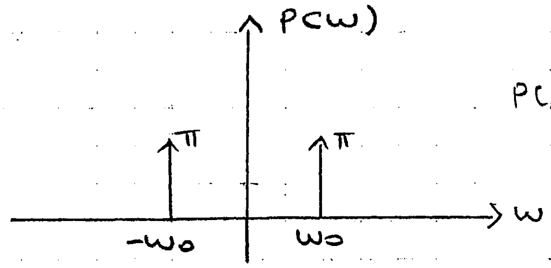
Sözlü ifadesi: Bir işaret herhangi iki işaretin çarpımı ise bu işaretin fourier dönüşümü diğer iki işaretin fourier dönüşümünün katlamasının 2π'ye bölümüdür.

$$x(t) = f(t) \cdot g(t) \Rightarrow X(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w-u) \cdot G(u) \, du = \frac{1}{2\pi} F(w) * G(w)$$

Example;



Şekil: s(t) işaretinin fourier dönüşümü



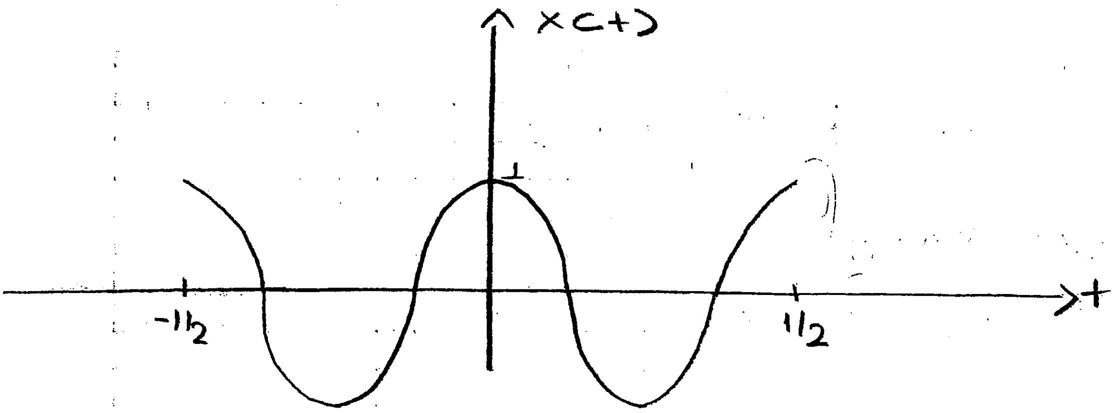
$$P(w) = \pi \left(\delta(w - w_0) + \delta(w + w_0) \right)$$

Şekil: $p(t) = \cos w_0 t$'nin fourier dönüşümü

$$r(t) = p(t) \cdot s(t) \Rightarrow R(w) = \frac{1}{2\pi} P(w) * S(w)$$

$$\Rightarrow R(w) = \frac{1}{2} \left(S(w - w_0) + S(w + w_0) \right)$$

ÖRNEK;



* Zamanda çarpma özelliğinden yararlanarak X(w)'yı bulalım.

$$x(t) = \underbrace{\cos 4\pi t}_{f(t)} \ \underbrace{(u(t+1/2) - u(t-1/2))}_{g(t)}$$

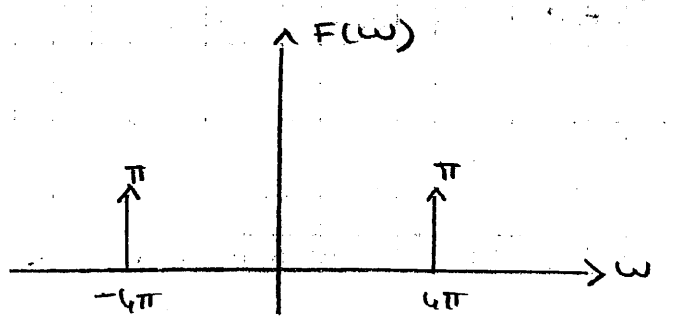
$$f(t) = \cos 4\pi t \Rightarrow F(w) = (e^{j4\pi t} + e^{-j4\pi t})/2$$

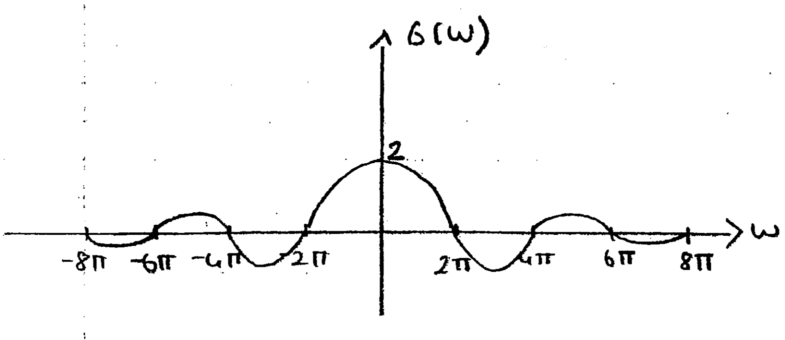
$$\Rightarrow F(w) = \frac{1}{2}\left(2\pi\,\delta(w-4\pi) + 2\pi\,\delta(w+4\pi)\right)$$

$$\Rightarrow F(w) = \pi\left(\delta(w-4\pi) + \delta(w+4\pi)\right)$$

$$g(t) = u(t+1/2) - u(t-1/2) \Rightarrow$$

$$G(w) = 2\sin(w/2)/w = \text{sincf}$$





$$X(\omega) = \frac{1}{2\pi} F(\omega) \times G(\omega)$$

$$\Rightarrow X(\omega) = \frac{1}{2\pi}\left(2\pi \sin\left(\frac{\omega-4\pi}{2}\right)/_{\omega-4\pi} + 2\pi \sin\left(\frac{\omega+4\pi}{2}\right)/_{\omega+4\pi}\right)$$

$$\Rightarrow X(\omega) = \sin\left(\frac{\omega-4\pi}{2}\right)/\omega-4\pi + \sin\left(\frac{\omega+4\pi}{2}\right)/\omega+4\pi$$

4) İkililik Özelliği

$f(t) \leftrightarrow F(\omega)$ $\qquad$ $f(t) \leftrightarrow f(f)$

$F(t) \leftrightarrow 2\pi f(-\omega)$ $\qquad$ $F(t) \leftrightarrow f(-f)$

5) Ölçeklendirme özelliği

$$f(\alpha t) \leftrightarrow \frac{1}{|\alpha|} F(\omega/\alpha)$$

* $f(t) \leftrightarrow F(\omega) \Rightarrow f(-t) \leftrightarrow F(-\omega)$

6) Türev özelliği

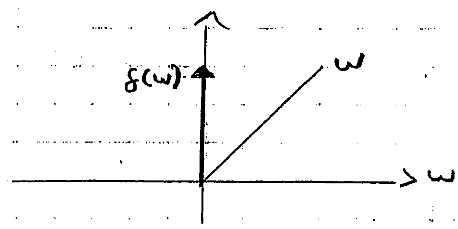
$$\frac{df(t)}{dt} = (j\omega)\cdot F(\omega) \quad , \quad \frac{d^n f(t)}{dt^n} = (j\omega)^n \cdot F(\omega)$$

Example; $x(t) = u(t) \Rightarrow \delta(\omega)$'yı türev özelliğinden yararlanarak bulunuz.

$$\delta(t) = \frac{dx(t)}{dt} \Rightarrow \delta(\omega) = F\left\{\int_{-\infty}^{\infty} \delta(t)\,dt\right\} = (j\omega)\cdot F(\omega)$$

$$\Rightarrow \delta(\omega) = j\omega\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) = 1$$

* $\omega\,\delta(\omega) = 0$



7) integral özelliği

$$F\left\{\int_{-\infty}^{t} f(z)\,dz\right\} \overset{F}{\longleftrightarrow} \frac{1}{j\omega} + \pi F(0)\,\delta(\omega)$$

$$\Rightarrow F(0) = \int_{-\infty}^{\infty} f(t)\,dt$$

Example; $x(t) = u(t) \Rightarrow$ $X(\omega)$'yı fourier dönüşümünün integral özelliğinden yararlanarak bulun.

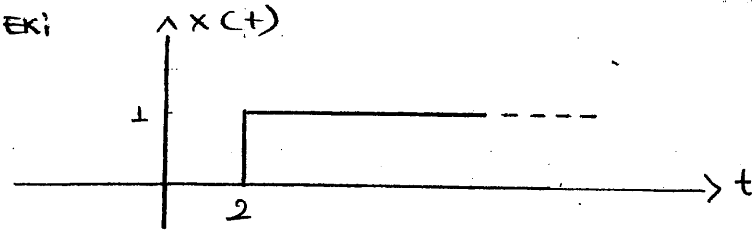
$$x(t) = \int_{-\infty}^{t} \delta(z)\, dz \qquad \delta(t) = f(t)$$

$$\Rightarrow X(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0)\,\delta(\omega)$$

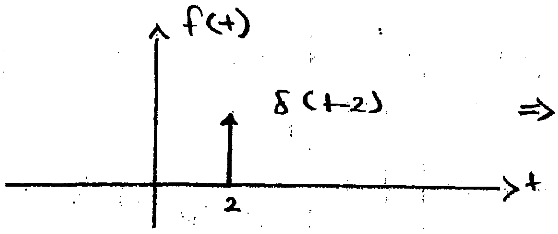
$$f(t) = \delta(t) \Rightarrow F(\omega) = 1 \;;\; F(0) = \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\Rightarrow X(\omega) = \frac{1}{j\omega} + \pi\,\delta(\omega)$$

ÖRNEK;



$X(\omega)$'yı integral özelliğinden yararlanarak bulalım;

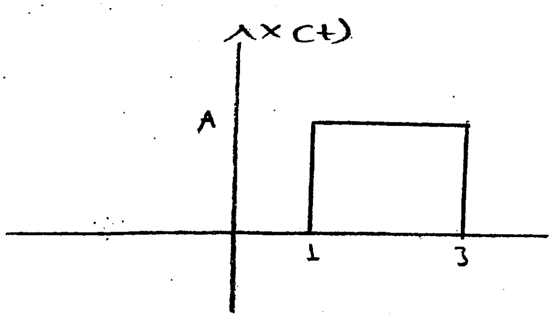


$$\Rightarrow x(t) = \int_{2}^{\infty} f(t)\,dt \qquad * \; F(\omega) = e^{-j2\omega}$$

$$\Rightarrow X(\omega) = \frac{F(\omega)}{j\omega} + \pi \underbrace{F(0)}_{1}\,\delta(\omega)$$

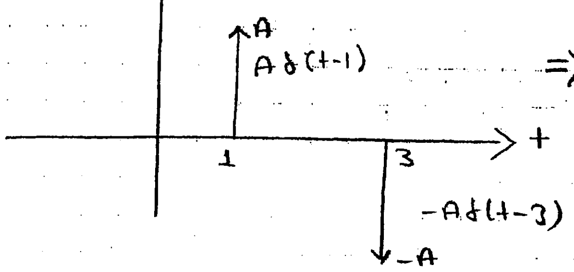
$$\Rightarrow X(\omega) = \frac{e^{-j2\omega}}{j\omega} + \pi\,\delta(\omega) = \frac{-j}{\omega} \times e^{-j2\omega} + \pi\,\delta(\omega)$$

ÖRNEK;



$X(w)$'yi ilk olarak integral özelliğinden yararlanarak bulalım;

$$f(t) = \frac{dx(t)}{dt}$$



$$\Rightarrow F(w) = A \cdot e^{-jw} - A e^{-j3w}$$

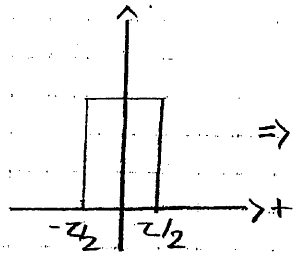
$$F(0) = \int_{-\infty}^{\infty} f(t) dt = 0$$

$$\Rightarrow X(w) = \frac{F(w)}{jw} + \pi F(0) \delta(w) \Rightarrow X(w) = \frac{A}{jw} \left(e^{-jw} - e^{-j3w} \right)$$

$$\Rightarrow X(w) = \frac{A}{jw} e^{-j2w} \left(e^{jw} - e^{-jw} \right)$$

$$\Rightarrow X(w) = 2A \cdot e^{-j2w} \sin w / w \Rightarrow X(w) = 2A e^{-j2w} \cdot \mathrm{sinc}(2f)$$

* ikinci yol;



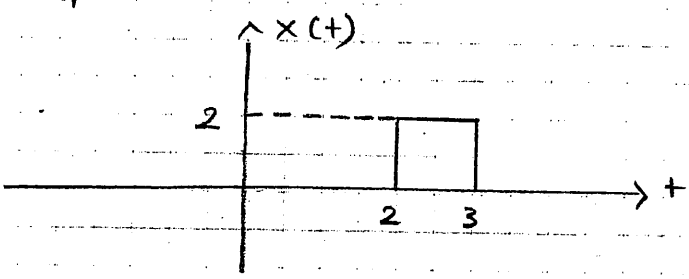
$\Rightarrow$ fourier dönüşümü $A\tau \, \mathrm{sinc}(f\tau)$

* Şimdi $x(t)$'yi buna benzetmeye çalışalım;

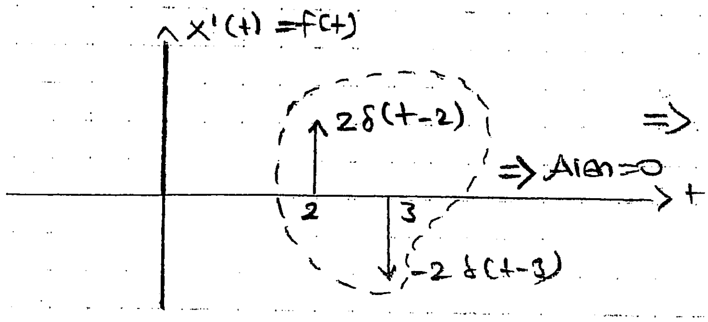
$\tau = 2$
$A = A$
kayma miktarı; 2

$$\Rightarrow X(w) = 2A e^{-j2w} \mathrm{sinc}(2f)$$

ÖRNEK;



$X(w)$'yi integral özelliğinden yararlanarak bulalım;



$$\Rightarrow F(w) = 2e^{-j2w} - 2e^{-j3w}$$

$$X(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

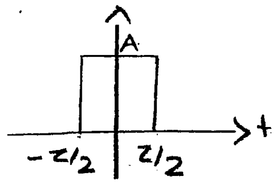
$$\Rightarrow X(\omega) = \left(2e^{-j2\omega} - 2e^{-j3\omega}\right)/j\omega$$

$$\Rightarrow X(\omega) = \frac{2}{j\omega} e^{-j2,5\omega}\left(e^{j0,5\omega} - e^{-j0,5\omega}\right)$$

$$\Rightarrow X(\omega) = \frac{4}{\omega} e^{-j2,5\omega} \sin 0{,}5\omega$$

$$\Rightarrow X(\omega) = \frac{2}{\pi f} e^{-j2,5\omega} \sin \pi f \Rightarrow X(\omega) = 2 \cdot e^{-j2,5\omega} \cdot \mathrm{sinc}(f)$$

* İkinci yol



$A\tau \, \mathrm{sinc}(f\tau)$

$$\left.\begin{array}{l} \tau = 1 \\ A = 2 \\ \text{kayma miktarı} = 2{,}5 \end{array}\right\} \quad X(\omega) = e^{-j2,5\omega} \cdot 2\,\mathrm{sinc}(f)$$

* Üçüncü yol (formülden)

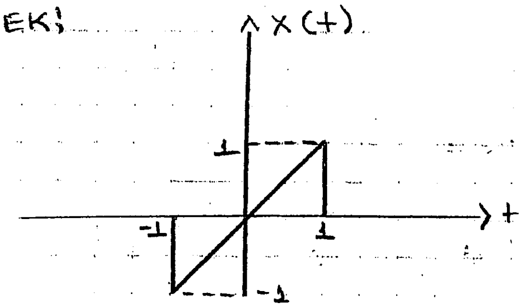
$$\Rightarrow X(\omega) = 2\int_2^3 e^{-j\omega t}\,dt = \frac{-2}{j\omega} e^{-j\omega t}\Big|_2^3$$

$$\Rightarrow X(\omega) = \frac{-2}{j\omega}\left(e^{-j3\omega} - e^{-j2\omega}\right)$$

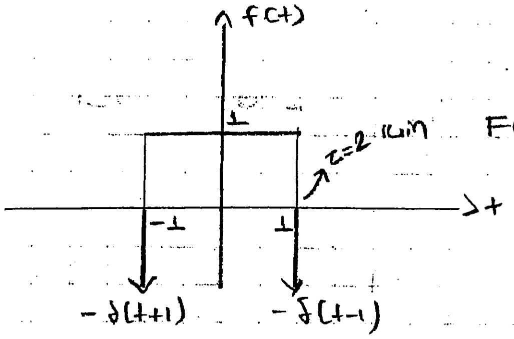
$$\Rightarrow X(\omega) = \frac{2}{j\omega} e^{-j2,5\omega}\left(e^{j0,5\omega} - e^{-j0,5\omega}\right)$$

$$\Rightarrow X(\omega) = \frac{4}{\omega} e^{-j2,5\omega} \sin \omega/2 \Rightarrow X(\omega) = 2e^{-j2,5\omega}\,\mathrm{sinc}(f)$$

ÖRNEK;



X(ω)'yı integral özelliğinden yararlanarak bulalım.



$F(\omega) = 2\,\mathrm{sinc}(2f) - e^{-j\omega} - e^{j\omega}$

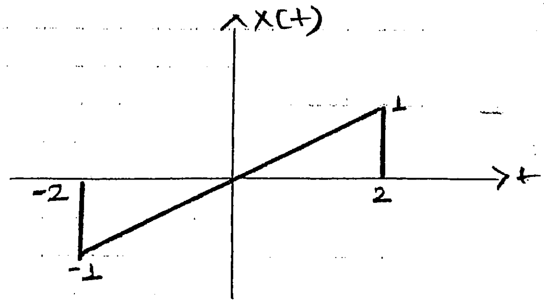
Alan=0

$$\Rightarrow F(\omega) = \frac{2\sin(\omega)/\omega}{j\omega} - (e^{j\omega} + e^{-j\omega})/j\omega$$

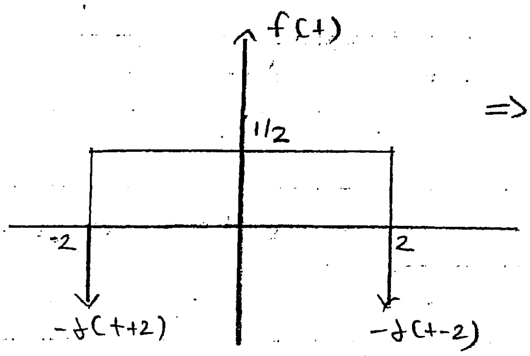
$$\Rightarrow F(\omega) = \frac{2\sin(\omega)}{j\omega^2} - \frac{2}{j\omega}\cos\omega$$

* Aynı sonuç formüllerde çıkar!

ÖRNEK;



X(ω)'yı integral özelliğinden yararlanarak bulalım;



$\Rightarrow F(\omega) = 2\,\mathrm{sinc}(4f) - e^{-j2\omega} - e^{j2\omega}$

Alan=0

$$\Rightarrow X(\omega) = \frac{\sin(2\omega)/\omega}{j\omega} - (e^{j2\omega} + e^{-j2\omega})/j\omega$$

$$\Rightarrow X(\omega) = \sin(2\omega)/j\omega^2 - \frac{2}{j\omega}\cos(2\omega)$$

* formülden çözelim;

$$X(\omega)=\int_{-2}^{2} t/2 \cdot e^{-j\omega t}dt \quad ; \quad t/2 = u \Rightarrow \frac{dt}{2}=du$$

$$e^{-j\omega t}dt = dv \Rightarrow v=\frac{-1}{j\omega}e^{-j\omega t}$$

$$\Rightarrow X(\omega)=-\left(\frac{t}{2j\omega}e^{-j\omega t}\right)\Big|_{-2}^{2}+\frac{1}{2j\omega}\int_{-2}^{2}e^{-j\omega t}dt$$

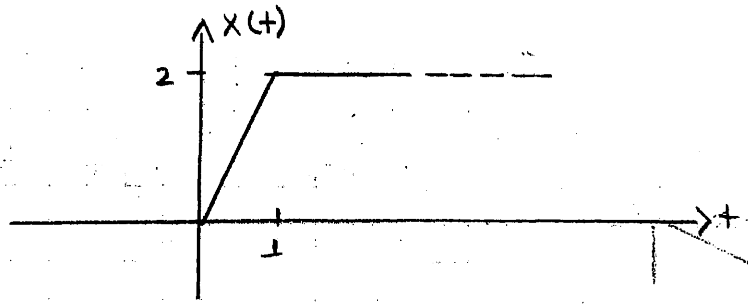
$$\Rightarrow X(\omega)=\frac{-1}{j\omega}\left(e^{j2\omega}+e^{-j2\omega}\right)+\frac{1}{2\omega^2}\left(e^{-j2\omega}-e^{j2\omega}\right)$$

$$\Rightarrow X(\omega)=\frac{-2}{j\omega}\cos(2\omega)-\frac{1\times j}{2j\omega^2}\left(e^{j2\omega}-e^{-j2\omega}\right)$$

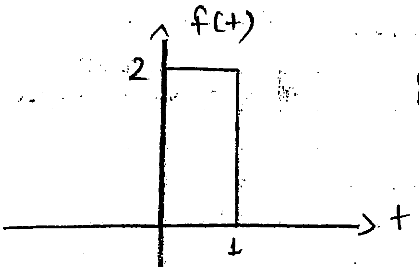
$$\Rightarrow X(\omega)=-\frac{2}{j\omega}\cos(2\omega)-\frac{j}{\omega^2}\sin 2\omega$$

$$\Rightarrow X(\omega)=\frac{\sin(2\omega)}{j\omega^2}-2\cos(2\omega)/j\omega$$

ÖRNEK;



$X(\omega)$'yı hesaplayalım;



$\boxed{\text{Alan}=2}$

$A=2$
$\tau=1$
kayma miktarı $=1/2$

$$\Big\}\ F(\omega)=2\,\text{sinc}(f)\cdot e^{-j\omega/2}$$

$$\Rightarrow X(\omega)=\left(4\sin(\omega/2)/\omega \cdot e^{-j\omega/2}\right)/j\omega+2\pi\delta(\omega)$$

$$\Rightarrow X(\omega)=\frac{4}{j\omega^2}\sin(\omega/2)\cdot e^{-j\omega/2}+2\pi\delta(\omega)$$

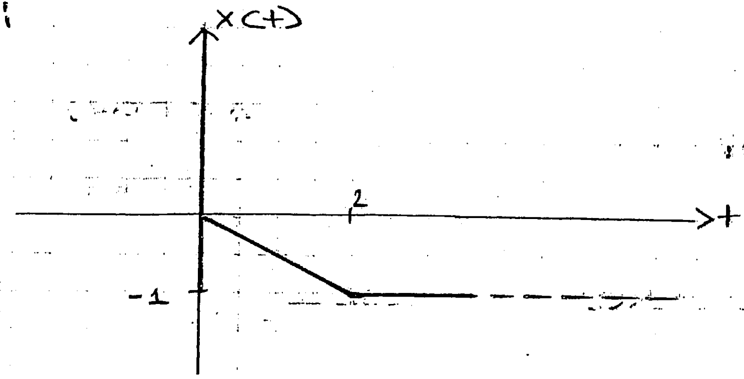
$$\Rightarrow X(w) = \frac{4}{jw^2} \cdot \frac{(e^{jw/2} - e^{-jw/2})}{2j} e^{-jw/2} + 2\pi\delta(w)$$

$$\Rightarrow X(w) = \frac{-2}{w^2}(1 - e^{-jw}) + 2\pi\delta(w)$$

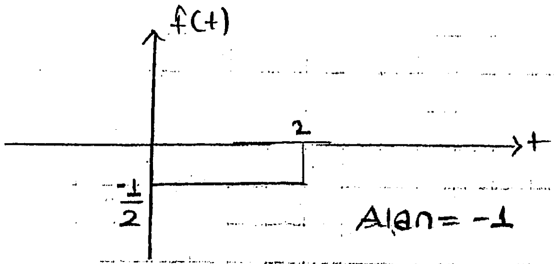
*formüldende çıkar.

$$\Rightarrow X(w) = \frac{-2}{w^2} + \frac{2e^{-jw}}{w^2} + 2\pi\delta(w)$$

ÖRNEK:



X(w)'yı bulalım:



$A = -1/2$
$\tau = 2$
kayma miktarı 1

$$\Rightarrow F(w) = -2A \operatorname{sinc}(2f) \cdot e^{-jw}$$

Alan $= -1$

$$\Rightarrow X(w) = (-2A\sin(w)/w \times e^{-jw})/jw - \pi\delta(w)$$

$$\Rightarrow X(w) = \frac{-2A}{jw^2}\sin w \times e^{-jw} - \pi\delta(w)$$

* formülden

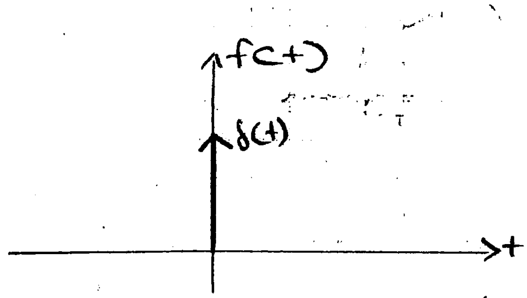
$$\Rightarrow X(w) = \frac{-2A}{jw^2}\left(\frac{e^{jw} - e^{-jw}}{2j}\right)e^{-jw} - \pi\delta(w) \qquad X(w) = -1/2\int_0^2 e^{-jwt}\,dt$$

$$\Rightarrow X(w) = \frac{A}{w^2}(1 - e^{-j2w}) - \pi\delta(w)$$

$$\Rightarrow X(w) = \frac{-1}{2w^2}e^{-j2w} + \frac{1}{2w^2} - \pi\delta(w)$$ bulunur.

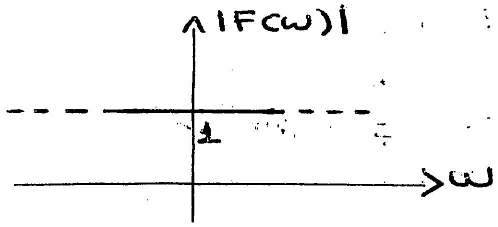
* Bazı önemli fonksiyonların fourier dönüşümleri

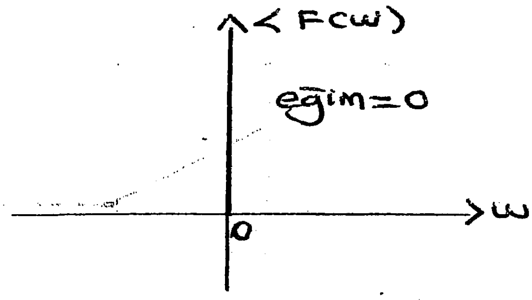
• Birim Vuruş İşareti

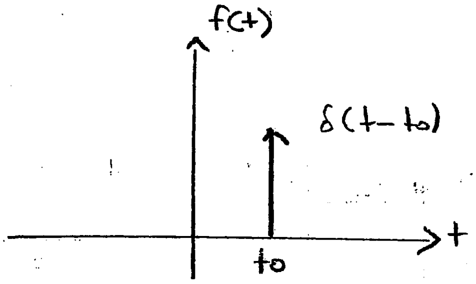


$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt \Rightarrow$$

$$F(\omega)=\int_{-\infty}^{\infty} \delta(t)\cdot e^{-j\omega t}\,dt = 1$$





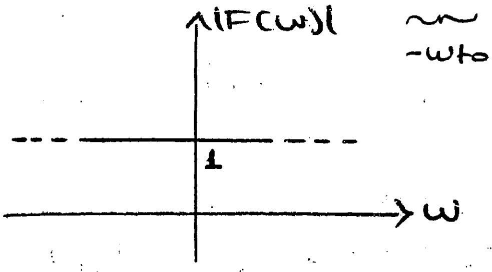


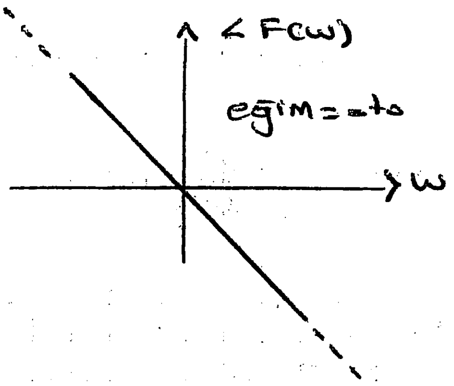
$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt$$

$$F(\omega)=\int_{-\infty}^{\infty} \delta(t-t_0)\, e^{-j\omega t}\,dt$$

$$\Rightarrow F(\omega)= \underbrace{e^{-j\omega t_0}} \Rightarrow |F(\omega)|=1$$

$$\underset{-\omega t_0}{\underbrace{-\tan\omega t_0}} \left\{ \frac{-\sin\omega t_0}{\cos\omega t_0} \right. \qquad \left\{ \cos\omega t_0 - j\sin\omega t_0 \right.$$



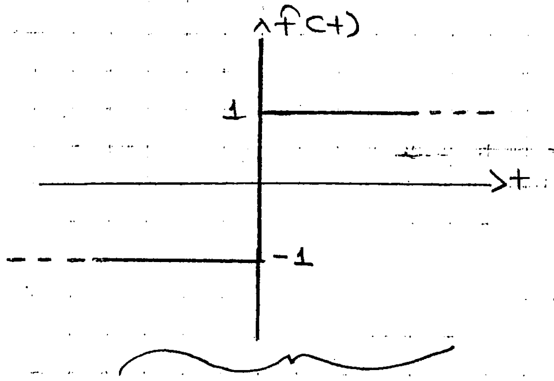


• 1'in fourier dönüşümü

$$1 = \underbrace{e^{j\omega_0 t}}_{2\pi\,\delta(\omega-\omega_0)} \quad (\omega_0=0)$$

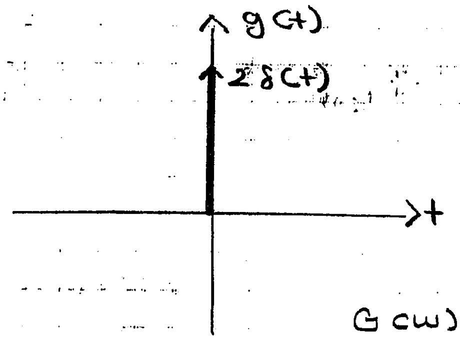
$$\Rightarrow \quad F\left\{\int_{-\infty}^{\infty} 1\,dt\right\} = \int_{-\infty}^{\infty} e^{-j\omega t}\,dt = 2\pi\,\delta(\omega) = \delta(f)$$

- **sgn(t)'nin fourier dönüşümü**



$f(t)=u(t)-u(-t)$

F(w)'yi integral özelliğinden yararlanarak bulalım;



$G(w)=2$

$\Rightarrow f'(t)=\delta(t)+\delta(t)=2\delta(t)=g(t)$

$* \; w\,\delta(w)=0$!

$$\Rightarrow F(w)=\frac{G(w)}{jw}+\pi G(0)\delta(w)=\frac{2}{jw}+2\pi\delta(w)=\frac{2}{jw}$$

- **cos(w₀t)'nin fourier dönüşümü**

$* \; \cos(w_0t)=\left(e^{jw_0t}+e^{-jw_0t}\right)/2=f(t)$

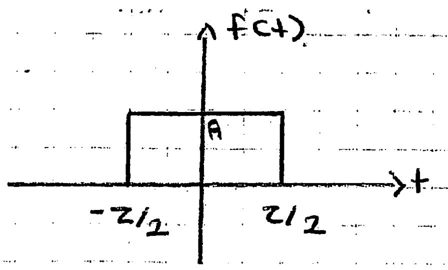
$$F(w)=\left(2\pi\delta(w-w_0)+2\pi\delta(w+w_0)\right)/2=\pi\left(\delta(w-w_0)+\delta(w+w_0)\right)$$

- **sin(w₀t)'nin fourier dönüşümü**

$* \; \sin(w_0t)=\left(e^{jw_0t}-e^{-jw_0t}\right)/2j=f(t)$

$$F(w)=\left(2\pi\delta(w-w_0)-2\pi\delta(w+w_0)\right)/2j=\frac{\pi}{j}\left(\delta(w-w_0)-\delta(w+w_0)\right)$$

- **Dikdörtgen Vuruş İşareti**



$$F(w)=\int_{-\infty}^{\infty} f(t)\,.\,e^{-jwt}\,dt$$

$$\Rightarrow F(w)=\int_{-\tau/2}^{\tau/2} A\,.\,e^{-jwt}\,dt=\frac{-A}{jw}\,e^{-jwt}\Big|_{-\tau/2}^{\tau/2}$$

$$\Rightarrow F(w)=\frac{-A}{jw}\left(e^{-jw\tau/2}-e^{jw\tau/2}\right)=\frac{A}{jw}\left(e^{jw\tau/2}-e^{-jw\tau/2}\right)$$

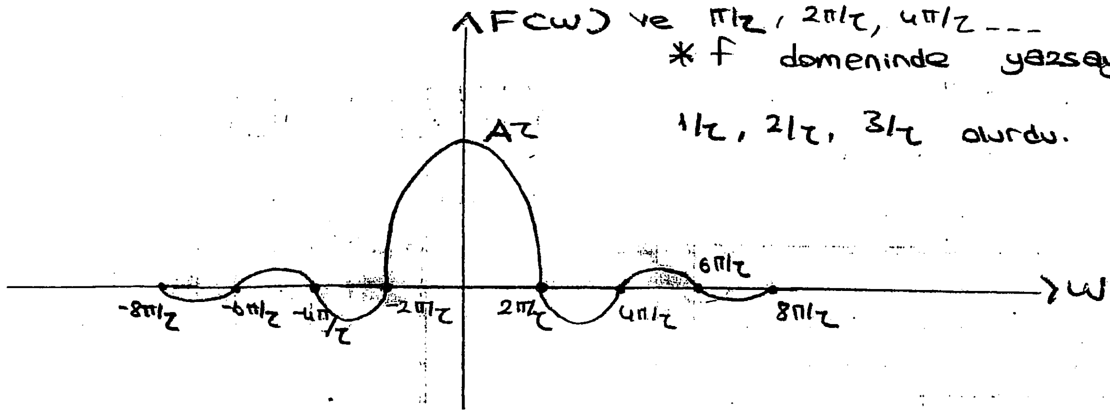
$$\Rightarrow F(w)=\frac{2A}{w}\sin w\tau/2=A\tau\,\frac{\sin(w\tau/2)}{w\tau/2}=A\tau\,\frac{\sin(\pi f\tau)}{\pi f\tau}$$

$\Rightarrow F(w)=A\tau\,\mathrm{sinc}(f\tau)$ 'dur.

* ... noktası

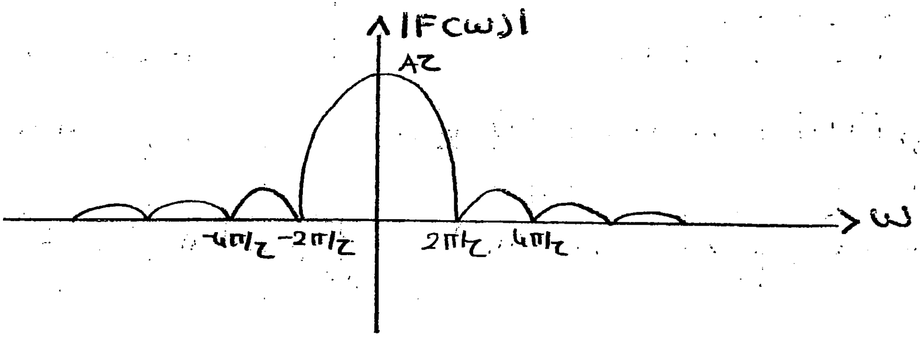
* $2A\tau \,\mathrm{sinc}(2f\tau)$

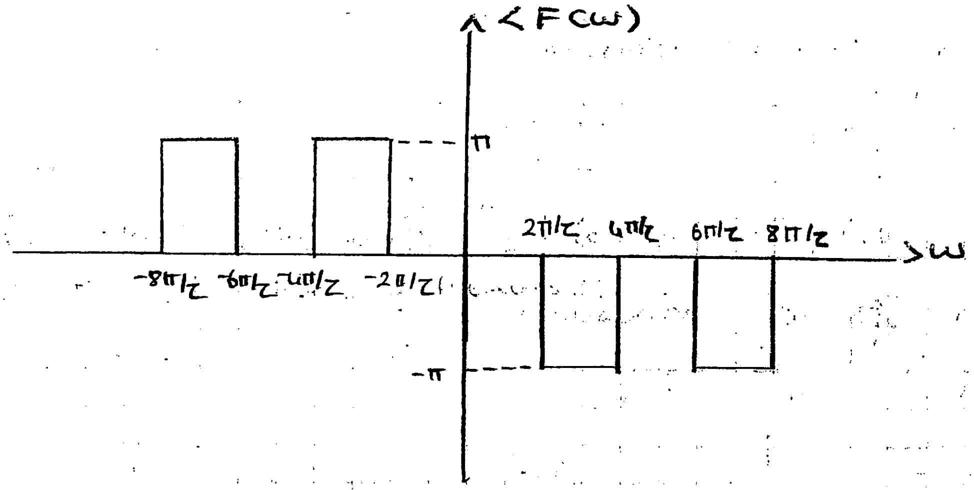
$1/2\tau,\ 2/2\tau,\ 4/2\tau$ ---



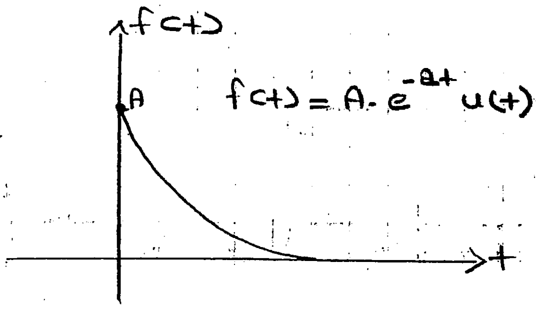
ve $\pi/\tau,\ 2\pi/\tau,\ 4\pi/\tau$ ---

* f domeninde yazsaydık $1/\tau,\ 2/\tau,\ 3/\tau$ olurdu.





- Tek yanlı Üstel Yarış İşareti



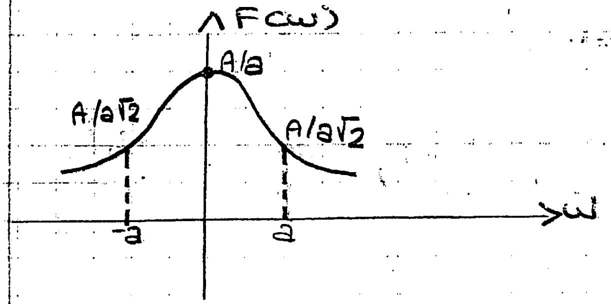
$$f(t) = A\cdot e^{-at}u(t) \qquad F(\omega) = \int_0^\infty A e^{-at} e^{-j\omega t}\,dt$$

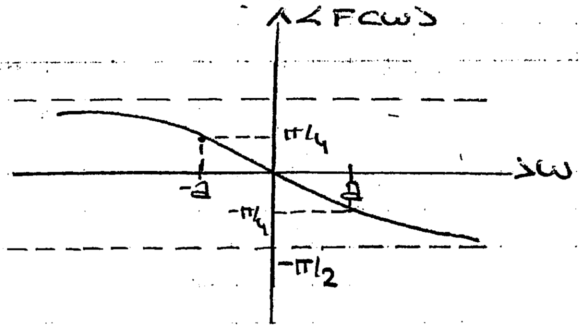
$$= A\int_0^\infty e^{-(a+j\omega)t}\,dt$$

$$= \frac{-A}{a+j\omega}\, e^{-(a+j\omega)t}\Big|_0^\infty$$

$$\Rightarrow F(\omega) = \frac{A}{a+j\omega} = \frac{A(a-j\omega)}{a^2+\omega^2} \quad \Rightarrow \angle F(\omega) = -\arctan(\omega/a)$$

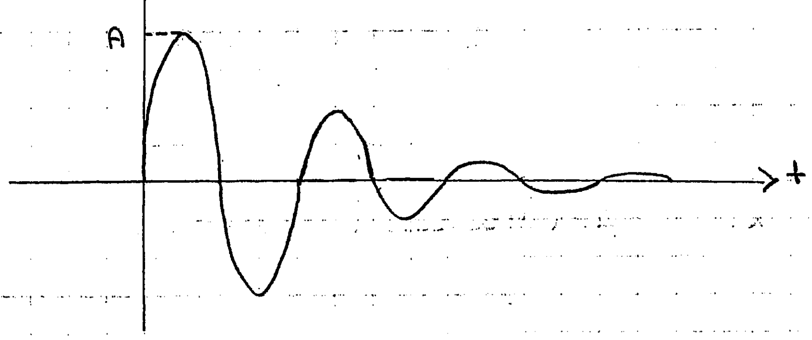
$$\Rightarrow |F(\omega)| = \frac{A}{\sqrt{a^2+\omega^2}}$$





• Sönümlü Sinüs İşareti

$$f(t) = A \cdot e^{-at} \cdot \sin(\omega_0 t) \cdot u(t)$$



$$F(\omega) = A\int_0^\infty e^{-at}\,\frac{\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)}{2j}\,e^{-j\omega t}\,dt$$

$$\Rightarrow F(\omega) = \frac{A}{2j}\int_0^\infty e^{-(a - j\omega_0 + j\omega)t}\,dt - \frac{A}{2j}\int_0^\infty e^{-(a + j\omega_0 + j\omega)t}\,dt$$

$$\Rightarrow F(\omega) = \frac{A}{2j} \times \frac{(-1)}{a + j(\omega - \omega_0)} \times e^{-(a + j(\omega - \omega_0))t}\Big|_0^\infty - \frac{A}{2j} \times \frac{(-1)}{a + j(\omega + \omega_0)} \times e^{-(a + j(\omega + \omega_0))t}\Big|_0^\infty$$

$$\Rightarrow F(\omega) = \frac{-A}{2j} \times \frac{1}{a + j(\omega - \omega_0)} \times (-1) + \frac{A}{2j} \times \frac{1}{a + j(\omega + \omega_0)} \times (-1)$$

$$\Rightarrow F(\omega) = \frac{A}{2j}\left(\frac{1}{a + j(\omega - \omega_0)} - \frac{1}{a + j(\omega + \omega_0)}\right)$$

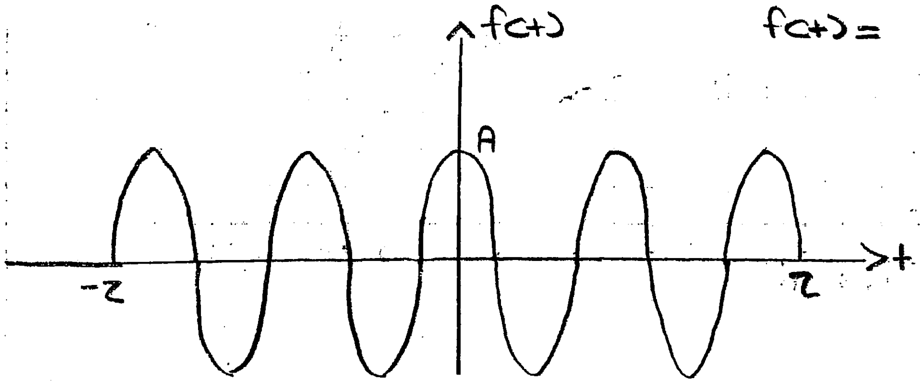
$$\Rightarrow F(\omega) = \frac{A}{2j}\,\frac{(\cancel{a} + \cancel{j\omega} + j\omega_0 - \cancel{a} - \cancel{j\omega} + j\omega_0)}{a^2 + a\omega j - a\cancel{j\omega_0} + ajw - \omega^2 + \cancel{\omega_0}\omega + a\cancel{j\omega_0} - \cancel{\omega_0}\omega + \omega_0^2}$$

$$\Rightarrow F(\omega) = \frac{A}{\cancel{2j}}\,(\cancel{2j}\omega_0) \Big/ a^2 + 2aj\omega - \omega^2 + \omega_0^2$$

$$\Rightarrow F(\omega) = \frac{A\omega_0}{a^2 - \omega^2 + \omega_0^2 + 2a\omega j} \qquad \Rightarrow |F(\omega)| = \frac{A\omega_0}{\sqrt{(a^2 - \omega^2 + \omega_0^2)^2 + 4a^2\omega^2}}$$

$$\Rightarrow \angle F(\omega) = \arctan\left(-2a\omega \big/ a^2 + \omega_0^2 - \omega^2\right)$$

• Yüksek frekanslı sinüs işareti



$$f(t) = \begin{cases} A\cos w_0 t & |t|<\tau \\ 0 & \text{diğer yerlerde} \end{cases}$$

Bu işaretin fourier dönüşümü, zamanda çarpma özelliğinden yararlanarak da bulunabilir.

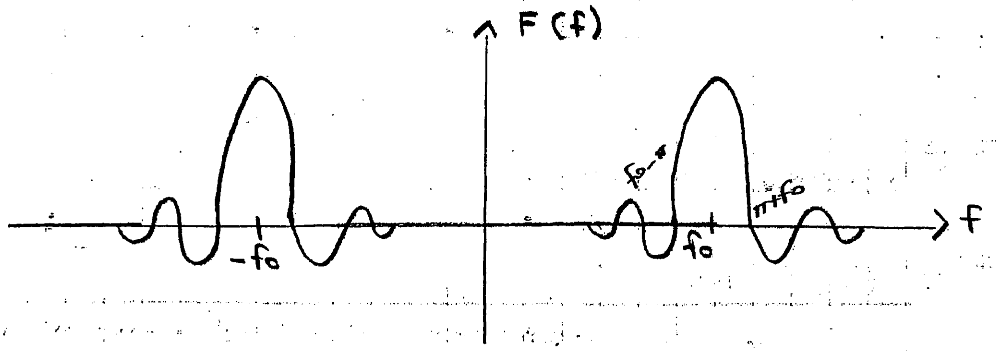
$$x(t) = A\cos w_0 t \Rightarrow X(w) = A\pi\left(\delta(w-w_0) + \delta(w+w_0)\right)$$

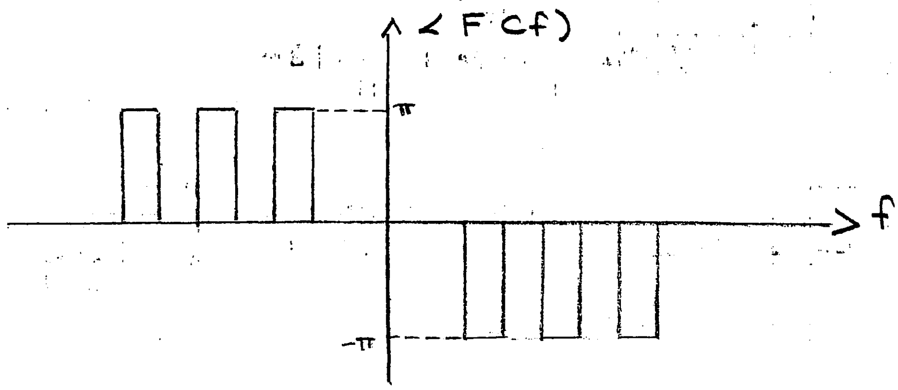
$$\underbrace{g(t) = u(t+\tau) - u(t-\tau)}_{\substack{\text{Burada } A=1 \\ T=2\tau}} \Rightarrow G(w) = 2\tau\,\mathrm{sinc}(2f\tau) = 2\tau \sin(w\tau)/w\tau$$

O hâlde $F(w)$, bu iki işaretin fourier dönüşümlerinin katlamasının 2π'ye bölümüdür.

$$F(w) = A\tau\left(\frac{\sin(w-w_0)\tau}{(w-w_0)\tau} + \frac{\sin(w+w_0)\tau}{(w+w_0)\tau}\right)$$

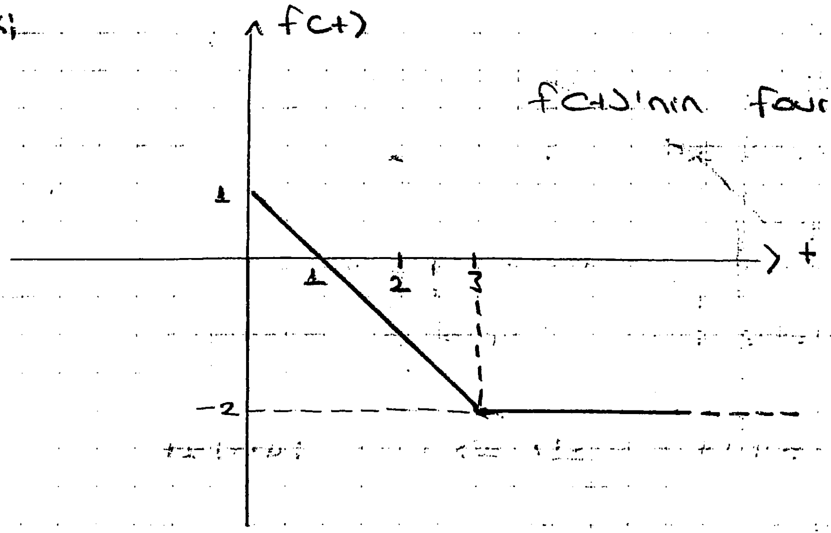
$$\Rightarrow F(f) = A\tau\left(\mathrm{sinc}\,(f-f_0)\tau + \mathrm{sinc}\,(f+f_0)\tau\right)$$





* Gauss vuruş işaretinin fourier dönüşümüde bir gausstur.

ÖRNEK;

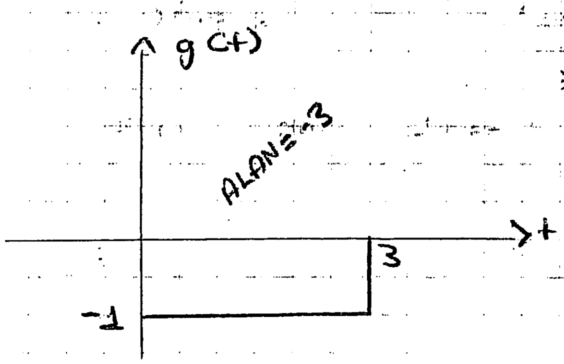


f(t)'nin fourier dönüşümünü bulunuz

* $\int_{-\infty}^{\infty} |f(t)|dt < \infty$ şartı sağlanmadığı için fourier dönüşümü yakınsamaz. Fakat fourier dönüşümünün integral özelliğinden yararlanarak fourier dönüşümünü hesaplayabiliriz.

* $g(t) = f'(t) = \frac{df(t)}{dt}$ olsun.

o hâlde; $F(w) = \frac{G(w)}{jw} + \pi G(0)\delta(w)$ olur. (integ. özel.)



* Şimdi ise fourier dönüşümünü bildiğimiz bir fonksiyon olan dikdörtgen vuruş işaretine benzetmeye çalışarak fourier dönüşümünü hesaplayalım:

A = -1
τ = 3
kayma miktarı; 1,5

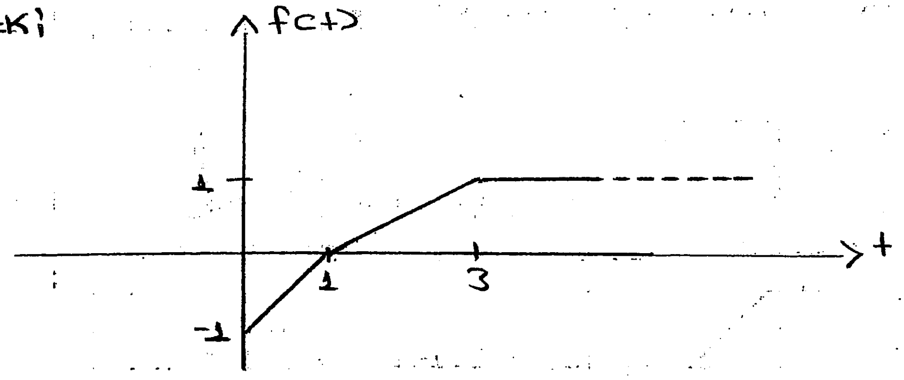
$\Rightarrow G(w) = -3\ \mathrm{sinc}(3f) \cdot e^{-j3w/2} = -3 \frac{\sin 3\pi f}{3\pi f} \times e^{-j3w/2}$

$\Rightarrow G(w) = -\frac{2}{w} \sin(3w/2) \times e^{-j3w/2}$

$\Rightarrow G(w) = \frac{-2}{2jw} (e^{j3w/2} - e^{-j3w/2}) e^{-j3w/2}$

$\Rightarrow G(w) = \frac{-2}{2j} 1 + \frac{2}{2j} e^{-j3w} \Rightarrow F(w) = \frac{1}{w^2} - \frac{1}{w^2} e^{-j3w} - 3\pi\delta(w)$

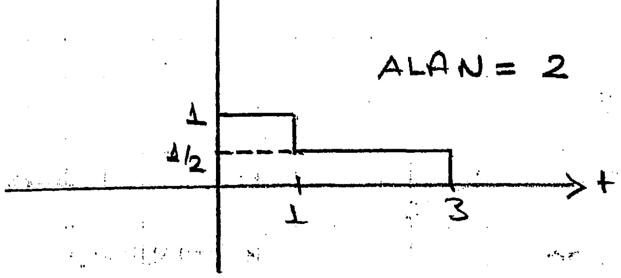
ÖRNEK:



F(w)'yı hesaplayalım; (integral özelliğiyle)

$$g(t) = f'(t) = \frac{df(t)}{dt} \Rightarrow f(t) = \int g(t)\,dt$$

ALAN = 2



$0-1$ için $A=1,\ Z=1,\ K.M=1/2 \Rightarrow e^{-jw/2}\,\mathrm{sinc}(f)$

$1-3$ için $A=1/2,\ Z=2,\ K.M=2 \Rightarrow e^{-j2w}\,\mathrm{sinc}(2f)$

$$\Rightarrow X_1(w) = e^{-jw/2}\frac{\sin \pi f}{\pi f} = \frac{2e^{-jw/2}}{w} \times \sin(w/2)$$

$$\Rightarrow X_2(w) = e^{-j2w}\frac{\sin 2\pi f}{2\pi f} = \frac{e^{-j2w}}{w} \times \sin(w)$$

$$\Rightarrow X_1(w) + X_2(w) = \frac{2}{w} \times \frac{e^{-jw/2}}{2j} \times \left(e^{jw/2} - e^{-jw/2}\right) + \frac{e^{-j2w}}{w} \times \frac{1}{2j}\left(e^{jw} + e^{-jw}\right)$$

$$\Rightarrow F(w) = \frac{X_1(w) + X_2(w)}{jw} + 2\pi\delta(w)$$

$$\Rightarrow F(w) = \frac{2}{2jw} \times \frac{1}{jw} \times 1 - \frac{2}{2jw} \times \frac{1}{jw} e^{-jw} + \frac{1}{2jw} \times \frac{1}{jw} e^{-jw} - \frac{e^{-j3w}}{2jw} \times \frac{1}{jw} + 2\pi\delta(w)$$

$$\Rightarrow F(w) = -1/w^2 + \frac{1}{2w^2} e^{-jw} + \frac{1}{2w^2} e^{-j3w} + 2\pi\delta(w)$$

* formülden de aynı sonucu buluruz.

$$G(w)=\int_0^1 e^{-jwt}dt+\frac{1}{2}\int_1^3 e^{-jwt}dt$$

$$\Rightarrow G(w)=\frac{-1}{jw}e^{-jwt}\Big|_0^1-\frac{1}{2jw}e^{-jwt}\Big|_1^3=\frac{-1}{jw}e^{-jw}+\frac{1}{jw}-\frac{1}{2jw}e^{-j3w}+\frac{1\times e^{-jw}}{2jw}$$

$$\Rightarrow \frac{G(w)}{jw}=\frac{e^{-jw}}{w^2}-\frac{1}{w^2}+\frac{1}{2w^2}e^{-j3w}-\frac{1}{2w^2}e^{-jw}$$

$$\Rightarrow F(w)=\frac{-1}{w^2}+\frac{1}{2w^2}e^{-jw}+\frac{1}{2w^2}e^{-j3w}+2\pi\delta(w)\text{'dır.}$$

8) İlinti özelliği

$$C_{fg}(z)=\int_{-\infty}^{\infty}g(t).f(t-z)\,dt=\int_{-\infty}^{\infty}f(t).g(t+z)\,dt\iff G(w).F(-w)$$
$$\iff G(f).F(-f)$$

* Periyodik olmayan işaretlerin genlik ve faz tayfı süreklidir.

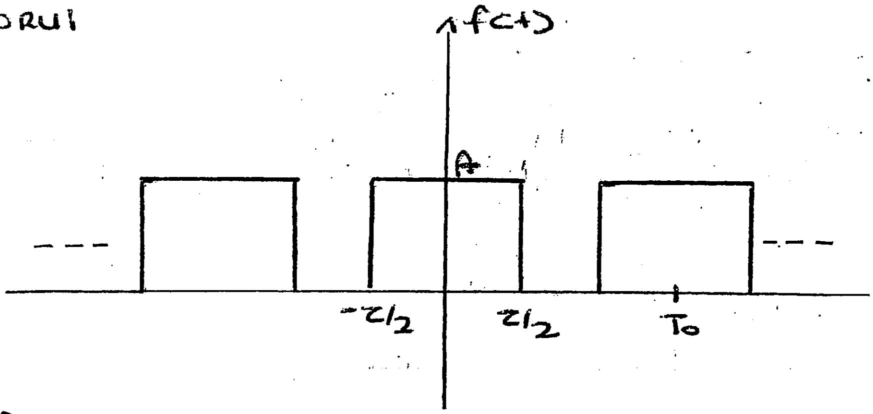
* Periyodik işaretlerin fourier dönüşümü hesaplanamaz, fakat dolaylı olarak hesaplanabilir. Bu işaretlerin genlik ve faz tayfı çizgiden oluşur.

$$x(t)=e^{\pm jw_0t}\Rightarrow X(w)=2\pi\,\delta(w\mp w_0)\Rightarrow X(f)=\delta(f\mp f_0)$$

$$x(t)=e^{\pm jkw_0t}\Rightarrow X(w)=2\pi\,\delta(w\mp kw_0)\Rightarrow X(f)=\delta(f\mp kf_0)$$

SORULAR

SORU1



fourier dönüşümünü hesaplayınız.

* Fourier serisi katsayıları ile fourier dönüşümü arasındaki ilişki:

$$F_k = \frac{1}{T_0} F(\omega)\Big|_{k\omega_0=\omega} = \frac{1}{T_0} F(k\omega_0)$$

periyot!

Bu işaretin fourier dönüşümü $F(\omega) = A\tau \,\mathrm{sinc}(f\tau)$

Yani $F(\omega) = A\tau \dfrac{\sin \pi f\tau}{\pi f\tau} \Rightarrow F(\omega) = 2A\tau \sin(\omega\tau/2)/\omega\tau$

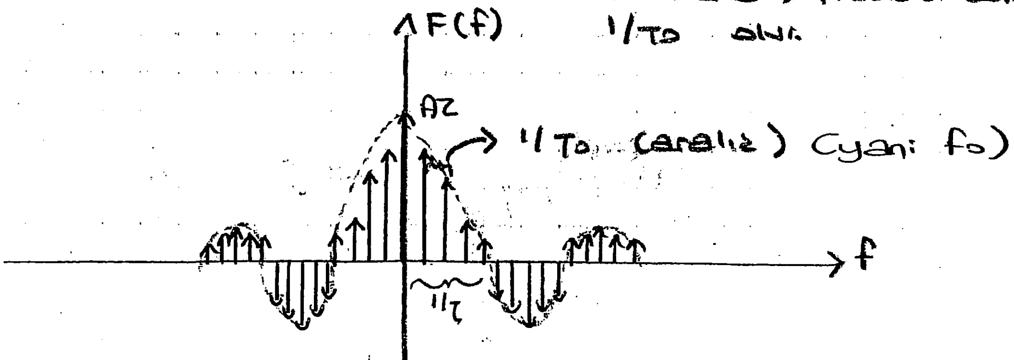
O hâlde $F_k = \dfrac{2A\tau}{T_0} \sin(k\omega_0\tau/2)/k\omega_0\tau$

$\Rightarrow F_k = \dfrac{2A\tau}{T_0 2\pi f_0 \tau k} \sin(\pi k f_0 \tau) \Rightarrow F_k = \dfrac{A\tau}{T_0} \mathrm{sinc}(kf_0\tau)$ olur.

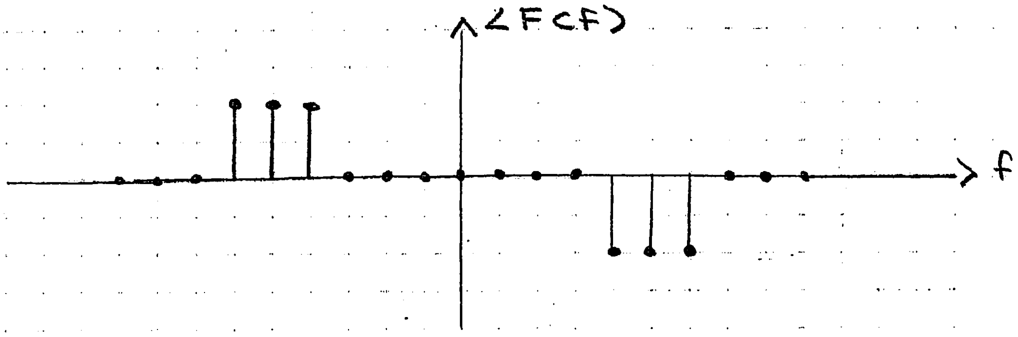
O hâlde: $f(t) = \displaystyle\sum_{k=-\infty}^{\infty} \frac{A\tau}{T_0} \mathrm{sinc}(kf_0\tau)\, e^{jk\omega_0 t}$

$\Rightarrow F(f) = \displaystyle\sum_{k=-\infty}^{\infty} \frac{A\tau}{T_0} \underbrace{\mathrm{sinc}(kf_0\tau)}_{\text{burayı sıfır değer yapan k'lı olur.}} \delta(f-kf_0)$ olur.

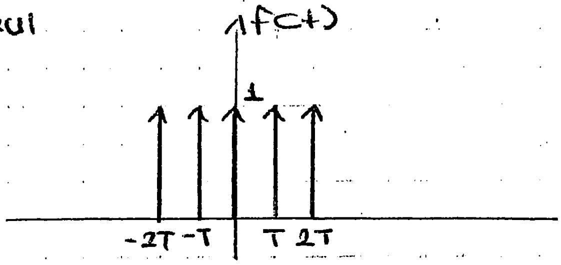
$T - kT_0 \Rightarrow$ zaman domeninde T_0 ileriler; frekans domeninde $1/T_0$ olur.



Periyot boyunca T_0/τ kadar çizgi olur.



? SORU



fourier dönüşümüne açalım

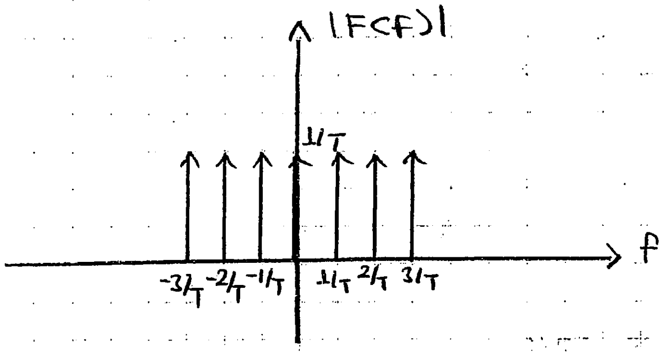
$$f(t) = \sum_{k=-\infty}^{\infty} F_k \cdot e^{jkw_0 t} \Rightarrow F_k = 1/T_0 \int_{T_0} f(t) \cdot e^{-jkw_0 t}\, dt$$

$$\Rightarrow F_k = \frac{1}{T} \int_0^T \delta(t) \cdot e^{-jk2\pi/T \, t}\, dt = 1/T$$

$*w_0 = 2\pi/T$ 'dir.

O hâlde $f(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} \overbrace{e^{jk \underbrace{2\pi/T}_{w_0} t}}^{e^{jkw_0 t}}$

$$F(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f - kf_0) \Rightarrow F(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f - k\frac{1}{T}\right) \text{ olur.}$$



* faz tayfı 0 olur.

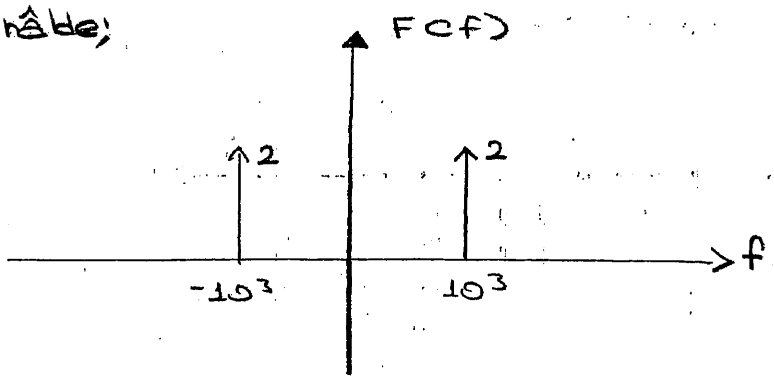
SORU: $f(t) = 4\sin[\underbrace{2\pi 10^3}_{w_0} t - \pi/3]$ fourier dönüşümünü hesaplayınız.

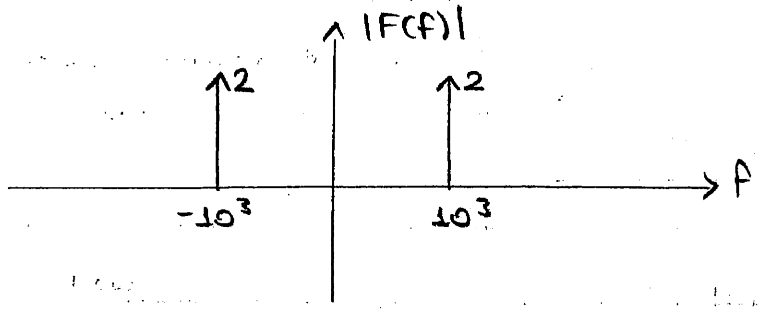
$$f(t) = \frac{4}{2j}\left(e^{j(2\pi 10^3 t - \pi/3)} - e^{-j(2\pi 10^3 t - \pi/3)}\right)$$

$$\Rightarrow F(f) = \frac{4}{2j} e^{-j\pi/3} \delta(f - 10^3) - \frac{4}{2j} e^{j\pi/3} \delta(f + 10^3)$$

$$\Rightarrow F(f) = -2j e^{-j\pi/3} \delta(f - 10^3) + 2j e^{j\pi/3} \delta(f + 10^3)$$

O hâlde;





Şimdide faz tayfını çizelim

$0-j \Rightarrow \arctan(-1/0) = \arctan(-\infty) \Rightarrow \theta = -\pi/2$

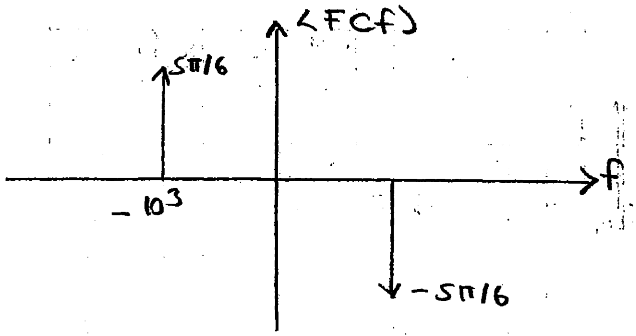
$0+j \Rightarrow \arctan(1/0) = \arctan(\infty) \Rightarrow \theta = \pi/2$

$e^{-j\pi/3} \to -\pi/3$ ve $e^{j\pi/3} \to \pi/3 \Rightarrow -\pi/3 - \pi/2 = -5\pi/6$

$\pi/3 + \pi/2 = 5\pi/6$

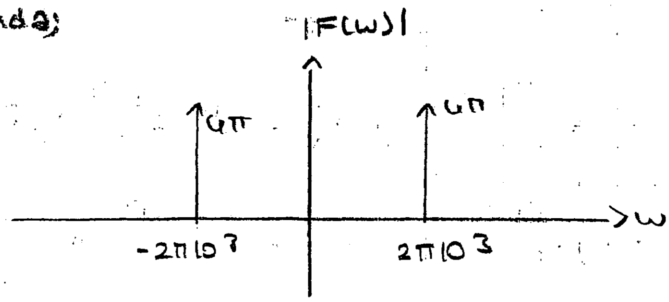
Yani: paydaki j + kısım için -90° olarak yukarı çizer

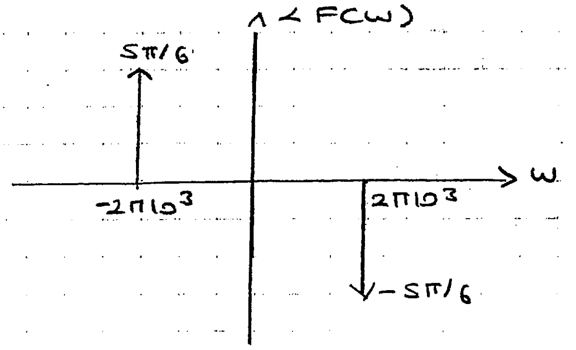
- kısım için 90° olarak yukarı çizer.



F(ω)'yı yazalım: $F(\omega) = -4\pi j e^{-j\pi/3}\,\delta(f-10^3) + 4\pi j e^{j\pi/3}\,\delta(f+10^3)$

Bu durumda;





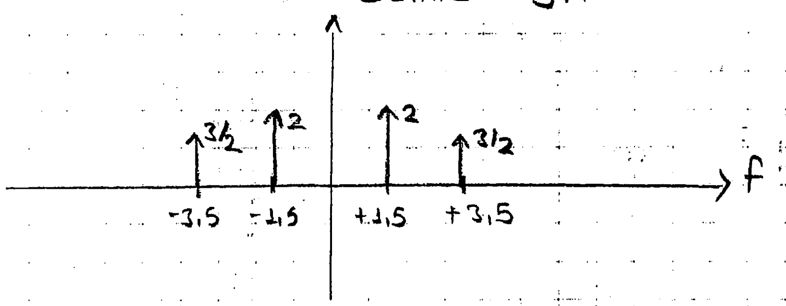
SORU: $x(t) = 3\sin(7\pi t - \pi/4) - 4\cos(3\pi t + \pi/6)$ 'nin genlik ve faz tayfını çiziniz.

$$x(t) = \frac{3}{2j}\left(e^{j(7\pi t - \pi/4)} - e^{-j(7\pi t - \pi/4)}\right) - 2\left(e^{j(3\pi t + \pi/6)} + e^{-j(3\pi t + \pi/6)}\right)$$

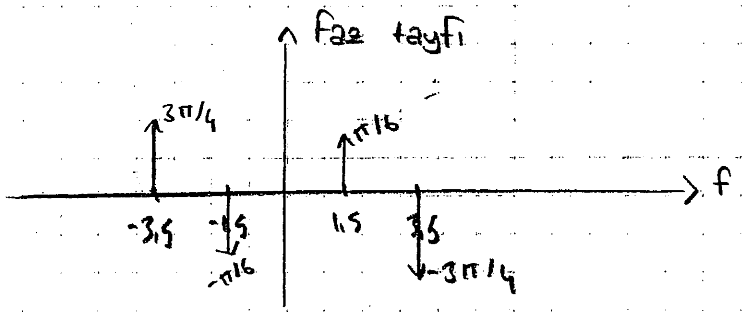
$\omega = 7\pi$, $f = 3{,}5$ $\qquad$ $\omega = 7\pi$, $f = -3{,}5$ $\qquad$ $f = 3/2$ $\qquad$ $f = -3/2$

$$x(t) = \frac{3}{2}(-j)\,e^{j(7\pi t - \pi/4)} + \frac{3}{2}(+j)\,e^{-j(7\pi t - \pi/4)}$$
$$-2\,e^{j(3\pi t + \pi/6)} + 2e^{-j(3\pi t + \pi/6)}$$

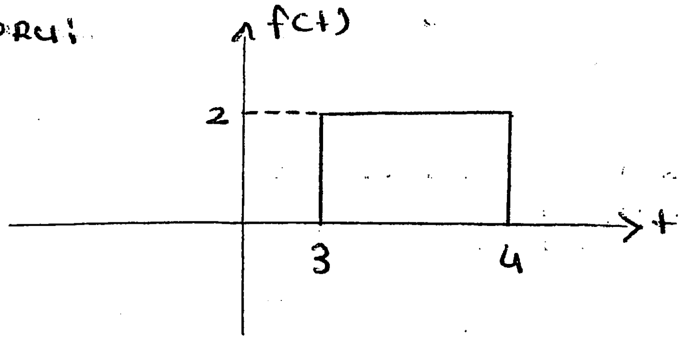
Genlik tayfı



Faz tayfı



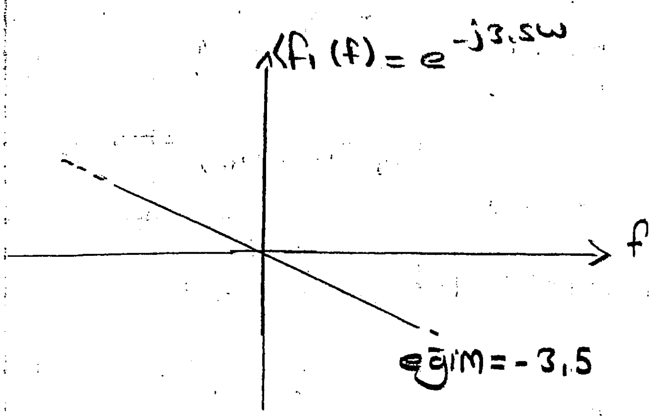
SORU:

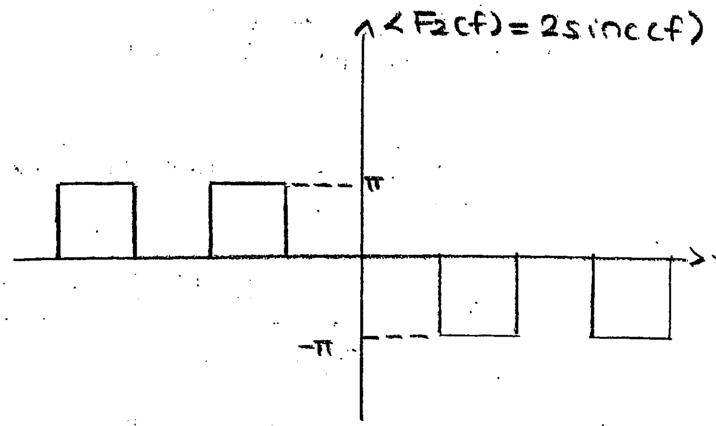


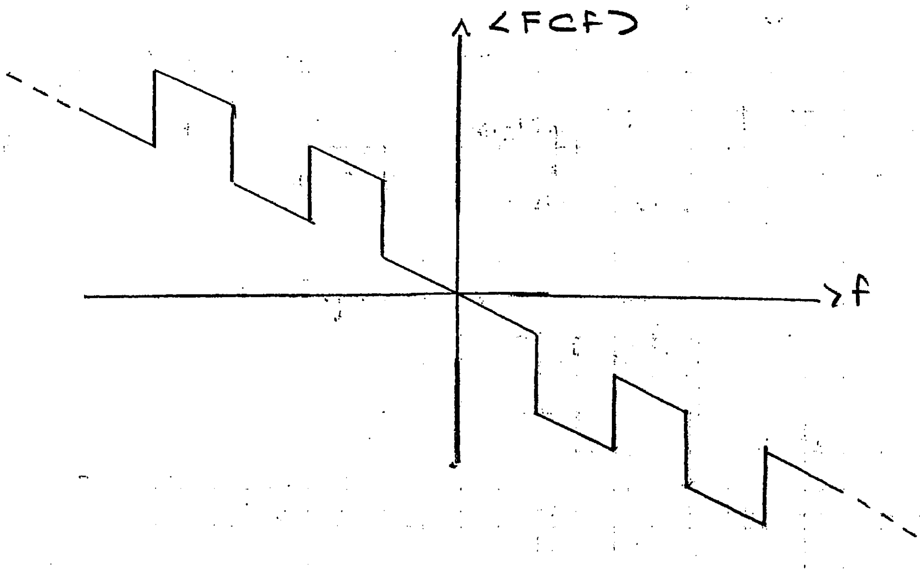
$F(\omega) = ?$

$$\left.\begin{array}{l} A = 2 \\ \tau = 1 \\ k.m = 3,5 \end{array}\right\} F(\omega) = 2\,\mathrm{sinc}(f) \cdot e^{-j3,5\omega}$$

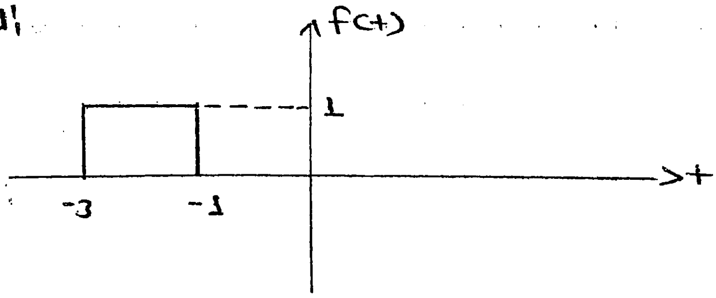
* formüllerle hesaplanır! (uzun yol)







SORU:



$F(\omega) = ?$

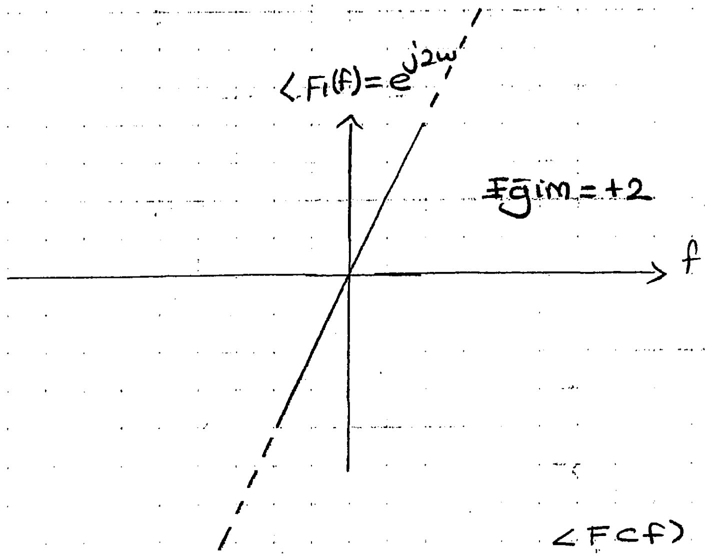
A:1
z:2
K.M:-2 } $F(w) = 2\,\text{sinc}(2f)\cdot e^{j2w}$

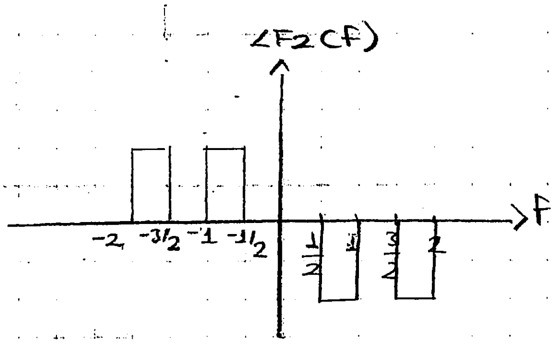
*formülden de hesaplayalım;

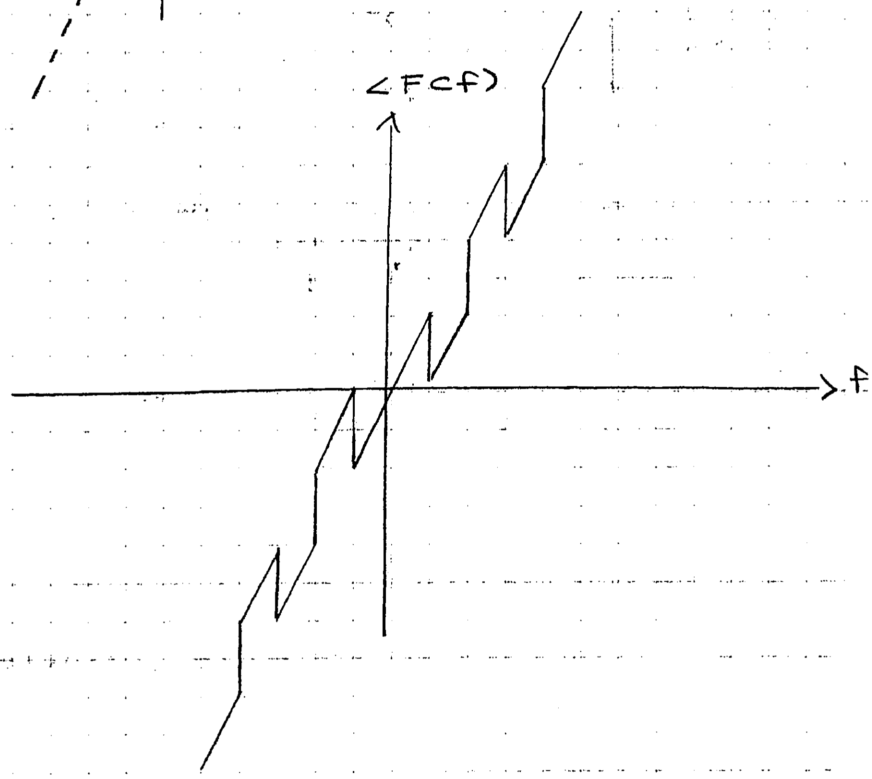
$$F(w) = \int_{-3}^{1} e^{-jwt}\,dt = -\frac{1}{jw} e^{-jwt}\Big|_{-3}^{1} = \frac{-1}{jw}\left(e^{jw} - e^{j3w}\right)$$

$$\Rightarrow F(w) = \frac{-e^{j2w}}{jw}\left(e^{-jw} - e^{jw}\right) = \frac{2e^{j2w}}{2jw}\left(e^{jw} - e^{-jw}\right)$$

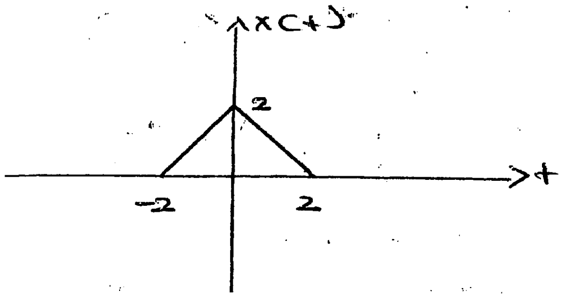
$$\Rightarrow F(w) = 2e^{j2w}\,\frac{\sin(w)}{w} = 2e^{j2w}\,\text{sinc}(2f)$$





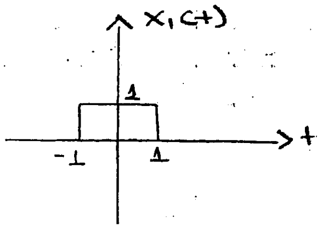


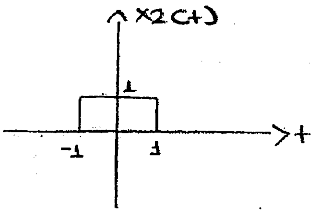
SORU!



Fourier dönüşümünü hesaplayınız.

* Bu işaret için katlama özelliğini kullanabiliriz.





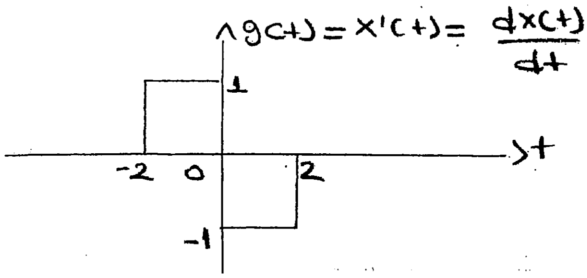
$x(t) = x_1(t) * x_2(t)$

$\Rightarrow X(w) = X_1(w) \cdot X_2(w)$ 'dır.

$X_1(w) = X_2(w) = 2\,\mathrm{sinc}(2f)$

$\Rightarrow X(w) = 4\,\mathrm{sinc}^2(2f)$

* X(w)'yi şimdide integral özelliğiyle çözelim



ALAN = 0

$-2 \rightarrow 0$ için $\left.\begin{matrix} \tau = 2 \\ A = 1 \\ k.m = -1 \end{matrix}\right\} \; 2\,\mathrm{sinc}(2f) \cdot e^{jw}$

$2 \rightarrow 0$ için $\left.\begin{matrix} \tau = 2 \\ A = -1 \\ k.m = 1 \end{matrix}\right\} \; -2\,\mathrm{sinc}(2f) \cdot e^{-jw}$

$\Rightarrow X(w) = [2\,\mathrm{sinc}(2f)\,(e^{jw} - e^{-jw})]\,/jw$

$\Rightarrow X(w) = [2\,\mathrm{sinc}(2f)\,\frac{2}{w}\,\sin(w)] = 2\,\mathrm{sinc}(2f) \times 2\,\mathrm{sinc}(2f)$

$\Rightarrow X(w) = 4\,\mathrm{sinc}^2(2f)$

SORU: $x(t) = \underbrace{3\,\mathrm{sinc}(2t)}_{f(t)} \cdot \underbrace{\cos 2\pi 10^3 t}_{g(t)}$ $\Rightarrow$ $x(w)$'yi bulunuz.

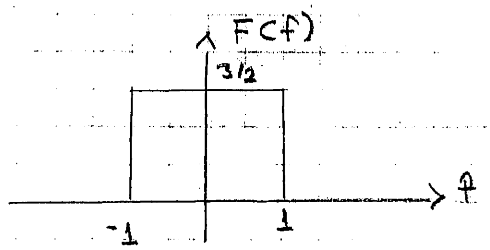
$x(t) = f(t)\cdot g(t) \Rightarrow \quad X(w) = \frac{1}{2\pi} F(w) * G(w)$

* Ters fourier dönüşümünü kullanalım:

$$F(f) = \int_{-B}^{B} A\, e^{j2\pi f t}\, df = \frac{A}{j2\pi t} e^{j2\pi t f}\Big|_{-B}^{B} = \frac{A}{j2\pi t}\left(e^{j2\pi B t} - e^{-j2\pi B t}\right)$$

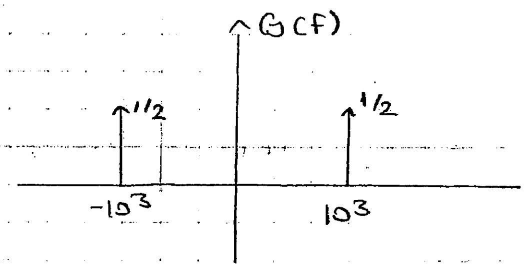
$$\Rightarrow \quad F(f) = \frac{A}{\pi t} \sin 2\pi B t = 2AB\,\mathrm{sinc}(2Bt)$$

$A = 3/2 \qquad B = 1$

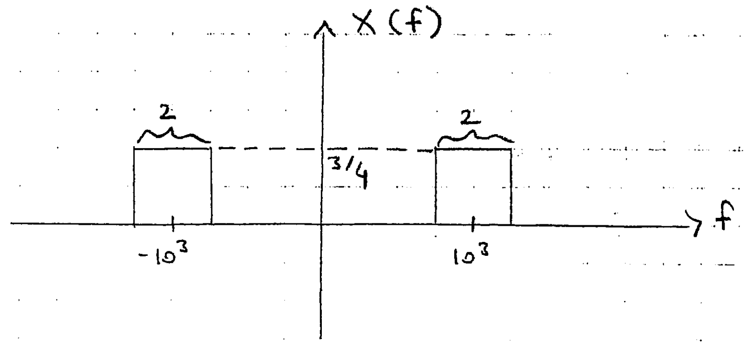


$g(t) = \cos 2\pi 10^3 t \Rightarrow \quad G(w) = \pi\left(\delta(w - 2\pi 10^3) + \delta(w + 2\pi 10^3)\right)$

$$\Rightarrow G(f) = \frac{1}{2}\left(\delta(f - 10^3) + \delta(f + 10^3)\right)$$



$\Rightarrow X(f) = F(f) * G(f)$



SORU! $f(t) \leftrightarrow F(\omega)$

$$\Rightarrow \frac{d}{dt}\left[e^{j2\pi 10^3 t} f(2t-3)\right]$$ 'in fourrier dönüşümünü hesaplayınız.

* Bu örnekte parantezin içinden başlayarak dışa doğru gideceğiz.

$$f(t) \leftrightarrow F(\omega) \Rightarrow f(2t) \leftrightarrow \frac{1}{2} F(\omega/2)$$

$$\Rightarrow f(2t-3) \leftrightarrow \frac{1}{2} F(\omega/2)\, \underbrace{e^{-j\omega/2 \times 3}}$$

zamanda kayma özelliği!

$$f(t-t_0) \leftrightarrow e^{-j\omega t_0} F(\omega)$$

Şimdi ise modulasyon özelliğini kullanalım;

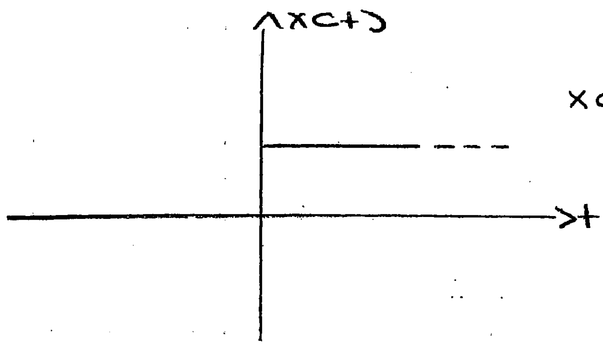
$e^{j\omega_0 t} f(t) \leftrightarrow F(\omega-\omega_0)$ idi.

Burada $\omega_0 = 2\pi \times 10^3$ 'tür.

O halde;

$$\frac{d}{dt}\left[e^{j2\pi 10^3 t} f(2t-3)\right] \leftrightarrow (j\omega)\frac{1}{2} F\left(\frac{\omega-2\pi 10^3}{2}\right) \times e^{-j\left(\frac{\omega-2\pi 10^3}{2}\right)\times 3}$$

SORU!



$x(t) = u(t) \Rightarrow X(\omega) = ?$

$u(t) = \int_0^\infty \delta(t)\,dt$ 'dir. $\delta(t) = f(t) \Rightarrow F(\omega) = 1$

$$\Rightarrow X(\omega) = \frac{1}{j\omega} + \pi \underbrace{F(0)}_{1} \delta(\omega)$$

$$\Rightarrow X(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

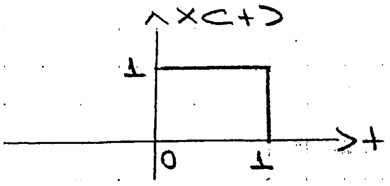
$$\Rightarrow X(f) = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

SORU: $x(t) = u(t) - u(t-1)$ işareti veriliyor.

a) $x(t)$'nin fourier dönüşümü $X(f)$'yi bulunuz.

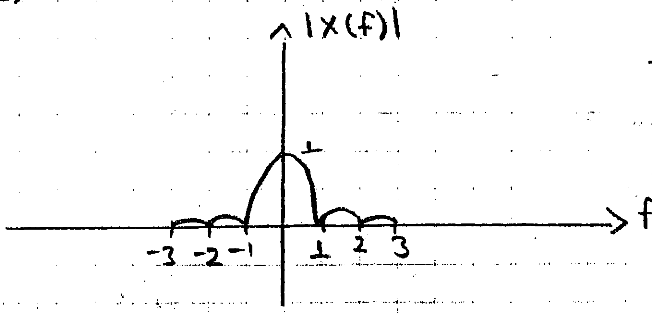
b) $X(f)$'nin iki yanlı genlik tayfı $|X(f)|$'yi ölçekli olarak çiziniz.

c) $X(f)$'nin iki yanlı faz tayfı $\angle X(f)$'yi ölçekli olarak çiziniz.



$$\left.\begin{array}{l} A: 1 \\ \tau: 1 \\ K.M: 1/2 \end{array}\right\} \text{a) } X(f) = \underbrace{\text{sinc}(f)}_{\frac{\sin \pi f}{\pi f}} \cdot e^{-jw/2}$$

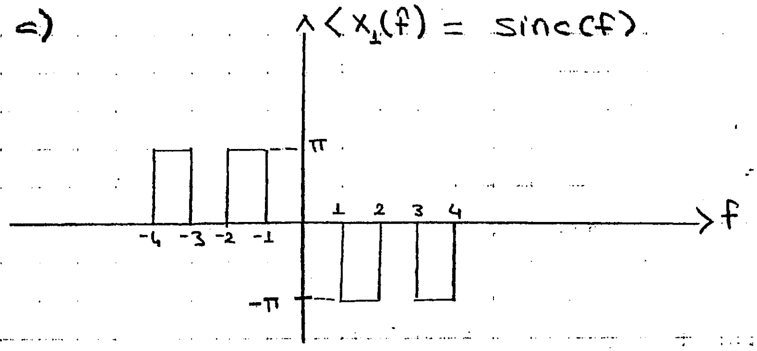
b)

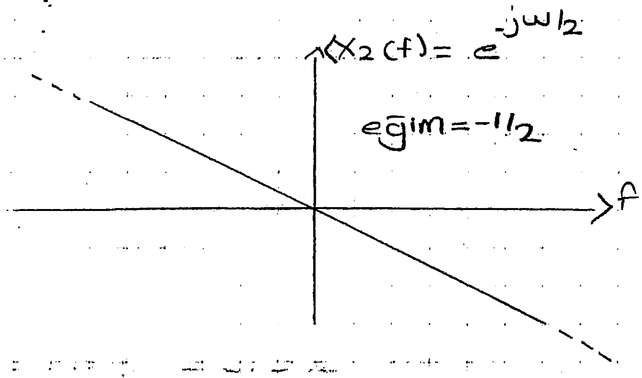


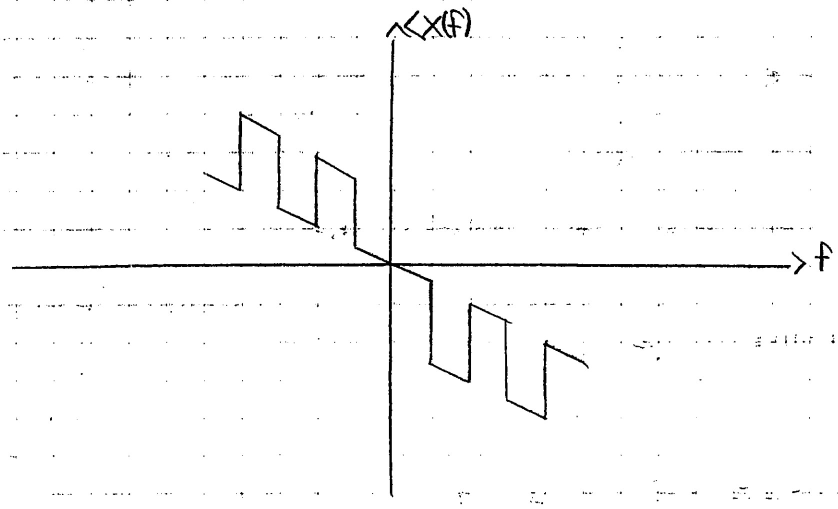
* $|e^{-jw/2}| = 1$

* Başka bir ifade olsaydı tepe değerini değiştirirdi!

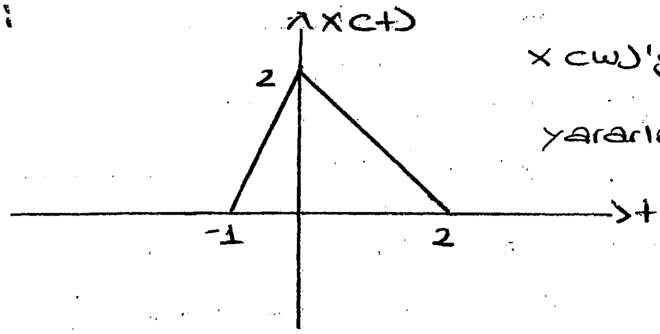
c)



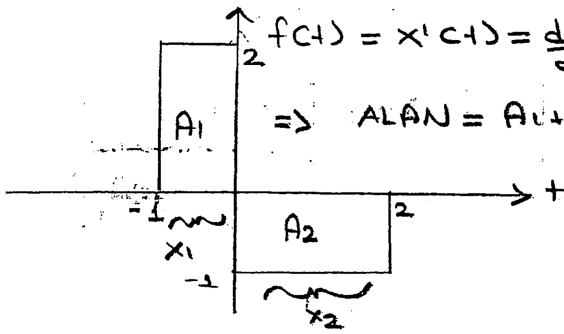




SORU:



$x(w)$'yi integral özelliğinden yararlanarak bulun.



$f(t) = x'(t) = \frac{dx}{dt} \Rightarrow x(t) = \int f(t)dt$

$\Rightarrow$ ALAN $= A_1 + A_2 = 2 - 2 = 0$

$$x_1(w) = \int_{-1}^{0} 2e^{-jwt}dt = \frac{-2}{jw} e^{-jwt}\Big|_{-1}^{0} = \frac{-2}{jw} + \frac{2}{jw}e^{jw}$$

$$x_2(w) = -\int_{0}^{2} 1\, e^{-jwt}dt = \frac{1}{jw} e^{-jwt}\Big|_{0}^{2} = \frac{1}{jw}e^{-j2w} - \frac{1}{jw}$$

$$\Rightarrow F(w) = \frac{-3}{jw} + \frac{e^{-j2w}}{jw} + \frac{2}{jw}e^{jw}$$

$$\Rightarrow X(w) = \frac{3}{w^2} - \frac{e^{-j2w}}{w^2} - \frac{2}{w^2} \times e^{jw}$$

SORU; $x(t) = \underbrace{3\sin(6\pi 10^3 t + \pi/6)}_{x_1(t)} - \underbrace{j\cos(12\pi 10^3 t)}_{x_2(t)} + \underbrace{2}_{x_3(t)} \quad -\infty < t < \infty$ veriliyor.

a) $x(t)$-nin fourier dönüşümü $x(f)$'yi bulun.

b) iki yanlı genlik tayfı $|x(f)|$'yi ölçekli olarak çizin.

c) İki yanlı faz tayfı $\angle x(f)$'yi ölçekli olarak çizin.

a) $x_1(t) = \frac{3}{2j}\left(e^{j(6\pi 10^3 t + \pi/6)} - e^{-j(6\pi 10^3 t + \pi/6)}\right)$

$$\Rightarrow x_1(t) = \frac{3}{2j} \times e^{j\pi/6} \times e^{j6\pi 10^3 t} - \frac{3}{2j} \times e^{-j\pi/6} \times e^{-j6\pi 10^3 t}$$

$$\Rightarrow X_1(f) = \frac{3}{2}(-j)e^{j\pi/6} \times \delta(f - 3\times 10^3) + \frac{3}{2}(j) \times e^{-j\pi/6}\delta(f + 3\times 10^3)$$

$$x_2(t) = -\frac{j}{2} \times \left(e^{j12\pi \times 10^3 t} + e^{-j12\pi 10^3 t}\right)$$

$$\Rightarrow X_2(f) = -\frac{j}{2}\,\delta(f-6\times10^3) - \frac{j}{2}\,\delta(f+6\times10^3)$$

$$\Rightarrow x_3(t) = 2 \Rightarrow X_3(f) = 2\delta(f)$$

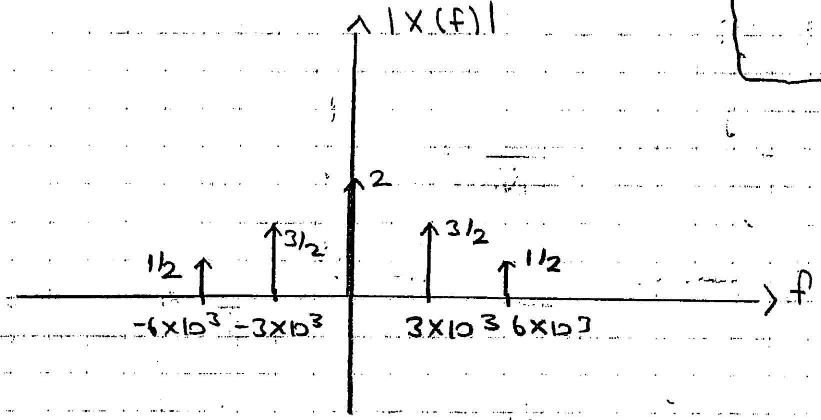
O hâlde $X(f) = X_1(f) + X_2(f) + X_3(f)$

$$\Rightarrow X(f) = \frac{3}{2}(-j)\,e^{j\pi/6}\,\delta(f-3\times10^3) + \frac{3}{2}(j)\,e^{-j\pi/6}\,\delta(f+3\times10^3)$$

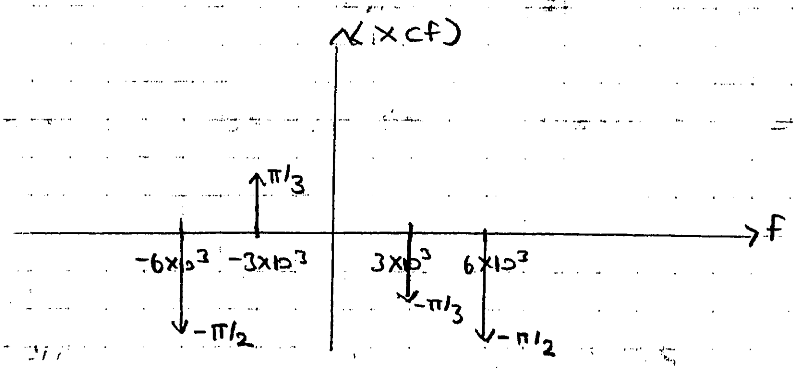
$$-\frac{j}{2}\,\delta(f-6\times10^3) - \frac{j}{2}\,\delta(f+6\times10^3) + 2\delta(f)$$

$$\boxed{* \; 2e^{-j0t} = 4\pi\,\delta(\omega) = 2\,\delta(f)}$$

b)



c)



* Ters fourier dönüşümü

$$F(t) = \int_{-\infty}^{\infty} F(f) \cdot e^{j2\pi f t}\, df$$

$$\Rightarrow x(t) = \int_{-\infty}^{\infty} 2\delta(f) \cdot e^{j2\pi f t}\, df = 2\, e^{j2\pi t 0} = \underline{\underline{2}}$$

SORU: Bir sistemin girişi x(t) ve çıkışı y(t) arasındaki ilişki $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$ diferansiyel denklemi ile verilmektedir.

a) Sistemin frekans transfer fonksiyonunu bulunuz.

b) Sistemin girişindeki işaret $x(t) = 5\sin(5t+20°) + 2\cos(2t-50°)$ ise çıkış işareti y(t)'yi H(w)'dan yararlanarak bulun.

a) $$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$\Rightarrow (jw)^2 Y(w) + 3(jw)Y(w) + 2Y(w) = X(w)$$

$$\Rightarrow \boxed{H(w) = \frac{Y(w)}{X(w)} = \frac{1}{(jw)^2 + 3jw + 2}}$$

b)
* İlk olarak x(t)'nin fourier serisi katsayılarını bulalım;

$$x(t) = \frac{5}{2j}\left(e^{j(5t+20)} - e^{-j(5t+20°)}\right) + e^{j(2t-50)} + e^{-j(2t-50°)}$$

$$\Rightarrow x(t) = \frac{5}{2j} \times e^{j20} \times e^{j5t} - \frac{5}{2j} \times e^{-j20} \times e^{j5t} + e^{-j50} \times e^{j2t} + e^{j50} \times e^{-j2t}$$

$$\Rightarrow X_2 = \frac{5}{2j} e^{j20}, \quad w=5 \qquad \Rightarrow X_1 = e^{-j50}, \quad w=2$$

$$\Rightarrow X_{-2} = \frac{-5}{2j} e^{-j20}, \quad w=-5 \qquad \Rightarrow X_{-1} = e^{j50}, \quad w=-2$$

O halde;
$$H(5) = \frac{1}{-23+j15} = \frac{-23-j15}{754} = 0{,}0364 \times e^{-j146{,}88}$$

$$H(-5) = \frac{1}{-23-j15} = \frac{-23+j15}{754} = 0{,}0364 \times e^{j146{,}88}$$

$$H(2) = \frac{1}{-2+j6} = \frac{-2-j6}{40} = 0{,}158 \times e^{-j108{,}44}$$

$$H(-2) = \frac{1}{-2-j6} = \frac{-2+j6}{40} = 0{,}158 \times e^{j108{,}44}$$

$Y_k = H(k\omega_0) X_k$

$$\Rightarrow Y_k = \frac{-5}{2j} e^{-j20} \times 0{,}0364 \times e^{j146{,}88} + e^{j50} \times 0{,}158 \times e^{j108{,}44}$$

$$+ e^{-j50} \times 0{,}158 \times e^{-j108{,}44} + \frac{5}{2j} e^{j20} \times 0{,}0364 \times e^{-j146{,}88}$$

$$\Rightarrow Y_{-2} = \frac{-0{,}182}{2j} e^{j126{,}88}, \quad Y_{-1} = 0{,}158\, e^{j158{,}44}, \quad Y_1 = 0{,}158\, e^{-j158{,}44}$$

$$Y_2 = \frac{0{,}182}{2j} e^{-j126{,}88}$$

ve $y(t) = \sum_{k=-2}^{2} Y_k \cdot e^{j\omega k t} \quad (k \neq 0)$

$$\Rightarrow y(t) = -\frac{0{,}182}{2j} e^{j126{,}88} e^{-j5t} + 0{,}158\, e^{j158{,}44} \times e^{-j2t} + 0{,}158 \times e^{-j158{,}44}$$

$$\times e^{j2t} + \frac{0{,}182}{2j} \times e^{-j126{,}88} \times e^{j5t}$$

$$\Rightarrow \quad y(t) = 0{,}182 \times \sin(5t - 126{,}88) + 0{,}316 \times \cos(2t - 158{,}44)$$

SORU; Bir sistemin birim vuruş tepkesi $h(t) = [e^{-t} - e^{-2t}]u(t)$'dir

a) Sistemin frekans transfer fonksiyonunu bulunuz.

b) Sistemin girişindeki işaret $x(t) = 5\sin(t+10^\circ) + 3\cos(2t-40^\circ)$ ise çıkış işareti $y(t)$'yi $H(\omega)$'dan yararlanarak bulun.

a) Transfer fonksiyonu $H(\omega)$'yı bulalım;

$$h(t) = (e^{-t} - e^{-2t})u(t) \Rightarrow H(\omega) = \int_0^\infty (e^{-t} - e^{-2t}) \cdot e^{-j\omega t}\, dt$$

$$\Rightarrow H(\omega) = \int_0^\infty e^{-(1+j\omega)t}\, dt - \int_0^\infty e^{-(2+j\omega)t}\, dt$$

$$\Rightarrow H(\omega) = \frac{-1}{1+j\omega} e^{-(1+j\omega)t}\Big|_0^\infty + \frac{1}{2+j\omega} e^{-j(2+j\omega)t}\Big|_0^\infty$$

$$\Rightarrow \boxed{H(\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}}$$

b) x(t)'nin fourier serisi katsayılarını bulalım:

$$x(t) = 5\sin(t+10^\circ) + 3\cos(2t-40^\circ)$$

$$\Rightarrow x(t) = \frac{5}{2j}\left(e^{j(t+10^\circ)} - e^{-j(t+10^\circ)}\right) + \frac{3}{2}\left(e^{j(2t-40^\circ)} + e^{-j(2t-40^\circ)}\right)$$

$$\Rightarrow x(t) = \underbrace{\frac{5}{2j}e^{j10}\cdot e^{jt}}_{w=1} - \underbrace{\frac{5}{2j}e^{-j10}\cdot e^{-jt}}_{w=-1} + \underbrace{\frac{3}{2}e^{-j40}\cdot e^{j2t}}_{w=2} + \underbrace{\frac{3}{2}e^{j40}\cdot e^{-j2t}}_{w=-2}$$

$$y_1 \begin{cases} X_1 = \frac{5}{2j}e^{j10}, \quad w=1 \\ H(1) = \frac{1}{1+j} - \frac{1}{2+j} = \frac{1-j}{2} - \frac{2-j}{5} = \frac{1-j3}{10} = 0{,}316 \times e^{-j71{,}56} \end{cases}$$

$$y_{-1} \begin{cases} X_{-1} = -\frac{5}{2j}e^{-j10}, \quad w=-1 \\ H(-1) = \frac{1}{1-j} - \frac{1}{2-j} = \frac{1+j}{2} - \frac{2+j}{5} = \frac{1+j3}{10} = 0{,}316 \times e^{j71{,}56} \end{cases}$$

$$y_2 \begin{cases} X_2 = \frac{3}{2}e^{-j40}, \quad w=2 \\ H(2) = \frac{1}{1+j2} - \frac{1}{2+j2} = \frac{1-j2}{5} - \frac{2-j2}{8} = \frac{-2-j6}{40} = 0{,}158 \times e^{-j108{,}43} \end{cases}$$

$$y_{-2} \begin{cases} X_{-2} = \frac{3}{2}e^{j40}, \quad w=-2 \\ H(-2) = \frac{1}{1-j2} - \frac{1}{2-j2} = \frac{1+j2}{5} - \frac{2+j2}{8} = \frac{-2+j6}{40} = 0{,}158 \times e^{j108{,}43} \end{cases}$$

$*\ y_1 = \frac{1{,}58}{2j}e^{-j61{,}56}$ $\quad *\ y_{-1} = \frac{-1{,}58}{2j}e^{j61{,}56}$

$*\ y_2 = \frac{0{,}474}{2}e^{-j148{,}43}$ $\quad *\ y_2 = \frac{0{,}474}{2}e^{j148{,}43}$

$$y(t) = \sum_{k=-2}^{2} y_k \cdot e^{jw_k t} \quad (k \neq 0)$$

o hâlde: $y(t)=\frac{1,58}{2j}e^{-j61,56}e^{jt}-\frac{1,58}{2j}e^{j61,56}e^{-jt}+\frac{0,474}{2}e^{-j148,43}\cdot e^{j2t}$

$+\frac{0,474}{2}e^{j148,43}\cdot e^{-j2t}$

$$\Rightarrow \boxed{y(t)=1,58\sin(t-61,56)+0,474\cos(2t-148,43^\circ)}$$

SORU! $x(t)=\sin(2\pi 10^3 t-\pi/6)+u(t)-u(t-1)$; $-\infty<t<\infty$ sinyali veriliyor. $x(t)$-nin fourier dönüşümü $X(f)$'yi bulunuz.

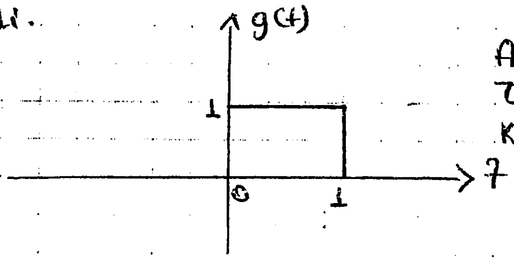
* $x(t)=f(t)+g(t)$ olsun. $f(t)=\sin(2\pi 10^3 t-\pi/6)$, $g(t)=u(t)-u(t-1)$

$f(t)=\left[e^{j(2\pi 10^3 t-\pi/6)}-e^{-j(2\pi 10^3 t-\pi/6)}\right]/2j$

$\Rightarrow f(t)=\frac{1}{2}(-j)e^{-j\pi/6}e^{j2\pi 10^3 t}+\frac{1}{2}(j)e^{j\pi/6}e^{-j2\pi 10^3 t}$

$$\Rightarrow \boxed{F(f)=\frac{1}{2}(-j)e^{-j\pi/6}\delta(f-10^3)+\frac{1}{2}(j)e^{j\pi/6}\delta(f+10^3)}$$

$g(t)=u(t)-u(t-1)$ -idi.



A:1
τ:1
K.M: 1/2

$\Big\}\ G(f)=\text{sinc}(f).e^{-j\omega/2}$

o hâlde: $X(f)=\frac{1}{2j}e^{-j\pi/6}\delta(f-10^3)-\frac{1}{2j}e^{j\pi/6}\delta(f+10^3)+\text{sinc}(f)e^{-j\omega/2}$

SORU! $x(t)=3\sin(6\pi 10^3 t+\pi/6)+j\cos(12\pi 10^3 t)+2$ $-\infty<t<\infty$ sinyali veriliyor.

a). $x(t)$-nin fourier dönüşümü $X(f)$-i bulunuz.

b) $x(t)$-nin ortalama gücünü zaman ve frekans domeninde ayrı ayrı hesaplayarak parseval teoreminin geçerli olduğunu kanıtlayınız.

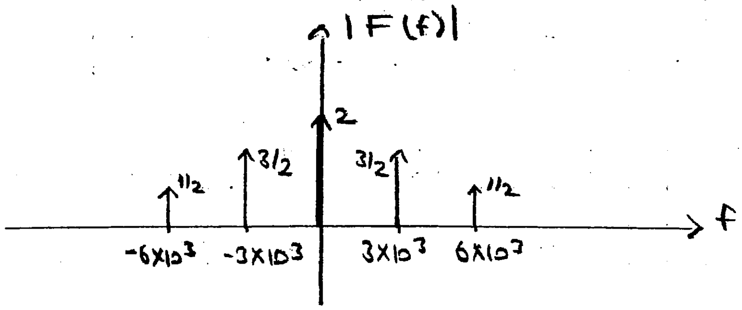
a) $x(t)=\frac{3}{2j}e^{j(6\pi 10^3 t+\pi/6)}-\frac{3}{2j}e^{-j(6\pi 10^3 t+\pi/6)}+\frac{j}{2}e^{j(12\pi 10^3 t)}+\frac{j}{2}e^{-j(12\pi 10^3 t)}$

$+2$

$\Rightarrow X(f)=\frac{3}{2j}e^{j\pi/6}\delta(f-3\times 10^3)-\frac{3}{2j}e^{-j\pi/6}\delta(f+3\times 10^3)+\frac{j}{2}\delta(f-6\times 10^3)$

$+\frac{j}{2}\delta(f+6\times 10^3)+2\delta(f)$

b) zaman domeninde ortalama güç F_k'ların genlik kareleri toplamıdır. o hâlde $P_t = \frac{9}{4} + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + 4 = 9$ //



Pf = Genlik kareler toplamı

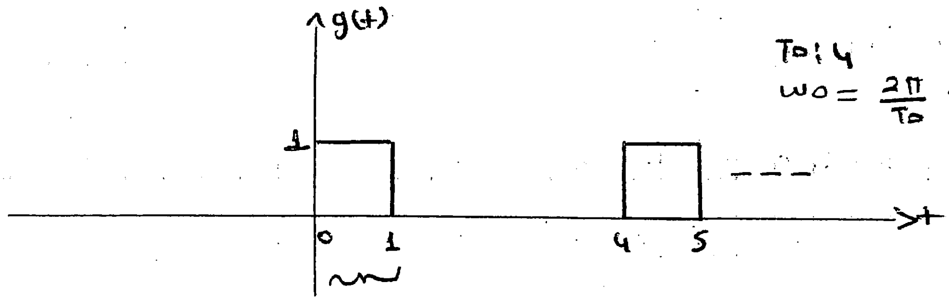
$Pf = 9/4 \times 2 + \frac{1}{4} \times 2 + 4 = 9 \Rightarrow P_t = P_f$

SORU! Fourier serisi ile fourier dönüşümü arasındaki ilişkiden yararlanarak

a) $g(t) = \sum_{n=-\infty}^{\infty} (u(t-4n) - u(t-1-4n))$ periyodik işaretinin üstel fourier serisi katsayılarını bulun. (G_k)

b) Bulduğunuz G_k lardan yararlanarak, genlik ve faz tayfını k'nın fonksiyonu olarak ölçekli çiziniz.

a)



$T_0 = 4$

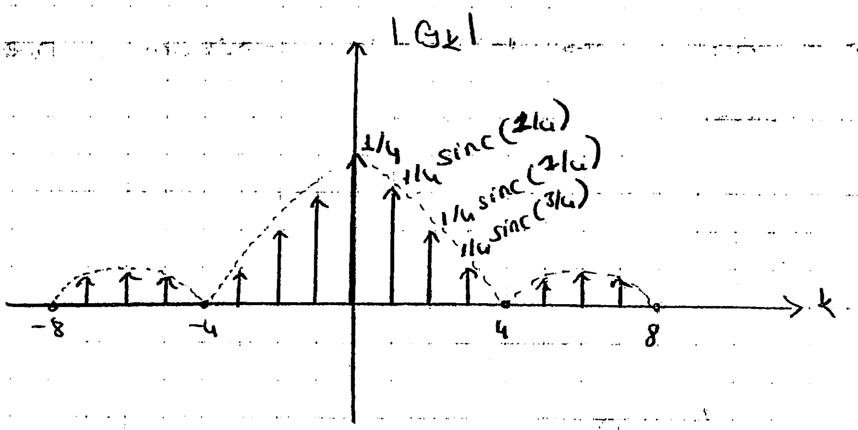
$w_0 = \frac{2\pi}{T_0} = \pi/2$

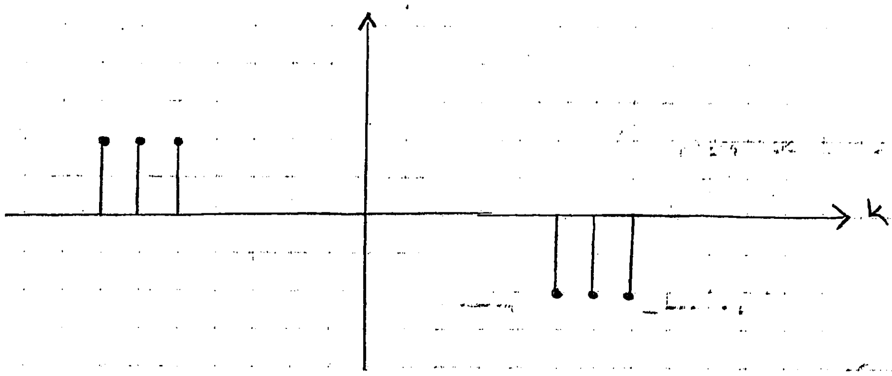
$f_0 = 1/4$

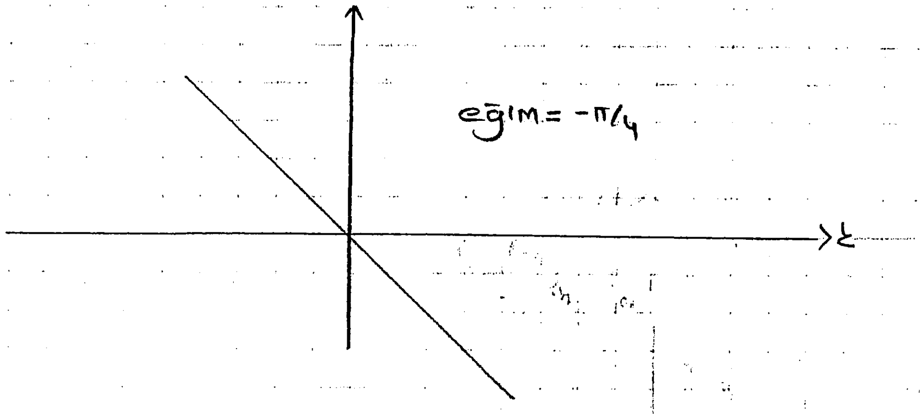
$A=1,\ \tau=1,\ \text{kim}=1/2$

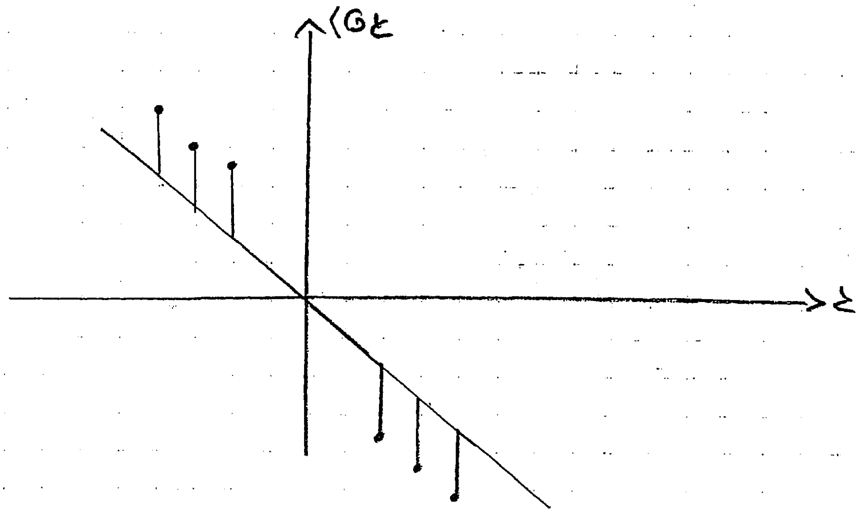
$G(f) = \text{sinc}(f)\, e^{-jw/2}$; $G_k = \frac{1}{T_0} G(kw_0)$

$\Rightarrow G_k = \frac{1}{4} \text{sinc}(k/4)\, e^{-j\pi k/4}$


|Gk|
1/4
1/4 sinc(1/4)
1/4 sinc(2/4)
1/4 sinc(3/4)
k
-8
-4
4
8


k


eğim = -π/4
k


∠Gk
k

SORU; Fourier serisi ve fourier dönüşümü arasındaki ilişkiden yararlanarak

a) $g(t) = \sum_{n=-\infty}^{\infty} \left(u(t+2-6n) - u(t-6n)\right)$ periyodik işaretinin fourier serisi katsayıları G_k'ları bulunuz.

b) Bulduğunuz G_k'lardan yararlanarak genlik ve faz tayfını k-nın fonksiyonu olarak ölçekli olarak çiziniz.

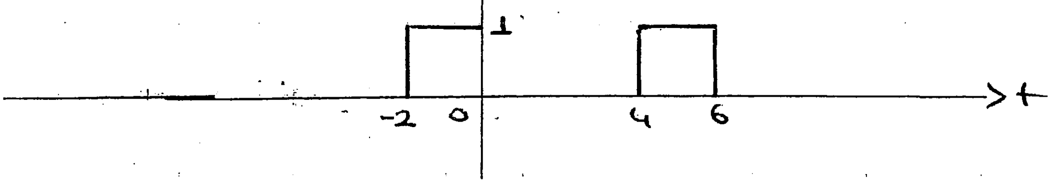
a)

$* A=1 \quad * \tau=2 \quad * k_m=-1$

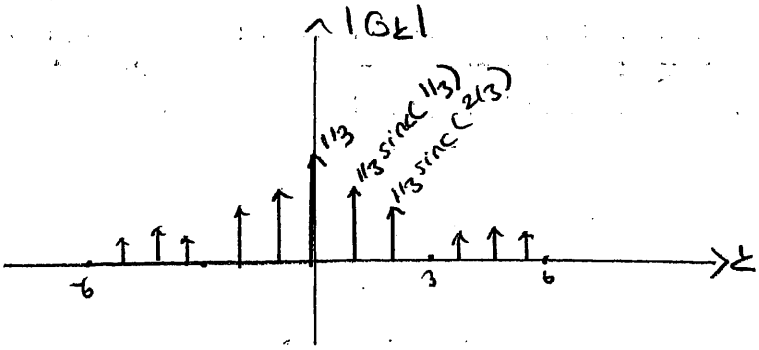
$\Rightarrow G(w) = 2\,\text{sinc}(2f) \cdot e^{jw}$

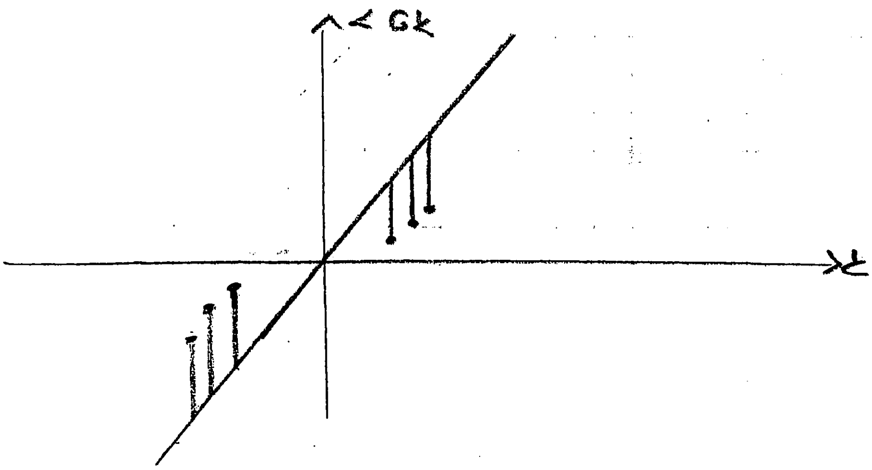
$T_0 = 6 \; ; \; w_0 = \frac{2\pi}{6} = \pi/3$

$\Rightarrow f_0 = 1/6$

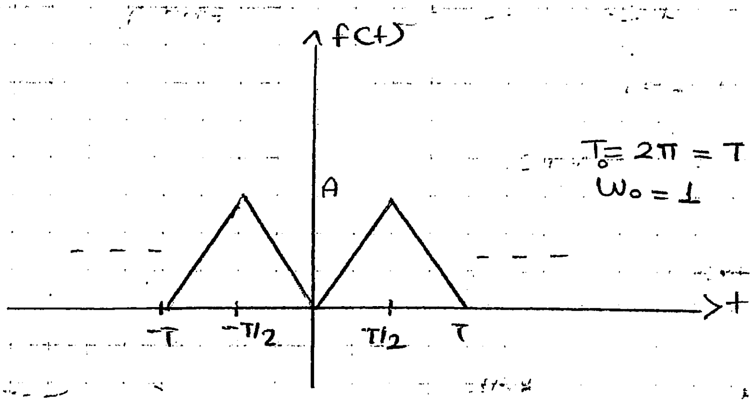


$\Rightarrow \quad G_k = \frac{1}{3}\,\text{sinc}(k/3) \cdot e^{jk\pi/3}$ eğim: $\pi/3$





SORU:



$T_0 = 2\pi = T$
$\omega_0 = 1$

* Üstel fourier serisi katsayılarını bulalım:

$$F_k = \frac{1}{T_0}\int_{T_0} f(t).e^{-jkw_0t}\,dt$$

$$\Rightarrow F_k = -\frac{1}{2\pi}\times\frac{A}{\pi}\int_{-\pi}^{0} te^{-jkt}\,dt + \frac{1}{2\pi}\times\frac{A}{\pi}\int_{0}^{\pi} te^{-jkt}\,dt$$

$t = u \Rightarrow dt = du$

$e^{-jkt}dt = dv \Rightarrow v = \frac{-1}{jk}e^{-jkt}$

$$\Rightarrow F_k = \frac{-A}{2\pi^2}\left(\frac{-t}{jk}e^{-jkt}\Big|_{-\pi}^{0} + \frac{1}{jk}\int_{-\pi}^{0} e^{-jkt}\,dt\right) + \frac{A}{2\pi^2}\left(\frac{-t}{jk}e^{-jkt}\Big|_{0}^{\pi} + \frac{1}{jk}\int_{0}^{\pi} e^{-jkt}\,dt\right)$$

$$\Rightarrow F_k = \frac{-A}{2\pi^2}\times\frac{(-\pi)}{jk}e^{j\pi k} - \frac{A}{k\pi^2 j2}\times\frac{(-1)}{jk}e^{-jkt}\Big|_{-\pi}^{0} - \frac{A\pi}{k\pi^2 j2}e^{-j\pi k}$$

$$+ \frac{A}{k\pi^2 j2}\times\frac{(-1)}{jk}e^{-jkt}\Big|_{0}^{\pi}$$

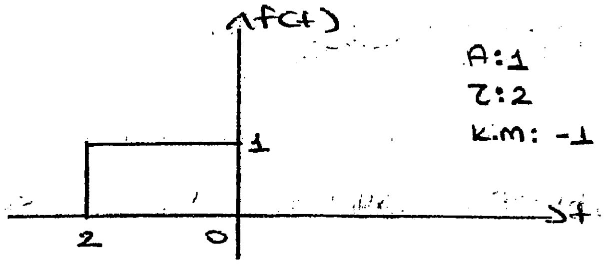
$$\Rightarrow F_k = \frac{A\pi}{k\pi^2}\sin\pi k + \frac{(-A)}{2\pi^2k^2}\left(1 - e^{j\pi k}\right) + \frac{A}{2\pi^2k^2}\left(e^{-j\pi k} - 1\right)$$

$$\Rightarrow F_k = A\,\text{sinc}(k) + \frac{A}{2\pi^2k^2}\left(e^{j\pi k} + e^{-j\pi k}\right) - \frac{A}{\pi^2k^2}$$

$$\Rightarrow F_k = A\,\text{sinc}(k) + \frac{A}{\pi^2k^2}\cos\pi k - \frac{A}{\pi^2k^2}$$

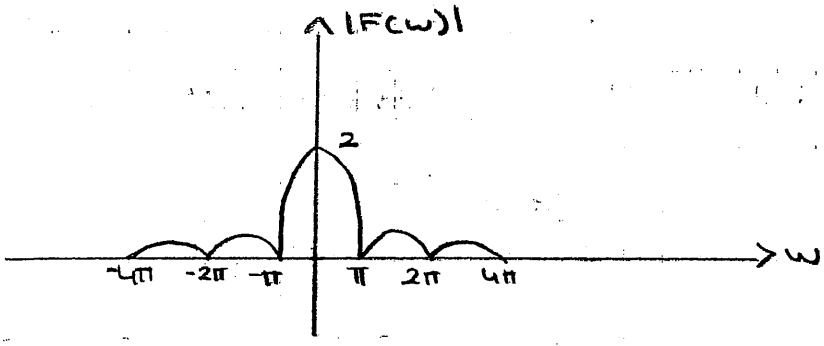
$$\Rightarrow f(t) = \sum_{k=-\infty}^{\infty}\frac{A}{2\pi^2k^2}\left(-2 + 2\cos\pi k + 2k\pi\sin(\pi k)\right).e^{jkt}$$ olur!

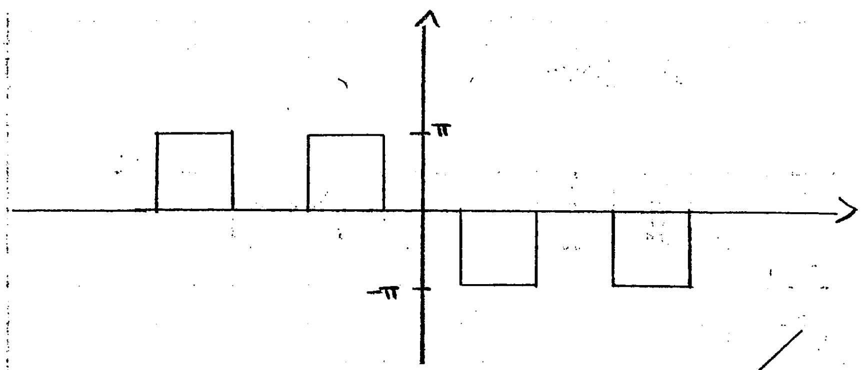
SORU| $f(t) = u(t+2) - u(t)$ işaretinin fourier dönüşümü $F(w)$-yı bularak iki yanlı genlik ve faz tayfını çiziniz

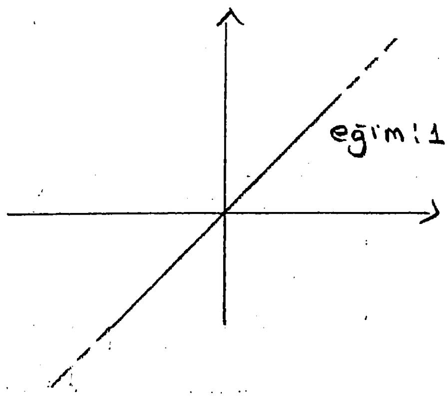


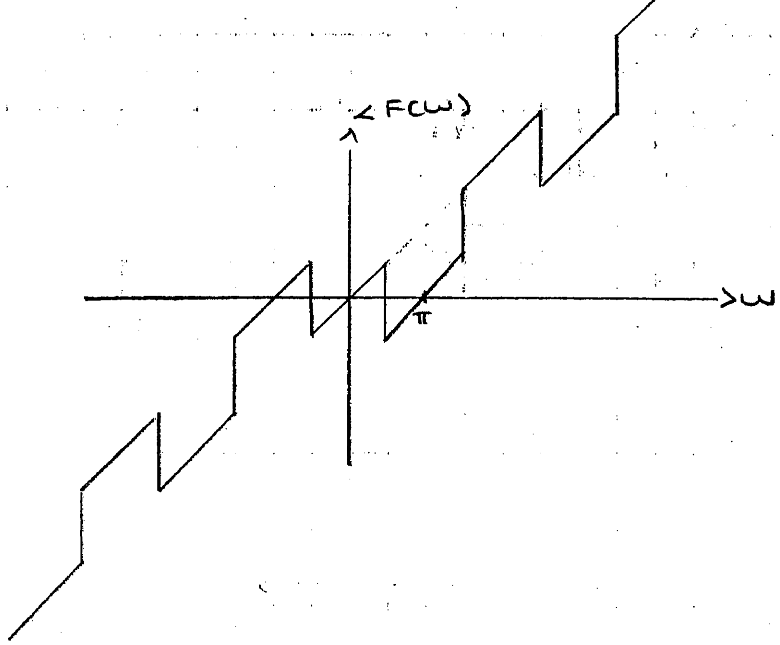
A:1
τ:2
k.m: -1

$$\Rightarrow F(w) = \underbrace{2\,\mathrm{sinc}(2f)\cdot e^{jw}}_{2\,\mathrm{sinc}\left(\frac{w}{\pi}\right)e^{jw}}$$









SORU; $x(t) = 3\sin(6\pi 10^3 t + \pi/6) - j\cos(12\pi 10^3 t) + 2, \quad -\infty < t < \infty$

sinyali veriliyor.

a) $x(t)$-nin fourier dönüşümü $X(f)$'yi bulunuz.

b) iki yanlı genlik tayfı $|X(f)|$'i ölçekli olarak çiziniz.

c) iki yanlı faz tayfı $\angle X(f)$'yi ölçekli olarak çiziniz.

d) a'da verilen $x(t)$ sinyali, frekans transfer fonksiyonu $H(f) = \dfrac{1}{1+j2\pi 10^{-4} f}$ olan bir sistemin girişine veriliyor. Sistemin çıkışındaki sinyalinin ortalama gücünün girişteki sinyalin ortalama gücüne oranını bulunuz.

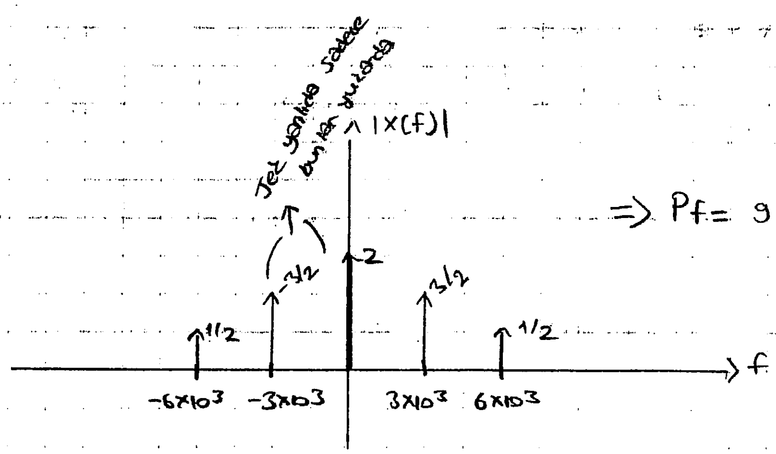
a) $$x(t) = \frac{3}{2j} e^{j(6\pi 10^3 t + \pi/6)} - \frac{3}{2j} e^{-j(6\pi 10^3 t + \pi/6)}$$

$$-\frac{j}{2} e^{j(12\pi 10^3 t)} - \frac{j}{2} e^{-j(12\pi 10^3 t)} + 2$$

$$\Rightarrow X(f) = \frac{3}{2}(-j) e^{j\pi/6} \delta(f - 3\times 10^3) + \frac{3}{2}(j) e^{-j\pi/6} \delta(f + 3\times 10^3)$$

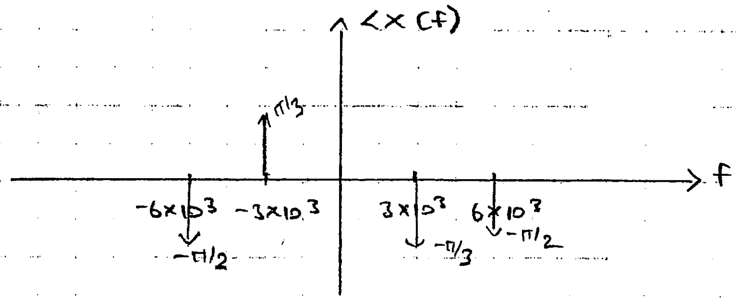
$$-\frac{j}{2}\delta(f - 6\times 10^3) - \frac{j}{2}\delta(f + 6\times 10^3) + 2\delta(f)$$

b)



$\Rightarrow P_f = 9$ //

c)



d) x(t)'nin üstel fourier serisi katsayılarını bulalım.

$$x(t) = \frac{3}{2j} e^{j\pi/6} \times \underbrace{e^{j6\pi 10^3 t}}_{f=3\times10^3} - \frac{3}{2j} e^{-j\pi/6} \times \underbrace{e^{-j6\pi 10^3 t}}_{f=-3\times10^3}$$

$$-j/2 \underbrace{e^{j12\pi 10^3 t}}_{f=6\times10^3} - \frac{j}{2} \underbrace{e^{-j12\pi 10^3 t}}_{f=-6\times10^3} + \underbrace{2e^{j0t}}_{f=0}$$

$$X_{-2} = -j/2 \ , \quad X_{-1} = \frac{3j}{2} e^{-j\pi/6} \ , \ X_1 = -\frac{3}{2} j e^{j\pi/6} \ , \ X_2 = -j/2 \ , \ X_0 = 2$$

o hâlde $Y_{-2} = -\frac{j}{2} \times \frac{1}{j-1{,}2\pi} = -\frac{j}{2} \ 0{,}256 \times e^{j75{,}14}$

$Y_2 = -\frac{j}{2} \times \frac{1}{j+1{,}2\pi} = -\frac{j}{2} \times 0{,}256 \times e^{-j75{,}14}$ $\qquad Y_0 = 2$

$Y_{-1} = \frac{-3}{2j} \times e^{-j30} \times \frac{1}{j-0{,}6\pi} = \frac{-1{,}407}{2j} e^{j32{,}05}$

$Y_1 = \frac{3}{2j} \times e^{j30} \times \frac{1}{j+0{,}6\pi} = \frac{1{,}407}{2j} \times e^{-j32{,}05}$

o hâlde $y(t) = \sum_{k=-2}^{2} Y_k . e^{jw_k t}$

$$\Rightarrow y(t) = \left(0{,}256 \times e^{j75{,}14} \times e^{-j12\pi 10^3 t} + 0{,}256 \times e^{-j75{,}14} \times e^{j12\pi 10^3 t}\right)/2j$$
$$+ \left(-\frac{1{,}407}{2j} e^{j32{,}05} e^{-j6\pi 10^3 t} + \frac{1{,}407}{2j} e^{-j32{,}05} \times e^{j6\pi 10^3 t}\right) + 2$$

$$\Rightarrow y(t) = 0{,}256\ (-j)\ \cos(12\pi 10^3 t - 75{,}14) + 1{,}407 \sin(6\pi 10^3 t - 32{,}05) + 2$$

$$\Rightarrow P = \left(\frac{16}{125}\right)^2 \times 2 + \frac{(1{,}407)^2}{2} + 4 \Rightarrow P = 5 \Rightarrow \text{Oran} = 1{,}8$$

SORU | Fourier dönüşümü

$$F(f) = \frac{1}{2j} \delta(f-\sqrt{3}) e^{-j\pi/6} - \frac{1}{2j} \delta(f+\sqrt{3}) e^{j\pi/6} + \delta(f-5) + \delta(f+5)$$

olarak bir sinyal veriliyor.

a) Üstel Fourier serisi katsayıları F_k'ları bulun.

b) İki yanlı genlik tayfını ve iki yanlı faz tayfını çizin.

c) Trigonometrik fourier serisi katsayıları a_k ve b_k ları bulun.

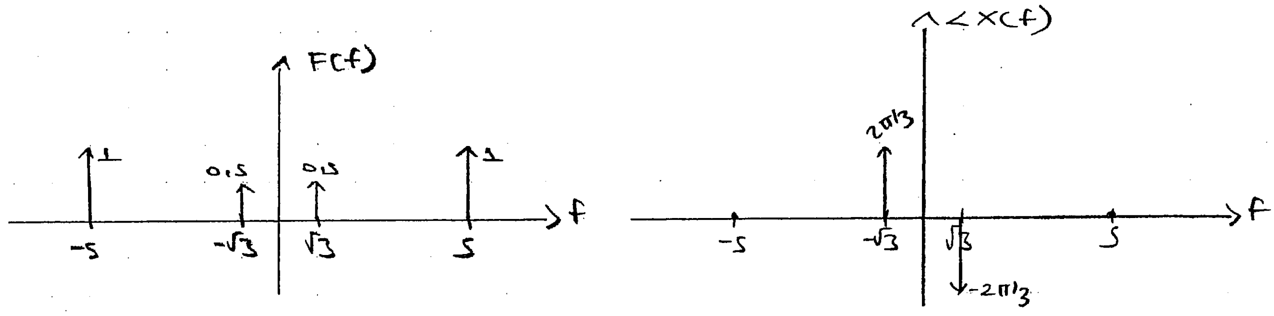
d) Tek yanlı genlik ve faz tayfını çizin.

e) F(f)'nin ters fourier dönüşümü f(t)'yi bulun.

f) Parseval teoreminden yararlanarak bu sinyalin gücünü zaman domeninde ve frekans domeninde ayrı ayrı hesaplayınız.

a) $F_1 = \frac{1}{2j} e^{-j\pi/6}$, $F_{-1} = \frac{-1}{2j} e^{j\pi/6}$, $F_2 = 1$, $F_{-2} = 1$

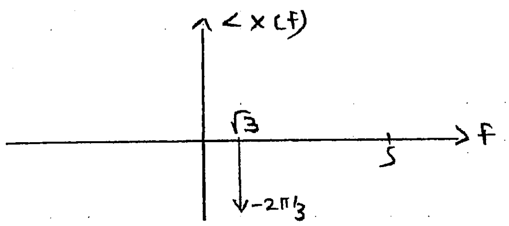
b)



c) $F_k = \frac{a_k - jb_k}{2} \Rightarrow a_k = 2\,Re\{F_k\}$

$b_k = 2jF_k = 2\,Im\{F_k\}$

d)



e) $F(f) = \frac{1}{2j}\delta(f-\sqrt{3})\,e^{-j\pi/6} - \frac{1}{2j}\delta(f+3)\,e^{j\pi/6}$

$+\delta(f-5) + \delta(f+5) \Rightarrow$

$$\Rightarrow f(t) = \frac{1}{2j} e^{j2\pi\sqrt{3}t} e^{-j\pi/6} - \frac{1}{2j} e^{-j2\pi\sqrt{3}t} e^{j\pi/6} + e^{j2\pi 5t} + e^{-j2\pi 5t}$$

$$\Rightarrow f(t) = \sin(2\pi\sqrt{3}t - \pi/6) + 2\cos(10\pi t)$$

f) $P_t = \frac{1}{4} + \frac{1}{4} + 1 + 1 = 2.5$ (Fourier serisi kat. kar. top)

$P_t = \frac{1}{4} + \frac{1}{4} + 1 + 1 = 2.5$ $\left(\sum_{k=-2}^{2} |F(f)|^2\right)$