Final project, deadline for code and write-up: December 11, 2012 at 9:45
Deadline for presentation (in pdf form): the time of your presentation.

Part I.1

Program a single cylindrical object in a field of uniform flow. Write the program in Matlab®, capable of plotting the imaginary part of the complex potential, i.e., the stream function, for uniform flow of any given velocity $\underset{0}{v}$ in any direction α . The cylinder is to be placed at any point z_1 and has radius $\underset{1}{R}$. Produce contour plots for flow at an arbitrary angle of your choice, but for a non-integer number of degrees.

Part I.2

Modify your program to add a second cylinder. The second cylinder is centered at an arbitrary point z_2 and has radius $\underset{2}{R}$. Model the second cylinder by using only a single term in the expansion, i.e., a term of the form

$$\underset{2}{\Omega} = \underset{2}{\alpha}_1 \frac{1}{\underset{2}{Z}}$$

where $\underset{2}{\alpha}$ is a complex constant. Your solution will now have the form

$$\Omega = -\underset{0}{v}z + \underset{1}{\alpha}_1 \frac{1}{\underset{1}{Z}} + \underset{2}{\alpha}_1 \frac{1}{\underset{2}{Z}}$$

Determine the two constants such that you obtain the best solution, i.e., a solution for which the stream function along each cylinder wall is as close to a constant as possible. Choose the two values of the constants $\underset{1}{\alpha}_1$ and $\underset{2}{\alpha}_1$ for your first estimate such that the boundary condition would be met exactly along each cylinder wall, if the other one would not be present. Investigate how the solution becomes less accurate as the cylinders are moved closer together. Finally, develop a scheme for determining the optimum value such that the stream function along the two objects is close to a constant; we will define a precise criterium for this in part II.

Part 1.3

Show results graphically of your analysis done for part 1.2

Part II

Part II.1

Program the expansions for the individual cylinders, passing to your routine the coefficients in the expansions. Write a routine that sums the contributions of all cylinders.

Part II.2

Write a routine that represents the contributions due to all cylinders, except the one currently being considered, say cylinder m (i.e., the one whose coefficients you are solving for), in terms of an expansion about the center of the cylinder considered.

Part II.3

Write a routine that computes the coefficients in the expansion for each individual cylinder, valid outside the cylinder, in terms of the combined expansion of all other cylinders about the center of the cylinder considered.

Part II.4

Write the routine that will compute all unknown coefficients in your solution iteratively, by considering one cylinder at a time. Write your program such that a check is made as to whether the values of the coefficients change more than some pre-defined amount (e.g., 1 percent of the value of the each coefficient).

Part II.5

Make contour plots of the stream function (i.e., of the streamlines) for an assembly of cylinders of your choice. Verify visually, by plotting the cylinders as circles, that the boundaries of the cylinders are indeed streamlines.

Part II.6

Make contour plots of the pressure distribution for the assembly chosen for Part II.5.

Deliverable

Write a report describing your approach and the equations used; include in your report listings of your program (in an appendix), and the contour plots. Keep your report brief, including only the essential details and graphical results.

Create a run file (or several run files) which I can use to run your program. Please make sure that all necessary routines are included with your submission.