Groundwater Modeling Project 1

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Problem Statement

The attached data file contains the layout of a river and of a plot. You are required to design a well field for a city. The wells must all be inside the plot, and the wells must pump at 80% of the maximum discharge, that is, the discharge at which just no river water is captured. An observation well (marked observation well 1 on the data file) has been used to establish an average head value of 22 m. The average head measured at a second observation well that was installed inside the plot is 20.25 m.

Your first step is to estimate flow field without the wells operating, but with the river included in your solution. The coordinates of the two observation points mentioned above are included in the attached data file. You have three parameters to choose: the direction of the field of uniform flow, the constant in the solution, and the magnitude of the uniform flow field. You may estimate the direction of the field of uniform flow by determine the average orientation of the river and assume that the direction of the field of uniform flow is normal to the river.

The head along the river bank is 20 m, the aquifer is unconfined, and the hydraulic conductivity is 10 m/day. You must first determine an approximate maximum total discharge of the well field by hand, then use your Matlab program to determine the maximum discharge for two scenarios: with a single well, and with a maximum of five wells. You must also estimate the heads at each of the wells for all scenarios. The wells cannot be closer than 10 m from one another.

There are not many data available for this problem; it is therefore necessary to try to establish how much variation in the solution is possible without violating the data. The parameters that you can vary are the both the direction and the magnitude of the field of uniform flow. Please report on the range of possible solutions, and give a recommendation as to what measurement(s) you recommend be taken in addition to the sparse data available. Note that the reference point in the solution should be chosen as the point farthest away from the river.

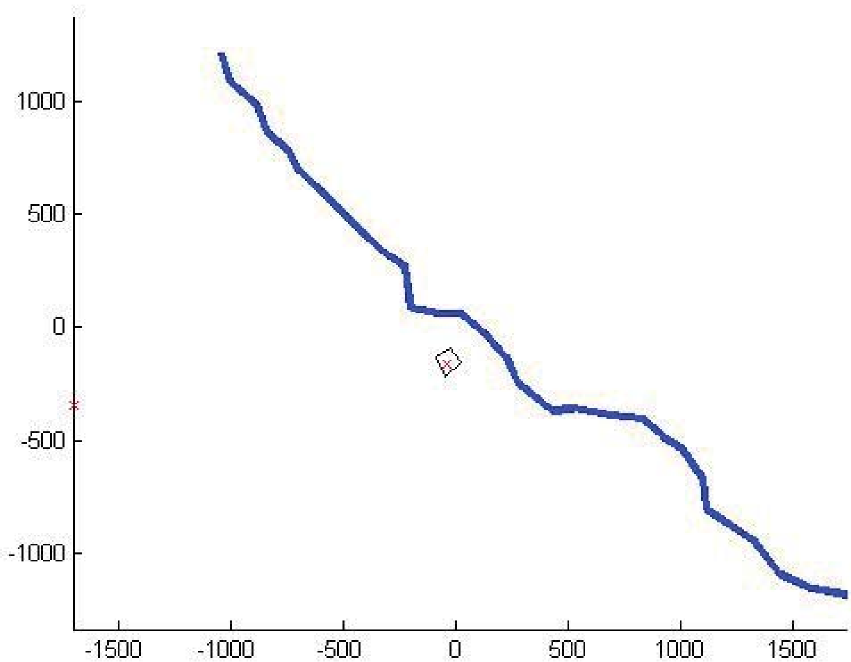

Figure 1: The field of interest. The blue line represents the river, the black polygon represents the well field, and the red x's show where the observation wells are.

Approximation of the Uniform Flow

Assuming that the orientation of the uniform flow is approximately perpendicular to the river, the first task becomes finding a good approximation of the river. This was done using a Microsoft Excel spreadsheet. The datafile that was attached to the project contained the locations of the start and end points of streams. These points were graphed and the line of best fit was found using Excel's regression line feature. The equation $y = -0.806x + 126.2801$ was found to be the line with the largest r^2 value: $r^2 = 0.97701$.

As a side note, this method is valid for a rough estimation because there are no reaches of the river that are much longer or much shorter than the rest of the reaches. If there were outliers in length, then the regression line would not be valid because in it, all points (and therefore reaches) have the exact same weight in calculating the r^2 value.

The slope of the line of best fit was found to be -0.806. This was converted into an angle and was added to $\frac{\pi}{2}$. In this way, the angle of the uniform flow was determined to be perpendicular to the simplified river. The angle was determined to be $\alpha = 0.8924$ radians or 51.1 degrees.

After the orientation of the uniform flow was determined, the magnitude of the uniform flow was calculated. This was done simply by finding the difference in potential between the distant observation well and the river and dividing by the distance between them.

$$Q_0 = \frac{\Phi_1 - \Phi_r}{dw1 - r}$$

The value for d_{w1-r} was found by inspection. The values for Φ_1 and Phi_r were computed by finding the potential, given the head in an unconfined aquifer:

$$\Phi = \frac{1}{2}k\phi^2$$

Where ϕ is the head in meters and k is the hydraulic conductivity.

The magnitude of the uniform flow was found to be 0.291 m/day. Along with the direction of uniform flow, this was all that was needed to find the complex potential from the uniform flow at any point in the system.

$$\Omega = -Q_0 e^{-\alpha i} z$$

Modeling the River

The river in this system is described by a series of points connected to form a line. Each of these lines (or "reaches") were divided up into sub-lines (or "segments"). Each line was modeled by a line-sink. The potential of a single line of unit strength is described as:

$$\Gamma(z) = \Re\left(\frac{L}{4\pi}\left[(Z+1)\ln(Z+1) - (Z-1)\ln(Z-1) + 2\ln\left(\frac{L}{2} - 2\right)\right]\right)$$

Where L is the length of the line-sink and Z is a dimensionless variable that is a function of the locations of the start and end points of the line-sink and the location at which the complex potential is to be computed.

$$Z = \frac{z - \frac{1}{2}(z_s + z_e)}{\frac{1}{2}(z_e - z_s)}$$

Where z_s is the start of the line-sink, z_e is the end of the line-sink, and z is the location at which the complex potential is to be computed.

The strength of a line-sink (or the infiltration caused by the line-sink) is represented by the symbol σ. Each segment of each reach will have a different strength that needs to be solved for. Because all the line-sinks influenced the potential at all the other line-sinks, it was necessary to set up a system of linear equations to solve for this system. The equation for a single line-sink in a field of uniform flow is expressed as:

$$\Omega = \sigma\Gamma + -Q_0 z e^{-i\alpha} + C$$

Where C is a constant that is added to the entire system.

Expanding this to multiple line-sinks, the system of equations in matrix form would look like this:

$$\begin{bmatrix} \Phi_r - \Phi_u(z_1) \\ \Phi_r - \Phi_u(z_2) \\ \Phi_1 - \Phi_u(z_3) \end{bmatrix} = \begin{bmatrix} \Gamma_1(z_1) & \Gamma_2(z_1) & 1 \\ \Gamma_1(z_2) & \Gamma_2(z_2) & 1 \\ \Gamma_1(z_3) & \Gamma_2(z_3) & 1 \end{bmatrix} * \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ C \end{bmatrix}$$

Where z_1, z_2, and z_3 are three locations at which the potential is known, Γ_1 and Γ_2 are the potentials for a line-sink of unit infiltration.

Each reach of the river was divided up into three segments, producing a total of 81 values of σ. The value for the constant, which is added to the potential at every point in the field, was found to be $C = 2040.7\text{m}^3/\text{day}$.

Adding a Single Well

Once the uniform flow, the constant, and the strengths of each river segment are determined, the well can be added. To begin with, the simplest situation was used. The additional well was placed at the reference point within the proposed well field. The shortest distance from the well to the river was determined. The orientation of the path from the well to the river was determined, by inspection, to be roughly the same as the orientation of the uniform flow. This meant that the orientation could be redefined such that uniform flow had only an x-component and the well could be placed on the x-axis at a distance d away from the river. d was defined as the distance found to be the shortest distance between the well and the river.

The resulting system could then be described with a well and a mirror well in a field of uniform flow.

$$\Phi = Q_{x0}x + \frac{Q}{2\pi}\ln\left((x - x_w) + y\right) - \frac{Q}{2\pi}\ln\left((x + x_w) + y\right)$$

The discharge was found using the following relationship:

$$Q_x = \frac{\partial \Phi}{\partial x}$$

The stagnation point will be located on the x-axis, so the value of y is set to 0.

$$Q_x = -Q_{x0} + \frac{Q}{2\pi}\frac{1}{x + d} - \frac{Q}{2\pi}\frac{1}{x - d}$$

Where Q is the discharge of the well, d is the location of the well, Q_x is the magnitude of the uniform flow, and x is the point at which the flow is to be calculated. When the well is drawing water up to the very edge of the river, the stagnation point will be located at $x = 0$, meaning that, at the maximum discharge, x will equal 0 and Q_x will equal 0. These two values were put into the previous equation and the equation was solved for Q. The initial value of Q was found to be $Q = 196.36$ m^3/day.

A well of discharge $Q = 196.36$ m^3/day was added to the field at the location where the observation well was. Figure 2 depicts this situation.

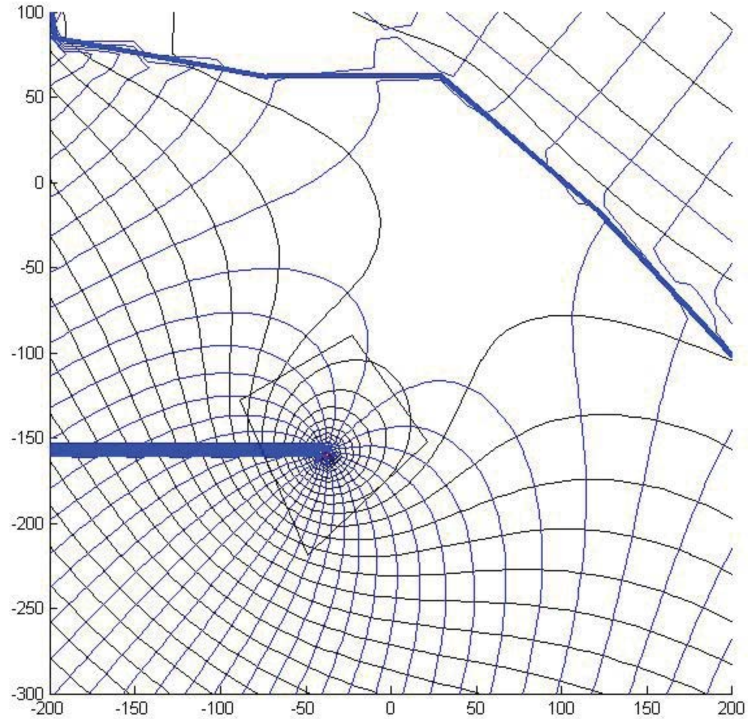

Figure 2: A single well at the location of the second observation well pumping at the estimated rate.

The flow lines in blue do not reach the river. This is a result of the approximation that the river is straight and that the uniform flow was parallel to the shortest distance between the observation well and the river. By varying the location of the well within the well field and the discharge of the well, the maximum discharge was found. Figure 3 shows the configuration and the flow net of the resulting scenario.

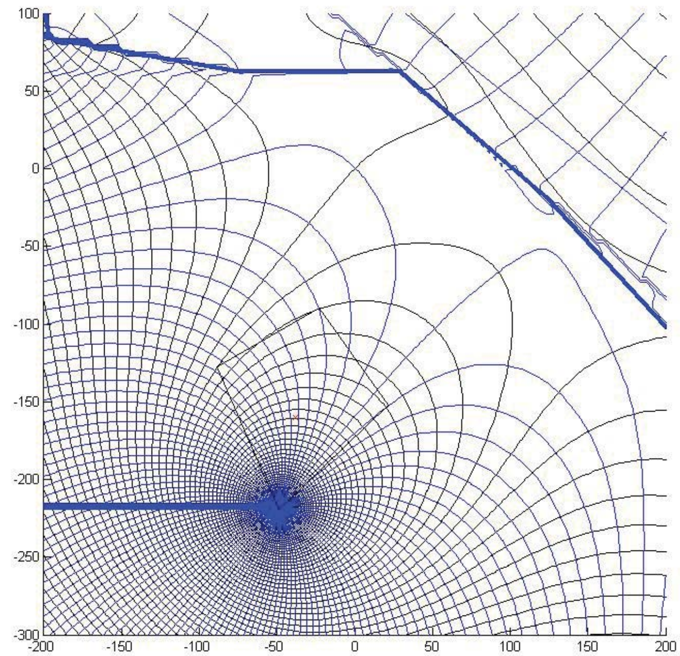

Figure 3: A single well at the optimal location pumping the maximum amount of water.

The discharge of the well shown in Figure 3 is $Q = 360\text{m}^3/\text{day}$. For safety reasons, the discharge of this well is taken to be only 0.8 times the maximum discharge. The new discharge is $Q = 288\text{m}^3/\text{day}$. Figure 4 shows this.

The head at this well was determined to be 20.35 m. This value is high, suggesting that draw-down of the aquifer will not be a major concern with this project.

Adding Multiple Wells

Once the single well was optimized, additional wells were added. The number, location, and discharge of these wells were varied until the optimal solution was obtained. The result was a system of three wells that pumped a total of 399.6 m^3/day between them. The reason this configuration was optimal, as opposed to one in which there were five wells, was because any additional wells would have to be placed nearer to the river. The closer a well is to the river, the more likely it is to draw water in. The trade-off was between pumping less water per well and placing wells closer to the river. The ideal configuration is shown in Figure 5.

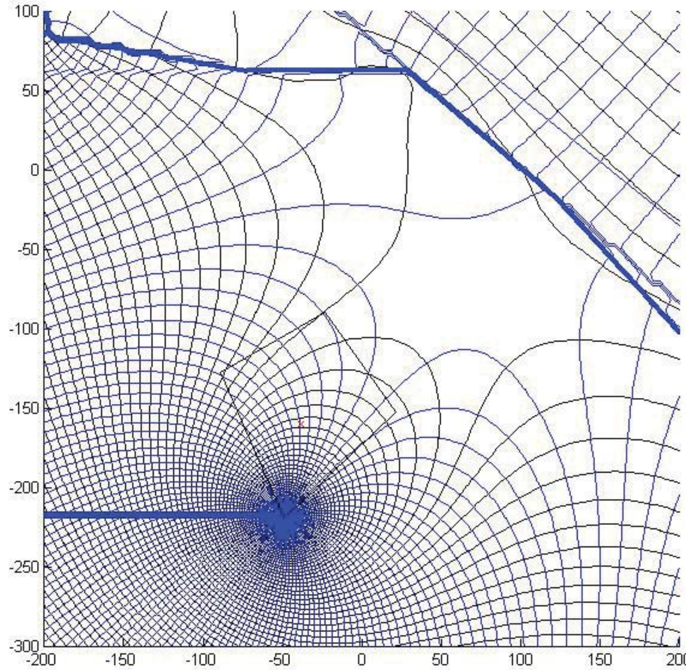

Figure 4: A single well pumping 0.8 times the maximum discharge.

Again, the total discharge of the wells needed to be 0.8 times the discharge at which the river water is just drawn in. Figure 6 shows this scenario.

The heads at these wells are 21.08m, 21.05m, and 21.1m. These numbers indicate that there is no problem with draw-down of the aquifer. This is an indication that the problem most likely to be facing us when pumping is drawing water from the river, which has already been seen.

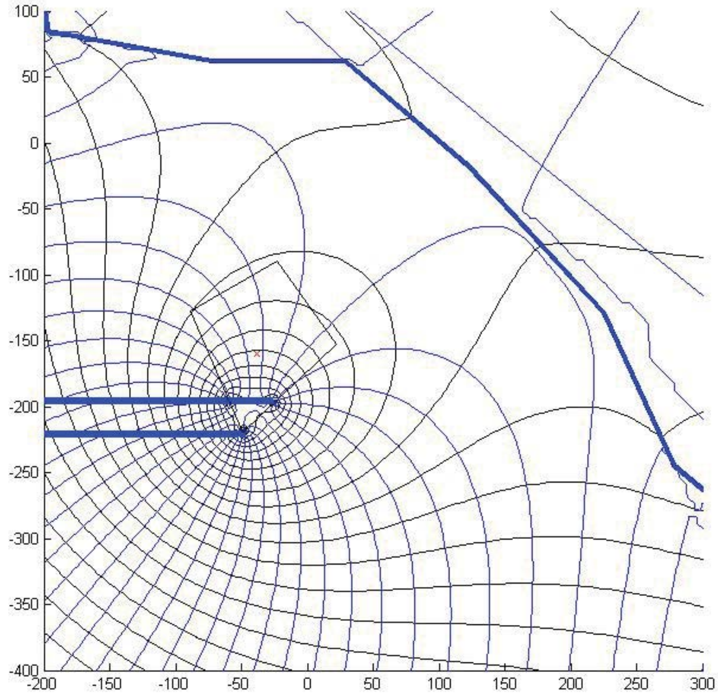

Figure 5: The optimal configuration of wells pumping the optimal amount of water.

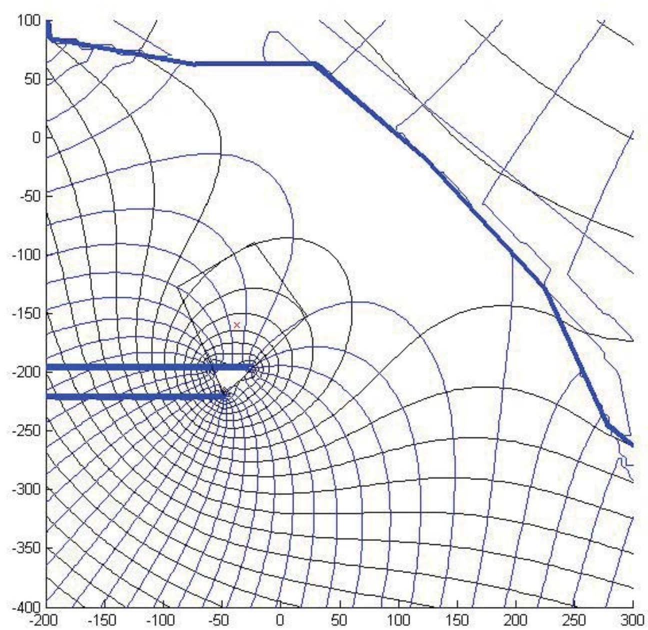

Figure 6: The optimal configuration of wells pumping 0.8 times the maximum discharge.

Variability

The values of the uniform flow and its orientation are only rough estimations. The best estimates of both values were presented in this report and used in the modeling of this system, but they are in no way certain. Therefore, an analysis of the variation of the values of the values of Q_0 and α was conducted and the results are presented in this section.

When the value of alpha is varied slightly, with the uniform flow staying at the same magnitude, there is a chance that river water will be drawn into the well. Figure 7 depicts the situation where the orientation is changed from 51.1 degrees to 56.1 degrees. This is only a very slight change, but, as can be seen in the figure, would result in river water being drawn into the well. Figure8 depicts the same situation with the orientation rotating in a different direction. In the clockwise direction, the orientation can vary until it reaches 25.1 degrees. At this point, water will be drawn from the river.

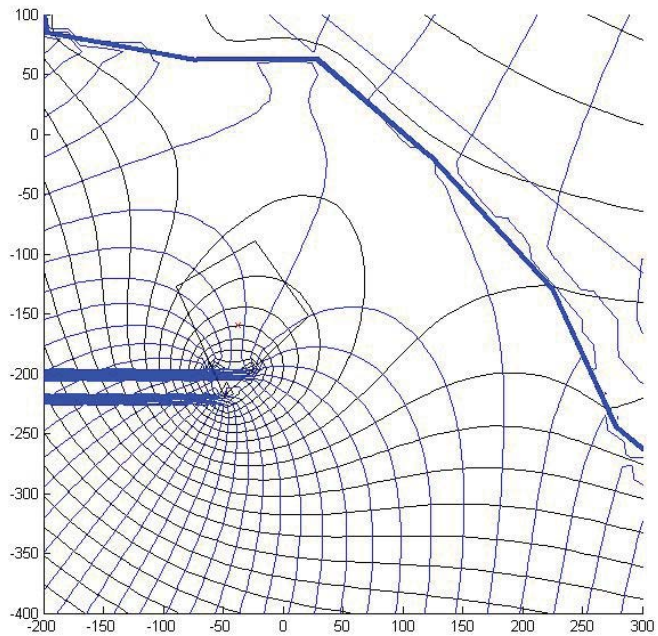

Figure 7: The scenario where the flow is oriented at 56.1 degrees from horizontal.

Similarly, we can examine the results of a small change in the magnitude of the uniform flow. If the magnitude of the uniform flow was 0.87 times the original magnitude found earlier in the project, the well field would draw water from the river. Therefore, the minimum value for the magnitude of the uniform flow is $Q = 0.253\text{m/day}$. Figure 9 shows the point at which the wells just start drawing river water.

There is little data available in this project. The best way to more accurately determine the magnitude of the uniform flow and its orientation would be to take two more head measurements. These measurements would be far away from the river, far away from each

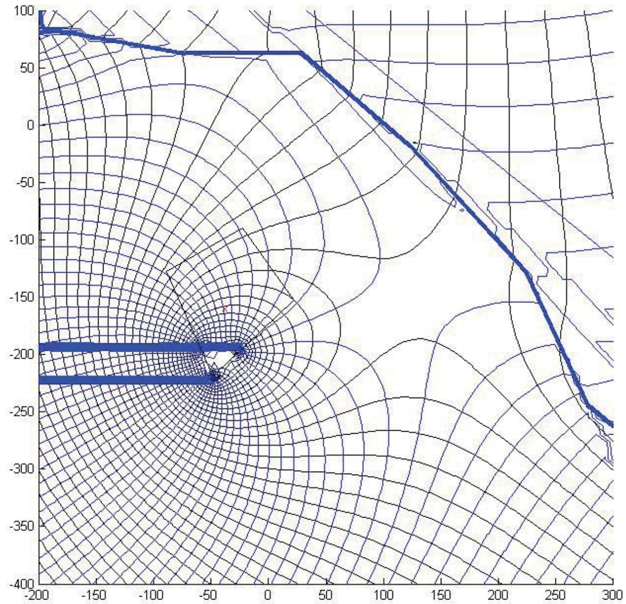

Figure 8: The scenario where the flow is oriented at 25.1 degrees from horizontal.

other, and not on a line with observation well 1. With three head measurements, the magnitude and direction of the unifrom flow can be determined. The reason they must be far away from each other is so that small inaccuracies in the measurements do not result in large changes in the uniform flow.

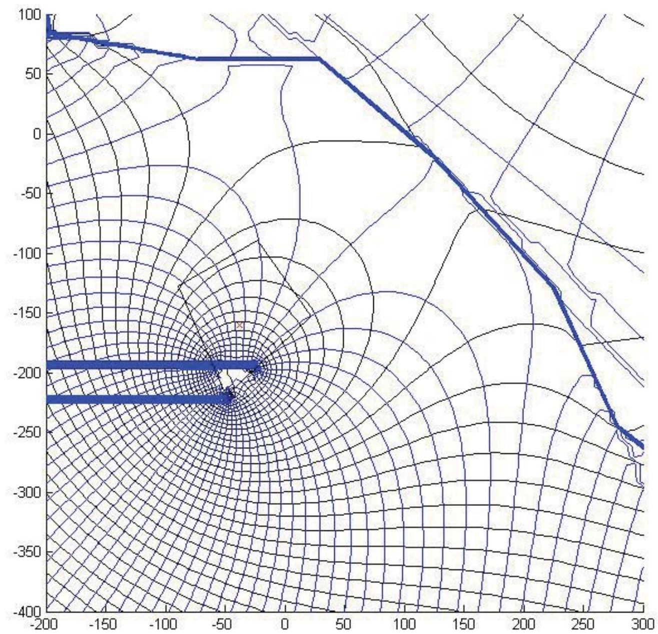

Figure 9: The well field when the magnitude of the uniform flow is just beyond the minimum.

Appendix: Matlab Code (note: after the run script, the functions are listed in alphabetical order)

Script: Project1

```
%make sure the data is in there
clear
load('datafile.mat');

%create a plot of the area of interest
MakingtheFieldDynamic(-2000,2000,-1500,1500,Project_1_datafile);

%These are for-sure values
k = 10; %m/day
H = 1000; %arbitrary- aquifer must be unconfined
phi1 = 22; %m
phi2 = 20.25; %m
phir = 20; %m
b = 0; %m
z1 = Project_1_datafile(1,1)+1i*Project_1_datafile(1,2);
z2 = Project_1_datafile(2,1)+1i*Project_1_datafile(2,2);
Phi1 = PotentialfromHead(k,H,b,phi1); %The potential at z1
Phi2 = PotentialfromHead(k,H,b,phi2); %the potential at z2
Phir = PotentialfromHead(k,H,b,phir); %The potential at the river
zw = 0;
rw = .3;
Qw = 0; %For the first part


%Estimation values were found using a least-squares regression performed with
Microsoft
%Excel. When the points on the river were plotted, Excel found a line with
%the equation y = -0.806x+126.2801 to fit the data the best with a R^2
%value of .97701.


alpha = atan(-0.806)+pi/2;%The orientation of the line of best fit, as
determined by Excel.

dist1 = 1443.2; %m this is approximately the distance (by inspection) between
the far observation well and the river.
Q0 = (Phi1-Phir)/dist1;


FuncKnown = @(z)Omega_uniform(z, Q0,alpha);
constants = [z1,Phi1];
elend = size(Project_1_datafile,1);
linescoords = Project_1_datafile(3:(elend-8),1:2);
spaces = ConstructSpaces(linescoords, 3);
[A,bm] = ConstructingMatrices2( spaces, FuncKnown, Phir, constants);
x = A\bm;
lengthx = size(x,1);
sigmas = x(1:lengthx-1,1);
C = x(lengthx,1);

Phi1 = real(OmegaAll(0,z1,spaces,sigmas,Q0,alpha,C,zw,rw));
```

```
phi1 = HeadfromPotential(k,H,b,Phi1);
Phi2 = real(OmegaAll(0,z2,spaces,sigmas,Q0,alpha,C,zw,rw));
phi2 = HeadfromPotential(k,H,b,Phi2);
Phir = real(OmegaAll(0,spaces(1,3),spaces, sigmas, Q0, alpha, C, zw, rw));
phir = HeadfromPotential(k,H,b,Phir);


%With a single well at the location of the observation well 1 with the
discharge found by hand
%  zw = z2;
%  Qw = 196.36;
%  figure;
%  ContourMe_flow_net(-200,200,50,-
300,100,50,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,zw,rw),30);
%  MakingtheFieldDynamic(-200,200,-300,100,Project_1_datafile);


% %With a single well at the location furthest away from the river
%  zw = complex(-48.65,-219);
%  Qw = 360;%max without getting any river water into the well
%  figure;
%  ContourMe_flow_net(-200,200,100,-
300,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,zw,rw),100);
%  MakingtheFieldDynamic(-200,200,-300,100,Project_1_datafile);


% %With a single well at the location furthest away from the river
 zw = complex(-48.65,-219);
 Qw = 288;%max without getting any river water into the well*0.8
 figure;
 ContourMe_flow_net(-200,200,100,-
300,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,zw,rw),100);
 MakingtheFieldDynamic(-200,200,-300,100,Project_1_datafile);

 Omega1 = OmegaAll(Qw,zw,spaces,sigmas,Q0,alpha,C,zw,rw);
 Headw1 = HeadfromPotential(k,H,b,real(Omega1));


wellLineStart = complex(Project_1_datafile(59,1),Project_1_datafile(59,2));
wellLineEnd = complex(Project_1_datafile(60,1),Project_1_datafile(60,2));
wellSpacing = linspace(wellLineStart,wellLineEnd,5);
dist = abs(wellSpacing(1)-wellSpacing(2));%Check to be sure that the distance
between wells will be <= 10m.

% Qw_indiv = 360/5;
% Qw = [Qw_indiv, Qw_indiv, Qw_indiv, Qw_indiv, Qw_indiv];
% figure;
% ContourMe_flow_net(-200,300,100,-
400,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,wellSpacing,rw),30);
% MakingtheFieldDynamic(-200,300,-400,100,Project_1_datafile);

%Three wells on back line
wellSpacing2 = wellSpacing(3:5);
% Qw_indiv = 360/3;
% Qw = [Qw_indiv, Qw_indiv, Qw_indiv];
```

```
% figure;
% ContourMe_flow_net(-200,300,100,-
400,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,wellSpacing2,rw),30);
% MakingtheFieldDynamic(-200,300,-400,100,Project_1_datafile);

%Three wells
% wellLine2Start =
complex(Project_1_datafile(62,1),Project_1_datafile(62,2));
% wellLine2End = complex(Project_1_datafile(61,1),Project_1_datafile(61,2));
% PossibleWellSpaces = linspace(wellLine2End, wellLine2Start,4);
% wellSpacing3 = [wellSpacing(4:5),PossibleWellSpaces(2)];
% figure;
% ContourMe_flow_net(-200,300,100,-
400,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw),30);
% MakingtheFieldDynamic(-200,300,-400,100,Project_1_datafile);

%This seems to be the ideal configuration
% Qw_total = 370;
% Qw_indiv = 370/4;
% Qw = [.35*Qw_total, .40*Qw_total, .33*Qw_total, 0*Qw_total];
% wellLine2Start =
complex(Project_1_datafile(62,1),Project_1_datafile(62,2));
% wellLine2End = complex(Project_1_datafile(61,1),Project_1_datafile(61,2));
% WellLine2Spaces = linspace(wellLine2End, wellLine2Start,4);
% wellSpacing3 = [wellSpacing(4:5),WellLine2Spaces(2:3)];
% figure;
% ContourMe_flow_net(-200,300,100,-
400,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw),30);
% MakingtheFieldDynamic(-200,300,-400,100,Project_1_datafile);

% %this is 0.8* the ideal configuration
Qw_total = 370*.8;
Qw_indiv = 370/4;
Qw = [.35*Qw_total, .40*Qw_total, .33*Qw_total, 0*Qw_total];
wellLine2Start = complex(Project_1_datafile(62,1),Project_1_datafile(62,2));
wellLine2End = complex(Project_1_datafile(61,1),Project_1_datafile(61,2));
WellLine2Spaces = linspace(wellLine2End, wellLine2Start,4);
wellSpacing3 = [wellSpacing(4:5),WellLine2Spaces(2:3)];
figure;
ContourMe_flow_net(-200,300,100,-
400,100,100,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw),30);
MakingtheFieldDynamic(-200,300,-400,100,Project_1_datafile);
%
%
% %%%Heads at wells
zw1 = wellSpacing3(1);
zw2 = wellSpacing3(2);
zw3 = wellSpacing3(3);
%zw4 = wellSpacing3(4);
Omega1 = OmegaAll(Qw,zw1,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw);
Omega2 = OmegaAll(Qw,zw2,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw);
Omega3 = OmegaAll(Qw,zw3,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw);
%Omega4 = OmegaAll(Qw,zw4,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw);
Headw1 = HeadfromPotential(k,H,b,real(Omega1));
Headw2 = HeadfromPotential(k,H,b,real(Omega2));
Headw3 = HeadfromPotential(k,H,b,real(Omega3));
```

```
%Headw4 = HeadfromPotential(k,H,b,real(Omega4))


%%%Analysis of change in alpha

% alphdeg = alpha*180/pi;
% %alphadeg = alphdeg + 5
% alphadeg = alphdeg - 26
% alpha = alphadeg*pi/180;

%%%Analysis of change in Q0
% Q0 = Q0*0.87;
% Qw_total = 370*.8;
% Qw_indiv = 370/4;
% Qw = [.35*Qw_total, .40*Qw_total, .33*Qw_total, 0*Qw_total];
% wellLine2Start =
complex(Project_1_datafile(62,1),Project_1_datafile(62,2));
% wellLine2End = complex(Project_1_datafile(61,1),Project_1_datafile(61,2));
% WellLine2Spaces = linspace(wellLine2End, wellLine2Start,4);
% wellSpacing3 = [wellSpacing(4:5),WellLine2Spaces(2:3)];
% figure;
% ContourMe_flow_net(-200,300,70,-
400,100,70,@(z)OmegaAll(Qw,z,spaces,sigmas,Q0,alpha,C,wellSpacing3,rw),50);
% MakingtheFieldDynamic(-200,300,-400,100,Project_1_datafile);
```

```
function [ Z ] = CapZ( z, z1,z2 )
%Returns the value of capital Z- a dimesionless variable with value -1 at
%z1 and 1 at z2
Z = (z-.5*(z1+z2))/(.5*(z2-z1));
end
```

```
function [A,bm] = ConstructingMatrices2( spaces, func, Phim, constants)
%CONSTRUCTINGMATRICES2 Takes a list of line-sinks (spaces), a known function,
the potential at the line-sinks, and a list of constants and returns the
%matrices for solving for the strengths of the line-sinks and the constant
%in the equation.
n = size(spaces,1);
A = zeros(n+1,n+1);
bm = zeros(n+1,1);

%Constructing A
for m = 1:n+1
    if m == n+1
        zcm = constants(1);
        bm(m,1)=constants(2)-real(func(zcm));
    else
        zcm = spaces(m,3);
        bm(m,1)=Phim-real(func(zcm));
    end
    %let's construct bm while we're here.

    for j = 1:n
        z1j = spaces(j,1);
        z2j = spaces(j,2);
        Z = CapZ(zcm,z1j,z2j);
        L = abs(z2j-z1j);
        A(m,j)=real(Omega_LineSink(Z,L));
    end
    A(m,(n+1))=1;
end
end
```

```matlab
function [spaces] = ConstructSpaces( linescoords,  b)
%CONSTRUCTDIFFERENT takes a series of lines and creates the matrix for
%finding the strengths of the line-sinks and the constant potential. There
%is also opportunity for using this function to create a matrix that will
%find the uniform flow rate or the discharge of a well, if a unit discharge
%or flow is used.
%b will be the number of sub-sinks that each length of river will be
%divided into
%This part constructs a matrix that includes the start and end points of a
%series of line-sinks and the centers of those line-sinks.

r = size(linescoords,1);
n= r/2;
spaces = NaN(n*b,3);

%set up a matrix with all the start and end points in the first and second
%columns and the mid point in the third column.

for i = 1:n
    z1 = linescoords(2*i-1,1)+1i*linescoords(2*i-1,2);
    z2 = linescoords(2*i,1)+1i*linescoords(2*i,2);
    row = linspace(z1, z2, b+1);
    for j = 1:b
        z_one = row(1,j);
        z_two = row(1,j+1);
        middle = (z_one+z_two)/2;
        spaces((((i-1)*b)+j),[1,2,3])= [z_one,z_two,middle];
        if (((i-1)*b)+j)>44
            spaces((((i-1)*b)+j),[1,2,3])= [z_two,z_one,middle];
        end
    end
end

end
```

```
function [Grid] = ContourMe_flow_net(xfrom, xto, Nx, yfrom, yto, Ny,
func,nint)
%==========================================================================
% ContourMe_I(xfrom, xto, Nx, yfrom, yto, Ny, func)                (01.23.09)
%
%   Contour the imaginary part of the specified complex function.
%
% Arguments:
%
%   xfrom   starting x-value for the domain
%   xto     ending x-value for the domain
%   Nx      number of grid columns
%
%   yfrom   starting y-value for the domain
%   yto     ending y-value for the domain
%   Ny      number of grid rows
%
%   func    function to contour;  must take one complex argument.
%
% Returns:
%
%   Grid    Ny x Nx matrix of values of func at the rid nodes.
%
% Example Usage:
%
%   G = ContourMe_I(1,2,11,1,2,11,@(z)Omega(1,-1,z));
%==========================================================================
Grid = zeros(Ny,Nx);

X = linspace(xfrom, xto, Nx);
Y = linspace(yfrom, yto, Ny);

for row = 1:Ny
    for col = 1:Nx
        Grid(row,col) = func( complex( X(col), Y(row) ) );
    end
end
Bmax=max(imag(Grid));
Bmin=min(imag(Grid));
Cmax=max(Bmax);
Cmin=min(Bmin);
D=Cmax-Cmin;
del=D/nint;
Bmax=max(real(Grid));
Bmin=min(real(Grid));
Cmax=max(Bmax);
Cmin=min(Bmin);
D=Cmax-Cmin;
nintr=round(D/del);


axis equal
%figure;
hold on
contour(X, Y,real(Grid),nintr,'k');
contour(X, Y,imag(Grid),nint,'b');
```

```
hold off
```

```matlab
function [ phi] = HeadfromPotential( k, H , b , Phi )
%HeadfromPotential Takes a head (phi), a conductivity (k), a height of an
%aquifer (H), and a constant which represents the distance from the base of
%the aquifer from which we are measuring head (b). This returns the
%potential.
%
if(Phi > 0.5*k*H^2)
    phi = Phi/(k*H)+.5*H+b;
elseif (Phi<0)
    phi = b;
else
    phi = sqrt(2*Phi/k)+b;
end
```

```
function [  ] = MakingtheFieldDynamic( xmin, xmax, ymin, ymax,
Project_1_datafile )
%MAKINGTHEFIELDDYNAMIC Creates a plot of the data from project 1.

%create a plot of all the elements given
well1 = Project_1_datafile(1,:);
well2 = Project_1_datafile(2,:);
linesinks = Project_1_datafile(3:56,:);
wellfield = Project_1_datafile(57:64,:);
hold off
hold on
plot(linesinks(:,1),linesinks(:,2),'b', 'LineWidth', 3);
plot(wellfield(:,1),wellfield(:,2),'k');
plot(well1(1,1),well1(1,2),'xr');
plot(well2(1,1),well2(1,2),'xr');
z1 = linesinks(1,1)+1i*linesinks(1,2);
z2 = linesinks(54,1)+1i*linesinks(54,2);
plot([1737.961;-999.755],[-1274.5;932.0827]);
axis([xmin xmax ymin ymax]);
%axis equal
hold off

end
```

```
function [ omega ] = Omega_LineSink( Z,L )
%LINESINK Gives the complex potential for a line sink of unit infiltration
%rate
omega = L/(4*pi)*((Z+1)*log(Z+1)-(Z-1)*log(Z-1)+2*log(L/2)-2);
end
```

```
function [ omega ] = Omega_uniform(z, Qo, alpha)
%OMEGA_UNIFORM returns the complex potential of a uniform flow
%
omega = -Qo*exp(1i*-alpha)*z;
end
```

```
function [ omega ] = Omega_well( z,zw,rw )
%Omega_well returns the complex potential of a well of unit discharge
%
if((z - zw) * conj(z - zw) < rw^2)
    omega = 0;
else
    omega = 1/(2*pi)*log((z-zw)/rw);
end
end
```

```
function [ omega ] = OmegaAll( Qw, z, spaces, strengths, Qo, alpha, const,
zw,rw)
%OMEGAALL Designed specifically for Project 1. Gives the complex potential
%for all elements in the field
omega = Omega_uniform(z, Qo, alpha);
omega = omega + const;
n_ls = size(spaces, 1);
num_wells = length(zw);
for i = 1:n_ls
    z1 = spaces(i,1);
    z2 = spaces(i,2);
    Z = CapZ(z,z1,z2);
    L = abs(z1-z2);
    sigma = strengths(i,1);
    omega = omega + sigma * Omega_LineSink(Z,L);
end
    for j = 1:num_wells
        omegawell = Qw(j)*Omega_well(z,zw(j),rw);
        omega = omega + omegawell;
    end
end
```