Quantum Multi-string Matching

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Presentation Overview

1. Problem Definition

2. Classical Algorithms
   - Existing Algorithms
   - Polynomial Matching
   - Efficient Polynomial Multiplication

String Matching

- Text $S[0...n-1]$
- Pattern (or key) $P[0...m-1]$
- Strings from alphabet Σ
- Find set of indices i such that $S[i..i+m] = P$

Example:

- T = "GTAT GATC TC" (ignore spaces)
- P_1 = "ATCT"
- P_2 = "TGAT"
- P_3 = "ACCC"
- P_1 matches at 5, P_2 matches at 3, P_3 has no matches

- Multi-string matching: search $P_1, P_2, ...$ in S simultaneously

Classical Algorithms

- Brute Force
- Boyer-Moore
- Suffix Tree/Array
- Karp-Rabin

TODO - add details (runtime), to above. more for karp-rabin, which polynomial is similar to

- Polynomial Matching

Polynomial Matching

Calculate a fingerprint for P and every m character sequence in S; matching fingerprints suggest pattern match!

$$A \mapsto -3$$
$$C \mapsto 5$$
$$G \mapsto -7$$
$$T \mapsto 11$$

$$\text{"ATCT"} \longrightarrow 11x^3 + 5x^2 + 11x - 3$$
$$\text{"GTAT GATC TC"} \longrightarrow -7x^{15} + 11x^{14} - 3x^{13} + ... + 11x^7 + 5x^6$$

Note that P encoding is reversed (similar to convolution)

Polynomial Matching

Given polynomials

$$P(x) = \sum_{i=0}^{m} a_i x^i, S(x) = \sum_{j=0}^{n} b_j x^j$$

then

$$R(x) = \sum_{k=0}^{m+n} c_k x^k$$

where

$$c_k = \sum_{i=0}^{k} a_i b_{k-i}, \quad \text{for } 0 \leq k \leq m+n,$$

with the convention that $a_i = 0$ for $i > m$ and $b_j = 0$ for $j > n$.

Polynomial Matching

Coefficients of $R(x)$ correspond to "dot products" between substrings.

$$P(x) = 11x^3 + 5x^2 + 11x - 3$$
$$S(x) = -7x^{15} + 11x^{14} - 3x^{13} + ... + 11x^7 + 5x^6$$
$$R(x) = ... + 71x^8 + 0x^9 + 276x^{10} + 98x^{11} - 4x^{12} + ...$$

Degrees map to index:

15 yields index 0

14 yields index 1

...

11 yields index 4

10 yields index 5

9 yields index 6

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Notice that $\|P\|^2 = 11^2 + 5^2 + 11^2 + (-3)^2 = 276$

We get exact fingerprint match for single patterns. Let's extend to multiple patterns...

Add patterns together:

$$P(x) = P_1(x) + P_2(x)$$
$$S(x) = -7x^{15} + 11x^{14} - 3x^{13} + ... + 11x^7 + 5x^6$$
$$R(x) = ... + 94x^8 + 240x^9 + 272x^{10} + 64x^{11} + 296x^{12} - 60x^{13}...$$

Notice that $\|P\|^2 = 11^2 + 5^2 + 11^2 + (-3)^2 = 276$

No more exact matches, but highest values correspond to likeliest index.

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Efficient Polynomial Multiplication

Talk about naive $O(n^2)$, Karatsuba, and FFT

Lists
Bullet Points and Numbered Lists

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus
  - Lorem ipsum dolor sit amet, consectetur adipiscing elit
  - Nam cursus est eget velit posuere pellentesque
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero

1. Nam cursus est eget velit posuere pellentesque
2. Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block Title

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue.

Example Block Title

Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan.

Alert Block Title

Pellentesque sed tellus purus. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos.

Suspendisse tincidunt sagittis gravida. Curabitur condimentum, enim sed venenatis rutrum, ipsum neque consectetur orci.

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table
Subtitle

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Definitions & Examples

Definition

A prime number is a number that has exactly two divisors.

Example

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (three divisors: 1, 2, and 4).

You can also use the `theorem`, `lemma`, `proof` and `corollary` environments.

Theorem, Corollary & Proof

Theorem (Mass–energy equivalence)

$E = mc^2$

Corollary

$x + y = y + x$

Proof.

$\omega + \phi = \epsilon$ $\qquad\square$

Equation

$$\cos^3 \theta = \frac{1}{4} \cos \theta + \frac{3}{4} \cos 3\theta \tag{1}$$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Slide without title.

Citing References

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2022, Kennedy, 2023].

References

📄 John Smith (2022)
Publication title
Journal Name 12(3), 45 – 678.

📄 Annabelle Kennedy (2023)
Publication title
Journal Name 12(3), 45 – 678.

Acknowledgements

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The End

Questions? Comments?