1 QT On The Fly

There are two ways to interpret the tomography formula, $\rho = \sum_\alpha |A_\alpha >< A_\alpha|\rho >$. The usual way is geometrical, where $= \sum_\alpha |A_\alpha >< A_\alpha| = 1$ is a complete set, and $< A_\alpha|\rho >$ is the projection onto each basis vector. As a geometrical idea this does not specifically refer to any notion of "probability." A different interpretation comes from thinking of $< A_\alpha|\rho > \to P(A_\alpha|\rho)$ being the probability to find A_α given ρ. Then $\rho = \sum_\alpha |A_\alpha > P(A_\alpha|\rho)$ instructs one to add up basis vectors weighted by the Born-rule probability they are measured. As a probability concept it does not specifically refer to the geometrical idea. We have a new approach taking the second interpretation seriously: Add up tensor basis elements by the probability they appear in experimental data generated by the Born rule.

1.1 Lepton Pair Production

Let $d\sigma(\theta) \propto tr(XL(\theta)) = 1/3 + tr(\tilde{X}\tilde{L}(\theta))$, where $\tilde{X}$ symbols are traceless, and θ stands for θ, ϕ. For now concentrate on the symmetric part of the unknown density matrix, denoted X^S. Expand the lepton DM $\tilde{L}_{ab}$ in Cartesian tensors, where the symmetric part $\tilde{L}^S_{ab} = (\delta_{ab}/3 - x_a x_b)$ (ignoring a factor of 1/2), and $x_a(\theta)$ is the unit vector of lepton momentum usually called $\hat{\ell}$. Note that all these symmetric tensors have zero trace, which is a general fact of representation theory.

We now seek a tensor $\underset{\sim}{L}^{jk}(\theta)$, which up to a constant inverts $\tilde{L}_{ab}$ when averaging over the angles. One solution is just the same tensor. Show by computing

$$\underset{\sim}{L}^{jk} = \delta_{jk}/3 - x_j x_k;$$

$$\int \frac{d\Omega}{4\pi}\, \tilde{L}_{ab}(\theta)\underset{\sim}{L}^{jk} = Q_{abjk} \to \text{“}\mathbb{1}\text{''} \times \text{constant};$$

$$\int \frac{d\Omega}{4\pi}\, (\delta_{ab}/3 - x_a x_b)(\delta_{jk}/3 - x_j x_k) = \frac{-2}{45}\delta_{ab}\delta_{jk} + \frac{1}{15}(\delta_{aj}\delta_{bk} + \delta_{ak}\delta_{bj}) = Q_{abjk}.$$

The integrals are done by knowing they are proportional to invariant tensors and computing contractions. The coefficients on the right make the $j = k$ and $a = b$ traces zero, as needed for traceless X^S.

It follows that

$$\int \frac{d\Omega}{4\pi} tr(\tilde{X}^S\tilde{L}^S(\theta))\underset{\sim}{L}^{jk}(\theta) = X^S_{ab}Q_{abjk} = \frac{2}{15}\tilde{X}^S_{jk};$$

$$\tilde{X}^S_{jk} = \frac{15}{2}\int \frac{d\Omega}{4\pi} tr(\tilde{X}^S\tilde{L}^S(\theta))\underset{\sim}{L}^{jk}(\theta) \qquad (1)$$

A similar expression exists for each type of tensor. For the 3×3 case the antisymmetric imaginary tensor is $\tilde{L}^A_{ab} = \imath\epsilon_{abc}x_c$, up to a factor involving c_A. Under angular averaging it is orthogonal to all tensors except itself. Then

$$\tilde{X}^A_{ab} \propto \imath\epsilon_{abc} \int \frac{d\Omega}{4\pi} tr(\tilde{X}\tilde{L})(\theta) x_c.$$

Similarly there are n-th rank tensors with n powers like $x_a x_b ... x_n$ that work the same way.

Perform the angular integration numerically with unit vectors x_a that are distributed by the Born rule, $d\sigma = 3tr(XL(\theta))/4\pi$. Make the statistically-weighted calculation by *simply averaging over a sample of experimental data.* Then restoring the unit matrix and including the normalization gives the QT on the fly formula:

$$X^S_{ab} = \frac{\delta_{ab}}{3} + \frac{5}{N} \sum_J^N \frac{\delta_{ab}}{3} - x^J_a x^J_b,$$
$$= 2\delta_{ab} - \frac{5}{N} \sum_J^N x^J_a x^J_b \tag{2}$$

Eq. 2 shows what to add up from a sample of events named J.

With $\vec{S}$ being the spin parameter of X, one will also find $\vec{S} \propto < \vec{x} >$, where $< ... >$ is the average in the quantum mechanically distributed data.

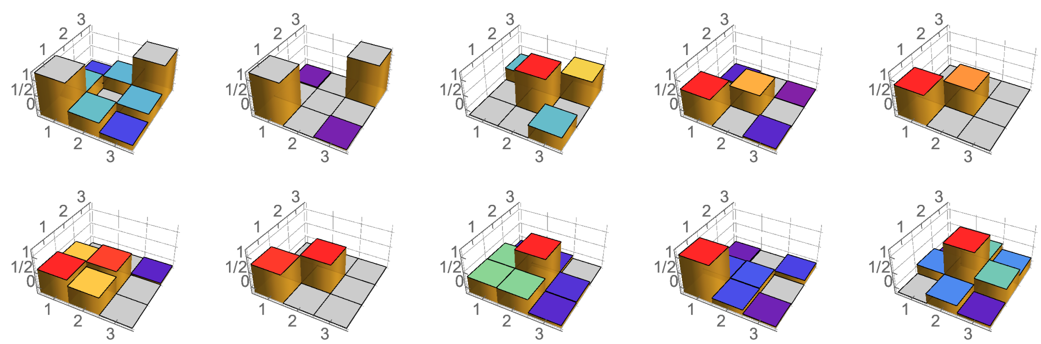

Figure 1: The first 10 estimated symmetric density matrices X^S from a sample of 10 dileptons distributed by the differential cross section $d\sigma$. The order of events is left to right, top to bottom. The exact X^S happens to appear after 7 events, namely the second case in the bottom row.

The traditional process of expanding operators in a complete orthonormal or equiangular (SIC) basis, as well as generating expectation values has been completely bypassed. A single event of dimension d spontaneously provides an estimate of the entire density matrix with d^2 parameters. As more events are added, their distribution in the data sample makes an estimator of the system density matrix from the mean of the single-event density matrices. The process converges by the central limit theorem, which has been verified by numerical simulations.

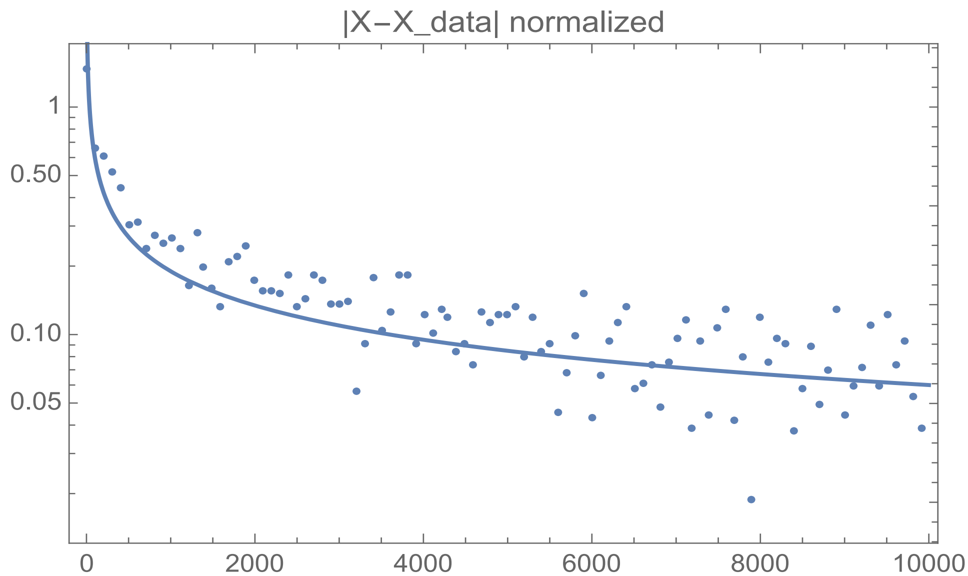

Figure 2: The Hilbert-Schmidt distance $\sqrt{tr\left((X^S - X^S_{data})\cdot(X^S - X^S_{data})\right)}/|X||X^S_{data}|$ as a function of the number of events N contributing to the average. The curve is $\log_e(6/\sqrt{N})$.