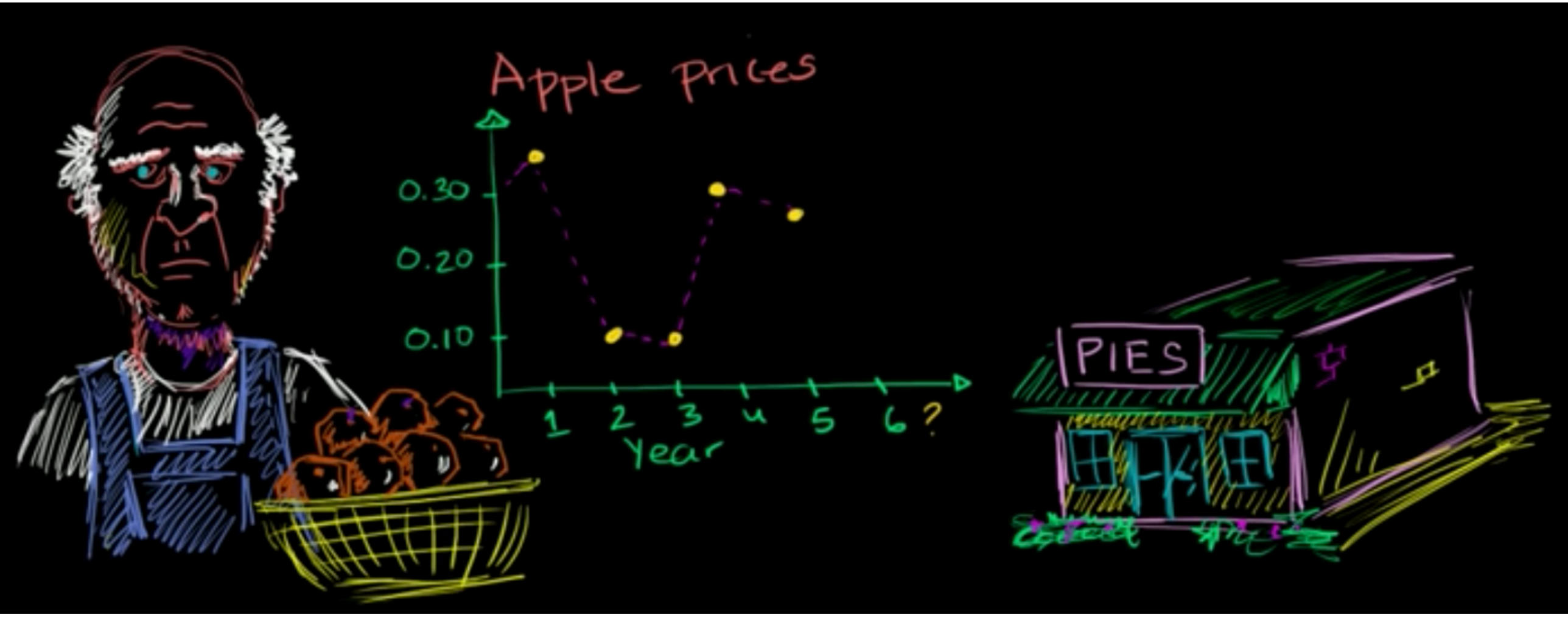
Forward and Futures Contracts

Instructor: Paruj Ratanaworabhan

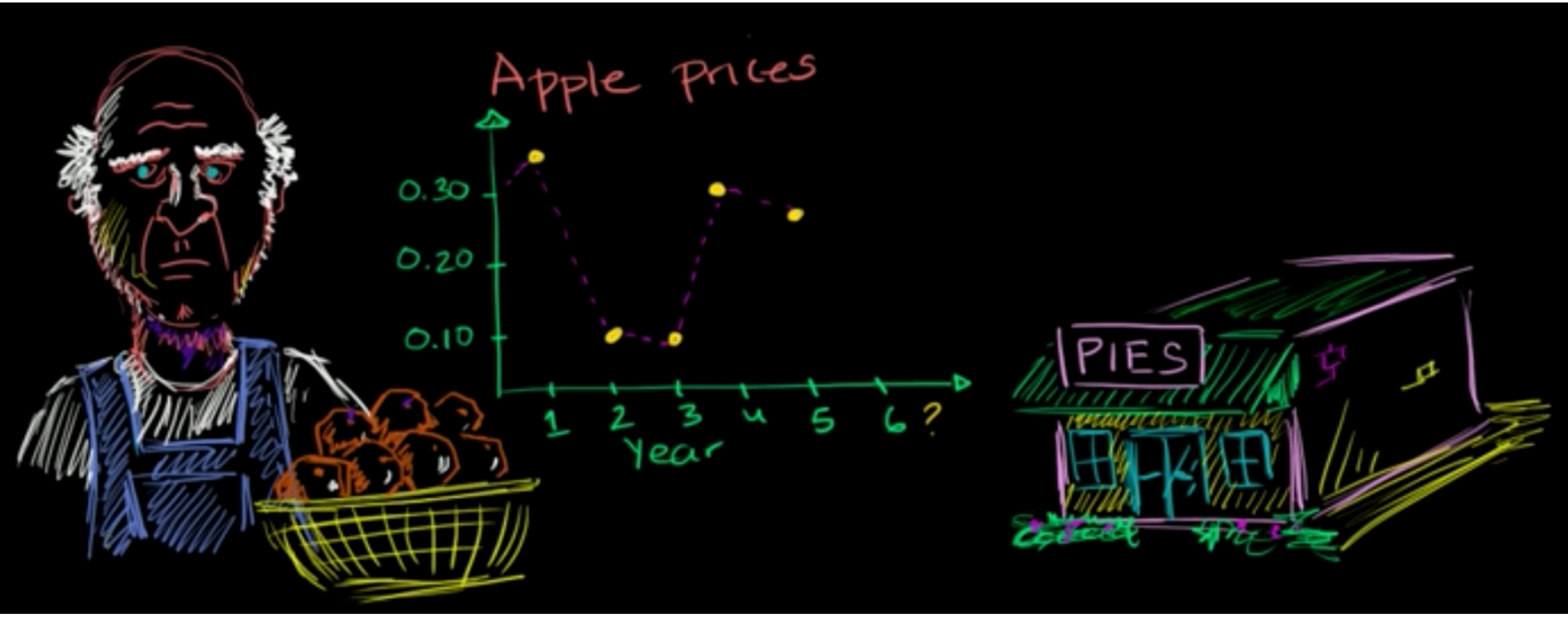
Sources:
“Forward and Futures Contracts”, Khan Academy
“Forward and Futures Contracts”, Andrew W. Lo, MIT Sloan School
“Understanding Futures Margin”, TD Ameritrade

Fluctuation in Prices



This is bad for both the producers and the consumers of apples.

Forward Contract



The Pies Chain agrees to buy 1M pounds of apples from the farmers @ harvest time for a price of $0.2 / pound

Another Example

- Current price of soybeans is $160/ton
- Tofu manufacturer needs 1,000 tons in 3 months
- Wants to make sure that 1,000 tons will be available
- 3-month forward contract for 1,000 tons of soybeans at $165/ton
- **Long side** will buy 1,000 tons from **short side** at $165/ton in 3 months

Forward Contracts

- Definition: A forward contract is a commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today

- The price fixed now for future exchange is the forward price
- The buyer of the underlying is said to be “long” the forward

Features of Forward Contracts

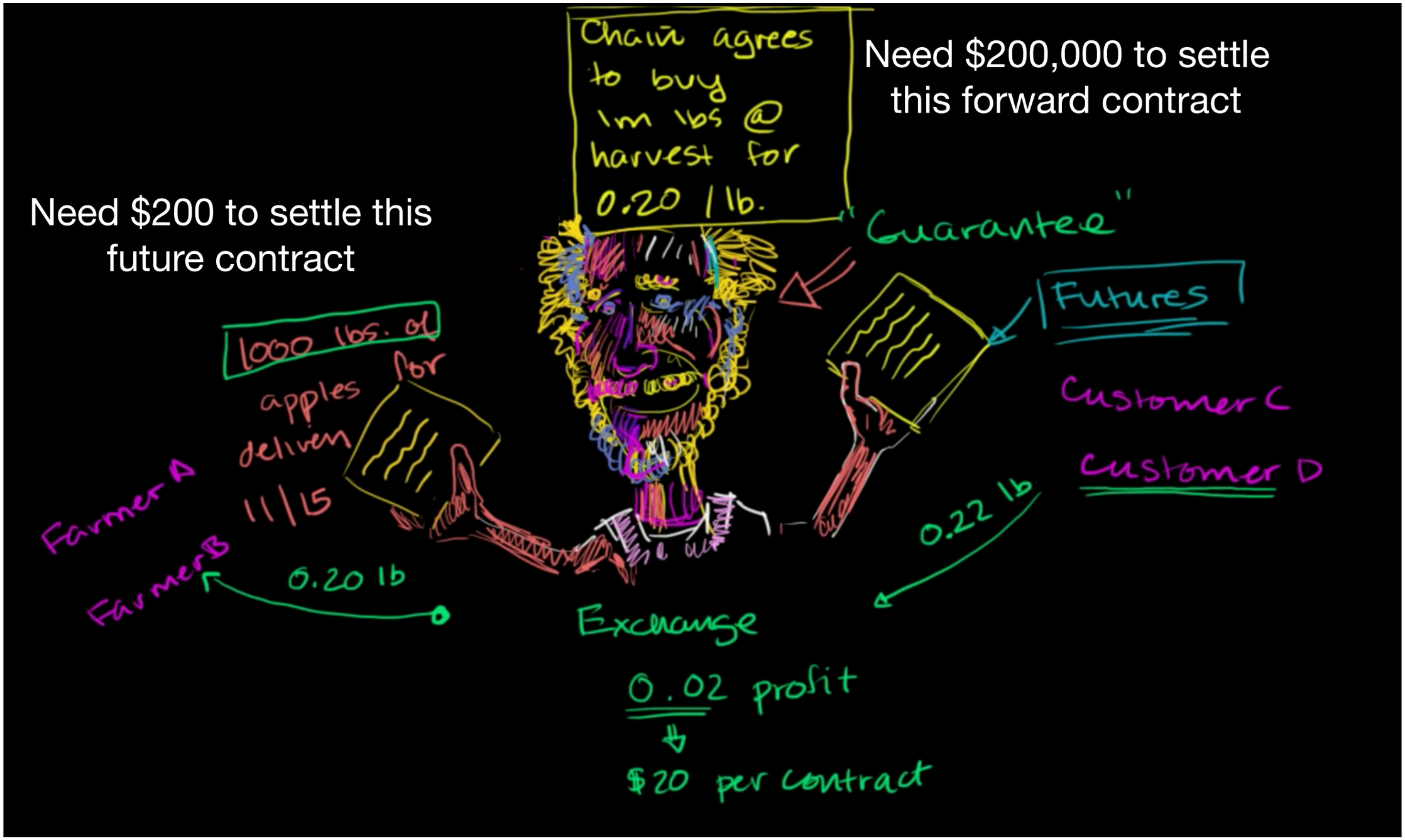
- Customized
- Non-standard and traded over the counter (not on exchanges)
  - Illiquidity
- No money changes hands until maturity
- Non-trivial counterparty risk

Futures Contracts

- Forward contracts have two limitations:
  - Illiquidity
  - Counterparty risk
- A futures contract is an exchange-traded, standardized, forward-like contract that is marked to market daily. This contract can be used to establish a long (or short) position in the underlying asset

Features of Futures Contracts

- Standardized contracts:
  - Underlying commodity or asset
  - Quantity
  - Maturity
- Exchange traded
- Guaranteed by the clearing house—no counter-party risk
- Gains/losses settled daily (marked to market)
- Margin required as collateral to cover losses



- An exchange that splits the original forward contract for 1M pounds of apples to smaller future contracts of 1K pounds of apples
- The exchange profits from the spread and other fees and commissions

Understanding Margin

Initial margin	$20.00
Maintenance margin	$10.00

Futures contract:
For **1,000 pounds** of apples delivery on 1 Nov. 2022 at **$0.2 per pound**
Total contract value = $200

	Seller margin account value	Contract market price	Buyer margin account value	Notes
Day 1	$20.00	$200.00	$20.00	
Day 2	$30.00	$190.00	$10.00	
Day 3	$35.00	$185.00	$5.00	
This is marked to market settlement			$20.00	Buyer deposits $15 to his margin account
Day 4	$25.00	$195.00	$30.00	
			$15.00	Buyer withdraws $15 from his margin account

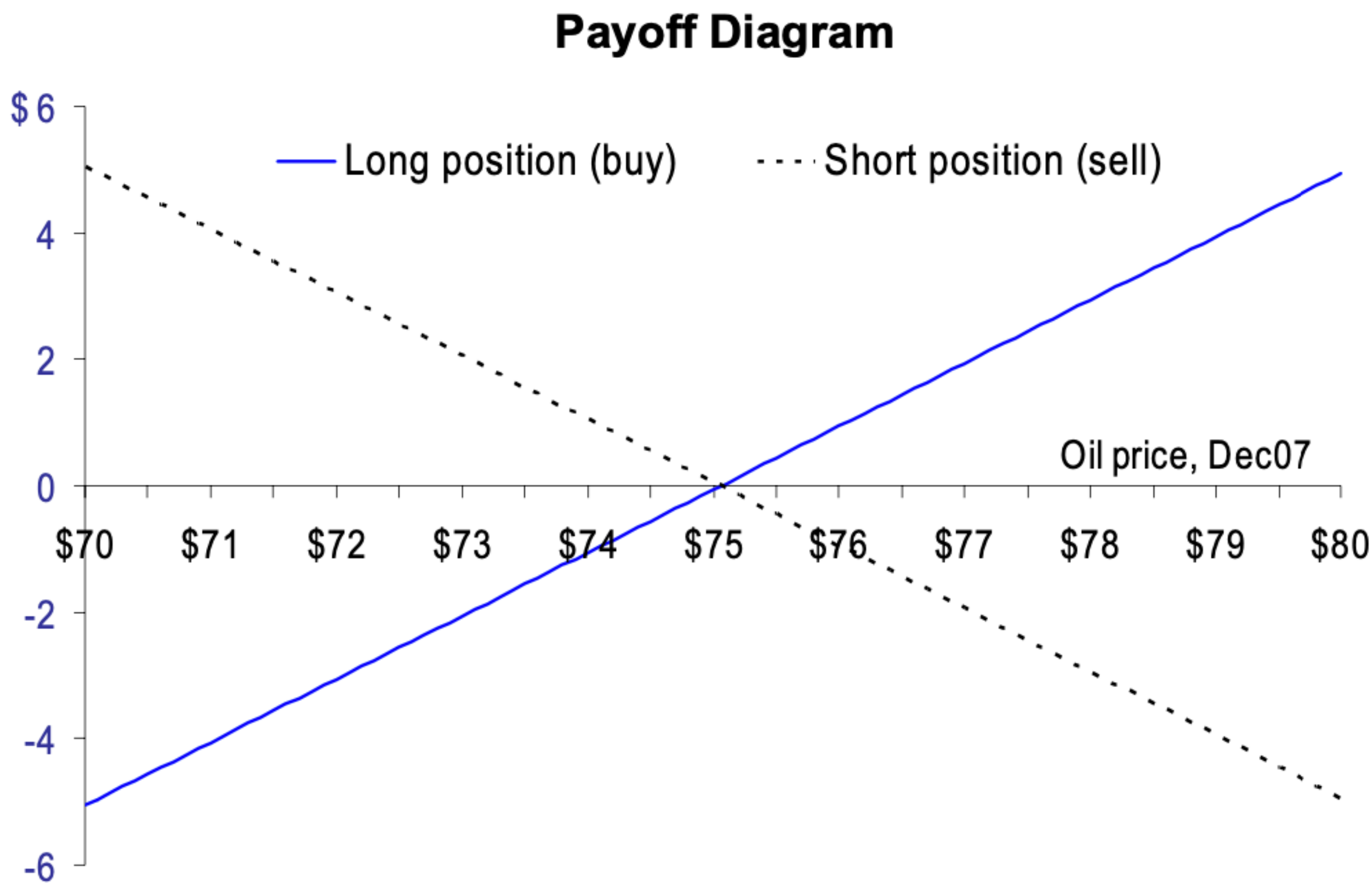
On 1 Nov. 2022, if the contract market price is $100:

- the seller gets $100 from the sale of 1,000 pound of apples and another $100 net transferred from the buyer account - net gain $200
- the buyer pays $100 for 1,000 pounds of apples and will have transferred $100 to the seller - net payout $200

Another Example

NYMEX crude oil (light) futures with delivery in Dec. 2007 at a price of $75.06 / bbl. on July 27, 2007 with 51,475 contracts traded

- Each contract is for 1,000 barrels
- Tick size: $0.01 per barrel, $10 per contract
- Initial margin: $4,050
- Maintenance margin: $3,000
- No cash changes hands today (contract price is $0)
- Buyer has a “long” position (wins if prices go up)
- Seller has a “short” position (wins if prices go down)


Payoff Diagram
$6
4
2
0
-2
-4
-6
Long position (buy)
Short position (sell)
Oil price, Dec07
$70
$71
$72
$73
$74
$75
$76
$77
$78
$79
$80

That has been the use of future contracts for **hedging**

Cash Settlement Futures Contracts

- No physical goods transferred
- Cash-settled only

https://www.barchart.com/futures

S&P 500 E-Mini Jun '22 (ESM22)

4,455.50s -3.75 (-0.08%) 04/20/22 [CME]

QUOTE OVERVIEW for Wed, Apr 20th, 2022

Try Excel Alerts Watch Help

Day Low	Day High
4,432.75	4,484.25

Open 4,441.00

Previous Close	4,459.25
Volume	1,391,211
Open Interest	2,134,690
Stochastic %K	30.41%
Weighted Alpha	+5.81
5-Day Change	+70.50 (+1.60%)
52-Week Range	4,020.50 - 4,800.00

Contract Specifications

See More

Contract	E-Mini S&P 500 Index	Exchange	CME
Contract Size	$50 times Index	Point Value	$50
Tick Size	0.25 points ($12.50 per contract)	Margin/Maintenance	$13,200/12,000
Trading Hours	5:00p.m. - 4:00p.m. (Sun-Fri) (Settles 3:00p.m.) CST	First Notice Date	06/17/22 (58 days)
		Expiration Date	06/17/22 (58 days)

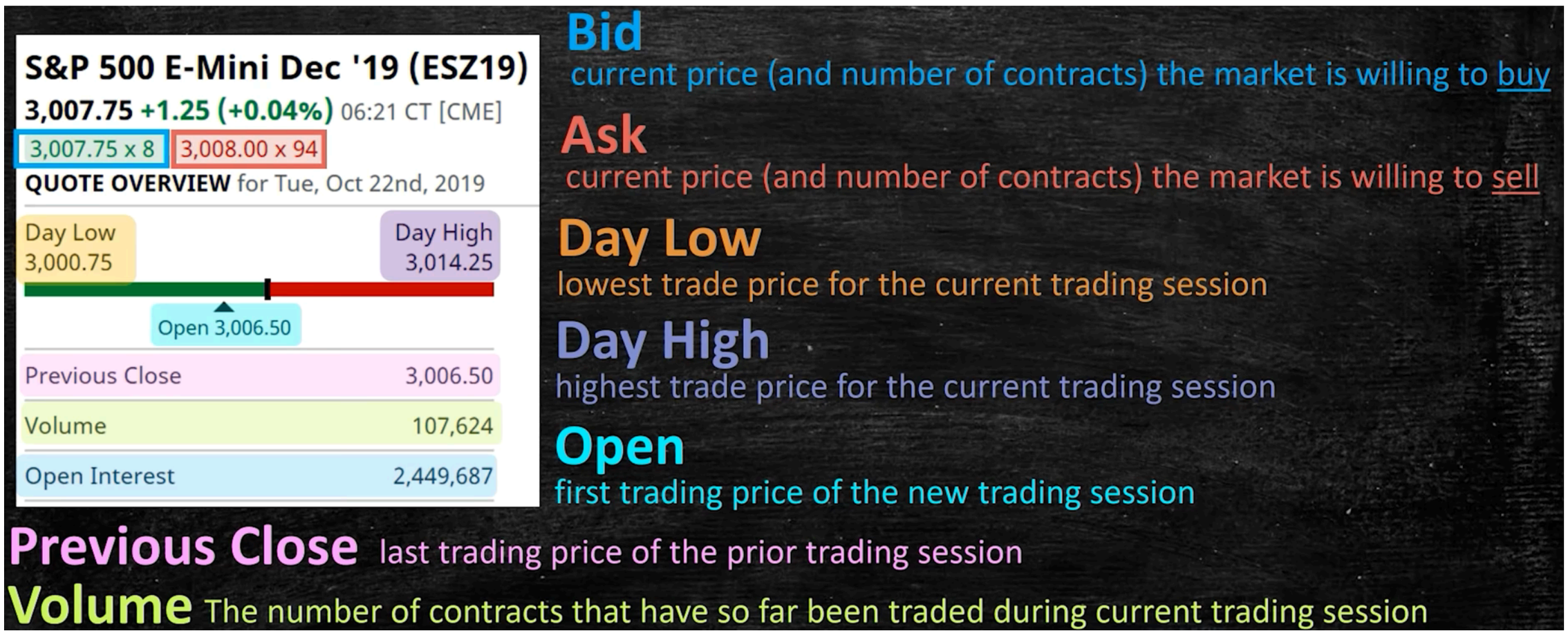
Price is measured in points, which can be translated to dollars by multiplying the points with its value, in this futures contract case, $50

Understanding Futures Quotes

Futures Contracts Months Codes

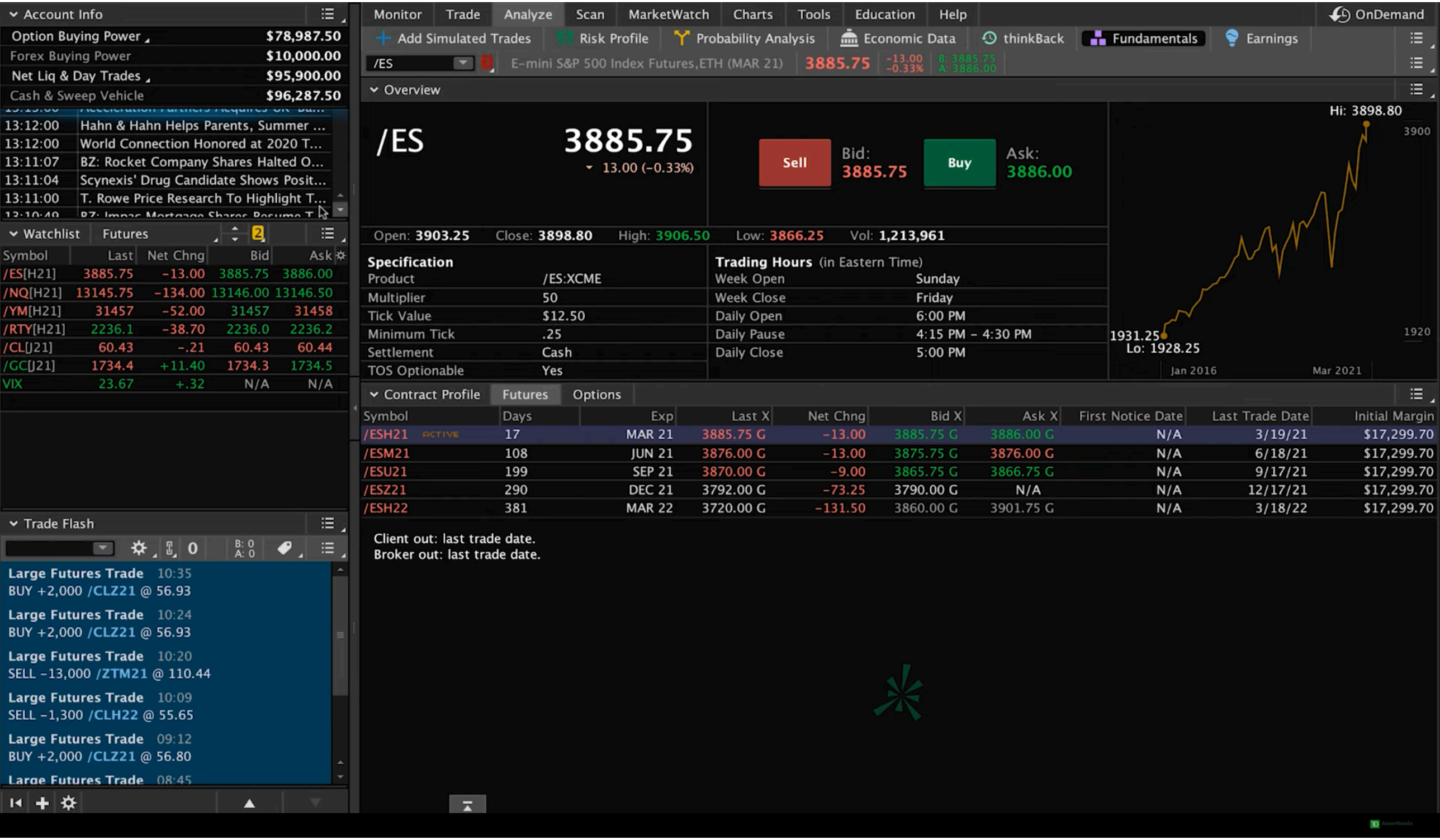
JAN	FEB	MAR	APR	MAY	JUN
F	G	H	J	K	M

JUL	AUG	SEP	OCT	NOV	DEC
N	Q	U	V	X	Z



Open Interest the total number of contracts that exist on the commodity/financial security

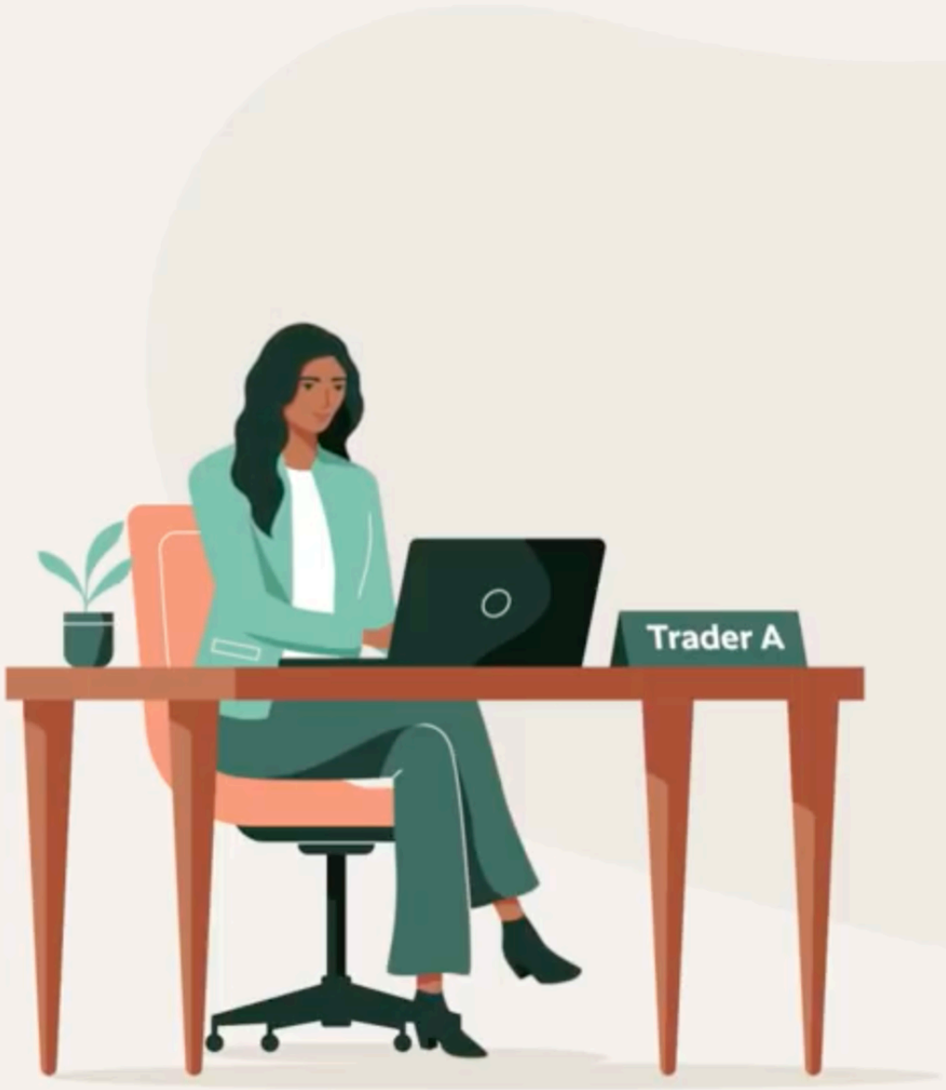
A Futures Trading Platform



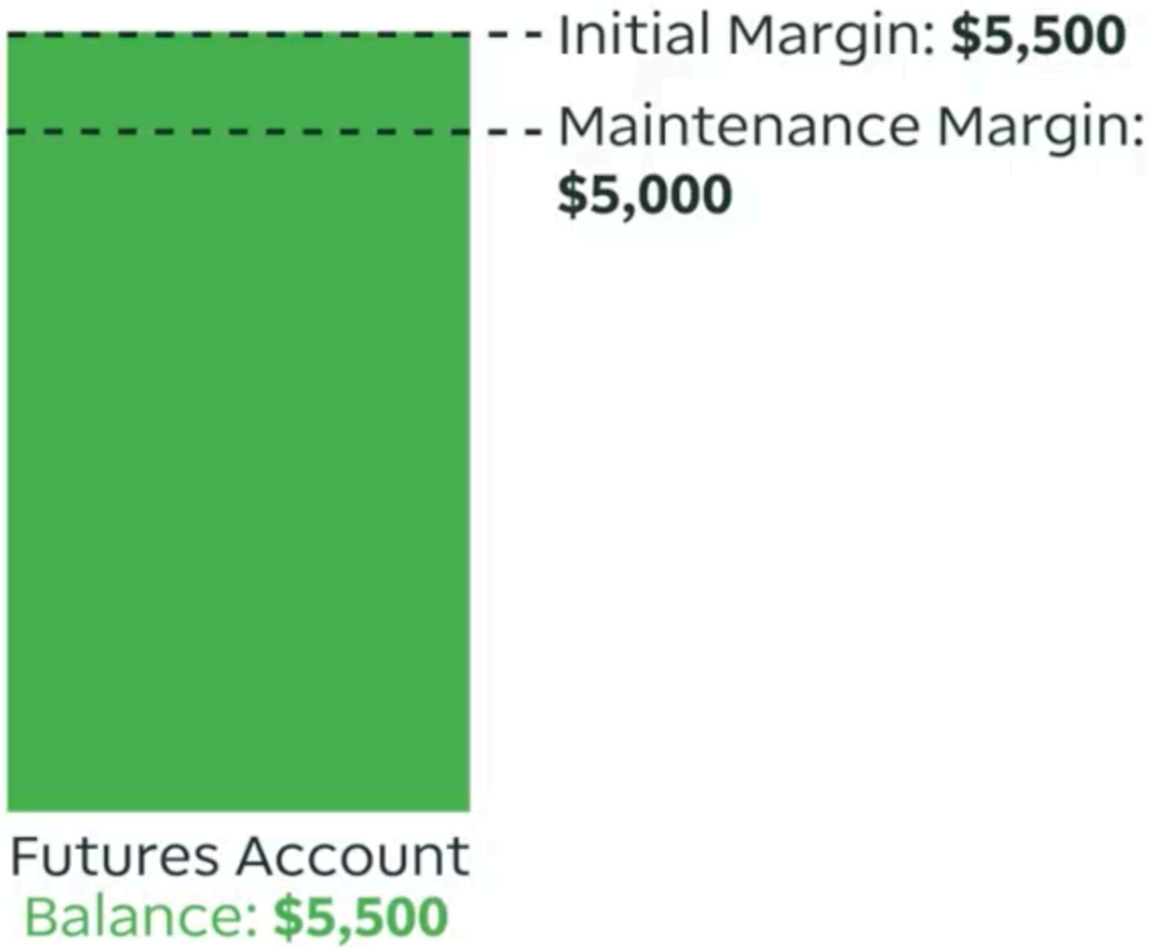
Trader A
Futures
/ES
Position: Long
Price: 2,800

Notional Value

Margin:
3% - 12% of
notional value



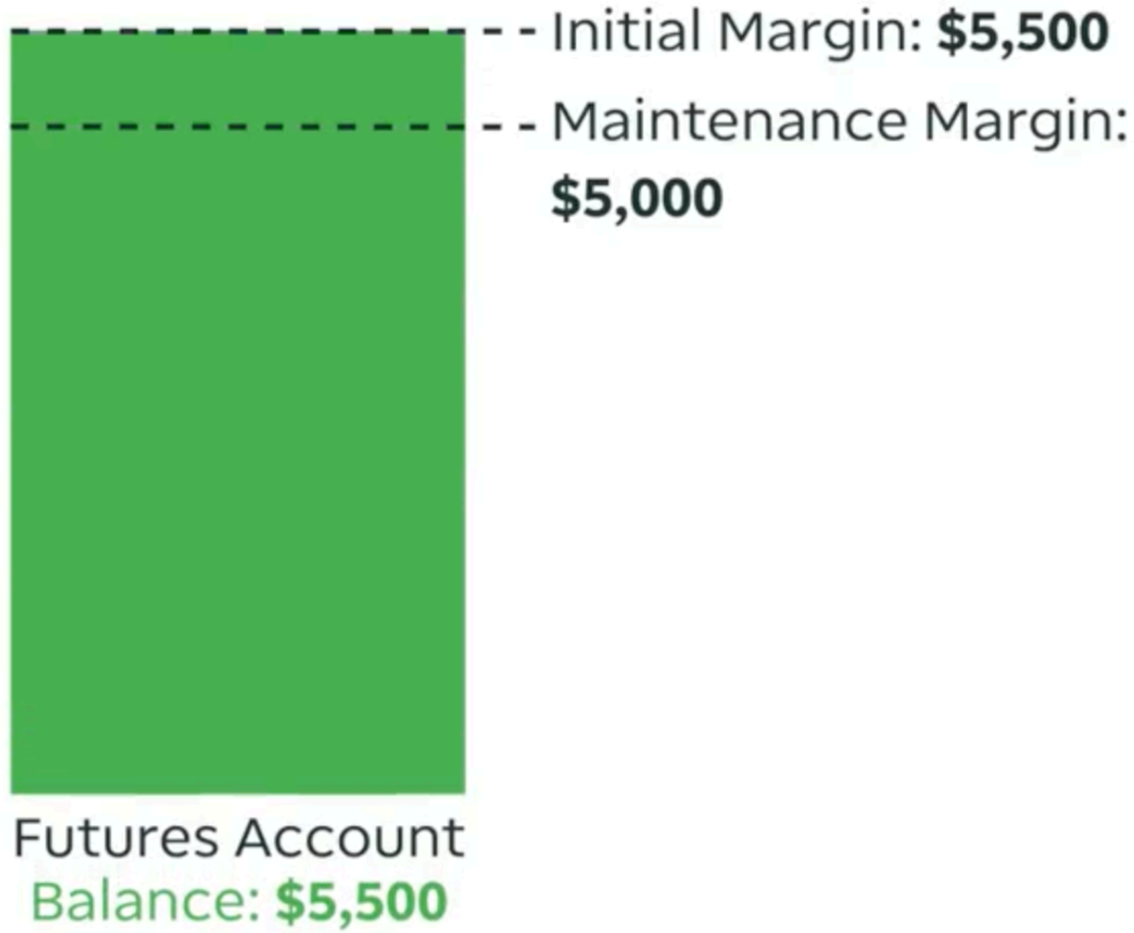
How Margin Works

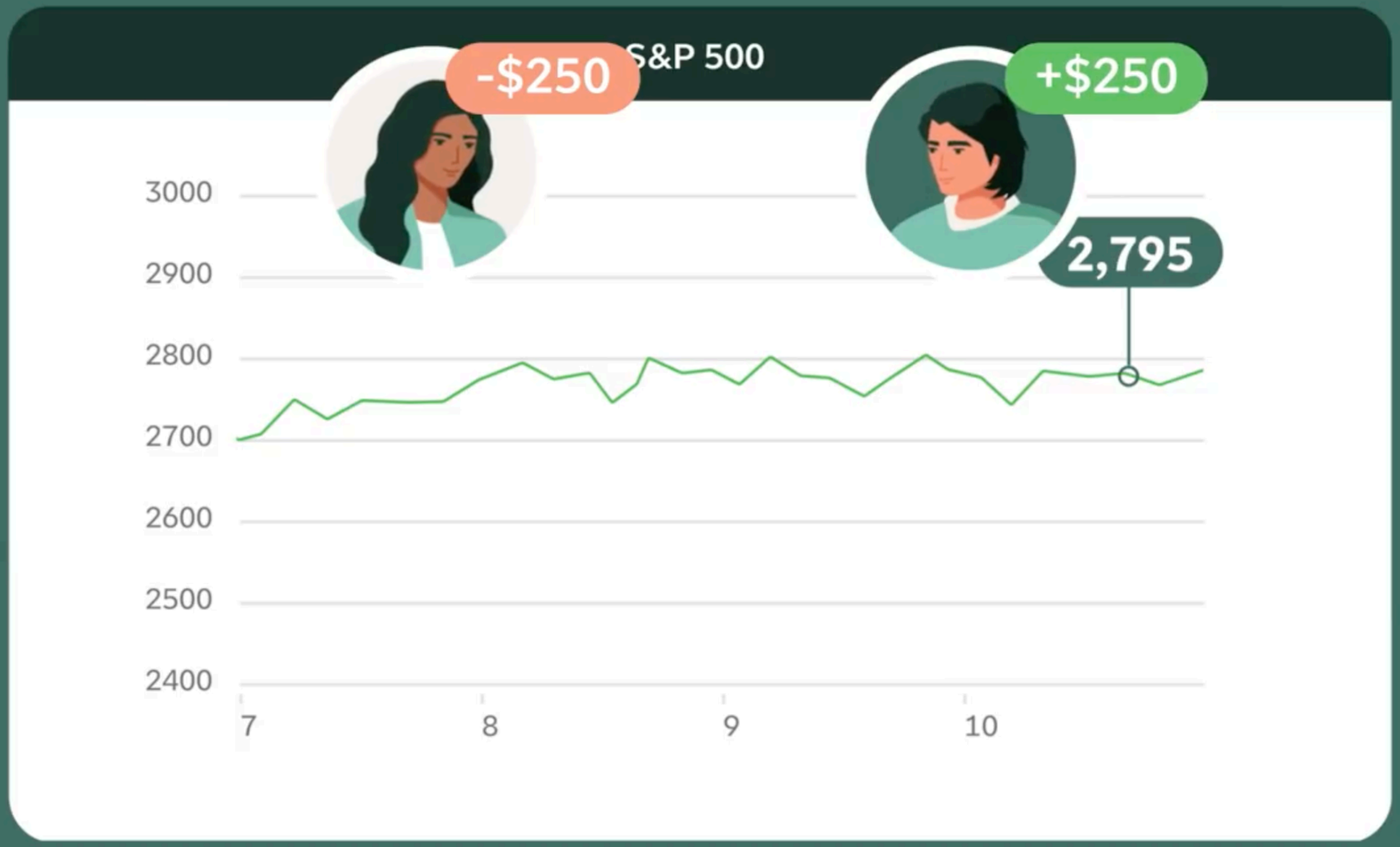


Futures Exchange
Buy
Trader A
Sell
Trader B

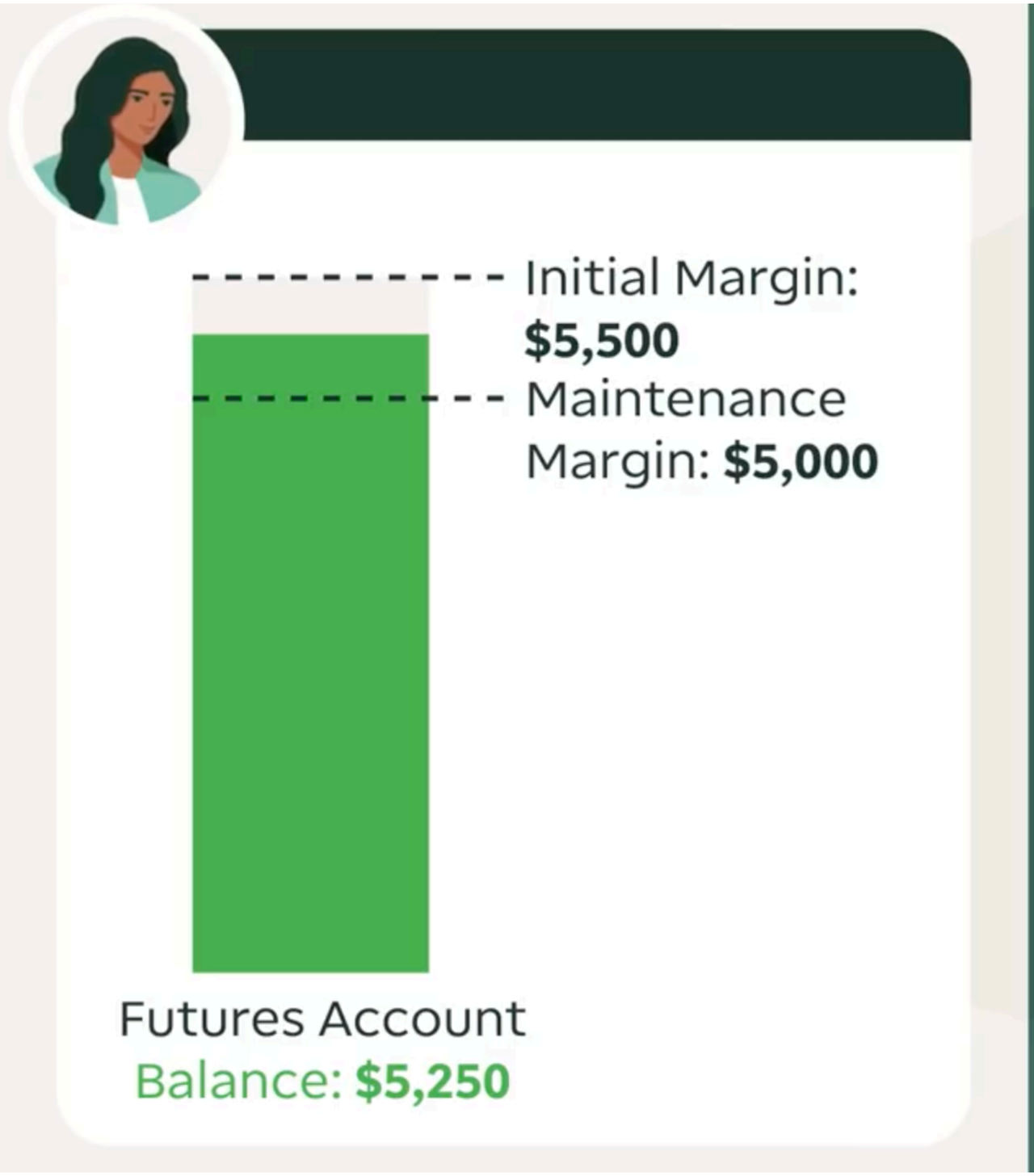
Futures
/ES
Position: Short
Price: 2,800
Trader B

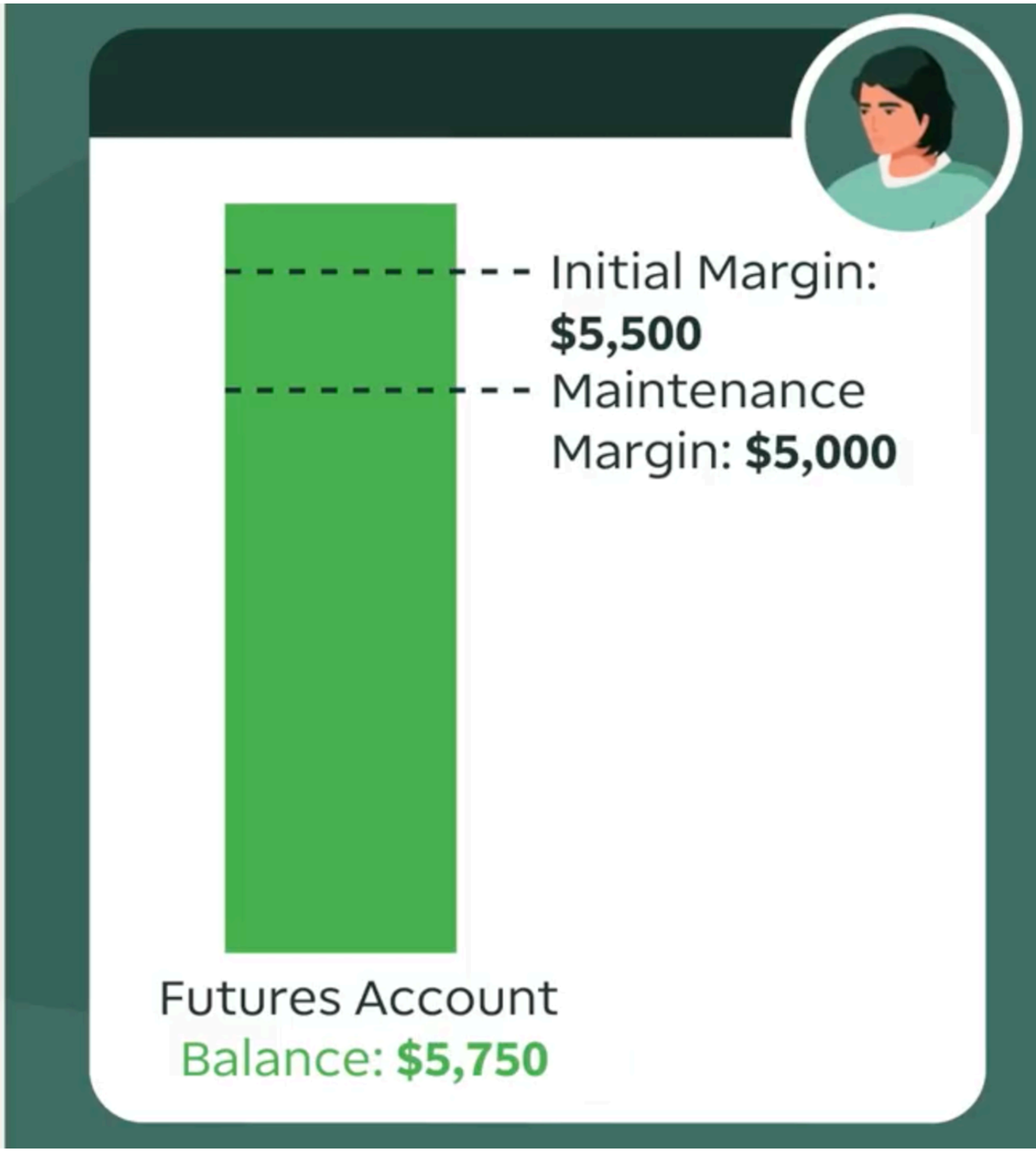
How Margin Works

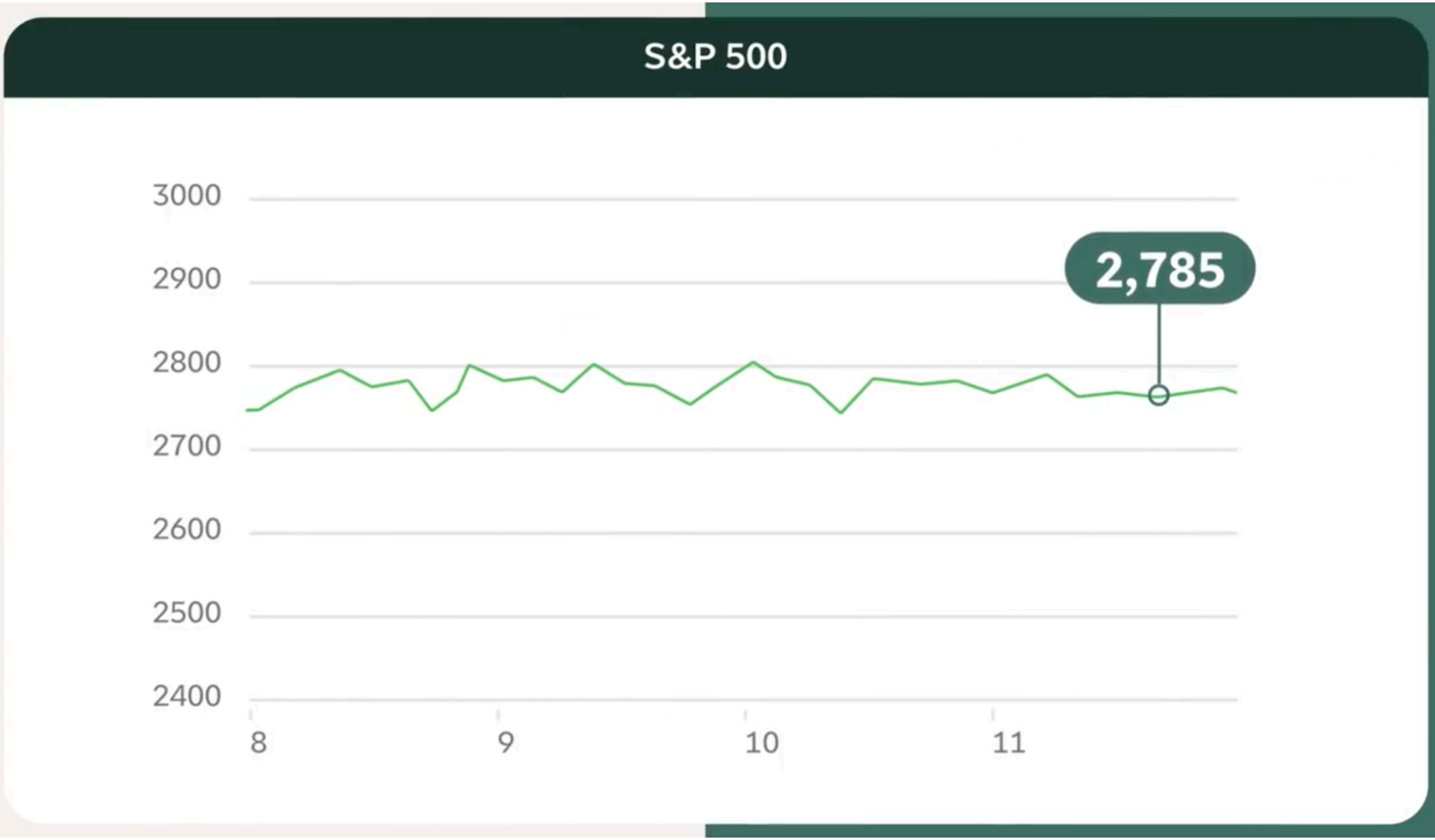


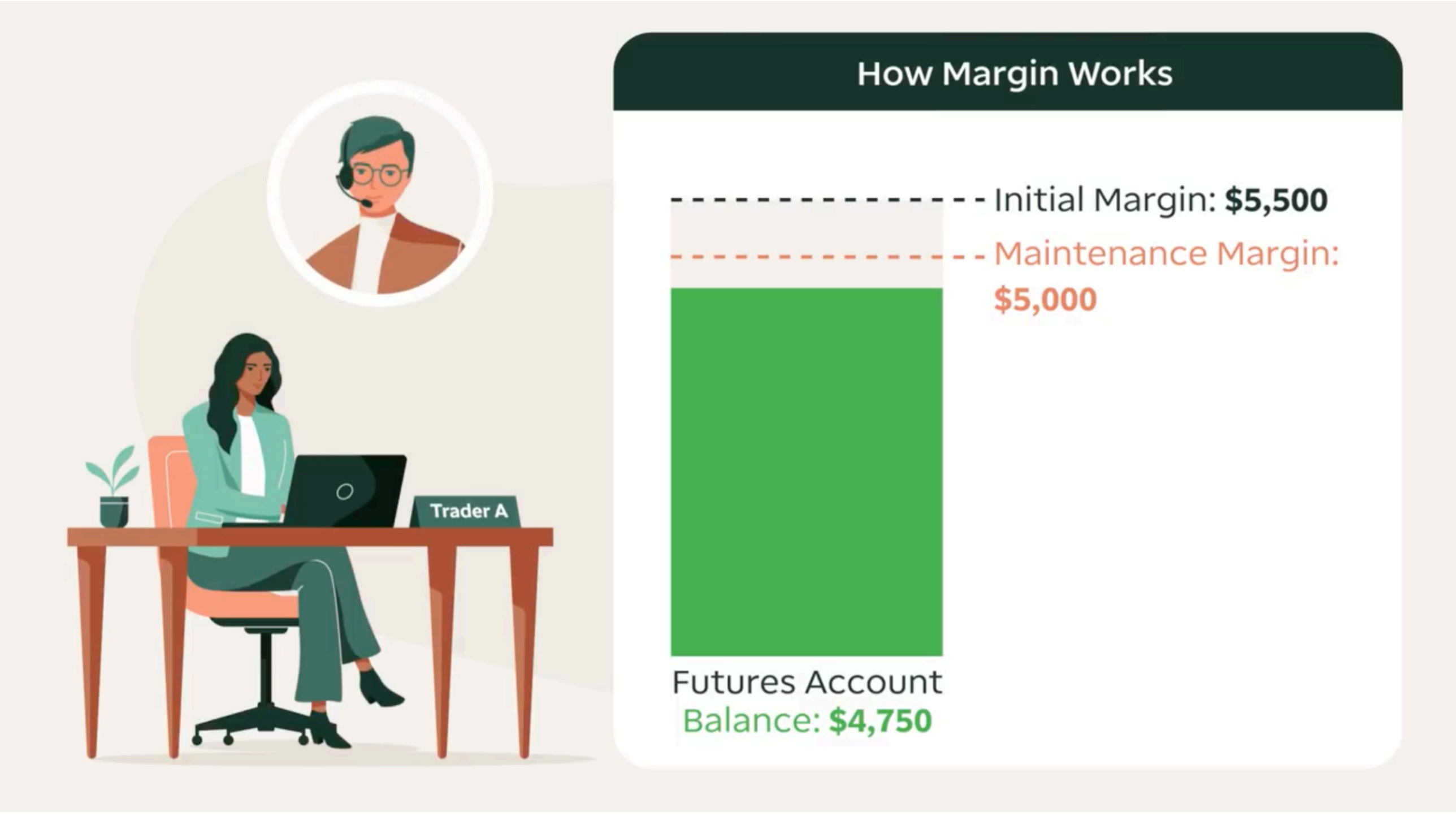


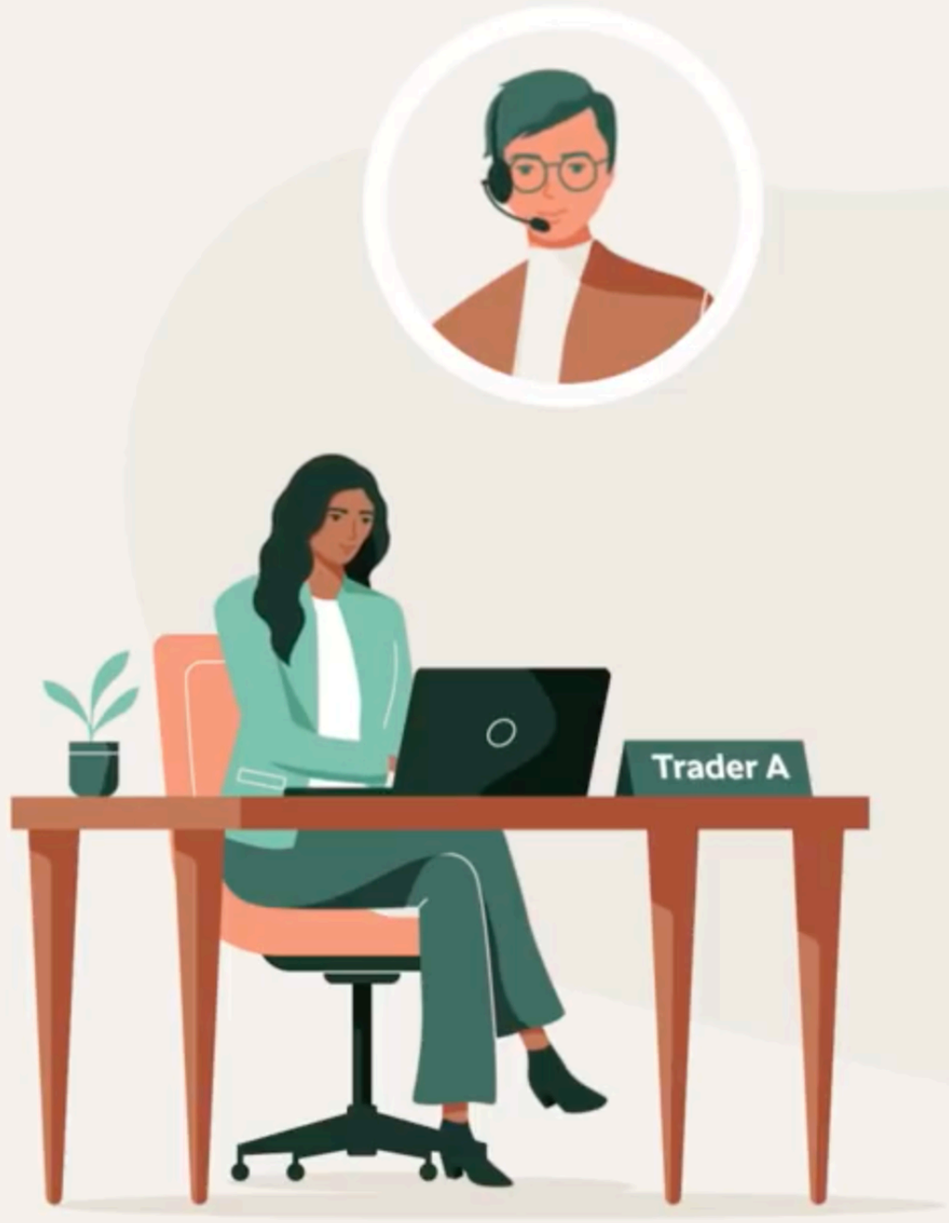
Index moves from 2,800 to 2,795 = 5/2,800 = 0.18%
Gain (or loss) = 250/5,500 = 4.545%
Leveraging = 4.545/0.18 = 25.25 = 140,000/5,500


Initial Margin: $5,500
Maintenance Margin: $5,000
Futures Account
Balance: $5,250

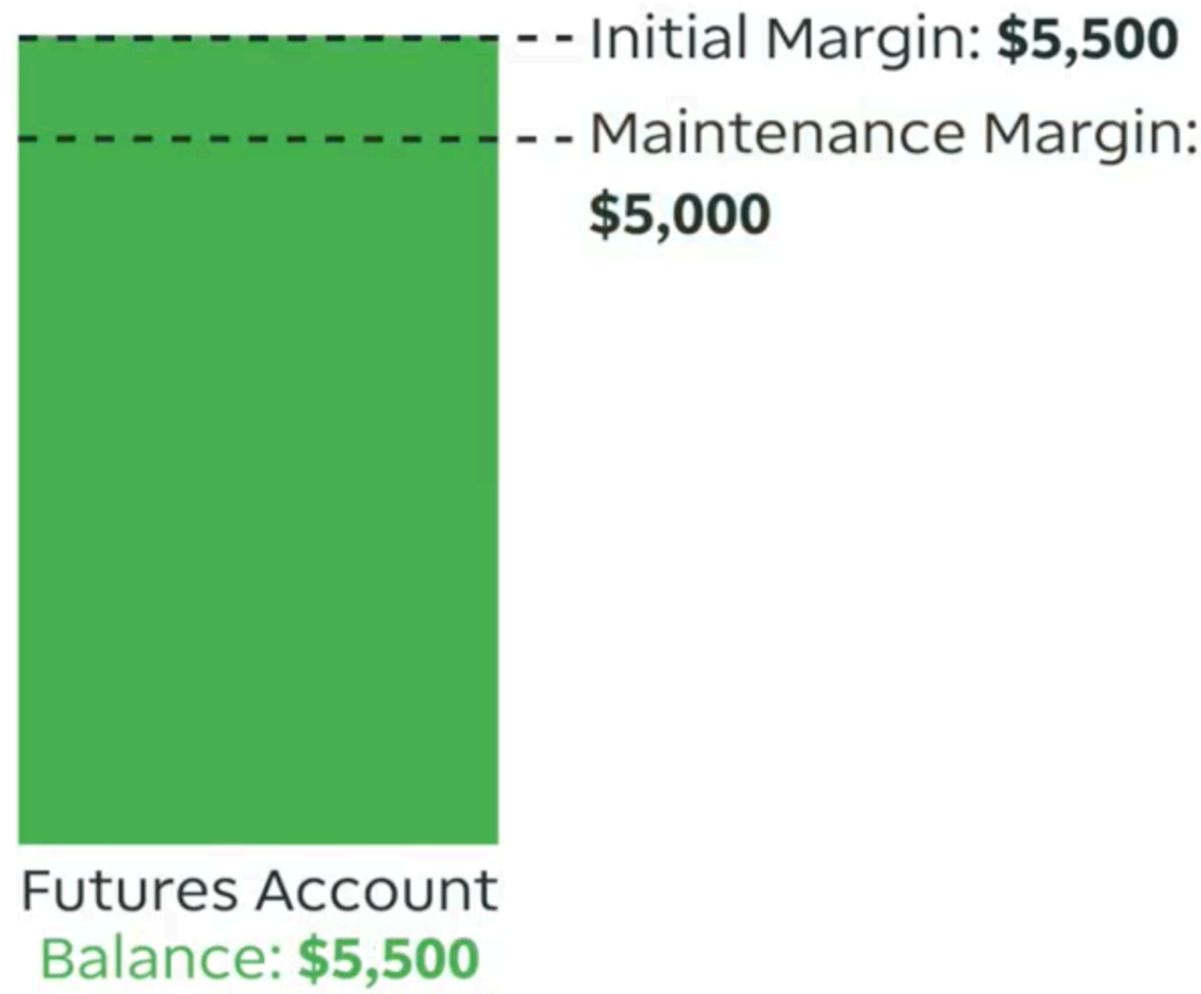

Initial Margin: $5,500
Maintenance Margin: $5,000
Futures Account
Balance: $5,750


S&P 500
3000
2900
2800
2700
2600
2500
2400
2,785
8
9
10
11

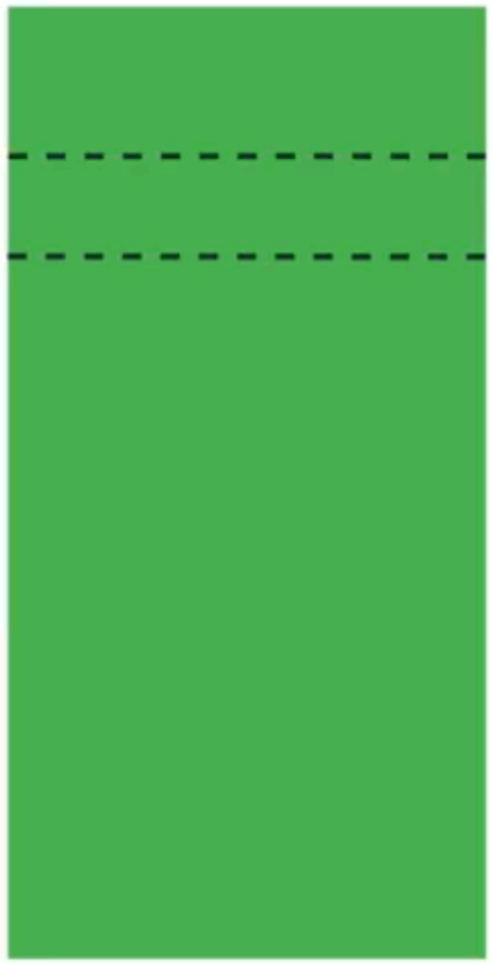

How Margin Works
Initial Margin: $5,500
Maintenance Margin: $5,000
Futures Account
Balance: $4,750
Trader A

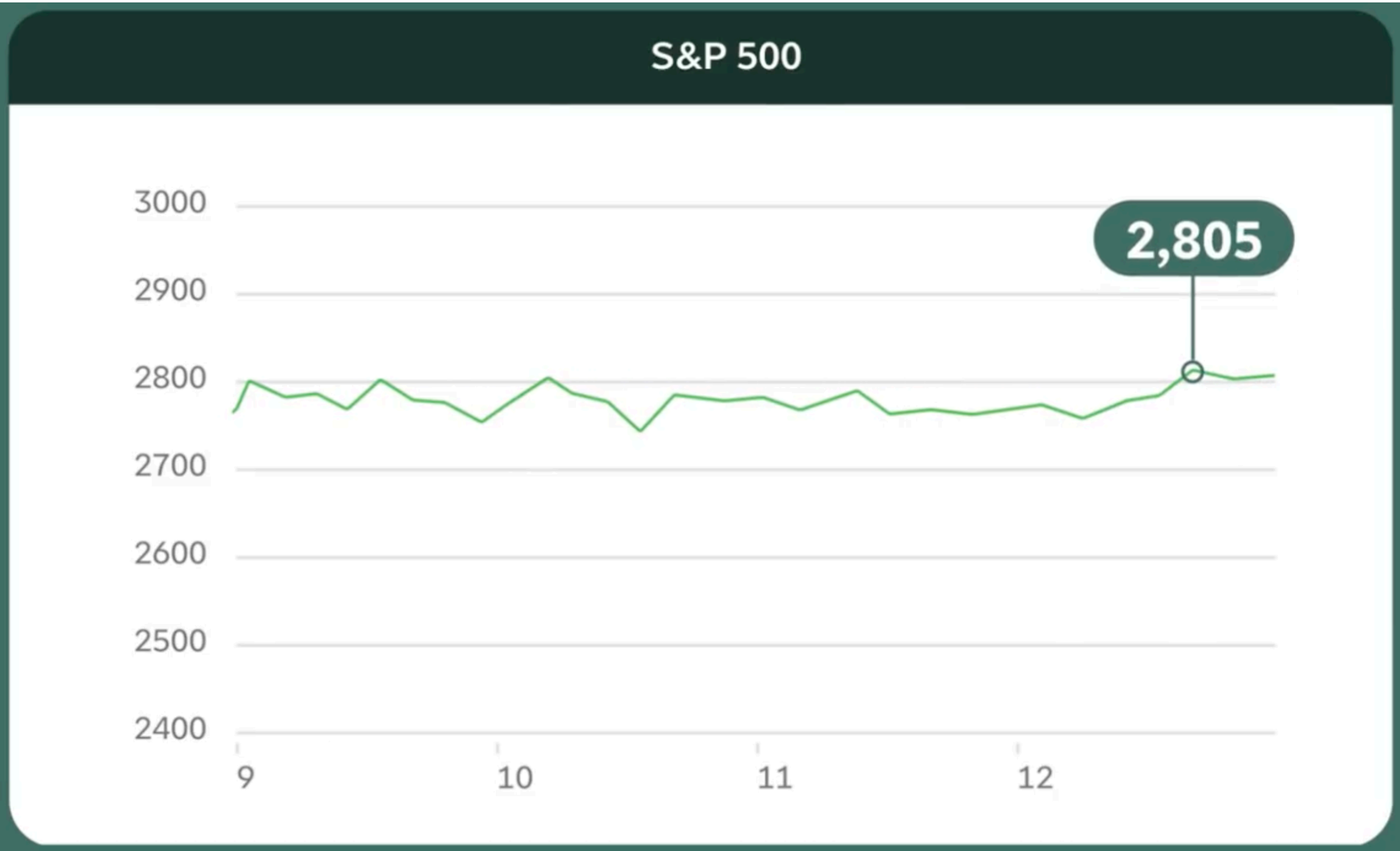


How Margin Works

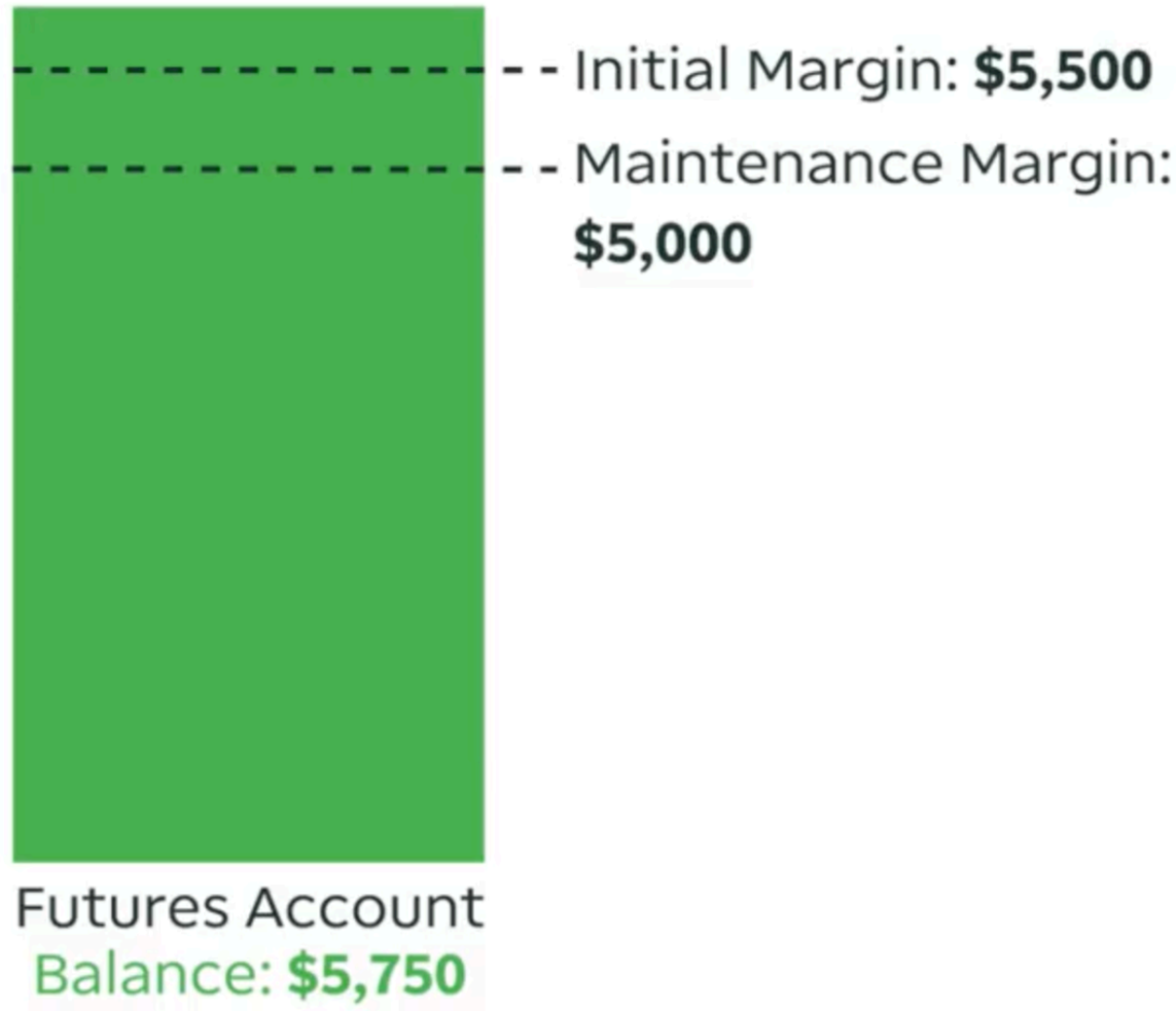


How Margin Works

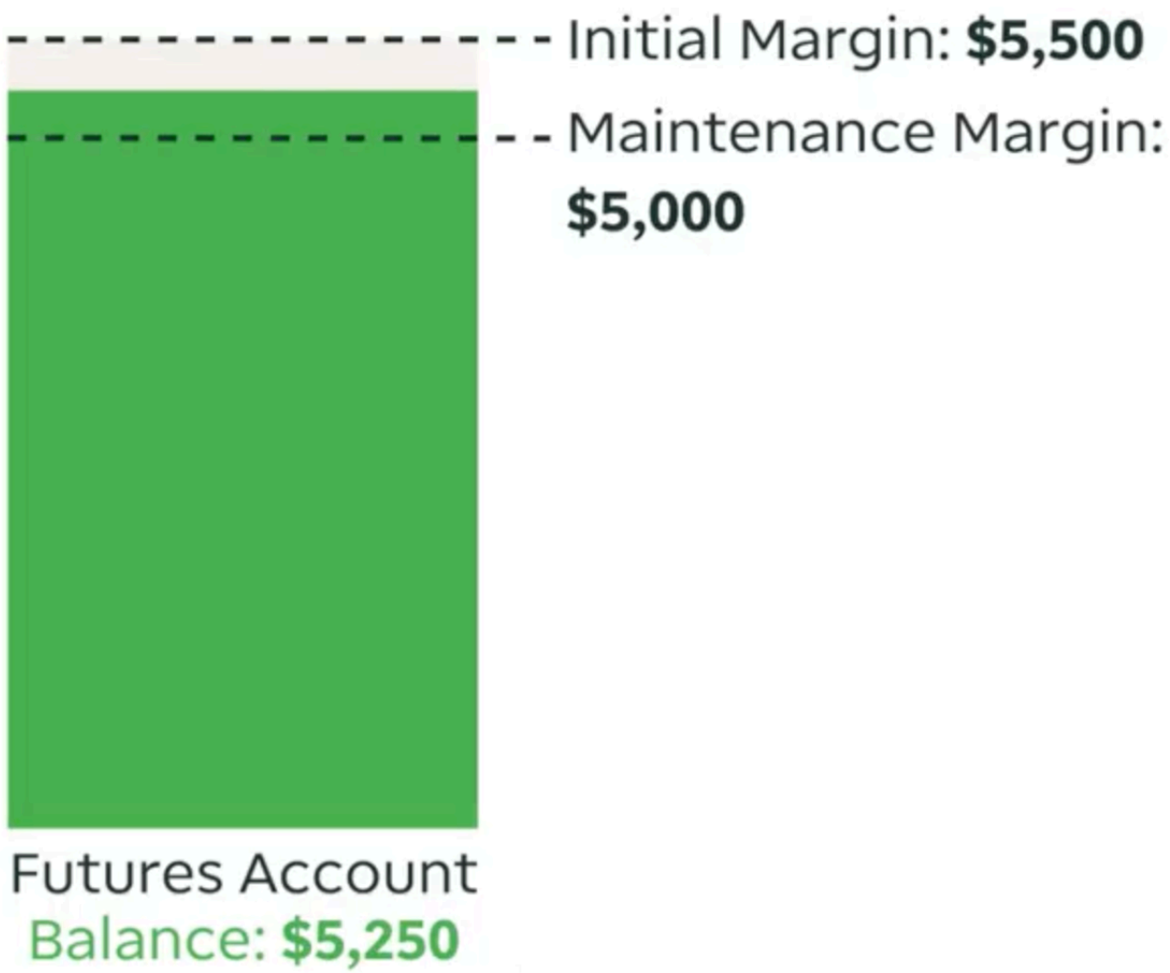


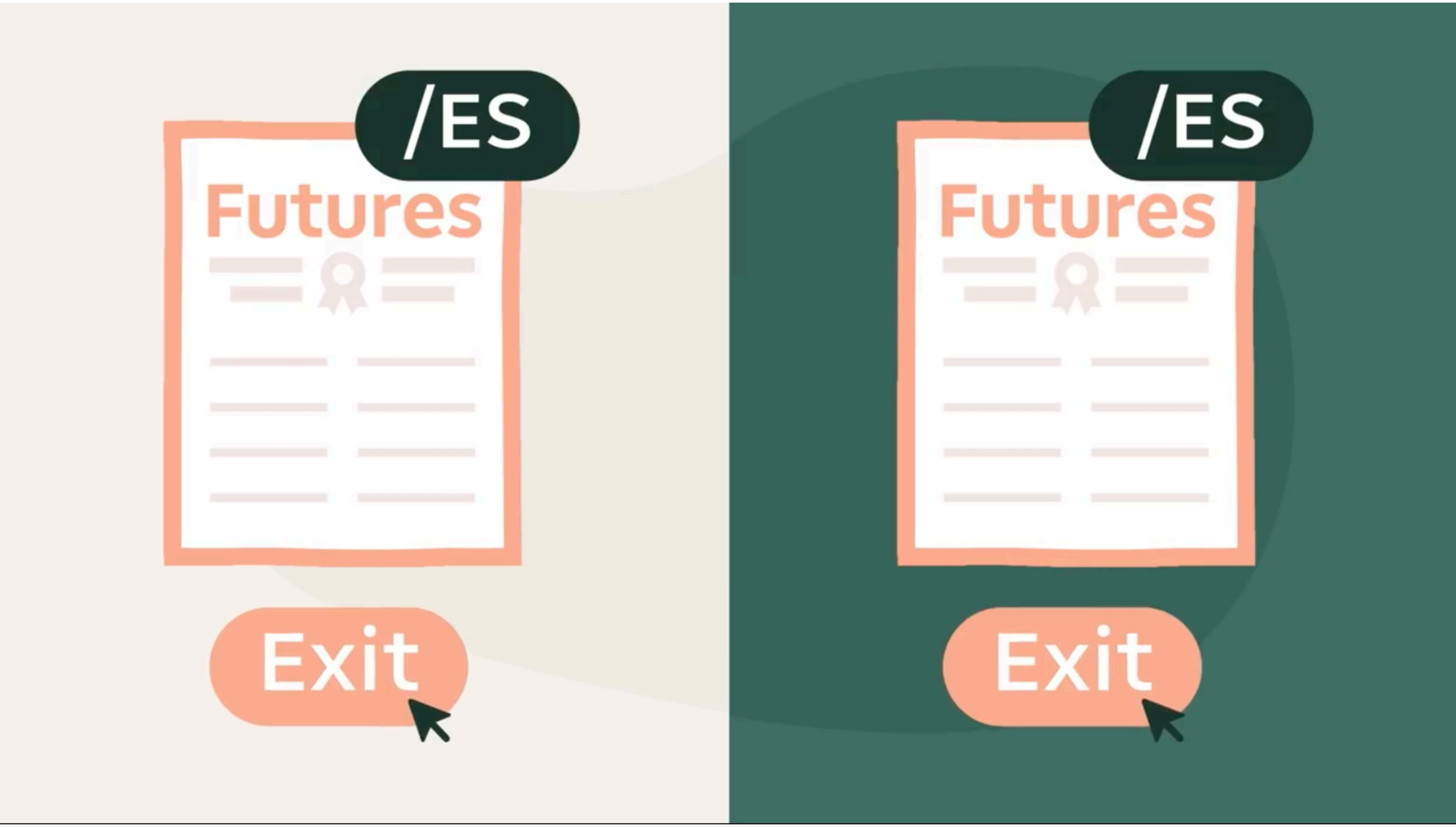

S&P 500
3000
2900
2800
2700
2600
2500
2400
2,805
9
10
11
12

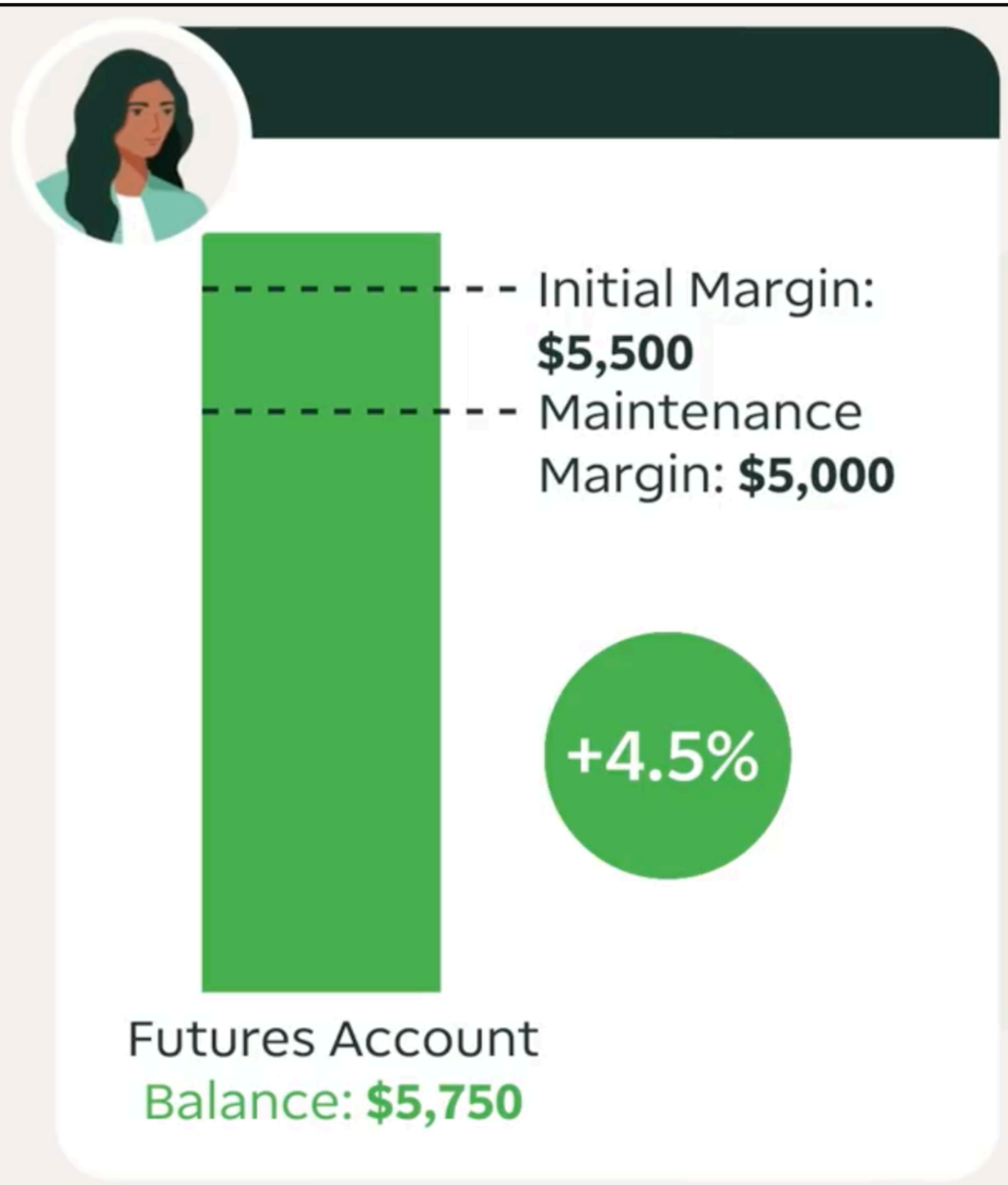
How Margin Works

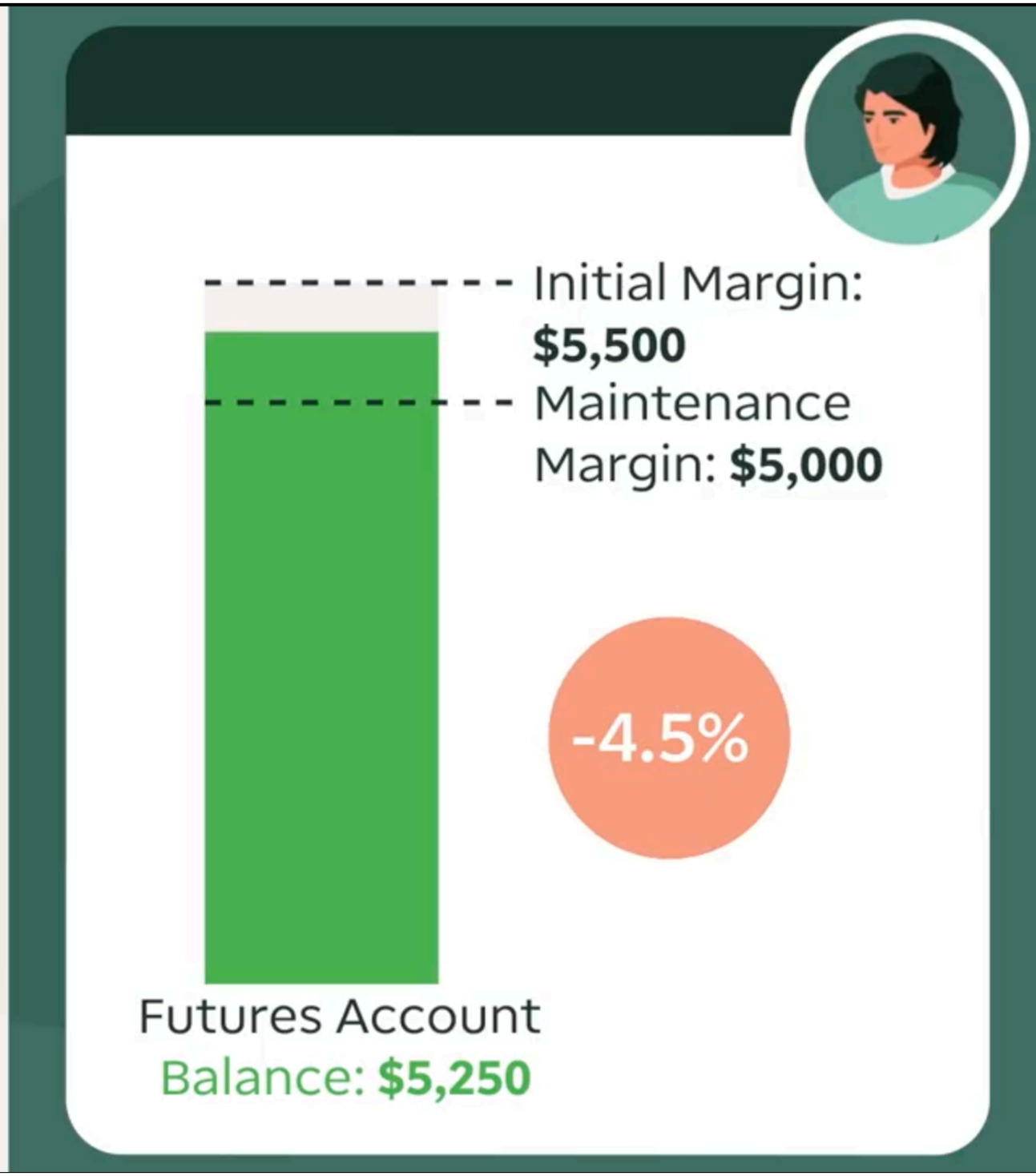


How Margin Works




/ES
Futures
Exit
/ES
Futures
Exit


Initial Margin:
$5,500
Maintenance
Margin: $5,000
+4.5%
Futures Account
Balance: $5,750


Initial Margin:
$5,500
Maintenance
Margin: $5,000
-4.5%
Futures Account
Balance: $5,250

That has been the use of future contracts for **speculating**

Valuation of Forwards and Futures

What Determines Forward and Futures Prices?

- Forward/futures prices ultimately linked to future spot prices
- Notation:

Contract	Spot at t	Forward	Futures
Price	S_t	$F_{t,T}$	$H_{t,T}$

- Ignore differences between forward and futures price for now

$$F_{t,T} \approx H_{t,T}$$

- Two ways to buy the underlying asset for date-*T* delivery
  1. Buy a forward or futures contract with maturity date *T*
  2. Buy the underlying asset and store it until *T*

Valuation of Forwards and Futures

Date	Forward Contract	Outright Asset Purchase
0	▪ Pay \$0 for contract with forward price $\$F_{0,T}$	▪ Borrow $\$S_0$ ▪ Pay $\$S_0$ for Asset
T	▪ Pay $\$F_{0,T}$ ▪ Own asset	▪ Pay back $\$S_0(1+r)^T$ ▪ Pay cumulative storage costs (if any) ▪ Deduce cumulative "convenience yield" (if any) ▪ Own asset
Total Cost at T	**$\$F_{0,T}$**	**$\$S_0(1+r)^T$ + net storage costs**

$$F_{0,T} \approx H_{0,T} = (1+r_f)^T S_0 + \text{FV}_T(\text{net storage costs})$$

$$\frac{F_{0,T}}{(1+r)^T} \approx \frac{H_{0,T}}{(1+r)^T} = S_0 + \text{PV}_0(\text{net storage costs})$$

Valuation of Forwards and Futures

Date	Forward Contract	Outright Asset Purchase
t	▪ Pay \$0 for contract with forward price $\$F_{t,T}$	▪ Borrow $\$S_t$ ▪ Pay $\$S_t$ for Asset
T	▪ Pay $\$F_{t,T}$ ▪ Own asset	▪ Pay back $\$S_t(1+r)^{T-t}$ ▪ Pay cumulative storage costs (if any) ▪ Deduce cumulative "convenience yield" (if any) ▪ Own asset
Total Cost at *T*	**$\$F_{t,T}$**	**$\$S_0(1+r)^{T-t}$ + net storage costs**

$$F_{t,T} \approx H_{t,T} = (1+r_f)^{T-t} S_t + \text{FV}_T(\text{net storage costs})$$

$$\frac{F_{t,T}}{(1+r)^{T-t}} \approx \frac{H_{t,T}}{(1+r)^{T-t}} = S_t + \text{PV}_t(\text{net storage costs})$$

Valuation of Forwards and Futures

What Determines Forward/Futures Prices?

- Difference between the two methods:
  - Costs (storage for commodities, not financials)
  - Benefits (convenience for commodities, dividends for financials)
- By no arbitrage (**Principal P1)**, these two methods must cost the same

Gold

- Easy to store (negligible costs of storage)
- No dividends or benefits
- Two ways to buy gold for T
  - Buy now for S_t and hold until T
  - Buy forward at t, pay $F_{t,T}$ at T and take delivery at T
- No-arbitrage requires that

$$F_{t,T} \approx H_{t,T} = (1 + r_f)^{(T-t)} S_t$$

Valuation of Forwards and Futures

Gasoline

- Costly to store (let c be percentage cost per period)
- Convenience yield does exist (let y be percentage yield per period)
- Not for long-term investment (like gold), but for future use
- Two ways to buy gasoline for T
  - Buy now for S_t and hold until T
  - Buy forward at t, pay $F_{t,T}$ at T and take delivery at T
- No-arbitrage requires that

$$F_{t,T} \approx H_{t,T} = (1 + r_f + c - y)^{(T-t)} S_t$$

Valuation of Forwards and Futures

Financials

- Let underlying be a financial asset
  - No cost to store (the underlying asset)
  - Dividend or interest on the underlying
- Example: Stock index futures
  - Underlying are bundles of stocks, e.g., S&P, Nikkei, etc.
  - Futures settled in cash (no delivery)
  - Let the annualized dividend yield be *d;* then:

$$F_{t,T} \approx H_{t,T} = (1 + r_f - d)^{(T-t)} S_t$$

Arbitraging Opportunity

- Spot price of Platinum is $1,000/oz
- 1-year future settlement price is $1,200/oz
- Borrowing interest rate is 10% per year
- Storage cost: $50 per oz. per year
- Is there a risk-free profit to be had here?

What We Have Learned

- Forward contracts and their limitations
- Futures contracts, solving the illiquidity and counterparty risk problems
- How margins work
- Valuations for forward and futures