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Trim Equations of Motion For Aircraft Design: Steady State Straight-Line Flight

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ABSTRACT

The development of performance-optimal stability and control design solutions for advanced conventional and unconventional aircraft configuration shapes pose a particular challenge during the conceptual design phase. This design challenge can be attributed to a lack of design methods available, chronic data shortage when dealing with novel configuration shapes, and permanent time pressure during the early conceptual design segment. The situation is particularly apt when evaluating symmetric or asymmetric aircraft shapes in asymmetric flight conditions.

When evaluating the stability and control characteristics of a flight vehicle, all design decision-making needs to be based on a trimmed flight vehicle, ideally in six degrees-of-freedom. The task of trimming the vehicle in symmetric and asymmetric flight conditions represents, however, a non-trivial task during the conceptual design phase. This paper presents the development and discusses the application of the trim equations of motion (steady state equations of motion) for steady state straight-line flight applicable to conventional and unconventional aircraft shapes of symmetric and asymmetric layout. The fully developed six-degree-of-freedom formulation aims to trim those flight conditions key to sizing the vehicle's control effectors during the conceptual design stage. The asymmetric-oblique-wing aircraft has been selected as the development 'benchmark vehicle', because of its unequaled development potential, the inherent inclusion of the range of symmetric aircraft types, and the fact that the majority of critical flight conditions for the design of controls are asymmetric flight conditions. Overall, the primary research aim is the development of a generic analytical framework with the capability to trim the aircraft in six degrees-of-freedom for the assessment of control power required/available.

NOMENCLATURE

b	=	span
C_D	=	drag coefficient (aircraft)
C_{D_0}	=	drag coefficient (aircraft) for zero angle-of-attack
C_{D_α}	=	variation of aircraft drag coefficient with angle of attack
C_l	=	rolling moment coefficient (aircraft)
C_{l_0}	=	rolling moment coefficient (aircraft) for zero angle-of-attack
C_{l_β}	=	variation of aircraft rolling moment coefficient with angle of sideslip
C_L	=	lift coefficient (aircraft)
C_{L_0}	=	lift coefficient (aircraft) for zero angle-of-attack
C_{L_α}	=	variation of aircraft lift coefficient with angle of attack
C_m	=	pitching moment coefficient (aircraft)
C_{m_0}	=	pitching moment coefficient (aircraft) for zero angle-of-attack
C_{m_α}	=	variation of aircraft pitching moment coefficient with angle of attack
C_n	=	yawing moment coefficient (aircraft)
C_{n_0}	=	yawing moment coefficient (aircraft) for zero angle-of-attack
C_{n_β}	=	variation of aircraft yawing moment coefficient with angle of sideslip
C_Y	=	side force coefficient (aircraft)
C_{Y_0}	=	side force coefficient (aircraft) for zero angle-of-attack
C_{Y_β}	=	variation of aircraft side force coefficient with angle of sideslip
F_B	=	body axes: F_B (*c.g., x, y, z*)
F_E	=	frame of reference (inertial system) attached to the Earth: F_E (O_E, x_E, y_E, z_E)
$F_{T_x}, F_{T_y}, F_{T_z}$	=	scalar components of $\vec{T}$

g = acceleration due to gravity

$\vec{G}$ = resultant external moment vector, about the mass center

$\vec{h}'$ = angular momentum vector of spinning rotors with respect to rotor mass center

h'_x, h'_y, h'_z = scalar components of $\vec{h}'$ in F_B

i = aerodynamic control effector variable incidence stabilizer angle (trimmable CE)

I_B = inertia matrix

I_x, I_y, I_z = moments of inertia

I_{xy}, I_{yz}, I_{xz} = products of inertia

L, M, N = scalar components of $\vec{G}$ in F_B, thrust moments

m = mass

p, q, r = scalar components of $\vec{\omega}$ in F_B

$\dot{p}, \dot{q}, \dot{r}$ = scalar components of $\dot{\vec{\omega}}$ in F_B, rate of change of aircraft angular velocity

$\bar{q}$ = aircraft dynamic pressure

S = area; wing reference area

$\vec{T}$ = thrust vector

T_x, T_y, T_z = scalar components of $\vec{T}$

u, v, w = scalar components of $\vec{V}$ in F_B, perturbed values of *U, V and W*

U, V, W = scalar velocity components of $\vec{V}$

W = weight (aircraft)

$\vec{V}$ = aircraft velocity vector

x_E, y_E, z_E = coordinates of aircraft mass center relative to fixed axes (inertial system F_E)

x_T, y_T, z_T = thrust line coordinates relative to the aircraft c.g.

X, Y, Z = components of resultant force

α , β = angle of attack, angle of sideslip

δ = control effector deflection angle

γ = flight path angle

Δ = increment (perturbation) of a parameter; non-zero reference value

τ_X, τ_M, τ_N = corrections for propulsive installation

ϕ, θ, ψ = Euler angles

$\vec{\omega}$ = angular velocity vector

Subscripts

A = aerodynamic

B, b = body frame F_B

D = drag increment due to inoperative engine

DiCE = Directional CE

i = variation with CE incidence angle

i,j,k/l,m,n = variable indices

LaCE = Lateral CE

LoCE = Longitudinal CE

s = stability axes

T = thrust

δ = variation with CE deflection

Superscripts

E = inertial system

INTRODUCTION

CONTEXT. A study has been conducted over a four-year period to develop a generic stability and control (s&c) methodology *AeroMech* to consistently size the control effectors of fixed-wing aircraft of conventional but in particular unconventional configuration shapes at the aircraft conceptual design level, see [1] and [2]. Overall, the true complexity of the research undertaking has been hidden in the objective, to develop a generic (vehicle configuration independent) methodology concept and algorithm. The inclusion of asymmetric flight conditions and asymmetric aircraft configurations, in addition to the reasonably well-known symmetric aircraft types, contributed significantly to the overall technical complexity and level of abstraction. In addition, control effectors (CEs) are not designed for the cruise design point. Instead, they are sized in the 'grey-areas' of the flight envelope, where non-linear aerodynamics prevails, demanding specific information during the initial conceptual design phase. Overall, it has been a clear development target to develop a methodology concept for conceptual design application with the consequent intent *"... things should be as simple as possible, but no simpler ..."*.

DESIGN FOR STABILITY AND CONTROL. In *AeroMech*, all design decision-making is based on the evaluation of the trimmed aircraft in six degrees-of-freedom (6-DOF), an exception being the take-off rotation maneuver. The trim (steady state) EOM are solved to determine control power available/required for a range of pre-defined critical flight conditions (DCFC). The dynamic (small perturbation) EOM are solved to estimate the gain constants required to restore stiffness and damping for reduced stability to unstable airframes. The dynamic stability characteristics are analyzed using the results generated by the dynamic EOM. An off-line analysis sequence evaluates the dynamic stability characteristics of the vehicle with reduced order models. This trend-information enables the designer to gain physical insights into the mode drivers. The output file finally contains design-relevant information, which ensures the balance between control power and static-, maneuver-, and dynamic stability. Control power available/required is defined with (a) the volume coefficient (geometry), (b) stability derivative coefficients (aerodynamics), and (c) the CE deflection angle required (operation). For more information, see [1].

BECHMARK VEHICLE. The term 'generic' implies, that the asymmetric aircraft type is considered to be the most general aircraft arrangement, whereby symmetric types represent rather 'simplified' or special cases where certain simplifying assumptions are valid. Clearly, functionality of the methodology concept for asymmetric aircraft ascertains functionality for the range of symmetric aircraft. An important by-product of this approach is the capability, to enable handling of asymmetric flight conditions of symmetric and asymmetric aircraft configurations, a non-typical ability for a conceptual design method.

Technically, asymmetric aircraft configurations resemble the most demanding aircraft type, which might, however, provide an unmatched performance potential. Asymmetric aircraft types, in particular the OFWC, are the single correct choice for minimum wave drag and minimum induced drag due to lift. In addition, the structural efficiency of the OFWC is superior due to its span-loader concept and volumetric efficiency. However, the real complications are their inherent stability and control characteristics. In contrast to symmetric aircraft types, asymmetric aircraft represent highly coupled systems due to inertia coupling and aerodynamic coupling effects. For introductory reading, Nelms [3] has produced an excellent summary of oblique-wing technology programmes. A more recent summary of oblique flying wing studies is presented by Li, Seebass, and Sobieczky [4].

MODELING APPROACH. There are two general flight conditions for which solutions of the equations of motion (EOM) are of primary interest. The steady state EOM form the basis for studying vehicle *controllability* problems (control power), whereby the perturbed state EOM form the basis for studying aircraft *dynamic stability and response* problems and *automatic flight control theory and application*. The present context is only concerned with studying controllability aspects.

There exist three principal approaches in analytically modelling the asymmetric aircraft type:

(a) Decoupling of the longitudinal and lateral-directional motions and neglecting the cross-coupling terms finally leads to the classical three degree-of-freedom (3-DOF) approach.
(b) Separation of the analysis into the longitudinal and the lateral-directional motions without decoupling (inclusion of cross-coupling terms), see Thelander [5] and Maine [6].
(c) Formulation of the fully coupled 6-DOF EOM including primary cross-coupling effects.

The implications of the above three schemes are briefly discussed. All three approaches do not demand any particular computing power. The primary issue of interest is simplicity. It is a constant quest in aircraft conceptual design that one strives for an analytical model complicated enough to adequately represent the system. Once a certain complexity level has been surpassed, extra complication in the model almost invariably degrades the result. The major complications one can foresee are twofold:

i. The aerodynamic estimation is not adequate for the complexity level selected.
ii. Excessive computation difficulties arise.

A combined longitudinal and lateral-directional model seems, at a first glance, far more complicated than two separate ones. However, it must be realized that both approaches might use the same aerodynamic data available, thus the only complication left is of computational character. If we assume that modern numerical methods are able to solve three equations simultaneously, then six equations do not pose a specific problem. Still, the 6-DOF approach provides more opportunity for things to go wrong. However, it should be recalled that modern CFD methods, finite element (FE) methods, or simulation software, are far more complex than the method developed here. The argumentation therefore has to concentrate on the issue, of how well the 3-DOF approach represents the physics of interest, or whether or not the 6-DOF approach provides more trustworthy information with more inherent potential in its approach for future applications (provision of a properly trimmed aircraft, consideration of all flight conditions of interest, consistent static and dynamic investigations, etc.).

It has been decided, to derive the underlying mathematical framework of the generic stability and control methodology *AeroMech* based on the 6-DOF static- and dynamic equations of motion (EOM). With this analytical framework in place it is possible, to evaluate the range of design-constraining flight conditions (DCFCs) for sizing CEs, see [1].

STEADY STATE EQUATIONS OF MOTION

GENERAL EULER EOM. Steady state flight is characterized by having zero rates of change of the linear and angular velocity components with time relative to the body-fixed axis system in an atmosphere of constant density.

$$\dot{\vec{V}} = 0 \tag{1}$$

$$\dot{\vec{\omega}} = 0 \tag{2}$$

The following flight cases have been modeled for the asymmetric aircraft type, see [1]:

1. *Steady State Straight-Line Flight*;
2. *Steady State Turning Flight*;
3. *Steady State Pull-Up and Push-Over Flight*;
4. *Steady State Rolling Performance*;
5. *Quasi-Steady Take-Off Rotation Maneuver*.

The quasi-steady take-off rotation flight case cannot be considered a steady state flight case, since $\dot{q} \neq 0$. However, the instant at which the rotation is evaluated ($q=0$) permits the grouping of this flight case with the steady state flight cases. This paper discusses the *Steady State Straight Line Flight* case. The underlying equations for modeling the above steady state flight conditions are the <u>General Euler Equations Of Motion With Spinning Rotors</u> (derivation see [1]).

$$X_A + X_T - mg\sin\theta = m\left(\dot{u}^E + qw^E - rv^E\right) \tag{3a}$$
$$Y_A + Y_T + mg\cos\theta\sin\phi = m\left(\dot{v}^E + ru^E - pw^E\right) \tag{3b}$$
$$Z_A + Z_T + mg\cos\theta\cos\phi = m\left(\dot{w}^E + pv^E - qu^E\right) \tag{3c}$$

$$L_A + L_T = I_x\dot{p} - I_{yz}\left(q^2 - r^2\right) - I_{zx}\left(\dot{r} + pq\right) - I_{xy}\left(\dot{q} - rp\right) - \left(I_y - I_z\right)qr + qh'_z - rh'_y \tag{4a}$$
$$M_A + M_T = I_y\dot{q} - I_{zx}\left(r^2 - p^2\right) - I_{xy}\left(\dot{p} + qr\right) - I_{yz}\left(\dot{r} - pq\right) - \left(I_z - I_x\right)rp + rh'_x - ph'_z \tag{4b}$$
$$N_A + N_T = I_z\dot{r} - I_{xy}\left(p^2 - q^2\right) - I_{yz}\left(\dot{q} + rp\right) - I_{zx}\left(\dot{p} - qr\right) - \left(I_x - I_y\right)pq + ph'_y - qh'_x \tag{4c}$$

$$p = \dot{\phi} - \dot{\psi}\sin\theta \tag{5a}$$
$$q = \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \tag{5b}$$
$$r = -\dot{\theta}\sin\phi + \dot{\psi}\cos\phi\cos\theta \tag{5c}$$

$$\dot{\phi} = p + q\left(\sin\phi + r\cos\phi\right)\tan\theta \tag{6a}$$
$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{6b}$$
$$\dot{\psi} = \left(q\sin\phi + r\cos\phi\right)\sec\theta \tag{6c}$$

$$\dot{x}_E = u^E\cos\theta\cos\psi + v^E\left(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi\right) + w^E\left(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi\right) \tag{7a}$$
$$\dot{y}_E = u^E\cos\theta\sin\psi + v^E\left(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi\right) + w^E\left(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi\right) \tag{7b}$$
$$\dot{z}_E = -u^E\sin\theta + v^E\sin\phi\cos\theta + w^E\cos\phi\cos\theta \tag{7c}$$

The above equations contain the following assumptions:

1. The Earth is treated flat and stationary in inertial space, thus rotational velocity is neglected.
2. The equations are valid for any orthogonal axis system fixed at the c.g. of the aircraft (body axes).
3. The aircraft is a rigid body ($\dot{I}_B = 0$), having attached to it any number of rigid spinning rotors.
4. The spinning rotors have constant angular speed relative to the body axes ($\dot{h}'_B = 0$). The axis of any spinning rotor is fixed in direction relative to the body axes. This assumption is valid for thrust vectoring with a movable nozzle (usual), where the thrust vector alters direction but the axes of the spinning rotors stay constant. The assumption of spinning rotors with fixed axes requires to be reviewed, when applied to the OFWC with engines pivoted dependent on wing sweep adjustment during flight.
5. The wind velocity is zero, so that $\vec{V}^E = \vec{V}$.

The usual assumptions like, (i) the existence of a plane of symmetry (C_{xz}), (ii) neglection of aerodynamic cross-coupling, (iii) the absence of rotor gyroscopic effects, have not been accepted in the present context.

THRUST FORCES AND MOMENTS. At first it is required to model the thrust forces and moments acting on the airframe.

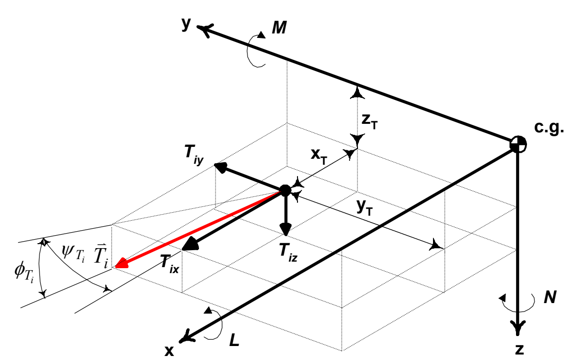

Fig. 1. *Thrust force component break-down.*

The thrust terms in stability axis in their final form (derivation see [1]):

$$F_{T_{x_s}} = \sum_{i=1}^{n} T_i\left(\cos\phi_{T_i}\cos\psi_{T_i}\cos\alpha + \sin\phi_{T_i}\sin\alpha\right) \tag{8a}$$

$$F_{T_{y_s}} = \sum_{i=1}^{n} T_i\left(\cos\phi_{T_i}\sin\psi_{T_i}\right) \tag{8b}$$

$$F_{T_{z_s}} = \sum_{i=1}^{n} T_i\left(-\cos\phi_{T_i}\cos\psi_{T_i}\sin\alpha + \sin\phi_{T_i}\cos\alpha\right) \tag{8c}$$

$$L_s = \sum_{i=1}^{n}\begin{bmatrix} T_i\cos\alpha\left(-\cos\phi_{T_i}\sin\psi_{T_i}z_T + \sin\phi_{T_i}y_T\right) \\ +T_i\sin\alpha\left(\cos\phi_{T_i}\sin\psi_{T_i}x_T - \cos\phi_{T_i}\cos\psi_{T_i}y_T\right)\end{bmatrix} \tag{9a}$$

$$M_s = \sum_{i=1}^{n}\left[T_i\left(\cos\phi_{T_i}\cos\psi_{T_i} z_T - \sin\phi_{T_i} x_T\right)\right] \quad (9b)$$

$$N_s = \sum_{i=1}^{n}\begin{bmatrix} T_i \sin\alpha\left(\cos\phi_{T_i}\sin\psi_{T_i} z_T - \sin\phi_{T_i} y_T\right) \\ + T_i\cos\alpha\left(\cos\phi_{T_i}\sin\psi_{T_i} x_T - \cos\phi_{T_i}\cos\psi_{T_i} y_T\right)\end{bmatrix} \quad (9c)$$

Note, the angle ϕ_{T_i} in Figure 1 is negative.

STEADY STATE STRAIGHT LINE FLIGHT. The form of the EOM required for steady state straight line flight is given with Equations (3) to (7). Steady state rectilinear flight is characterized by zero perturbation, $\Delta = 0$, and $\bar{\omega} = 0$, which in turn implies $p = q = r = 0$. The kinematic equations become trivial and the force and moment equations become

Steady state straight line flight is the simplest steady flight case, since all time derivatives are zero and there is no angular velocity of the body about its c.g. ($\bar{\omega} = 0$). The kinematic equations become trivial leading to the non-linear _6-DOF Trim EOM for Steady State Straight Line Flight_ written in stability axes (derivation see [1]).

$$0 = -mg\sin\theta + X_A + X_T \quad (10a)$$
$$0 = mg\sin\phi\cos\theta + Y_A + Y_T \quad (10b)$$
$$0 = mg\cos\theta\cos\phi + Z_A + Z_T \quad (10c)$$

$$0 = L_A + L_T \quad (11a)$$
$$0 = M_A + M_T \quad (11b)$$
$$0 = N_A + N_T \quad (11c)$$

The corresponding aerodynamic and thrust forces and moments are substituted into (10) to (11), to give the following set of _Non-Linear 6-DOF Trim EOM for Steady State Straight Line_ flight written in stability axes ($\theta = \gamma$). A thorough check of the units in Equations (12) and (13) has been performed, see [1].

X-Force:

$$mg\sin\gamma = -\begin{bmatrix} C_{D_0} + C_{D_\alpha}\alpha + \sum_{j=1}^{n} C_{D_{i_{LoCE_j}}} i_{LoCE_j} + \sum_{k=1}^{m} C_{D_{\delta_{LoCE_k}}}\delta_{LoCE_k} \\ + \sum_{j=1}^{n} C_{D_{i_{DiCE_j}}} i_{DiCE_j} + \sum_{k=1}^{m} C_{D_{\delta_{DiCE_k}}}\delta_{DiCE_k} \\ + \sum_{j=1}^{n} C_{D_{i_{LaCE_j}}} i_{LaCE_j} + \sum_{k=1}^{m} C_{D_{\delta_{LaCE_k}}}\delta_{LaCE_k}\end{bmatrix}\bar{q}S + \sum_{i=1}^{n} T_i\left(\cos\phi_{T_i}\cos\psi_{T_i}\cos\alpha + \sin\phi_{T_i}\sin\alpha\right) - \sum_{i=1}^{n}\Delta X_{D_i} \quad (12a)$$

Y-Force:

$$-mg\sin\phi\cos\gamma = \begin{bmatrix} C_{y_0} + C_{y_\beta}\beta + \sum_{j=1}^{n} C_{y_{i_{LoCE_j}}} i_{LoCE_j} + \sum_{k=1}^{m} C_{y_{\delta_{LoCE_k}}}\delta_{LoCE_k} \\ + \sum_{j=1}^{n} C_{y_{i_{DiCE_j}}} i_{DiCE_j} + \sum_{k=1}^{m} C_{y_{\delta_{DiCE_k}}}\delta_{DiCE_k} \\ + \sum_{j=1}^{n} C_{y_{i_{LaCE_j}}} i_{LaCE_j} + \sum_{k=1}^{m} C_{y_{\delta_{LaCE_k}}}\delta_{LaCE_k}\end{bmatrix}\bar{q}S + \sum_{i=1}^{n} T_i\cos\phi_{T_i}\sin\psi_{T_i} \quad (12b)$$

Z-Force:

$$-mg\cos\gamma\cos\phi = -\begin{bmatrix} C_{L_0} + C_{L_\alpha}\alpha + \sum_{j=1}^{n} C_{L_{i_{LoCE_j}}} i_{LoCE_j} + \sum_{k=1}^{m} C_{L_{\delta_{LoCE_k}}}\delta_{LoCE_k} \\ + \sum_{j=1}^{n} C_{L_{i_{DiCE_j}}} i_{DiCE_j} + \sum_{k=1}^{m} C_{L_{\delta_{DiCE_k}}}\delta_{DiCE_k} \\ + \sum_{j=1}^{n} C_{L_{i_{LaCE_j}}} i_{LaCE_j} + \sum_{k=1}^{m} C_{L_{\delta_{LaCE_k}}}\delta_{LaCE_k}\end{bmatrix}\bar{q}S + \sum_{i=1}^{n} T_i\left(-\cos\phi_{T_i}\cos\psi_{T_i}\sin\alpha + \sin\phi_{T_i}\cos\alpha\right) \quad (12c)$$

L-Moment:

$$0=\left[\begin{array}{l} C_{l_0}+C_{l_\beta}\beta+\sum_{j=1}^{n}C_{l_{i_{LoCE_j}}}i_{LoCE_j}+\sum_{k=1}^{m}C_{l_{\delta_{LoCE_k}}}\delta_{LoCE_k} \\ \qquad +\sum_{j=1}^{n}C_{l_{i_{DiCE_j}}}i_{DiCE_j}+\sum_{k=1}^{m}C_{l_{\delta_{DiCE_k}}}\delta_{DiCE_k} \\ \qquad +\sum_{j=1}^{n}C_{l_{i_{LaCE_j}}}i_{LaCE_j}+\sum_{k=1}^{m}C_{l_{\delta_{LaCE_k}}}\delta_{LaCE_k} \end{array}\right]\bar{q}Sb+\sum_{i=1}^{n}\mathrm{T_i}\cos\alpha\left(-\cos\phi_{T_i}\sin\psi_{T_i}z_T+\sin\phi_{T_i}y_T\right)$$

$$+\sum_{i=1}^{n}\mathrm{T_i}\sin\alpha\left(\cos\phi_{T_i}\sin\psi_{T_i}x_T-\cos\phi_{T_i}\cos\psi_{T_i}y_T\right) \tag{13a}$$

M-Moment:

$$0=\left[\begin{array}{l} C_{m_0}+C_{m_\alpha}\alpha+\sum_{j=1}^{n}C_{m_{i_{LoCE_j}}}i_{LoCE_j}+\sum_{k=1}^{m}C_{m_{\delta_{LoCE_k}}}\delta_{LoCE_k} \\ \qquad +\sum_{j=1}^{n}C_{m_{i_{DiCE_j}}}i_{DiCE_j}+\sum_{k=1}^{m}C_{m_{\delta_{DiCE_k}}}\delta_{DiCE_k} \\ \qquad +\sum_{j=1}^{n}C_{m_{i_{LaCE_j}}}i_{LaCE_j}+\sum_{k=1}^{m}C_{m_{\delta_{LaCE_k}}}\delta_{LaCE_k} \end{array}\right]\bar{q}Sb+\sum_{i=1}^{n}\mathrm{T_i}\left(\cos\phi_{T_i}\cos\psi_{T_i}z_T-\sin\phi_{T_i}x_T\right)\pm\sum_{i=1}^{n}\Delta\mathrm{M_{D_i}} \tag{13b}$$

N-Moment:

$$0=\left[\begin{array}{l} C_{n_0}+C_{n_\beta}\beta+\sum_{j=1}^{n}C_{n_{i_{LoCE_j}}}i_{LoCE_j}+\sum_{k=1}^{m}C_{n_{\delta_{LoCE_k}}}\delta_{LoCE_k} \\ \qquad +\sum_{j=1}^{n}C_{n_{i_{DiCE_j}}}i_{DiCE_j}+\sum_{k=1}^{m}C_{n_{\delta_{DiCE_k}}}\delta_{DiCE_k} \\ \qquad +\sum_{j=1}^{n}C_{n_{i_{LaCE_j}}}i_{LaCE_j}+\sum_{k=1}^{m}C_{n_{\delta_{LaCE_k}}}\delta_{LaCE_k} \end{array}\right]\bar{q}Sb+\sum_{i=1}^{n}\mathrm{T_i}\sin\alpha\left(\cos\phi_{T_i}\sin\psi_{T_i}z_T-\sin\phi_{T_i}y_T\right)$$

$$+\sum_{i=1}^{n}\mathrm{T_i}\cos\alpha\left(\cos\phi_{T_i}\sin\psi_{T_i}x_T-\cos\phi_{T_i}\cos\psi_{T_i}y_T\right) \tag{13c}$$

$$\pm\sum_{i=1}^{n}\Delta N_{D_i}$$

Equations (12) and (13) are non-linear equations with respect to the state variables α, ϕ, and γ. The above two sets of steady state equations form the basis for studying vehicle controllability design-aspects in rectilinear (straight-line) flight. The mathematical model is able to simulate the following steady state straight-line flight conditions for the full range of symmetric and anti-symmetric aircraft configurations and concepts:

(A) $\gamma \gtrless 0$, $\beta = 0$, $\phi = 0$, *Thrust Symmetry*
[horizontal flight, shallow climbs & dives, glides]

(B) $\gamma \gtrless 0$, $\beta \gtrless 0$, $\phi \gtrless 0$, *Thrust Symmetry*
[horizontal flight, shallow climbs & dives, glides, simulated crosswind condition due to *β><0* during de-crab, crossed controls, certain systems failed]

(C) $\gamma \gtrless 0$, $\beta \gtrless 0$, $\phi \gtrless 0$, *Thrust Asymmetry*
[horizontal flight, shallow climbs & dives, glides, engine failure, simulated crosswind condition due to *β><0* during de-crab, crossed controls, certain systems failed]

Equations (12) and (13) can be solved for any combination of the following design- and state-variables for straight-line flight:

$$\alpha\ ,\ \beta\ ,\ \gamma\ ,\ \phi\ ,\ V\ ,\ \bar{\rho}\ ,\ \underbrace{\sum i_{LoCE,DiCE,LaCE}}_{\text{variable incidence stabilizer}}\ ,$$

$$\sum\delta_{LoCE,DiCE,LaCE}\ ,\ T_i\ ,\ x_{T_i}\ ,\ y_{T_i}\ ,\ z_{T_i}\ ,\ \phi_{T_i}\ ,\ \psi_{T_i}$$

Since there are six equations, several of the design- and state variables listed above have to be specified before the system can be solved using iterative matrix techniques. The following example flight cases indicate the potential of Equations (12) and (13).

CASE (A) ($\gamma \gtrless 0$, $\beta = 0$, $\phi = 0$, *Thrust Symmetry*)

(i) *Cruise Trim Drag – Utilizing Aerodynamic CEs*

Pre-Selection: $\beta = 0$, $\gamma = 0$, $\phi = 0$, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-1}\delta_i$, V, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, $\delta_{LoCE_{i_{trim}}}$, T_i

Problem Description: This longitudinal case simulates the cruise condition with emphasis on LoCE margins, trim drag, mis-trim when pre-selecting LoCE stabiliser settings, etc.

(ii) *Cruise Trim Drag – Utilizing Thrust Vectoring CEs*

Pre-Selection: $\beta = 0$, $\gamma = 0$, $\phi = 0$, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m} \delta_i$, $\delta_{LoCE_{i_{trim}}} = 0$, V, x_{T_i}, y_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, z_{T_i}, ϕ_{T_i}, T_i

Problem Description: Having trimmed the aircraft longitudinally the step before using aerodynamic CEs, the longitudinal trim drag is eliminated by trimming the aircraft altering the vertical position of the thrust line and the thrust-line inclination angle. Again, this case simulates the cruise condition with emphasis on LoCE margins, trim drag, mis-trim when pre-selecting a longitudinal CE setting.

(iii) *Minimum Control Speed*

Pre-Selection: α, $\beta = 0$, $\gamma = 0$, $\phi = 0$, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-1} \delta_i$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: $\delta_{LoCE_{i_{trim}}}$, V, T_i

Problem Description: This longitudinal case simulates minimum control speed, trim drag, etc.

CASE (B) ($\gamma \geq\leq 0$, $\beta \geq\leq 0$, $\phi \geq\leq 0$, *Thrust Symmetry*)

(i) *Trim Drag During Straight Sideslip*

Pre-Selection: β, $\gamma = 0$, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-3} \delta_i$, V, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, ϕ_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$, T_i

Problem Description: This asymmetric flight case evaluates the 6-DOF trim CE-settings required for straight sideslip to steepen, e.g., the glide slope. Depending on the constraints imposed, a characteristic diagram may be generated or the optimum solution for minimum trim drag is estimated iteratively.

(ii) *Crosswind Landing Using Aerodynamic CE*

Pre-Selection: β, γ, $\phi \geq\leq 0$, V, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-3} \delta_i$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$, T_i

Problem Description: This asymmetric flight case determines the CE deflections and thrust setting required for a pre-defined crosswind component.

(iii) *Crosswind Landing Using Thrust Vectoring System*

Pre-Selection: β, γ, $\phi \geq\leq 0$, V, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m} \delta_i$, $\delta_{LoCE_{i_{trim}}} = 0$, $\delta_{DiCE_{i_{trim}}} = 0$, $\delta_{LaCE_{i_{trim}}} = 0$, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, T_i, ϕ_{T_i}, ψ_{T_i}

Problem Description: This asymmetric flight case determines the thrust line angles required to trim the pre-defined crosswind component.

CASE (C) ($\gamma \geq\leq 0$, $\beta \geq\leq 0$, $\phi \geq\leq 0$, *Thrust Asymmetry*)

(i) *Trim Drag During One-Engine Inoperative (OEI) Cruise*

Pre-Selection: $\beta = 0$, $\gamma = 0$, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-3} \delta_i$, T_i, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, ϕ_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$, V

Problem Description: This asymmetric flight case with thrust asymmetry evaluates the 6-DOF trim CE-settings required for flight with $\beta=0$ (minimum drag configuration). The algorithm determines the maximum cruise speed possible for the remaining thrust at the altitude defined.

(ii) *Crosswind Landing With One-Engine Inoperative (OEI)*

Pre-Selection: $\beta \geq\leq 0$, $\gamma \geq\leq 0$, V, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-3} \delta_i$, T_i, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}, ψ_{T_i}

Numerical Solution: α_{trim}, ϕ_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{DiCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$

Problem Description: This asymmetric flight case (thrust asymmetry and crosswind component) evaluates the 6-DOF trim CE-settings required for this worst-case scenario. The algorithm determines the maximum cruise speed possible for the remaining thrust at the altitude defined.

(iii) *Rudder Hard-Over System Failure*

Pre-Selection: β, $\gamma = 0$, ϕ, V, $\bar{\rho}$, $\sum_{j=1}^{n} i_j$, $\sum_{i=1}^{m-2} \delta_i$, $\delta_{DiCE_{i_{\max}}}$, T_i, x_{T_i}, y_{T_i}, z_{T_i}, ϕ_{T_i}

Numerical Solution: α_{trim}, $\delta_{LoCE_{i_{trim}}}$, $\delta_{LaCE_{i_{trim}}}$, ψ_{T_i}

Problem Description: This asymmetric flight case simulates a 'rudder hard-over' system failure. The thrust line toe angle is determined to check the survivability of this system failure case with the availability of a thrust vectoring system.

Any other than the above example combinations of the design- and state variables can be handled with the mathematical model. Trade studies can be performed, to investigate the effects of design drivers[1] on meeting design guidelines and certification requirements.

In general, it is possible to model a non-zero flight-path angle, γ, which corresponds to an 'instantaneous' steady-state condition only, because of the density changes with varying altitude. As a consequence, only shallow climbs and dives are permissible. However, modeling the flight path angle is of particular relevance, since it is of interest to determine the rate of climb- and descent performance as a function of the design- and state variables. To remain within the band of validity, it is useful to define a rate-of-climb constrained simply by specifying $V\sin\gamma$, which is the z-component of the velocity vector.

Clearly, Equations (12) and (13) can be solved, in theory, for six unknowns. However, the number of solutions obtained may be infinite. Finding practical solutions depends on placing adequate constraints on the design- and state variables, and/or by reducing the number of unknowns to a smaller number. Overall, the task of finding a practical solution is an optimization task, requiring repeated solutions of the equations. The interpretation of the results may be performed by hand or may be supported (automated) using a mathematical optimizer environment to satisfy an objective function. Clearly, since the solution may not be unique, it is up to the design experience of the user to initially specify the steady state condition to the degree, so that the trim[2] algorithm converges to a practical, if not unique, solution.

For all example flight-cases presented above, a unique trim solution is attainable when defining/calculating realistic start values/solutions:

- The stabilizer incidence angles, i_k, and the flap deflection angles, δ_i, must be consistent with attached flow considerations (tail-stall, etc.).
- The aircraft angle of attack, α_{trim}, must be consistent with attached flow considerations over the wing.
- The flight path angle, γ, must be consistent with operational and model-validity constraints.
- The thrust force, T_i, must be consistent with power available and operational criteria.
- The airspeed, V, and density, $\overline{\rho}$, must be consistent with operational considerations.

The steady state asymmetric flight cases with and without thrust symmetry (Cases B and C), are of primary importance for the design of controls. When reviewing pertinent literature throughout aviation history related to asymmetric power conditions, crosswind effects, straight sideslipping flight, and adverse yaw compensation, the difficulty of evaluating the CE design problem at conceptual design level becomes obvious, see Table 1.

The design-oriented approach proposed by Roskam and Anemaat [14] is considered most promising for the conventional TAC. The approach developed in the present context is similar in philosophy but capable of considering the full 6-DOF problem, thus it avoids separating the longitudinal motion from the lateral-directional motion thereby including cross-coupling effects. Figure 2 illustrates possible CE design scenarios concerned with asymmetric flight with and without thrust symmetry.

The following distinct flight cases are discussed in Figure 2:

FLIGHT CASE (A): Lack of a sidewind component, $V_{wind} = 0$, and thrust symmetry result in performance-optimal flight from *cruise* to *flare*. The control authority required throughout is minimal.

FLIGHT CASE (B): The existence of a sidewind component, $V_{wind} \neq 0$, and thrust symmetry result in the 'crab-method' during cruise with zero sideslip, $\beta = 0$. Certification requirements require maintaining a straight course in crosswind conditions. The force-vector polygon indicates minimum CE deflection required. However, additional control power is required during the low dynamic pressure 'de-crab maneuver' crosswind landing, leading to $\beta \neq 0$.

[1] An example for a primary design driver is the positioning of the c.g. The trim solution is strongly influenced by altering this parameter, since all stability derivative coefficients change.

[2] *AeroMech* considers only irreversible control systems, where the overall trim state does usually not imply hinge-moment trim (trim drag), and where force trim is not relevant.

TABLE 1. Design-Oriented Approaches to the Analysis of Asymmetric Flight Conditions

Implementation	Reference, Year	Comments
Hartman	[7,1938]	Wind tunnel investigation of One Engine Inoperative (OEI) flight conditions. Discussion of relevant design parameters, asymmetric flight with and without angle of sideslip, and the influence of power. The effects on stability, controllability, lift, and drag are estimated. Wind-tunnel investigation only without delivering of an analytical approach.
Archbold *et al*	[8,1945]	Development of an analytical method to estimate the size of the DiCE (fin and rudder) using a 1-DOF approach of the yawing moment equation. The analysis is restricted, for simplicity, to zero bank angle throughout the motion. A curve of maximum sideslip against DiCE size is the final result.
Yates	[9,1947]	The aerodynamics of the problem of regaining and maintaining control after engine failure is discussed qualitatively. In a first part, the transient effects related to a sudden failure of an engine are discussed. The second part is concerned with steady flight under asymmetric power, and the final part evaluates the baulked landing with one or more engines dead. The complexity of the problem is presented without delivering the means to design for it. Some comments are made with respect to novel aircraft layouts.
Baker	[10,1948]	A mathematical exposition of working rules for the choice of key lateral-directional stability derivatives is presented. The semi empirical analytical framework presented is based on several assumptions valid for the TAC only. The 3-DOF approach presented evaluates the DiCE and the LaCE. Having presented the underlying assumptions, the balance equations are formulated, which are checked against quantified stability criteria.
Wright	[11,1950]	The problems involved with flight on asymmetric engine power are evaluated from a pilot's perspective. This primarly qualitative description thoroughly defines 'Flight on Asymmetric Power'. A following section discusses the importance of the 'Safety Speed'. Stability issues are evaluated for flight under asymmetric conditions. Flight techniques are presented for engine failure on TO, in flight, approach and landing. The complexity of the problem is presented without delivering the means to design for it.
Pinsker	[12,1967]	A criterion has been developed to define a minimum acceptable value for the directional stability derivative N_v. The theory has been in agreement with observations on the high-speed BAC 221. In particular, when the bank angle is constrained by aileron control, the lateral motion degenerates into a simple directional oscillation, dependent on the 'effective' directional stability parameter. The 3-DOF theory developed is based on the concept of a partially constrained motion.
Leyman *et al*	[13,1972]	The effects of engine failures in high-speed cruise are described. This highly configuration-specific discussion presents primarily wind tunnel and flight test results of Concorde. Any design-oriented interpretation depends primarily on the prediction quality of stability derivative information for asymmetric flight. Engine failures in supersonic flight of this FWC are described as innocuous. The multi-disciplinary complexity of asymmetric flight is vividly illustrated, but no generic design guidelines are presented.
Roskam *et al*	[14,1994]	A practical method is presented to analyse longitudinal and lateral-directional trim problems with All Engines Operating (AEO) and One Engine Inoperative (OEI). The analytical framework separates the longitudinal motion from the lateral-directional motion. The approach has been developed for conceptual design level, taking standard assumptions for the TAC into account. Clearly, this 3-DOF approach enables the evaluation of TAC-specific stability and control aspects.
Aly	[15,1997]	This study evaluates the effects of side wind on the aerodynamic characteristics of an aircraft model in the wind tunnel. Focus has been on estimating the effects on aircraft performance in terms of lift, drag, sideforce, pitching-, rolling-, and yawing moment, which are reproduced quantitatively. The effects of thrust asymmetry have not been investigated. No design guideline is presented, having outlined the aerodynamic coupling effects of asymmetric flight.
Grasmeyer	[16,1998]	This study describes the estimation of stability and control derivatives using primarily DATCOM, and the establishment of an engine-out constraint based on the required yawing moment coefficient. The use of thrust vectoring and circulation control to provide additional yawing moment is also described. The engine-out case is approached with a 2-DOF model. The aerodynamic data set is assembled with the classical component build-up technique. The method is non-generic in character, thus suitable for the TAC only.
Burcham *et al*	[17,1998]	A propulsion-controlled aircraft (PCA) system is presented in which computer-controlled engine thrust provides emergency flight control. Flight test results are presented of an F-15 and MD-11 landed without using any flight control surfaces. Studies have shown that engines on only one wing can provide some flight control capability if the lateral c.g. can be shifted towards the side of the aircraft that has the operating engines. The study illustrates the feasibility of PCA.

FLIGHT CASE (C): Thrust asymmetry (critical engine failed) results in an aerodynamic sideslip angle $\beta \neq 0$ and a CE trim deflection during cruise (sideslip method). This flight case requires additional control power for trim at low dynamic pressure (low-speed) flight conditions and high thrust setting, since the sideslip angle becomes a maximum. Supplementary control power is required to de-crab the aircraft before touchdown (elimination of sideslip angle).

FLIGHT CASE (D): This flight case represents the worst case scenario with respect to control power required and control power available. The control power required to trim thrust asymmetry and to compensate for the crosswind component during the de-crab maneuver during the landing flare, usually surpasses CE-design limits (The DiCE and LaCE of modern transonic aircraft are not designed to meet this flight case, since it would result in oversized CEs).

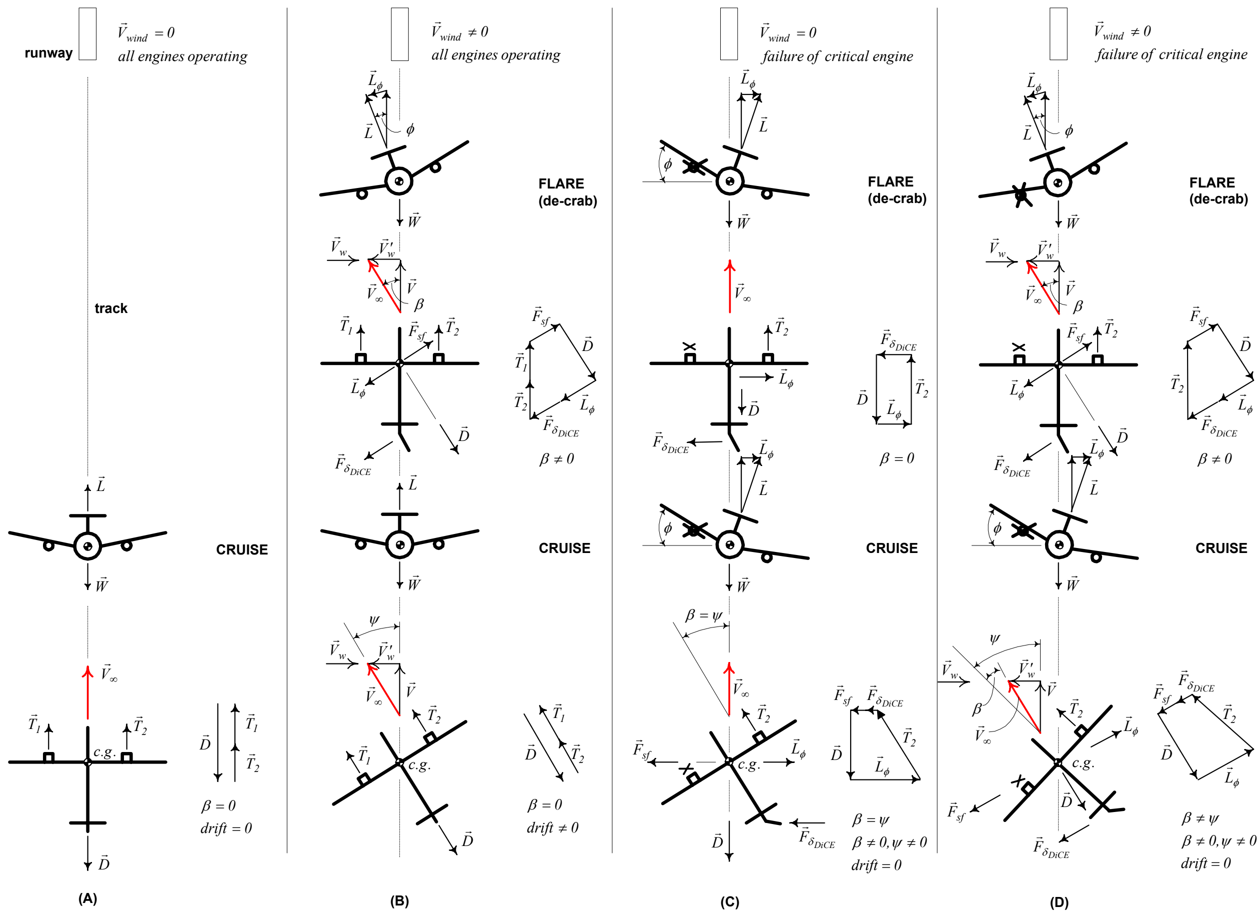

Fig. 2. Asymmetric-flight CE sizing scenarios qualitatively.

There are an infinite number of permutations of the sideslip parameters α, β, and ϕ, each individual combination influencing trim drag, ease of control, and comfort. The following flight cases with thrust asymmetry are of particular interest:

(a) $\beta = 0$, $\phi \neq 0$ [minimum drag solution but unfavorable comfort];
(b) $\beta \neq 0$, $\phi = 0$ [maximum drag solution with favorable comfort];
(c) $\beta \neq 0$, $\phi \neq 0$ [usually practiced solution].

Asymmetric thrust flight cases due to engine failure(s) on multi-engine aircraft result in drag increments on the inoperative engine(s), leading to additional forces and moments. The following force- and moment increments are taken into account in Equations (12) and (13):

$$\sum_{i=1}^{n} \Delta X_{D_i}\ ,\ \sum_{i=1}^{n} \Delta M_{D_i}\ ,\ \sum_{i=1}^{n} \Delta N_{D_i}$$

These force- and moment increments depend on the type of propulsive installation. It is usually acceptable to write for the total thrust-induced force and moments:

$$\underbrace{X_T}_{\substack{operative\\ engine(s)}} - \underbrace{\sum_{i=1}^{n} \Delta X_{D_i}}_{in-operative\ engine(s)} \approx X_T - \underbrace{\sum_{i=1}^{n} X_T(\tau_X - 1)}_{thrust\ fraction\ \text{of operative engine(s)}} \tag{14a}$$

$$M_T \pm \sum_{i=1}^{n} \Delta M_{D_i} \approx M_T \pm \sum_{i=1}^{n} M_T(\tau_M - 1) \tag{14b}$$

$$N_T \pm \sum_{i=1}^{n} \Delta N_{D_i} \approx N_T \pm \sum_{i=1}^{n} N_T(\tau_N - 1) \tag{14c}$$

The terms τ_X, τ_M, and τ_N are larger than 1.0 and cover the effects of the propulsive installation. Roskam presents in [18] information for several propulsive installation schemes.

Sudden engine failure(s) lead to dynamic airframe motion-transients. Coupled longitudinal and lateral-directional motions characterize these dynamic airframe responses. Before the steady state sideslip condition is attained, a dynamic overswing motion has to be arrested without risking CE stall. The extra control power demanded to cope with the dynamic overswing condition is accounted for by providing a sufficiently large control power margin.

It should be noted that the qualitative classification presented in Figure 2 looses some of its meaning when considering asymmetric aircraft, in particular the OFWC (Oblique Flying-Wing Configuration). For the OFWC, steady state sideslip is the dominating operational flight condition. As a result, crosswind landings may be performed without bank angle at all. Since the OFWC lacks the conventional fuselage and associated forces and moments (see force polygons in Figure 2), the control power demand expected for crosswind landings is expected to be low.

CONCLUSIONS

The presented formulation for steady state straight-line flight allows the aircraft designer to evaluate a range of flight and certification-critical flight conditions with a minimum set of input data at conceptual design level. This fully developed six-degree-of-freedom formulation of the equations of motion enables the designer to consistently evaluate conventional and unconventional fixed-wing flight vehicles in symmetric and asymmetric flight conditions. The design aim, to arrive at a simple but generic formulation valid for a range of flight vehicles, has been achieved.

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