2022 SUMMER

업데이터 통계학 스터디

Chapter 5 – Introduction to Statistical Inference

Introduction

Introduction

In many cases, populations are characterized by numerical descriptive

measure called parameters.

- Ω: parameter space = all possible values of θ
- θ : unknown parameter in a model
- $Y_1, \cdots, Y_n$: data $\Rightarrow$ $= d(Y_1, \cdots, Y_n)$: estimator of θ

Many different estimators may be obtained for the same parameter.

How can we establish criteria of goodness to compare statistical estimators?

Bias

Bias

- Point estimation is like firing a gunshot at a target.
- Drawing a single sample and compute an estimate for a parameter, firing a single shot at the target.

- We cannot evaluate the "goodness" of shooter based on only one shot. We must observe the results many times under the same setting. ⇒ We look at the frequency distribution of the values of estimates in repeated sampling.

Bias

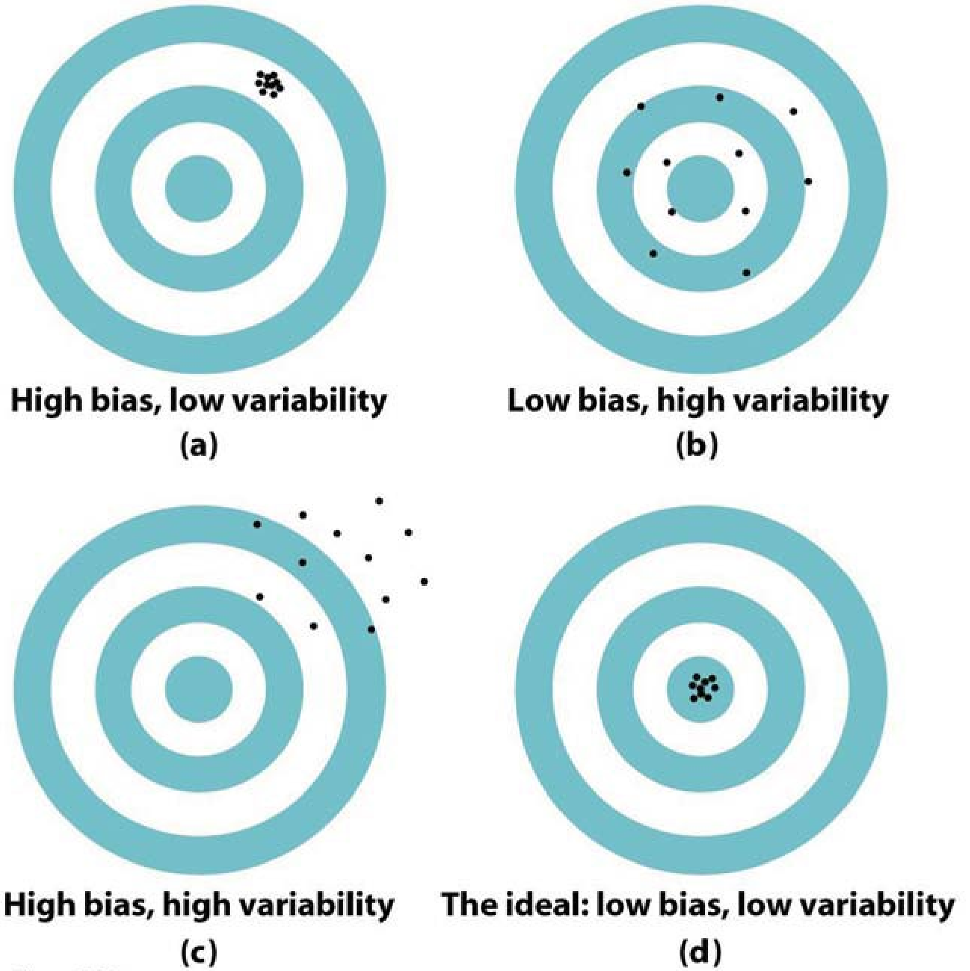

Figure 3-12
Introduction to the Practice of Statistics, Fifth Edition

Bias

θ: target parameter, $\hat{\theta}$: estimator

Definition

Bias of $\hat{\theta}$ as an estimator of θ is

$$B(\hat{\theta}) = E(\hat{\theta}) - \theta$$

$\hat{\theta}$ is an unbiased estimator of θ if $E(\hat{\theta}) = \theta$ for all $\theta \in \Omega$.
Otherwise, biased.

Example 6.1

$Y_1, \cdots, Y_n$: iid with mean μ and variance σ^2.
Evaluate the following two estimator.

1) An estimator for μ : $\bar{Y}$

2) An estimator for σ^2: S^2

Example 6.2

$S = \sqrt{S^2}$ is not an UE of σ.

Prove it.

MSE

MSE

Is unbiasedness enough?

Definition

MSE = Mean Square Error

$$MSE(\hat{\theta}) = E_{\theta}\left\{\left(\hat{\theta} - \theta\right)^2\right\} = V(\hat{\theta}) + \{B(\hat{\theta})\}^2$$

We want to use an estimator that has a small MSE.

Example 6.3

$Y_1, \cdots, Y_n$: $iid\ Unif(0, \theta)$, $\Omega = \{\theta : 0 < \theta < \infty\}$

$$\hat{\theta}_1 = 2\overline{Y}, \quad \hat{\theta}_2 = \frac{n+1}{n}\, max(Y_i)$$

Which estimator is the better estimator?

Confidence Intervals

Confidence Intervals

Definition

A level $1-\alpha$ confidence interval (CI) for parameter θ is an interval $[\hat{\theta}_L, \hat{\theta}_U]$, where $\hat{\theta}_L$, $\hat{\theta}_U$ are found from data s.t.

$$P_\theta(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 1 - \alpha$$

Confidence Intervals

<u>Pivotal Method</u>

A pivotal quantity is a function of the sample measurements and unknown parameter θ, (θ is the only unknown parameter) and its probability distribution does not depend on θ.

After finding a pivotal quantity and its distribution, only some algebra is needed to get CI.

Confidence Intervals

Pivotal Method

$g(y_1, \cdots, y_n; \theta)$: pivot ⇒ Find c_1 and c_2 s.t.

$$P_\theta(c_1 \le g(y_1, \cdots, y_n; \theta) \le c_2) = 1 - \alpha$$

⇒ Restate in the form of

$$P_\theta\left(\hat{\theta}_L \le \theta \le \hat{\theta}_U\right) = 1 - \alpha$$

Example 6.3

$Y \sim Exp(\theta)$. Use Y to form a 90% CI for θ.

Fundamental Sampling Theorem

<u>Theorem</u>

$Y_1, \cdots, Y_n$: $iid\ N(\mu, \sigma^2)$. Then,

1. $\overline{Y} \sim N(\mu, \frac{\sigma^2}{n})$
2. $S^2 = \frac{1}{n-1} \sum (Y_i - \overline{Y})^2 \perp \overline{Y}$
3. $(n-1)S^2/\sigma^2 \sim \chi^2(n-1)$

Example 6.4

$Y_1, \cdots, Y_n$: $iid\ Unif(0, \theta)$, a level $1-\alpha$ CI for θ ?

Example 6.5

$Y_1, \cdots, Y_n$: $iid\ N(\mu, \sigma^2)$, a level $1-\alpha$ CI for μ ?
(σ: unknown parameter)

Example 6.6

$Y_1, \cdots, Y_n$: $iid\ N(\mu, \sigma^2)$, a level $1 - \alpha$ CI for σ^2?
(μ, σ: unknown parameter)