2022 SUMMER

업데이터 통계학 스터디

Chapter 9 – Point Estimation 3

MVUE

MVUE

Definition (MVUE)

$\hat{\theta}$ is a minimum variance unbiased estimator (MVUE) of θ if

(a) $\hat{\theta}$ is unbiased.

(b) $V_\theta(\hat{\theta}) \leq V_\theta(\acute{\theta})$ for any θ if $\acute{\theta}$ is also an UE of θ.

Rao-Blackwell Theorem

Theorem

$\hat{\theta}$ is an UE of θ, and $V\left(\hat{\theta}\right) < \infty$. U is a SS for θ.

Let $\hat{\theta}^* = E(\hat{\theta}|U)$. Then for all θ.

$$E\left(\hat{\theta}^*\right) = \theta \quad \text{and} \quad V_\theta(\hat{\theta}^*) \leq V_\theta(\hat{\theta})$$

Completeness

Definition (CSS)

Let $f(y|\theta)$ be a family of PDFs or PMFs for a random variable Y.

The family of probability distributions is called complete if $E(g(Y)) = 0$ for all θ implies $P(g(Y) = 0) = 1$ for all θ.

If T is a SS for θ and the family of its PDFs is complete, T is a complete sufficient statistic(CSS).

Example 9.1

$Y_1, \cdots, Y_n$: $iid\ Bernoulli(p)$

Show that $Y_1 - Y_2$ is not a CSS, but $\sum_{i=1}^{n} Y_i$ is a CSS.

Example 9.2

$Y_1, \cdots, Y_n$: $iid\ Unif(0, \theta)$

Show that $\max(Y_i)$ is a CSS.

Lehmann-Scheffe Theorem

Theorem

$Y_1, \cdots, Y_n$: a random sample from $f(y|\theta)$ U is a CSS for θ.

If there is a UE of θ as a function of U, then it is the unique MVUE of θ.

Example 9.3

$Y_1, \cdots, Y_n$: random sample from $Unif(0, \theta)$

Find a MVUE of θ.

MLE

Maximum Likelihood Estimator (MLE)

Idea:

For a fixed value of θ, $L(\theta)$ is "probability" of getting $y_1, \cdots, y_n$ when θ is true. It represents how likely the value θ is as the true parameter given $y_1, \cdots, y_n$.

Example:

An urn has three balls. Balls may be red or white and we do not know the total number of either color. Take a random sample of size 2, without replacement. Suppose we draw 2 red balls.

Two possible scenarios: 1) 2R 1W 2)3R

Maximum Likelihood Estimator (MLE)

Seems reasonable to choose the 2nd scenario, i.e., we expect that there are three red balls based on our sample, because this maximizes the probability obtaining the observed sample.

ML estimates θ to be the value that makes the observation you got most probable.

Maximum Likelihood Estimator (MLE)

Definition

MLE $\hat{\theta}$ of θ is a value θ such that for given $y_1, \cdots, y_n$,

$$L\left(\hat{\theta}\right) = \max(L(\theta))$$

Example 9.4

4 Bernoulli trials. $y_1 = 0, y_2 = 0, y_3 = 1, y_4 = 0$.

Find a MLE $\hat{p}$.

Example 9.5

n Bernoulli trials. $y_1, \cdots, y_n$.

Find a MLE $\hat{p}$.

Example 9.6

$Y_1, \cdots, Y_n$: $iid\ Exp(\theta)$

Find a MLE $\hat{\theta}$.

Example 9.7

$Y_1, \cdots, Y_n$: $iid\ Unif(0, \theta)$ //non-regular case

Find a MLE $\hat{\theta}$.