

업데이터 통계학 스터디: ASSIGNMENT 4

1. A device contains three components, each of which has a lifetime in hours with the PDF

$$f(y) = \frac{2y}{10^2} e^{-(\frac{y}{10})^2}, \quad 0 < y < \infty$$

The device fails with the failure of one of the components. Assuming independent lifetimes, what is the probability the device fails in the first hour of operation?

2. Let Y have the PDF

$$f(y) = \frac{2}{\sqrt{2\pi}} e^{-y^2/2}, \quad 0 < y < \infty$$

Find the PDF of $U = Y^2$.

3. When Y follows $Exp(\theta)$, find the PDF of $U = (2Y/\theta)^{1/2}$.

4. When Y follows $Gamma(\alpha, \beta)$, find the PDF of $U = 1/Y$.

5. Find the PDF of $Z = X + Y$ when the joint PDF of X and Y is as follows.

$$f_{X,Y}(x,y) = 2(x+y), \quad 0 \leq x \leq y \leq 1$$

6. Find the PDF of $W := \min(Y_i)$ when Y_1, \dots, Y_n follow i.i.d. $Unif(0, \theta)$. And find c such that cW is an UE of θ .

7. When $Y \sim \text{Bernoulli}(p)$, we consider two estimators, $\hat{\theta}_1 = Y$ and $\hat{\theta}_2 = \frac{1}{2}$, for p with one observation Y . Compare their biases and MSE s.

8. Suppose that we have a random sample, Y_1, \dots, Y_n from the exponential distribution with mean $\frac{1}{\theta}$. Obtain a 90% confidence interval for

(a) θ , (b) θ^2 , (c) $\exp(-\theta)$.

9. Suppose that X_1, \dots, X_n and Y_1, \dots, Y_m are independent random samples from $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$, respectively. S_X^2 and S_Y^2 are sample variances.

(a) Show that $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2}$ is a pivotal quantity for estimating σ_X^2/σ_Y^2 .

(b) Provide a 95% confidence interval for $\frac{\sigma_X^2}{\sigma_Y^2}$ when $n = 10, m = 16, S_X^2 = 15$, and $S_Y^2 = 20$.

10. Let Y have PDF

$$f_Y(y) = \frac{2(\theta - y)}{\theta^2}, \quad 0 < y < \theta.$$

(a) Find the cumulative distribution function of Y .

(b) Show that Y/θ is a pivotal quantity.

(c) Use the pivotal quantity in (b) to find a 90% lower confidence limit for θ .