2022 SUMMER

업데이터 통계학 스터디

Day.1 – Elementary Statistics

Project Introduction

- 김재훈 (010-9854-0923, jaehoondata@korea.ac.kr)
- 매주 월/수 15시 ~ 17시, 백주년기념삼성관 2층 스터디룸
- 모든 강의 자료는 공유 드라이브에 등록될 예정임. (단체 대화방 참고)
- 매주 과제 있음. 공부를 위해 하는 것이니 개인 역량으로 풀어 주시기 바람.
- 7/6(업데이터 MT), 7/13(PM 학생회 업무), 8/15(광복절)

  위 날에는 공식적으로 스터디 쉽니다.

Project goal

- 엄밀하고 알찬 설명을 목표함. (근데 제가 수학을 못해서.. 미리 죄송합니다)
- 스터디원 모두가 방학 기간 갓생 사는 기분을 느끼길 목표함.
- 합응/고연전 기간, 공부를 놓을 우리의 학점을 위해 예습하는 것을 목표함.
- 입실렌티로 인해 확입/수통을 대충 공부한 21학번의 과거 반성을 목표함.
- 스터디원 사이의 친목 도모를 목표함.
- PM의 PPT 제작 능력 및 영어 작문 능력 함양을 목표함.

고등학교 수학 복습 (근데 영어로)

Random Experiment

Definition.

- A **random experiment** is any action or process whose outcome is subject to uncertainty.

Example

- Roll a (fair) die

Sample Space

Definition.

- The **sample space** associated with a random experiment is the set of all possible outcomes. Typically, denoted as $'\boldsymbol{S}'$.

Event

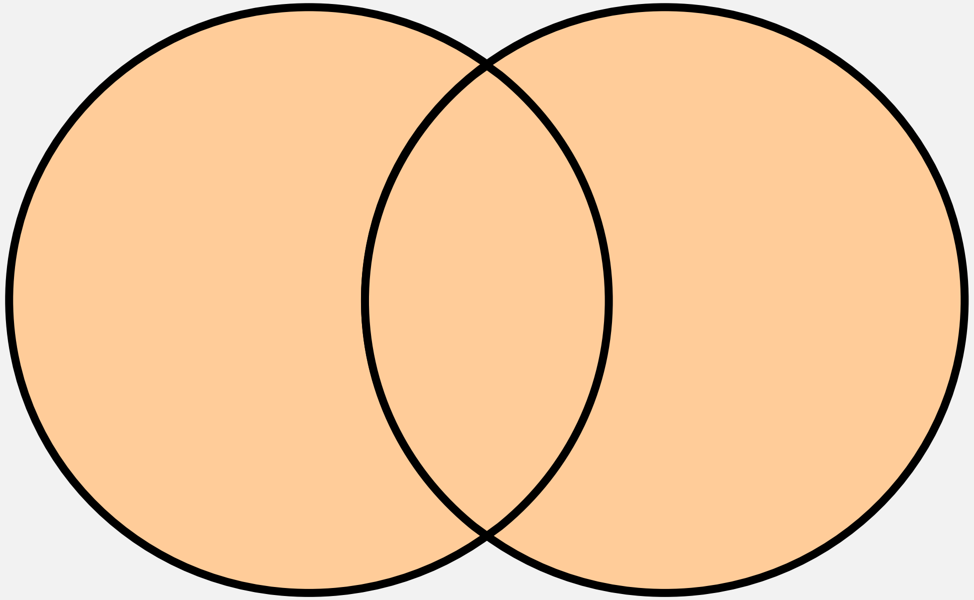
Definition.

- An **event** is a subset of the sample space.

Union (Set Theory)

Definition.

- The **union** of events A and B, denoted $A \cup B$, is the collection of all outcomes that are elements of one or the other of the sets A and B, or of both.



Intersection (Set Theory)

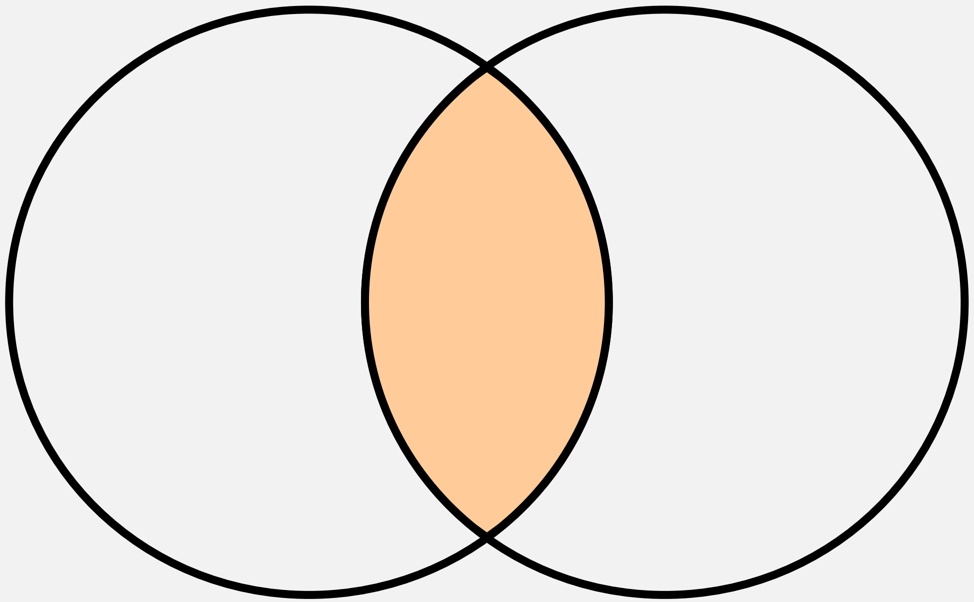
Definition.

- The **intersection** of events A and B, denoted $A \cap B$, is the collection of all outcomes that are elements of both sets A and B.

- If $A \cap B = \emptyset$,

  A and B are **mutually exclusive.**

  (It's also called **disjoint**)

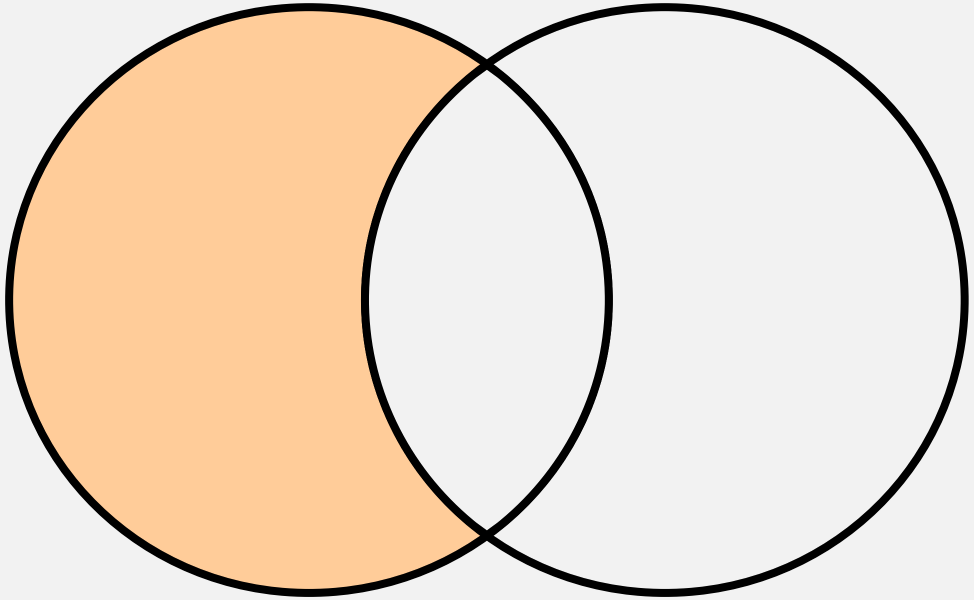


Complements (Set Theory)

Definition.

- The complement of an event A in a sample space S, denoted A^C, is the collection of all outcomes in S that are not elements of the set A.

- $A - \mathrm{B} = \mathrm{A} \cap B^C$



Example 1.1

Suppose that you roll a six-sided fair die twice.
Let S be the sample space of this random experiment.

1. List all elements in S.
2. Let A be an event that you roll the same number. A = ?
3. Let B be an event that the sum of two numbers is 8. B = ?
4. Are A and B mutually exclusive? What is $A \cap B$?

Set Operations

Commutative laws:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup \mathrm{C})$$
$$(A \cap B) \cap C = A \cap (B \cap \mathrm{C})$$

Set Operations

Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

De Morgan's laws:

$$(A \cup B)^C = A^C \cap B^C$$
$$(A \cap B)^C = A^C \cup B^C$$

Others

Factorial:

$$n! = n(n-1)(n-2)\cdots,\ \ 0! = 1$$

Permutations:

$${}_{n}P_{r} = \frac{n!}{(n-r)!}$$

Others

Combination:

$$ {}_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!} $$

같있순:

$$ \binom{n}{n_1, n_2, \cdots, n_k} = \frac{n!}{n_1!n_2!\cdots n_k!} $$

$$ where, \qquad n_1 + n_2 + \cdots + n_k = n $$

Example 1.2

- A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if

(a) there are no restrictions.

(b) A will serve only if he is president.

(c) B and C will serve together or not at all.

(d) D and E will not serve together?

Example 1.3

- How many different letter arrangements can be made from the letters in the word *STATISTICS* ?

Binomial Theorem

- Consider the polynomial $(x + y)^n$.

This product expands into a sum of 2^n terms, each of which has n factors, 0 to n of which are x's and the rest are y's.

We are asking "How many ways are there to choose k of the n factors to equal the x's?"

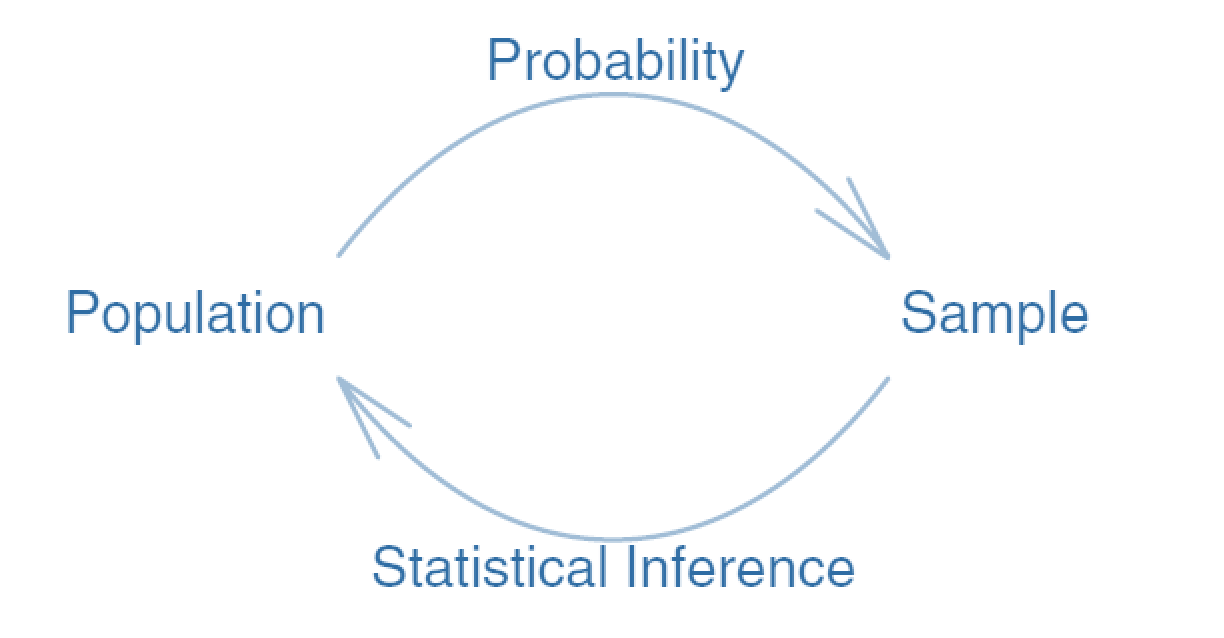
$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Combinatorial Identities

- For $n > 0$, $\binom{n}{1} - \binom{n}{2} + \binom{n}{3} - \cdots (-1)^{n+1}\binom{n}{n} = 1$
- $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, $\quad 1 \leq r \leq n$
- $n\binom{n-1}{k-1} = k\binom{n}{k}$ for any $\mathrm{n}, k \in \mathbb{Z}^{+}, k \leq n$
- Vandermonde's identity: $\binom{m+n}{k} = \sum_{i=0}^{k}\binom{m}{i}\binom{n}{k-i}$

Probability

Outline: Why Study Probability ?



Probability

- If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is

$$P(A) = \frac{n}{N}$$

Axioms of probability

Probability is a real-valued set function $P()$ that assigns, to each event A in the sample space S, a number $P(A)$ satisfying

- $P(S) = 1$
- If $A \subset S \;\rightarrow\; 0 \leq P(A) \leq 1$
- If $A_1, A_2, A_3 \cdots$ is a sequence of mutually exclusive events,

$$\mathrm{P}(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

Properties of probability

S is a sample space, and A, B, and C are subsets of S. Then the followings are true.

- $P(A) = 1 - P(A^C) \rightarrow P(\emptyset) = 0$
- If $A \subseteq B$, then $P(A) \leq P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- Inclusion-Exclusion Principle

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} A_i - \sum_{i<j} P\left(A_i \cap A_j\right)$$

$$+(-1)^{m+1} \sum_{\mathrm{i}<\mathrm{i}<\cdots m} P(A_i \cap A_j \cap A_m) + \cdots + (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

Example 1.4

- A coin is tossed twice. What is the probability that at least 1 head occurs?

Example 1.5

- In a poker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks.

Example 1.6

- A statistics class for engineers consists of 25 industrial, 10 mechanical, 10 electrical, and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is

(a) an industrial engineering major.

(b) a civil engineering or an electrical engineering major.

Example 1.7

- Given that $P(A \cup B) = 0.76$ and $P(A \cup B^C) = 0.87$. Find $P(A)$.

Conditional Probability

Conditional probability

- Let A and B are events. The conditional probability of A given B is the probability that we would (will) assign to A if (after) we learn that B occurs. We denote this $P(A|B)$. We also call this the probability of A conditional on B.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(If $P(B) > 0$)

Example 1.8

- A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Multiplication rule of probability

By definition of the conditional probability,

$$P(A \cap B) = P(B)P(A|B)$$

It is known as a **multiplication rule of probability**. And it can be extended to more than two events.

$$P(A_1 \cap \cdots \cap A_n) = P(A_1)P(A_2|A_1) \cdots P(A_n|A_1 \cap A_2 \cap \cdots \cap A_{n-1})$$

Example 1.9

- Suppose that an urn contains 8 red balls and 4 white balls. We draw 2 balls from the urn without replacement. If we assume that at each draw each ball in the urn is equally likely to be chosen, what is the probability that both balls drawn are red?

Independence

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B)$$

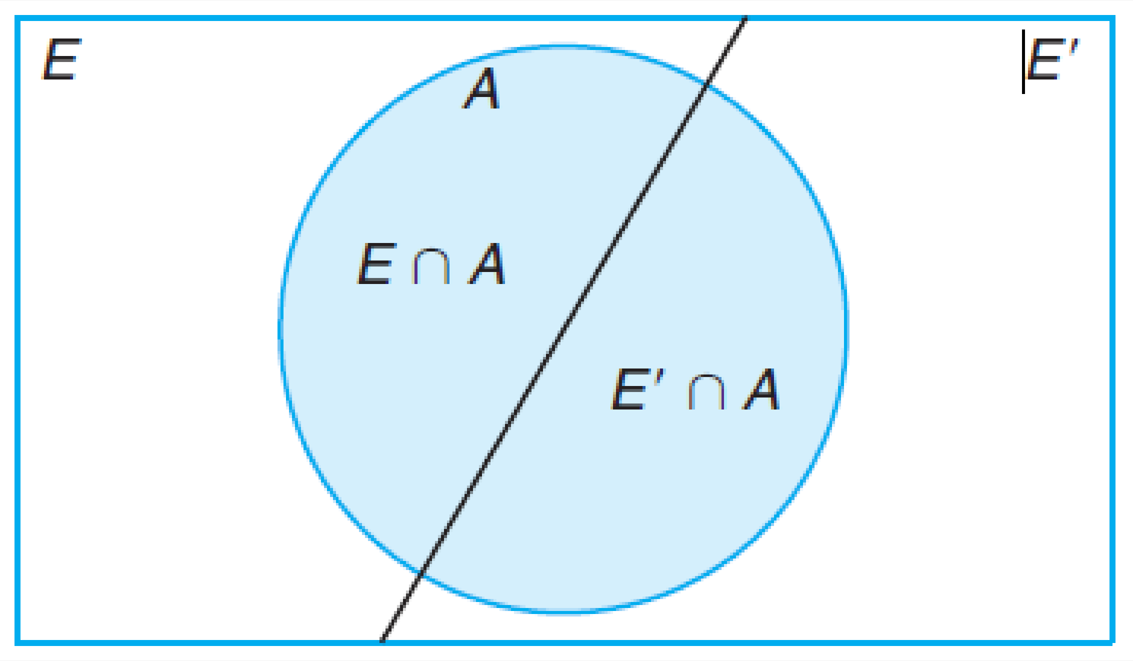
Therefore, to obtain the probability that two independent events will both occur, we simply find the product of their individual probabilities. And It's can be extended like,

$$P(A_1 \cap \cdots \cap A_n) = P(A_1)P(A_2)\cdots P(A_n)$$

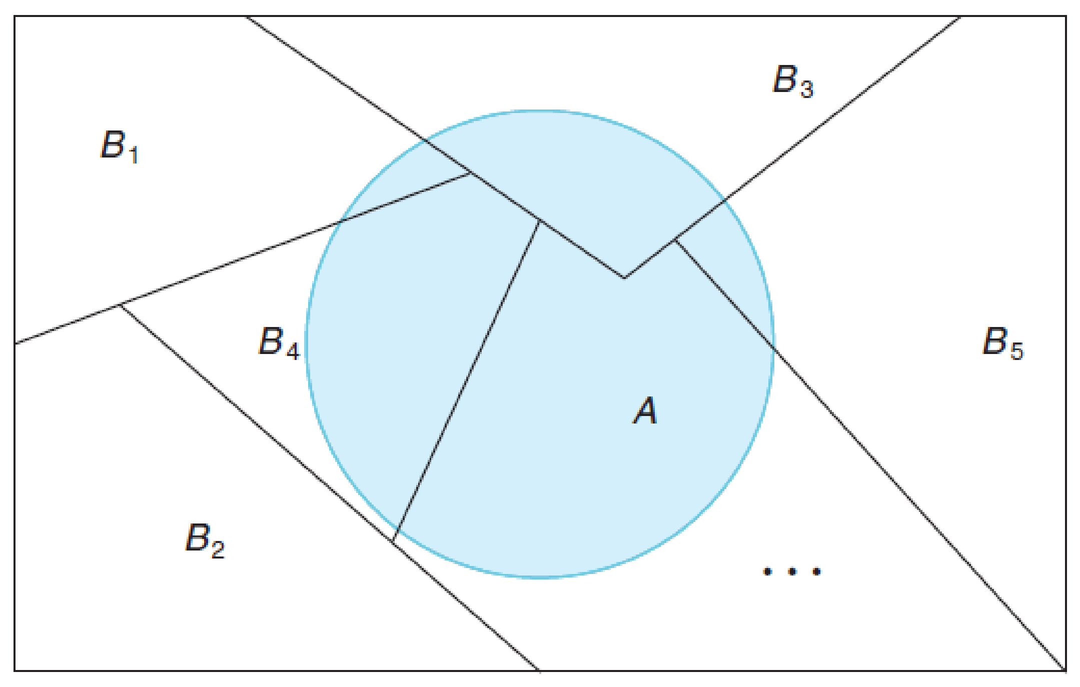
Bayes’ Theorem

$$P(A) = P(E \cap A) + P(E' \cap A)$$

$$= P(E)P(A|E) + P(E')P(A|E)$$



$$P(A)=\sum_{i=1}^{n}P(B_i\cap A)=\sum_{i=1}^{n}P(B_i)\,P(A|B_i)$$



Bayes' Theorem

Let the n events $A_1, A_2, \cdots, A_n$ constitute a partition of the sample space S. That is, A_i are mutually exclusive and exhaustive. In addition, $\mathrm{P}(A_i) > 0$. Then for any event B with $P(B) > 0$,

$$P(A_k|B) = \frac{P(A_k \cap B)}{P(B)} = \frac{P(A_k) \cdot P(B|A_k)}{\sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)}$$

Example 1.10

An urn contains two type A coins and one type B coin.

- When a type A coin is flipped, it comes up a head with probability
- 1/4, whereas
- When a type B coin is flipped, it comes up a head with probability 3/4.

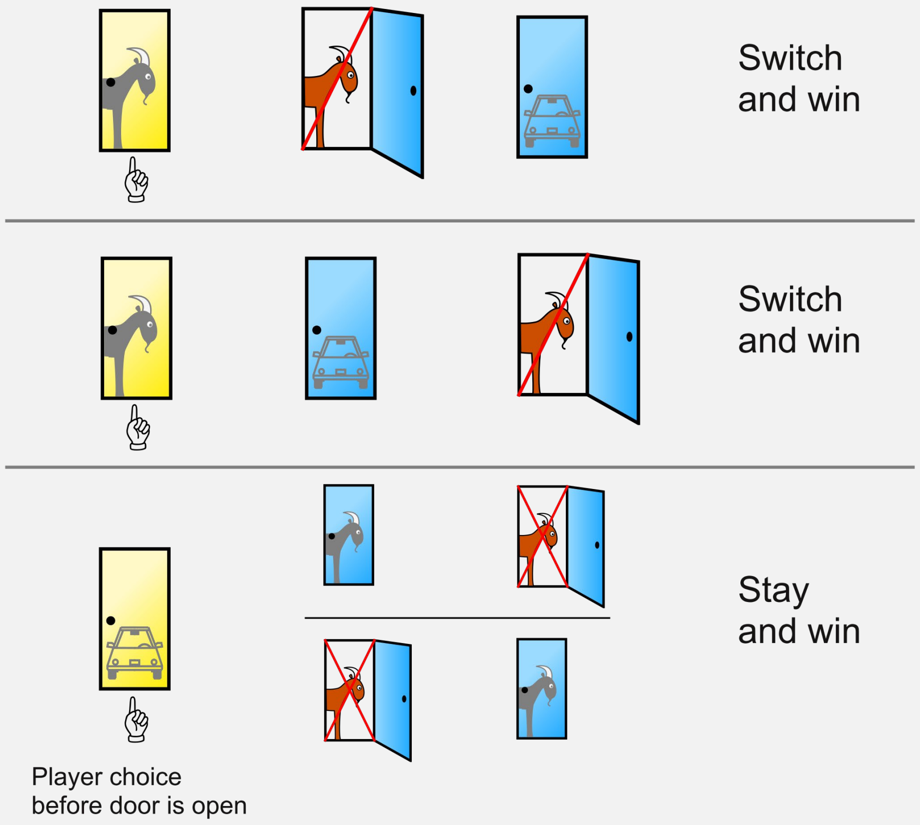
A coin is randomly chosen from the urn and flipped. Given that the flip landed on a head, what is the probability that it was a type A coin?

Monty-Hall problem

A: The door you choose.

B, C: The other doors.

D : The event that, Monty opens a door, except A, to show a goat



Monty-Hall problem

Monty cannot open the A door and must show the goat door between the B and C doors, so each probability can be calculated.

If Car is in A, $P(D|A) = \frac{1}{2}$

If Car is in B, $P(D|B) = 1$

If Car is in C, $P(D|C) = 0$

Monty-Hall problem

According to the Bayes theorem, the probability of a conditional event occurring is the same as the probability of a historical event occurring among all the number of historical events.

$$P(A|D) = \frac{P(D|A)}{P(D|A) + P(D|B) + P(D|C)} = \frac{1}{3}$$

$$P(B|D) = \frac{P(D|B)}{P(D|A) + P(D|B) + P(D|C)} = \frac{2}{3}$$

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고생하셨습니다.