업데이터 통계학 스터디: ASSIGNMENT 3

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1. $X \sim Exponential(\lambda)$. Find

   (a) PDF of X

   (b) MGF of X (Proof)

   (c) $E(X)$ and $Var(X)$, by using the MGF

   (d) Write what you know about the Exponential distribution.

2. $Y \sim Gamma(\alpha, \beta)$. Find

   (a) PDF of Y

   (b) MGF of Y (Proof)

   (c) $E(Y)$ and $Var(Y)$, by using the MGF

   (d) Write what you know about the Gamma distribution.

3. Compare the MGF of the Exponential distribution with the MGF of the Gamma distribution. And explain the relationship between the two distributions as you know it.

4. $U \sim \chi^2(v)$. Find

   (a) PDF of U

   (b) MGF of U

   (c) $E(U)$ and $Var(U)$, by using the MGF

   (d) Write what you know about the Chi-Square distribution.

   (e) Is there a relationship between the Chi-Square distribution and the Gamma distribution? If a relationship exists, write what you know about it.

5. $W \sim Beta(\alpha, \beta)$. Find

(a) PDF of W

(b) $E(W)$ and $Var(W)$

6. $X \sim N(\mu, \sigma^2)$. Find

(a) PDF of X

(b) MGF of X (Proof)

(c) $E(X)$ and $Var(X)$, by using the MGF

(d) Write what you know about the Normal distribution.

7. $Z \sim N(0,1)$. Find

(a) PDF of Z

(b) MGF of Z

(c) Write what you know about the Standard-Normal distribution.

8. Use the MGF method to prove that the description below is true. If,

(a) $X_1, \cdots, X_r$ are follow i.i.d $Geometric(p) \Rightarrow \sum_{i=1}^{r} X_i \sim NegBin(r, p)$

(b) $Y_1, \cdots, Y_n$ are follow i.i.d $Poisson(\lambda) \Rightarrow \sum_{i=1}^{n} Y_i \sim Poisson(n\lambda)$

(c) $W_1, \cdots, W_n$ are follow $Gamma(\alpha_i, \beta)$ by mutually independently $\Rightarrow$ $\sum_{i=1}^{n} W_i \sim Gamma(\sum \alpha_i, \beta)$

(d) $Z_1, \cdots, Z_n$ are follow i.i.d $N(0,1) \Rightarrow \sum_{i=1}^{n} Z_i \sim \chi^2(n)$

9. (a) Write what you know about the relationship between the Exponential distribution and the Gamma distribution. By using 8.(c)

(b) Write what you know about the relationship between the Standard-Normal distribution and the Chi-Square distribution. By using 8.(d)

10. Show that $T^2 \sim F(1,n)$ when $T \sim t(n)$.

11. Find the PDF of $Y := \tan(X)$ when $X \sim Unif(-\frac{\pi}{2}, \frac{\pi}{2})$

12. Find the PDF of $T := Max(G_i)$ when $G_1, \cdots, G_n$ follow i.i.d. $Gamma(1, \beta)$

13. Find the PDF of $W := min(U_i)$ when $U_1, \cdots, U_n$ follow i.i.d. $Unif(0, \theta)$