MA THEMA TICS DEPARTMENT

Model Examination - Parametrization and Conic Sections (MATH 2025)

Total Marks: 16

Time: 1.5 Hours

Part A

(Answer any 4 questions. Each carries 1 mark)

1. State the standard Cartesian equation for an ellipse centered at the origin with foci on the x-axis. Define the terms semimajor axis and semiminor axis. (Ref: Section 11.6)
2. Define the eccentricity e of an ellipse and a hyperbola in terms of a and c. (Ref: Section 11.7 Definition)
3. State the formula for the length L of a polar curve $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$. (Ref: Section 11.5, Eq. 3)
4. State one test for symmetry about the y-axis (the line $\theta = \pi/2$) for a polar graph $r = f(\theta)$. (Ref: Section 11.4, Symmetry Tests)
5. State the parametric formula for d^2y/dx^2. (Ref: Section 11.2, Eq. 2)
6. What is the relationship between polar coordinates (r, θ) and Cartesian coordinates (x, y)? (Ref: Section 11.3)

(4 x 1 = 4 Marks)

Part B

(Answer any 3 questions. Each carries 2 marks)

7. Given the parametric equations $x = -\sqrt{t}$, $y = t$, $t \geq 0$. Identify the particle's path by finding a Cartesian equation. Graph the Cartesian equation, indicating the portion traced and the direction of motion. (Ref: Exercise 11.1.2)
8. Given the parametric equations $x = 4 \cos t$, $y = 2 \sin t$, $0 \leq t \leq 2\pi$. Find a Cartesian equation for the path and identify the shape. Indicate the direction of motion. (Ref: Exercise 11.1.7)
9. Find d^2y/dx^2 as a function of t if $x = t - t^2$ and $y = t - t^3$. (Ref: Example 11.2.2)
10. Find the focus and directrix of the parabola $x^2 = 6y$. Sketch the parabola, including the focus and directrix. (Ref: Exercise 11.6.10)
11. Replace the polar equation $r^2 = 4r \cos \theta$ with an equivalent Cartesian equation and identify the graph. (Ref: Example 11.3.6b)

(3 x 2 = 6 Marks)

Part C

(Answer any 2 questions. Each carries 3 marks)

12. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$. (Ref: Example 11.5.2)
13. Find the length of the curve defined by $x = t^3$, $y = 3t^2/2$ for $0 \le t \le \sqrt{3}$. (Ref: Exercise 11.2.26)
14. The ellipse $(x^2/16) + (y^2/9) = 1$ is shifted 4 units to the right and 3 units up.
    a. Find an equation for the new ellipse.
    b. Find the center, foci, and vertices of the new ellipse.
    c. Sketch the new ellipse, showing its center, foci, and vertices. (Ref: Exercise 11.6.41)
15. Graph the lemniscate $r^2 = 4\cos\theta$. Discuss its symmetry and find the slopes at the origin (pole). (Ref: Example 11.4.2 & Slope discussion)

(2 x 3 = 6 Marks)

ANSWERS

Part A

1. **Ellipse Equation:** $x^2/a^2 + y^2/b^2 = 1$ (where $a > b$). a is the **semimajor axis**, b is the **semiminor axis**.
2. **Eccentricity:** For an ellipse, $e = c/a = \text{sqrt}(a^2 - b^2) / a$. For a hyperbola, $e = c/a = \text{sqrt}(a^2 + b^2) / a$.
3. **Polar Arc Length:** $L = \int[\alpha \text{ to } \beta] \text{ sqrt}(r^2 + (dr/d\theta)^2)\ d\theta$.
4. **Symmetry about y-axis (Polar):** If the point (r, θ) lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ also lies on the graph. (Replacing θ with $\pi - \theta$ or replacing r with $-r$ and θ with $-\theta$ leaves the equation unchanged).
5. **Parametric d^2y/dx^2:** $d^2y/dx^2 = (dy'/dt) / (dx/dt)$, where $y' = dy/dx$.
6. **Polar/Cartesian Relationship:** $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $\tan\theta = y/x$.

Part B

7. **Cartesian Equation for $x = -\sqrt{t}$, $y = t$:**
    - From $x = -\sqrt{t}$, we have $\sqrt{t} = -x$. Since $t \ge 0$, $\sqrt{t} \ge 0$, so $-x \ge 0$, which means $x \le 0$.
    - Squaring $x = -\sqrt{t}$ gives $x^2 = t$.
    - Substitute $t = y$: The Cartesian equation is $x^2 = y$ or $y = x^2$.

- **Path:** This is the parabola $y = x^2$.
- **Portion Traced:** Since $x \leq 0$, it's the left half of the parabola (including the vertex).
- **Direction:** As t increases from 0 to ∞, $x = -\sqrt{t}$ decreases from 0 to $-\infty$, and $y = t$ increases from 0 to ∞. The particle starts at (0,0) and moves up the left side of the parabola.
- **Graph:** Sketch the parabola $y=x^2$, highlight the left half, and draw an arrow starting at (0,0) pointing up along the left branch.

8. **Cartesian Equation for x = 4 cos t, y = 2 sin t:**

    - $x/4 = \cos t$, $y/2 = \sin t$.
    - Using $\cos^2 t + \sin^2 t = 1$: $(x/4)^2 + (y/2)^2 = 1$.
    - $x^2/16 + y^2/4 = 1$.
    - **Shape:** This is an ellipse centered at (0,0) with semimajor axis $a=4$ (along x-axis) and semiminor axis $b=2$ (along y-axis).
    - **Direction:** At $t=0$, point is $(4\cos 0, 2\sin 0) = (4,0)$. At $t=\pi/2$, point is $(4\cos(\pi/2), 2\sin(\pi/2)) = (0,2)$. At $t=\pi$, point is $(-4,0)$. At $t=2\pi$, back to $(4,0)$. The motion is counterclockwise, starting at (4,0).

9. **d^2y/dx^2 for $x = t - t^2$, $y = t - t^3$:**

    - $dx/dt = 1 - 2t$
    - $dy/dt = 1 - 3t^2$
    - $y' = dy/dx = (dy/dt) / (dx/dt) = (1 - 3t^2) / (1 - 2t)$
    - $dy'/dt = d/dt [(1 - 3t^2) / (1 - 2t)]$ (Use quotient rule)
      $= [(-6t)(1 - 2t) - (1 - 3t^2)(-2)] / (1 - 2t)^2$
      $= [-6t + 12t^2 + 2 - 6t^2] / (1 - 2t)^2$
      $= (6t^2 - 6t + 2) / (1 - 2t)^2 = 2(3t^2 - 3t + 1) / (1 - 2t)^2$
    - $d^2y/dx^2 = (dy'/dt) / (dx/dt)$
      $= [2(3t^2 - 3t + 1) / (1 - 2t)^2] / (1 - 2t)$
      $= 2(3t^2 - 3t + 1) / (1 - 2t)^3$

10. **Focus and Directrix for $x^2 = 6y$:**

    - Standard form is $x^2 = 4py$.
    - Comparing, $4p = 6$, so $p = 6/4 = 3/2$.
    - The parabola opens upwards (since $p > 0$ and it's x^2).
    - Vertex is $(0, 0)$.
    - Focus is $(0, p) = (0, 3/2)$.
    - Directrix is $y = -p$, which is $y = -3/2$.

- **Sketch:** Draw the parabola opening up, vertex at origin, mark focus at (0, 1.5) and draw the horizontal line $y = -1.5$ as the directrix.

11. **Cartesian Equation for $r^2 = 4r \cos \theta$:**
    - Substitute $r^2 = x^2 + y^2$ and $r \cos \theta = x$.
    - $x^2 + y^2 = 4x$.
    - Rearrange: $x^2 - 4x + y^2 = 0$.
    - Complete the square for x: $(x^2 - 4x + 4) + y^2 = 4$.
    - $(x - 2)^2 + y^2 = 4$.
    - **Graph:** This is a circle centered at $(2, 0)$ with radius $\sqrt{4} = 2$.

Part C

12. **Area inside r=1 and outside r = 1 - cos θ:**
    - **Intersection:** $1 = 1 - \cos \theta$ => $\cos \theta = 0$. This occurs at $\theta = -\pi/2$ and $\theta = \pi/2$.
    - The region is traced as θ goes from $-\pi/2$ to $\pi/2$.
    - Outer curve $r_2 = 1$. Inner curve $r_1 = 1 - \cos \theta$.
    - Area $A = (1/2) \int[\alpha \text{ to } \beta] (r_2^2 - r_1^2)\, d\theta$

      $A = (1/2) \int[-\pi/2 \text{ to } \pi/2] (1^2 - (1 - \cos \theta)^2)\, d\theta$

      $A = (1/2) \int[-\pi/2 \text{ to } \pi/2] (1 - (1 - 2 \cos \theta + \cos^2 \theta))\, d\theta$

      $A = (1/2) \int[-\pi/2 \text{ to } \pi/2] (2 \cos \theta - \cos^2 \theta)\, d\theta$
    - Use $\cos^2 \theta = (1 + \cos(2\theta))/2$:

      $A = (1/2) \int[-\pi/2 \text{ to } \pi/2] (2 \cos \theta - (1 + \cos(2\theta))/2)\, d\theta$

      $A = (1/2) \int[-\pi/2 \text{ to } \pi/2] (2 \cos \theta - 1/2 - (1/2)\cos(2\theta))\, d\theta$
    - Evaluate:

      $A = (1/2) [2 \sin \theta - (1/2)\theta - (1/4)\sin(2\theta)] |\_[-\pi/2 \text{ to } \pi/2]$

      $A = (1/2) [(2\sin(\pi/2) - (1/2)(\pi/2) - (1/4)\sin(\pi)) - (2\sin(-\pi/2) - (1/2)(-\pi/2) - (1/4)\sin(-\pi))]$

      $A = (1/2) [(2*1 - \pi/4 - 0) - (2*(-1) + \pi/4 - 0)]$

      $A = (1/2) [(2 - \pi/4) - (-2 + \pi/4)]$

      $A = (1/2) [2 - \pi/4 + 2 - \pi/4] = (1/2) [4 - \pi/2] = 2 - \pi/4$

13. **Length of $x = t^3$, $y = 3t^2/2$ for $0 \le t \le \sqrt{3}$:**
    - Formula: $L = \int[a \text{ to } b] \text{sqrt}((dx/dt)^2 + (dy/dt)^2)\, dt$
    - $dx/dt = 3t^2$
    - $dy/dt = (3/2)(2t) = 3t$

- $(dx/dt)^2 + (dy/dt)^2 = (3t^2)^2 + (3t)^2 = 9t^4 + 9t^2 = 9t^2(t^2 + 1)$
- $\text{sqrt}((dx/dt)^2 + (dy/dt)^2) = \text{sqrt}(9t^2(t^2 + 1)) = 3t * \text{sqrt}(t^2 + 1)$ (since $t \geq 0$)
- Integral: $L = \int[0 \text{ to } \sqrt{3}]\ 3t * \text{sqrt}(t^2 + 1)\ dt$
- Substitution: Let $u = t^2 + 1$, then $du = 2t\ dt$, so $3t\ dt = (3/2)\ du$.
  - When $t=0$, $u=1$. When $t=\sqrt{3}$, $u=(\sqrt{3})^2+1 = 3+1=4$.
- $L = \int[1 \text{ to } 4]\ \text{sqrt}(u) * (3/2)\ du = (3/2) \int[1 \text{ to } 4]\ u^{(1/2)}\ du$
- Evaluate:

  $L = (3/2)\ [\ (2/3)u^{(3/2)}\ ]\ |\_[1 \text{ to } 4]$

  $L = [\ u^{(3/2)}\ ]\ |\_[1 \text{ to } 4]$

  $L = 4^{(3/2)} - 1^{(3/2)} = (\sqrt{4})^3 - 1^3 = 2^3 - 1 = 8 - 1 = 7$

14. **Shifted Ellipse:** Original $x^2/16 + y^2/9 = 1$. Shift right 4, up 3.

- a. **New Equation:** Replace x with $(x-4)$ and y with $(y-3)$:

  $(x - 4)^2 / 16 + (y - 3)^2 / 9 = 1$
- b. **Original Features:** $a^2=16,\ b^2=9$. So $a=4,\ b=3$. Foci on x-axis.

  $c^2 = a^2 - b^2 = 16 - 9 = 7$, so $c = \sqrt{7}$.
  - Original Center: (0, 0)
  - Original Foci: (±√7, 0)
  - Original Vertices: (±4, 0) (major); (0, ±3) (minor)
- **New Features:** Apply shift (add 4 to x, add 3 to y):
  - New Center: $(0+4,\ 0+3) = (4,\ 3)$
  - New Foci: $(4 \pm \sqrt{7},\ 3)$
  - New Vertices: $(4 \pm 4,\ 3)$ -> $(8,\ 3)$ and $(0,\ 3)$ (major); $(4,\ 3 \pm 3)$ -> $(4,\ 6)$ and $(4,\ 0)$ (minor)
- c. **Sketch:** Draw axes. Mark the center (4, 3). Mark the major vertices (0, 3) and (8, 3). Mark the minor vertices (4, 0) and (4, 6). Mark the foci $(4-\sqrt{7},\ 3)$ and $(4+\sqrt{7},\ 3)$ (approx 1.35 and 6.65 for x). Sketch the ellipse passing through the vertices.

15. **Graphing $r^2 = 4 \cos \theta$:**

- **Symmetry:**
  - x-axis: $(r,\ -\theta)$ -> $r^2 = 4\cos(-\theta) = 4\cos\theta$. Yes, symmetric about x-axis.
  - y-axis: $(r,\ \pi-\theta)$ -> $r^2 = 4\cos(\pi-\theta) = -4\cos\theta$. No. Test $(-r,\ -\theta)$ -> $(-r)^2 = 4\cos(-\theta)$ -> $r^2 = 4\cos\theta$. Yes, symmetric about y-axis.
  - Origin: $(-r,\ \theta)$ -> $(-r)^2 = 4\cos\theta$ -> $r^2 = 4\cos\theta$. Yes, symmetric about origin.

    (Symmetry about x and y implies origin symmetry)
- **Domain:** Requires $\cos\theta \geq 0$, which occurs for $-\pi/2 \leq \theta \leq \pi/2$.

- **Shape:** Lemniscate (figure-eight shape).
- **Slopes at Origin:** The curve passes through the origin when $r=0$, which happens when $\cos\theta = 0$, i.e., $\theta = \pm\pi/2$.

  The slope at the origin for a polar curve $r=f(\theta)$ passing through at $\theta=\theta_0$ is $\tan(\theta_0)$.
  - At $\theta = \pi/2$, slope is $\tan(\pi/2)$ which is undefined (vertical tangent).
  - At $\theta = -\pi/2$, slope is $\tan(-\pi/2)$ which is undefined (vertical tangent).
- **Graph:** Draw the figure-eight shape, symmetric about the x-axis, passing through the origin with vertical tangents there, and extending along the x-axis. The maximum r is ± 2 when $\theta=0$.

MA THEMA TICS DEPARTMENT
Model Examination - Parametrization and Conic Sections (MATH 2025)

Total Marks: 15
Time: 1.5 Hours

Part A
(Answer any 2 questions. Each carries 1/2 mark)

1. Define a parametric curve and its parameter interval. (Ref: Section 11.1 Definition)
2. State the parametric formula for dy/dx if $x = f(t)$, $y = g(t)$. (Ref: Section 11.2, Eq. 1)
3. What is the eccentricity e of a parabola? (Ref: Section 11.7 Definition)

(2 x 1/2 = 1 Mark)

Part B
(Answer any 2 questions. Each carries 1.5 marks)

4. Given the parametric equations $x = 3t$, $y = 9t^2$, $-\infty < t < \infty$. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation and indicate the direction of motion. (Ref: Exercise 11.1.1)
5. Find the focus and directrix of the parabola $y^2 = 10x$. Sketch the parabola. (Ref: Example 11.6.1)
6. Find an equation for the line tangent to the curve $x = \sec t$, $y = \tan t$ at the point $(\sqrt{2}, 1)$ (corresponding to $t = \pi/4$). (Ref: Example 11.2.1)

(2 x 1.5 = 3 Marks)

Part C
(Answer any 2 questions. Each carries 2 marks)

7. Find the area under one arch of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$. (Ref: Exercise 11.2.21)
8. The parabola $y^2 = 8x$ is shifted down 2 units and right 1 unit to generate the parabola $(y + 2)^2 = 8(x - 1)$.
   a. Find the new parabola's vertex, focus, and directrix.
   b. Plot the new vertex, focus, and directrix, and sketch in the new parabola. (Ref: Exercise 11.6.39)
9. Find the length of the cardioid $r = 1 - \cos \theta$. (Ref: Example 11.5.3)
10. Find a Cartesian equation for the hyperbola centered at the origin that has a focus at $(3, 0)$ and the line $x = 1$ as the corresponding directrix. (Ref: Example 11.7.1)

(2 x 2 = 4 Marks)

(Note: The total marks add up to 8 based on the structure above. To reach 15, the number of questions to answer or the marks per question would need adjustment, similar to the variations in the sample papers. For example: Part A: 4x1=4, Part B: 3x2=6, Part C: 2x2.5=5)

ANSWERS

Part A

1. **Define parametric curve:** If x and y are given as continuous functions $x = f(t)$, $y = g(t)$ over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a **parametric curve**. The variable t is the **parameter**, and its domain I is the **parameter interval**.
2. **Parametric formula for dy/dx:** If $x = f(t)$ and $y = g(t)$ are differentiable, and $dx/dt \neq 0$, then $dy/dx = (dy/dt) / (dx/dt)$.
3. **Eccentricity of parabola:** The eccentricity e of a parabola is $e = 1$.

Part B

4. **Cartesian Equation for x = 3t, y = 9t^2:**
   - From $x = 3t$, we get $t = x/3$.
   - Substitute into $y = 9t^2$: $y = 9(x/3)^2 = 9(x^2/9) = x^2$.
   - The Cartesian equation is $y = x^2$. This is a parabola opening upwards with vertex at (0,0).
   - **Direction:** As t increases from $-\infty$ to ∞, $x = 3t$ increases from $-\infty$ to ∞.
     - When $t < 0$, $x < 0$, the particle moves down the left side towards the vertex.
     - When $t = 0$, $(x, y) = (0, 0)$.

- When $t > 0$, $x > 0$, the particle moves up the right side away from the vertex.
  - **Graph:** Sketch the parabola $y = x^2$. Indicate direction with arrows moving from left-to-right through the vertex.

5. **Focus and Directrix for $y^2 = 10x$:**
   - Standard form is $y^2 = 4px$.
   - Comparing, $4p = 10$, so $p = 10/4 = 5/2$.
   - The parabola opens to the right (since $p > 0$ and it's y^2).
   - Vertex is $(0, 0)$.
   - Focus is $(p, 0) = (5/2, 0)$.
   - Directrix is $x = -p$, which is $x = -5/2$.
   - **Sketch:** Draw the parabola opening right, vertex at origin, mark focus at (2.5, 0) and draw the vertical line $x = -2.5$ as the directrix.

6. **Tangent Line for x = sec t, y = tan t at t = π/4:**
   - Point: At $t = \pi/4$, $x = \sec(\pi/4) = \sqrt{2}$, $y = \tan(\pi/4) = 1$. The point is $(\sqrt{2}, 1)$.
   - Derivatives:
     - $dx/dt = d/dt\ (\sec t) = \sec t \tan t$
     - $dy/dt = d/dt\ (\tan t) = \sec^2 t$
   - Slope dy/dx:
     - $dy/dx = (dy/dt) / (dx/dt) = (\sec^2 t) / (\sec t \tan t) = \sec t / \tan t = (1/\cos t) / (\sin t / \cos t) = 1 / \sin t$
   - Slope at $t = \pi/4$:
     - $dy/dx \,|_{(t=\pi/4)} = 1 / \sin(\pi/4) = 1 / (1/\sqrt{2}) = \sqrt{2}$.
   - Tangent Line Equation (point-slope form): $y - y1 = m(x - x1)$
     - $y - 1 = \sqrt{2}\ (x - \sqrt{2})$
     - $y - 1 = \sqrt{2}\ x - 2$
     - $y = \sqrt{2}\ x - 1$

Part C

7. **Area under Cycloid Arch:**
   - Curve: $x = a(t - \sin t)$, $y = a(1 - \cos t)$. One arch corresponds to $0 \le t \le 2\pi$.
   - Area $A = \int y\ dx$. We need $dx = (dx/dt)\ dt$.
   - $dx/dt = a(1 - \cos t)$.

- Limits: When $t=0$, $x=0$. When $t=2\pi$, $x=a(2\pi - \sin(2\pi)) = 2\pi a$.
- Integral:

  $$A = \int_{t=0}^{2\pi} y(t)\,(dx/dt)\,dt$$

  $$A = \int_0^{2\pi} a(1 - \cos t) * a(1 - \cos t)\,dt$$

  $$A = a^2 \int_0^{2\pi} (1 - \cos t)^2\,dt$$

  $$A = a^2 \int_0^{2\pi} (1 - 2\cos t + \cos^2 t)\,dt$$

  $$A = a^2 \int_0^{2\pi} (1 - 2\cos t + (1 + \cos(2t))/2)\,dt$$

  $$A = a^2 \int_0^{2\pi} (3/2 - 2\cos t + (1/2)\cos(2t))\,dt$$
- Evaluate:

  $$A = a^2 \left[(3/2)t - 2\sin t + (1/4)\sin(2t) \right]\Big|_{0}^{2\pi}$$

  $$A = a^2 \left[\left((3/2)(2\pi) - 2\sin(2\pi) + (1/4)\sin(4\pi) \right) - \left((3/2)(0) - 2\sin(0) + (1/4)\sin(0) \right) \right]$$

  $$A = a^2 \left[(3\pi - 0 + 0) - (0 - 0 + 0) \right]$$

  $$A = 3\pi a^2$$

8. **Shifted Parabola $(y + 2)^2 = 8(x - 1)$:**

   - Original parabola: $y^2 = 8x$. Here $4p = 8$, so $p = 2$.
     - Original Vertex: (0, 0)
     - Original Focus: (p, 0) = (2, 0)
     - Original Directrix: $x = -p = -2$
   - Shift: Down 2 units (y replaced by $y+2$), Right 1 unit (x replaced by $x-1$).
   - a. **New Features:**
     - New Vertex: Shift (0, 0) right 1, down 2 -> $(1, -2)$
     - New Focus: Shift (2, 0) right 1, down 2 -> $(3, -2)$
     - New Directrix: Shift $x = -2$ right 1 -> $x = -1$
   - b. **Plot and Sketch:**
     - Mark the vertex at (1, -2).
     - Mark the focus at (3, -2).
     - Draw the vertical directrix line $x = -1$.
     - Sketch the parabola opening to the right, passing through the vertex (1, -2), symmetric about the line $y = -2$.

9. **Length of Cardioid $r = 1 - \cos\theta$:**

   - The curve is traced once as θ goes from 0 to 2π.
   - Formula: $L = \int_{\alpha}^{\beta} \sqrt{r^2 + (dr/d\theta)^2}\,d\theta$
   - $r = 1 - \cos\theta$
   - $dr/d\theta = \sin\theta$

- `r^2 + (dr/dθ)^2 = (1 - cos θ)^2 + (sin θ)^2`
  `= (1 - 2 cos θ + cos^2 θ) + sin^2 θ`
  `= 1 - 2 cos θ + (cos^2 θ + sin^2 θ)`
  `= 1 - 2 cos θ + 1 = 2 - 2 cos θ`
- Using identity `1 - cos θ = 2 sin^2(θ/2)`:
  `r^2 + (dr/dθ)^2 = 2(1 - cos θ) = 2(2 sin^2(θ/2)) = 4 sin^2(θ/2)`
- `sqrt(r^2 + (dr/dθ)^2) = sqrt(4 sin^2(θ/2)) = |2 sin(θ/2)|`
- For `0 ≤ θ ≤ 2π`, `0 ≤ θ/2 ≤ π`, so `sin(θ/2) ≥ 0`. Thus `|2 sin(θ/2)| = 2 sin(θ/2)`.
- Integral:
  `L = ∫[0 to 2π] 2 sin(θ/2) dθ`
- Evaluate:
  `L = [ -4 cos(θ/2) ] |_[0 to 2π]`
  `L = (-4 cos(2π/2)) - (-4 cos(0/2))`
  `L = (-4 cos(π)) - (-4 cos(0))`
  `L = (-4 * -1) - (-4 * 1) = 4 - (-4) = 8`

10. **Hyperbola from Focus/Directrix:**

- Focus `F = (c, 0) = (3, 0)`, so `c = 3`.
- Directrix `x = k = 1`.
- Relationship for hyperbola centered at origin: `k = a/e` and `c = ae`.
- From `k = 1`, we have `a/e = 1`, so `a = e`.
- Substitute `a = e` into `c = ae`: `3 = e * e = e^2`.
- So, `e^2 = 3`, and the eccentricity `e = √3`.
- Since `a = e`, we have `a = √3`. So `a^2 = 3`.
- For a hyperbola, `b^2 = c^2 - a^2`.
- `b^2 = 3^2 - (√3)^2 = 9 - 3 = 6`.
- The standard equation for a hyperbola with foci on the x-axis is `x^2/a^2 - y^2/b^2 = 1`.
- Substituting `a^2 = 3` and `b^2 = 6`:
  `x^2/3 - y^2/6 = 1`