Measures Of Central Tendancy.

Central Tendancy:

The tendancy of the observation to cluster around some central values of data is known as Central tendancies.

There are various measures of central tendancy. most of important measures of central tendancy are

(1) arithematic mean

(2) median

(3) mode

(4) Geometric mean

(5) Harmonic mean.

Characteristics

* it should be rigidly defined.
* it should be readily comprehensible and easy to calculate
* it should be based on all the observations
* it should be suitable for further mathematical treatment.
* it should be affected by as little as possible

by fluctuations of Sampling.
* it should not be affected much by extreme values.

Arithematic Mean.

Arithematic mean of a set of Observation is their Sum divided by the number of Observation
eg:- The arithematic mean $\bar{x}$ of n observations $x_1, x_2, x_3 \ldots x_n$ is given by.

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \cdots + x_n).$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

in case of frequency distribution x_i/f_i $= 1, 2, \ldots$
where f_i is the frequency of the variable x_i

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i x_i \qquad \left[\sum_{i=1}^{n} f_i = N\right]$$

in case of grouped or continous frequency distribution, x is taken as the mid value of the corresponding class.

eg ① The following data shows the family size corresponding to 10 students
3, 4, 4, 5, 4, 4, 5, 5, 4, 4

Ans

x	f	$x_i f_i$
3	1	3
4	6	24
5	3	15

The average family size

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{3+24+15}{10} = \frac{42}{10}$$

$$= \underline{\underline{4.2}}$$

eg ② The following data show the family size of 100 students. Compute the average family size.

family size	No of families
2	2
3	25
4	45
5	18
6	10

Ans

x	f	xf
2	2	4
3	25	75
4	45	180
5	18	90
6	10	60
	100	409

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{409}{100}$$

$$= \underline{\underline{4.09}}$$

The following frequency distribution shows the expenditure (per week) of 100 students. Obtain the average.

expenditure	No of students
0-100	6
100-200	24
200-300	60
300-400	8
400-500	2.

Here we have a grouped frequency distributio

∴ The formulae for arithematic mean is

$\overline{x} = \frac{\Sigma xf}{N}$ where x are the midvalue of th class.

f is the corresponding frequenc

N is the total frequency.

The calculation can be done in following way

x	f	xf
50	6	300
150	24	3600
250	60	15000
350	8	2800
450	2	900
	100	22600

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$$\bar{x} = \frac{\sum xf}{N} = \frac{300+3600+15,000+2800+900}{100}$$

$$= \frac{22600}{100} = \underline{\underline{226}}.$$

Properties Of Arithematic Mean.

1). The Sum of deviation of observation from its arithematic mean is zero.

Proof :-

Let $x_1, x_2, x_3 \dots x_n$ be n observation then its arithematic mean is

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i \qquad \boxed{\sum_{i=1}^{n} x_i = N\bar{x}}$$

$x_1 - \bar{x}, x_2 - \bar{x}, \dots, x_n - \bar{x}$ are known a deviations of the observation from the arithem mean.

∴ Sum of deviation of observation from arithem mean

$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \dots + (x_n - \bar{x})$$

$$= \sum_{i=1}^{n} (x_i - \bar{x})$$

$$= \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$$

$$= N\bar{x} - N\bar{x}$$

$$= 0$$

The above proof can easily be extended for a frequency distribution

$x_i f_i$, $i = 1, 2, \ldots n$.

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i f_i \qquad \boxed{N\bar{x} = \sum_{i=1}^{n} x_i f_i}$$

Sum of deviations of from $\bar{x}$

$$= \sum_{i=1}^{n} (x_i - \bar{x}) f_i$$

$$= \sum_{i=1}^{n} x_i f_i - \sum_{i=1}^{n} \bar{x} f_i$$

$$= \sum_{i=1}^{n} x_i f_i - \bar{x} \sum_{i=1}^{n} f_i$$

$$= N\bar{x} - N\bar{x}$$

$$= \underline{\underline{0}}$$

2. The Sum of Squares of deviations of the observations is a minimum when it is taken from arithematic mean.

Proof

let $g(a)$ = Sum of squares of deviation of observation from 'a'.

$$= \sum_{i=1}^{n} (x_i - a)^2$$

where a can be ~~only~~ any real number.

$$\cancel{g(a)} = g'(a) = \frac{d\,g(a)}{d\,(a)}$$

$$= \sum_{i=1}^{n} 2(x_i - a) \times -1$$

$$= -2 \sum_{i=1}^{n} (x_i - a).$$

$$g(a) = 0 \Rightarrow 0 = -2 \sum_{i=1}^{n} (x_i - a)$$

$$0 = \sum_{i=1}^{n} (x_i - a)$$

$$\sum_{i=1}^{n} x_i - \sum_{i=1} a = 0$$

$$N\bar{x} = \sum_{i=1}^{n} a$$

$$N\bar{x} = Na$$

$$\underline{\underline{\bar{x} = a}}.$$

$$g'(a) = -2 \sum_{i=1}^{n} (x_i - a)$$

$$g''(a) = -2 \times -1$$

$$= +2$$

$+2 > 0 \Rightarrow g(a)$ is minimum ~~at~~ when $a = \bar{x}$.

$\therefore$ Sum of Squares of deviation is a minimum when it is taken from its Arithematic Mean.

The above proof can be easily extented to the case of a frequency distribution as follows. let $x_i f_i$, $i = 1 \rightarrow n$ denotes a frequency distribution then $\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i f_i$ $\quad \boxed{N\bar{x} = \sum_{i=1}^{n} x_i f_i}$

~~$g(a)$~~

$$g(a) = \sum_{i=1}^{n} (x_i - a)^2 f_i$$

$$g'(a) = \sum_{i=1}^{n} 2 \times (x_i - a) \times -1 \cdot f_i$$

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$g'(a) = -2 \times \sum_{i=1}^{n} (x_i - a)\, f_i$

$g'(a) = 0 \implies -2 \times \sum_{i=1}^{n} (x_i - a) \cdot f_i = 0$

$= \sum_{i=1}^{n} (x_i - a) \cdot f_i = 0$

$\sum_{i=1}^{n} x_i f_i - \sum_{i=1}^{n} a f_i = 0$

$\sum_{i=1}^{n} x_i f_i = \sum_{i=1}^{n} a f_i$

$N\bar{x} = a \sum_{i=1}^{n} f_i$

$N\bar{x} = aN$

$\bar{x} = a$

$g''(a) = -2 \times \sum_{i=1}^{n} -1 \times f_i$

$= +2 \sum_{i=1}^{n} f_i$

$= 2N > 0$

$\Rightarrow$ $g(a)$ attains minimum when $a = \bar{x}$

$\therefore$ Sum of squares of deviation from their arithematic mean is minimum

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3. Let $x_1, x_2, x_3 \dots x_n$ be n observations. and a be any real constant and $c>0$, be any positive constant.
Now consider the transformed observations.

$u_1 = \frac{x_1 - a}{c}$ $u_2 = \frac{x_2 - a}{c}$ $\dots$ $u_n = \frac{x_n - a}{c}$.

in this case $\bar{x} = a + c\bar{u}$

This property is also known as a Shortcut method for the computation of arithematic ~~for~~ mean.

Proof:
We have

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i$$

$$\bar{u} = \frac{1}{N} \sum_{i=1}^{n} u_i$$

$$\bar{u} = \frac{1}{N} \sum_{i=1}^{n} \frac{x_i - a}{c}$$

$$= \frac{1}{c} \left[\frac{1}{N} \sum_{i=1}^{n} x_1 - \sum_{i=1}^{n} a \right]$$

$$= \frac{1}{c} \left[\frac{1}{N} \sum_{i=1}^{n} x_i - \frac{1}{N} \sum_{i=1}^{n} a \right]$$

$$\bar{u} = \frac{1}{c} \left[\bar{x} - \frac{1}{N} aN \right]$$

$$= \frac{1}{c} \left[\bar{x} - a \right]$$

$$c\bar{u} = \bar{x} - \bar{a}$$
$$\bar{x} = \bar{a} + c\bar{u}$$

The above result is useful when we have complicated calculations when the observations are large in size. We usually take 'a' as the some middle value of observation. and 'c' as the greatest common divisor of the deviation $x_1 - a, x_2 - a, \cdots x_n - a$.

Eg ① Compute the arithematic mean of the following numbers using shortcut method.
100, 150, 200, 250, 300.
here we may take 'a' = 200; 'c' = 50.
calculations are given below.

x	$x_i - 200$	$u = \frac{x_i - 200}{50}$
100	-100	-2
150	-50	-1
200	0	0
250	50	1
300	100	2
		0

$$\bar{u} = \frac{1}{n} \Sigma u_i = \frac{1}{5} \times 0 = 0$$

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$$\bar{x} = a + c\bar{u} = 200 + 0 = \underline{\underline{200}}.$$

Assignment

1) The following frequency distribution shows mark of 500 students in mathematics in their SSLC examination. Compute the average mark using shortcut method.

Marks	No of students
0-10	10
10-20	15
20-30	25
30-40	100
40-50	200
50-60	75
60-70	25
70-80	40
80-90	10

2) Suppose that $x_i\, f_i$ $i = 1 \ldots n$, represents a frequency distribution. Let a and c are real constant $c > 0$, then $u = \frac{x - a}{c}$. Show $\bar{x} = a + c\bar{u}$

swers

1)

x	f	$\frac{x-45}{10}$	$\frac{x-45}{10} \times f$
5	10	$= -4$	-40
15	15	-3	-45
25	25	-2	-50
35	100	-1	-100
45	200	0	0
55	75	1	75
65	25	2	50
75	40	3	120
85	10	4	40
	$N = 500$		$\Sigma u = 50$

$$\bar{u} = \frac{\Sigma u}{N} = \frac{50}{500} = \frac{1}{10}$$

$$\bar{x} = a + c\bar{u}$$

$$= 45 + 10 \times \frac{1}{10}$$

$$= 45 + 1$$

$$= 46$$

2) $\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i f_i$ $\qquad u = \frac{x_n - a}{c}$

$$\bar{u} = \frac{1}{N} \sum_{i=1}^{n} u_i f_i$$

$$\bar{u} = \frac{1}{N} \sum_{i=1}^{n} \left(\frac{x_i - a}{c}\right) f_i$$

$$= \frac{1}{c} \frac{1}{N} \sum_{i=1}^{n} (x_i - a) f_i$$

$$= \frac{1}{c} \left[\frac{1}{N} \sum_{i=1}^{n} x_i f_i - \frac{1}{N} \sum_{i=1}^{n} a f_i\right]$$

$$= \frac{1}{c} \left[\bar{x} - \frac{1}{\cancel{N}} a \cancel{N}\right]$$

$$= \frac{1}{c} [\bar{x} - a]$$

$$c\bar{u} = \bar{x} - a$$

$$\bar{x} = a + c\bar{u}$$

4. Combined Arithematic Mean (mean of the comparable ~~series~~ series).

Let $\bar{x}_1, \bar{x}_2, \dots \bar{x}_K$ are means of K series of observations of sizes $n_1, n_2, n_3, \dots, n_K$. then the mean $\bar{x}$ of the combined series is given by

$$\bar{x} = \frac{n_1 x_1 + n_2 x_2 + \dots + n_K x_K}{n_1 + n_2 + \dots + n_K} = \frac{\sum_{i=1}^{n} n_i \bar{x}_i}{\sum_{i=1}^{n} n_i}$$

Proof.

Let observations are in the following form

Series	Observation	Mean
1	$x_{11}, x_{12}, x_{13} \dots x_{1n_1}$	$\bar{x}_1$
2	$x_{21}, x_{22}, x_{23} \dots x_{2n_2}$,	$\bar{x}_2$
3	$x_{31}, x_{32}, x_{33} \dots x_{3n_3}$,	$\bar{x}_3$
.	. . . . . .	.
.	. . . . . .	.
K	$x_{K1}, x_{K2}, x_{K3} \dots x_{Kn_K}$	$\bar{x}_K$

Then the combined arithematic mean is given by

$$\bar{x} = \frac{(x_{11} + x_{12} + x_{13} + \dots + x_{1n_1}) + (x_{21} + x_{22} + x_{23} + \dots + x_{2n_2}) + \dots + (x_{K1} + x_{K2} + x_{K3} + \dots + x_{Kn_K})}{(n_1 + n_2 + n_3 + \dots + n_K)}.$$

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$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \cdots + n_K\bar{x}_K}{n_1+n_2+\cdots+n_K}.$$

eg:- The average weight of 10 students in a class is 55 Kg. the average weight of 20 students in another class is 60 Kg. find the combined average

Ans) The combined mean $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \cdots + n_K\bar{x}_K}{n_1+n_2+\cdots+n_K}$

$$= \frac{10\times55 + 20\times60}{10+20}$$

$$= \frac{550+1200}{30}$$

$$= \frac{175}{3}$$

$$= \underline{\underline{58.3}}$$

Assignment

1) The average salary of male employees in a firm was ₹520. that of female employee was ₹420. The mean salary of all employees is ₹500. find the % of male and female employees.

2) explain merits and demerits of Arithematic mean as a measure of central tendancy.

3Q) defined weighted arithematic mean and Write down the formulae for weighted arithematic mean

4Q) find the Simple and weighted arithematic mean of 1^{st} n natural numbers. the weights being the Corresponding numbers.

5Q) find the arithematic mean from the follow class interval Showing weekly pocket money Spent by 100 students in a college using Shortcut method.

	frequency.
0-100	5
100-200	20
200-300	50
300-400	15
400-500	10

6Q) The arithematic mean of mark of 10 stude was 60. However the mark of one student was wrongly entered as 45. instead of 54. find the actual arithematic mean of mark

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Median

Median is a positional average. it is defined as the middle value of Observation when the observation are arranged in ascending or descending order of magnitude

Suppose that $x_1, x_2, x_3 \cdots x_n$ are n observations arranged in ascending order of magnitude.

if n is an odd number. then

median = $\frac{n+1}{2}$ th observation

If n is an even number then

median = $\frac{\frac{n}{2}\text{th observation} + (\frac{n}{2}+1)\text{th observation}}{2}$

ie in this case. median is the arithematic mean of $n/2$ th observation and $\frac{n}{2}+1$ th observation.

Eg① Q) The following data shows family size of 7 houses of a panchayath find the median family size.

1 7 3 4 2 2 6.

to find median we first arrange observation in ascending order ie 1 2 2 3 4 6 7

here n=7 so median is $\frac{n+1}{2}$ th item ie 4th item = 3

So median = 3

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eg ② The following data shows the sleeping time of six students in a day. Obtain the median sleeping hours.

4 5 10 4 6 9

Ans. When observations are arranged in ascending order.

4 4 5 6 9 10

here $n = 6$. so. median $= \dfrac{\frac{n}{2}^{th} + (\frac{n}{2}+1)^{th}}{2}$

$= \dfrac{3^{rd} + 4^{th}}{2}$

Median $= \dfrac{5+6}{2}$

$= 5.5$

Computation of Median from a frequency distribution

in a discrete or ungrouped frequency distribution, median can be computed using following steps.

- find $\frac{N}{2}$, $N = \Sigma f$. = total frequency.
- Compute the Cumulative frequency (CF) just greater than (N/2)
- The corresponding value of x is the median.

eg ① compute median in the following discrete frequency distribution showing the family sizes of 20 college students

family size	2	3	4	5	6
No of students	2	5	10	2	1

Ans

x	F	CF
2	2	2
3	5	7
4	10	17
5	2	19
6	1	20

here $n/2 = \frac{20}{2} = 10$

The cumulative frequency just greater than 10 is 17. The corresponding x value is 4.

$\therefore$ Median = 4.

Computation of median from a continuous frequency distribution

in a continuous distribution has a following ways.

class	$x_1 - x_2$	$x_2 - x_3$	...	$x_K - x_{K+1}$
frequency	f_1	f_2	...	f_K.

in this case the median can be calculate using following formula.

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right) \text{ where}$$

$l \rightarrow$ lower limit of median class.
by median class we mean, the class corresponding to the C.f. just greater than N/2.
$f \rightarrow$ frequency of median class
$h \rightarrow$ Magnitude of median class.
$c \rightarrow$ C.f of preceding the median class
$N = \Sigma f$.

Eg Compute the median from the continuous frequency distribution showing the life length of 50 cancer patients after chemotherapy

Class	frequency.
0-5	20
5-10	10
10-15.	7
15-20	6
20-25	7

Ans The the formula of median $= l + \frac{h}{f}\left(\frac{N}{2} - c\right)$

$l \to$ lowerlimit of median class.
$F \to$ frequency of median class.
$h \to$ Magnitude of median class.
$C \to$ c.f of preceeding median class.

class	frequency	c.f.
0-5	20	20
5-10.	10	30
10-15	7	37
15-20	6	43
20-25	7	50

$\longrightarrow$ Median class.

here $N = 50$ so $\frac{N}{2} = \frac{50}{2} = 25$.
$l = 5$ $h = 5$ $F = 10$ $C = 20$

$\therefore$ Median $= l + \frac{h}{F}\left(\frac{N}{2} - C\right)$.

$= 5 + \frac{5}{10}\left(\frac{50}{2} - 20\right)$

$25 - 20 = 5$

$= 5 + \frac{5}{10} \times 5$

$= 5 + \frac{25}{10}$

$= 5 + 2.5$

$= \underline{\underline{7.5}}$

H.W

1Q) compute the median following discrete frequency distribution

x	f
0	5
5	20
10	32
15	45
20	10

57
45
102

2Q) Obtain the median from the following continous frequency distribution

class	frequency.
0-20	5
20-40	35
40-80	40
80-100	30
100-150	9

3Q) explain the merits and demerits of me as a measure of central tendancy.

1) Ans)

x	C.F	f
0	5	5
5	25	20
10	57	32.
15	102.	45
20	112	10

$n = 112$

$n/2 = \frac{112}{2} = 51$

The CF just greater than 51 is 57. corrs $x = 10$.
So median is 10

2 Ans)

Class	frequency	C.f
0-20	5	5
20-40	35	40
40-80	40	80
80-100	30	110
100-150	9	119

$N = \frac{119}{2} = 59.5$

~~$l = 40$~~ $l = 40$

$f = 40$ $N = 119$.

$c = 40$

$h = 40$

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$$= \cancel{80 + \frac{20}{30}(}$$

$$= 40 + \frac{40}{40}\left(\frac{119}{2} - 40\right)$$

$$= 40 + (19.5)$$

$$= \underline{\underline{59.5}}$$

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3ans) Merits and Demerits of Median

Merits

* it is rigidly defined.
* it is easily understood and is easy to calculate: in Some cases it can be located merely by inspection.
* it is not at all affected by extreme val
* it can be calculated for distributions with Open - end classes.

Demerits

* in case of even numbers of observations medi cannot be determined exactly, we merely estimate it by taking the mean of two middle terms
* it is not based on all observation. for eg:- Median of 10, 25, 50, 60 and 65 is 50. We can replace 10 & 25 by any two values. which are smaller than 50 and observations 60 & 65 by two values greater than 50 without affecting the value of median. This property is described by saying that Median is insensitive

* it is not amenable to algebraic treatment
* As compared with mean, it is affected much by fluctuation of sampling.

* it is not amenable to algebraic treatment
* As compared with mean, it is affected much by fluctuation of sampling

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Mode

Mode is the most frequently occured observation.

for eg:- Consider the following observation representing the shoe sizes of 10 male students

8 7 9 10 10 9 10 8 10 11

Here 10 is the most freauently ocauring observation. Therefore 10 is the mode of the observation.

In case of ungrouped freauency distribution the mode is the value of x for which the maximum frequency occurs. as an example.

Consider the following frequency distribution show the family sizes of 50 college students

family size	No of students
2	3
3	4
4	3
5	15
6	7
77	10
	8

In the case of ungrouped frequency distribution. We first locate the modal class which is the class for which frequency is maximum.

Then we have the following formula for the computation of mode.

$$\text{Mode} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

where l is lower limit of modal class.

h is the width of the modal class.

f_0 is the frequency of the class preceding modal class.

f_2 is the frequency of the class succeeding modal class.

following example is an illustration:

following data shows the pocket money spent by 100 college students. compute the mode.

Money spent	No of students	
0 - 50	5	$f_0 = 25$
50-100	25	$f_1 = 50$
$\boxed{100\text{-}150}$	$\boxed{50}$	$f_2 = 15$
150 - 200	15	$l = 100$
200 - 250	5	$h = 50$

here the modal class is the class with ~~class~~
100-150
The formulae of mode is

$$\text{Mode} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

l is lowerlimit of model class.
h is the classwidth of model class.
f_1 - is the frequency of model class.
f_0 - is the frequency of class preceding model class.
f_2 - is the frequency of class ~~pre~~ succeeding model class.

$$\text{Mode} = 100 + \frac{50(50-25)}{2\times 50 - 25 - 15}$$

$$= 100 + \frac{(50\times 25}{60}$$

$$= 100 + \cdot$$

$$= \underline{\underline{120.83}}$$

10. 25
15
40.

25
5
125

Q). find the mode of following frequency.

Class int :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
frequen :	5	8	7	12	28	20	10	10

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$\boxed{40-50 = 28}$

Ans) $Mode = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$

$l = 40$

$h = 10$

$f_1 = 28$

$f_0 = 12$

$f_2 = 20$

$Mode = 40 + \frac{10(28-12)}{2 \times 28 - 12 - 20}$

$= 40 + \frac{160}{}$

$=$

28
12
16

28
2
56
32
12
20
32

Q) The median and mode of the following wage distribution are known to be Median = ₹ 33.50. Mode = ₹ 34. find the values of F_3, F_4 & f_5.

Wages.	Frequency.
0-10	4
10-20	16
20-30	F_3
30-40	f_4
40-50	f_5
50-60	6
60-70	4
	230

Wages	f	c.f.
0-10	4	4
10-20	16	20
20-30	f_3	$20+f_3$
30-40	f_4	$20+f_3+f_4$
40-50	f_5	$20+f_3+f_4+f_5$
50-60	6	$26+f_3+f_4+f_5$
60-70	4	$30+f_3+f_4+f_5$
	230	

$\frac{\overset{115}{\cancel{230}}}{\cancel{2}} = 1$

~~Median~~

Since it is given median = 33.5 and mode is 34 respectively. both median class and modal class should be 30-40.

from the given information.

$\Sigma f = 230 = 30 + f_3 + f_4 + F_5$

$\therefore \quad f_3 + f_4 + f_5 = 200$

Now median $= l + \frac{h}{f}(n/_2 - c)$

from given information

$\therefore \quad 33.50 = 30 + \frac{10}{f_4}(115 - 20 - f_3).$

$$115 - 20 = 95.$$

$$33.50 - 30 = 3$$

$3.50 = \frac{10}{f_4}(95 - f_3).$

$0.350 f_4 = 95 - f_3.$

$f_3 = 95 - 0.35 f_4$

$$\text{Mode} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

$$34 = 30 + \frac{10(f_4 - f_3)}{2f_4 - f_3 - f_5}$$

$$4 = \frac{10(f_4 - f_3)}{2f_3 - f_2 - f_4}$$

$$0.4 = 0.65$$

$f_3 + f_4 + f_5 = 200$

$(f_3 + f_5) = 200 - f_4$

$2f_4 - 200 - f_4$

$\boxed{3f_4 - 200}$

$f_4 = 95 - 0.35f_4$

H.W Q) Explain the merits and demerits of mode as a measure of central tendency:

Geometric Mean

Geometric mean of set of positive observations is defined as the ~~and~~ n^{th} root ~~or~~ of product of the observations if $x_1, x_2, x_3 \ldots x_n$ are n positive observation. Then.

$$GM = (x_1 \cdot x_2 \cdots x_n)^{1/n}$$

for a frequency distribution of a positive values

x_i / f_i, $i = 1, 2, \ldots n$.

$$G.M = (x_1^{f_1} \cdot x_2^{f_2} \cdots x_n^{f_n})^{1/N}$$

where ~~N is~~ $N = \sum_{i=1}^{n} f_i$

$$GM = \left(\prod_{i=1}^{n} x_i^{f_i} \right)^{1/N}.$$

The computation of geometric mean ~~of~~ for large numbers is difficult. So we usually calculate the logarithm of G.M and then compute the Anti logarithm of G.M.

rather than computing the product of the ~~GM~~ numbers. We can compute sum of the logarithms of observation

We have

$$GM = (x_1 \cdot x_2 \cdots x_n)^{1/n}.$$

$$\log GM = \frac{1}{N} \log(x_1 \cdot x_2 \cdots x_n).$$

$$= \frac{1}{N} (\log x_1 + \log x_2 + \cdots + \log x_n)$$

$$= \frac{1}{N} \sum_{i=1}^{n} \log x_i$$

log G.M = Arithemetic of logarithms of observations

Antilogarithm of this quantity will give ~~yield~~ us GM of the observations.

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for frequency distribution

$$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot \cdot \cdot x_n^{f_n})^{1/N}$$

$$\log GM = \frac{1}{N}(f_1 \log x_1 + f_2 \log x_2 + \cdots + f_n \log x_n)$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i \log x_i$$

= AM of logarithms of observations

The Antilogarithm of this quantity will be the G.M ~~of cor~~ of the corresponding frequency distribution.

eg.

Q) find the geometric mean of the following numbers.

1. 2. 4 8

$$GM = (x_1 \cdot x_2 \cdots x_n)^{1/N}$$

$$= (1 \cdot 2 \cdot 4 \cdot 8)^{1/4}$$

$$= (64)^{1/4}$$

$$= (2^6)^{1/4}$$

$$= \underline{\underline{2^{3/2}}} = \underline{\underline{\cancel{8}}}\ 2\sqrt{2}$$

$\cancel{6}^{3} \times \frac{1}{\cancel{4}2}$

$\cancel{8}^{2} \times \frac{1}{4}$

Q) find the geometric mean from the following frequency distribution

x	f	$f \log x_i$
1	5	$5 \log 1$
2	10	$10 \log 2$
3	25	$25 \log 3$
4	15	$15 \log 4$
5	5	$5 \log 5$

Ans $GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot \ldots \cdot x_n^{f_n})^{1/n}$.

$$\log GM = \frac{1}{N} \sum_{i=1}^{n} f_i \log x_i$$

$$= \frac{1}{60} \times \sum_{i=1}^{n} f_i \log x_i$$

$$= \frac{1}{60} \times (5 \log 1 + 10 \log 2 + 25 \log 3 + 15 \log 4 + 5 \log 5$$

$$= \frac{1}{60} \times (0 + 3.01029 + 11.9280 + 9.0308 + 3.4948)$$

$$= \frac{1}{60} \times 27.4638$$

3.010299957
11.92803137
9.03089987
3.494850022
27.46408122

$\log GM = 0.457734687$

$GM = 2.868902773 41$

Q) Explain the merits and Demerits of mode as a measure of central tendancy.

Ans). Merits

* Mode is readily comprehensible and easy to calculate. Like median, mode can be located in some cases merely by inspection.
* mode is not at all affected by extreme values.
* mode can be ~~conve~~ conviently located even if the frequency distribution has class-intervals of unequal magnitude provided the modal class and the classes preceding & succeeding it are of same magnitude. Open ended classes also do not pose any problem in the location of mode.

Demerits

* Mode is ill defined. it is not always possible to find a clearly defined mode.
* it is not capable for further mathematical treatment
* it is not based on all observation
* As compared with mean, mode is affected to a

a greater extent by fluctuations of sampling

Harmonic

Harmonic mean of set of observations is defined as the reciprocal of the Arithmetic mean of the reciprocal of the observation.
if $x_1, x_2, x_3 \ldots x_n$ are n observation

$$H = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}$$

in case of frequency distribution. $x_i | f_i, (i = 1, 2, \ldots n)$

$$H = \frac{1}{\frac{1}{N}\sum_{i=1}^{n}(f_i / x_i)}$$

Q) find the harmonic mean of number 1, 2, 3.

$$\cancel{\frac{1}{1}} + \cancel{\frac{1}{2}} +$$

$$H = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}} = \frac{1}{\frac{1}{3}\times\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right)}$$

$$\frac{1}{\frac{1}{3}\times\frac{6+3+2}{6}} = \frac{1}{\frac{11}{18}}$$

$$= \frac{18}{11}$$

Merits

- it is rigidly defined.
- it is based on all the observations
- it is Suitable for further mathematical calculation.
- it is not much affected by fluctuation of Sampling.
- it gives greater importance to small items and is useful only when small items have to be given a greater weightage.

Demerit

- it is not easily ~~und~~ to understand
- it is difficult to Compute.

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Q) A cyclist pedals from his house to his college at a speed of 10 mph. and back from the college to his house at 15 mph. Find the average speed

Ans) Distance from home to school => x.

So x metere is covered in $\frac{x}{10}$ hr. from ho -> sch.

So x km is covered in $\frac{x}{15}$ hr from school -> home

total $2x$. covered in $\frac{x}{10} + \frac{x}{15}$ hrs.

Hence Average Speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{2x}{\frac{x}{10} + \frac{x}{15}}$$

$$= \frac{2\cancel{x}}{\cancel{x}\left(\frac{1}{10} + \frac{1}{15}\right)}$$

$$= \frac{2}{\frac{15+10}{150}} = \frac{2}{\frac{\cancel{25}\,5\,1}{\cancel{150}\,\cancel{30}\,6}} = \underline{\underline{12}}$$

Q) You can take a trip which entails travelling 900 km. by train at an average of 60 km per hour, 3000 kms by boat at an average of 25 kmph. 400 kms by plane at 350 kms per hr. and finally 15 kms by taxi at 25 kms per hour. What is the average speed.

Speed in km/h x	distance w	w/x
60	900	15
25	3000	120
350	400	1.43
15	25	0.60
	4315	137.03

$$\text{Average speed} = \frac{\Sigma w}{\Sigma w/x}$$

$$= \frac{4315}{137.03}$$

$$= \underline{\underline{31.489}}$$

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Q) find the minimum value of :

(1) $f(x) = (x-6)^2 + (x+3)^2 + (x-8)^2 + (x+4)^2 + (x-3)^2$

(2) $g(x) = |x-6| + |x+3| + |x-8| + |x+8| + |x-3|$

Ans) (1) $f(x) = (x-6)^2 + (x+3)^2 + (x-8)^2 + (x+4)^2 + (x-3)^2$

$$f'(x) = 2(x-6) + 2(x+3) + 2(x-8) + 2(x+4) + 2(x-3)$$
$$= 2x-12 + 2x+6 + 2x-16 + 2x+8 + 2x-6$$
$$= 10x - 12 - 8$$
$$= 10x - 20$$

$f'(x) = 0 \Rightarrow 10x - 20 = 0$
$$10x = 20$$
$$x = 2$$

Statistics $\Longrightarrow$

6, −3, 8, −4, 3.

$$\text{Mean} = \frac{6 + -3 + 8 - 4 + 3}{5} = \frac{10}{5} = 2$$

when $x = 2$.

$$f(x) = (2-6)^2 + (2+3)^2 + (2-8)^2 + (2+4)^2 + (2-3)^2$$
$$= 4^2 + 5^2 + 6^2 + 6^2 + 1^2$$
$$= 16 + 25 + 36 + 36 + 1$$
$$= \underline{\underline{114}}$$

Sum of ~~deviation~~ Squares of deviation is minimum when it is taken from $\bar{x}$.

$$\begin{array}{r} 36 \\ 36 \\ 25 \\ 16 \\ \hline 114 \end{array}$$