1. Determine whether or not the Language $L = \{a^n b^i c^j d^k, n+k \le i+j\}$ is context-free.

Ans) Assume that L is CFL. Then there exists a pumping length p for L.

Let's consider the string $w = a^p b^p c^p d^p$.

Then, i) w is in L because $p+p \le p+p$.

ii) According to pumping lemma, w can be decomposed into $uvxyz$, where

$uv^i x y^i z$ $(\because i \ge 0) \in L$, $|vy| \ge 1$, $|vxy| \le p$.

Because $|vxy| \le p$, vxy can only contain symbols from at most two blocks among the a-block, b-block, c-block, d-block.

This means vxy can not contain all four types symbols a, b, c, and d.

i) only affect one type of symbol, which can break the condition $n+k \le i+j$.

ii) only affect symbols in adjacent blocks, which can break the condition $n+k \le i+j$.

Then, we will consider three cases considering above conditions.

1) vxy only contains a's and b's.

→ pumping up will increase n without changing k, i and j.

It indicates that it violates the condition $n+k \le i+j$.

2) vxy only contains b's and c's

→ pumping up will increase i and j, i or j without changing n, k.

It indicates that it will not be possible to maintain the condition $n+k \le i+j$.

3) vxy only contains c's and d's

→ pumping up will be increase k without changing n, i, and j.

It indicates that it violates the condition $n+k \le i+j$.

In all cases, pumping v and y will result in a string that does not satisfy $n+k \le i+j$.

Therefore, the Language $L = \{a^n b^i c^j d^k, n+k \le i+j\}$ is NOT context-free.

2. Construct Turing machines that will accept the following languages on $\{a, b\}$:

$$L = \{w : n_a(w) \neq n_b(w)\}.$$

[Ans] Assume a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, F)$, where $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$, $F = \{q_5\}$, and transition function δ as follows:

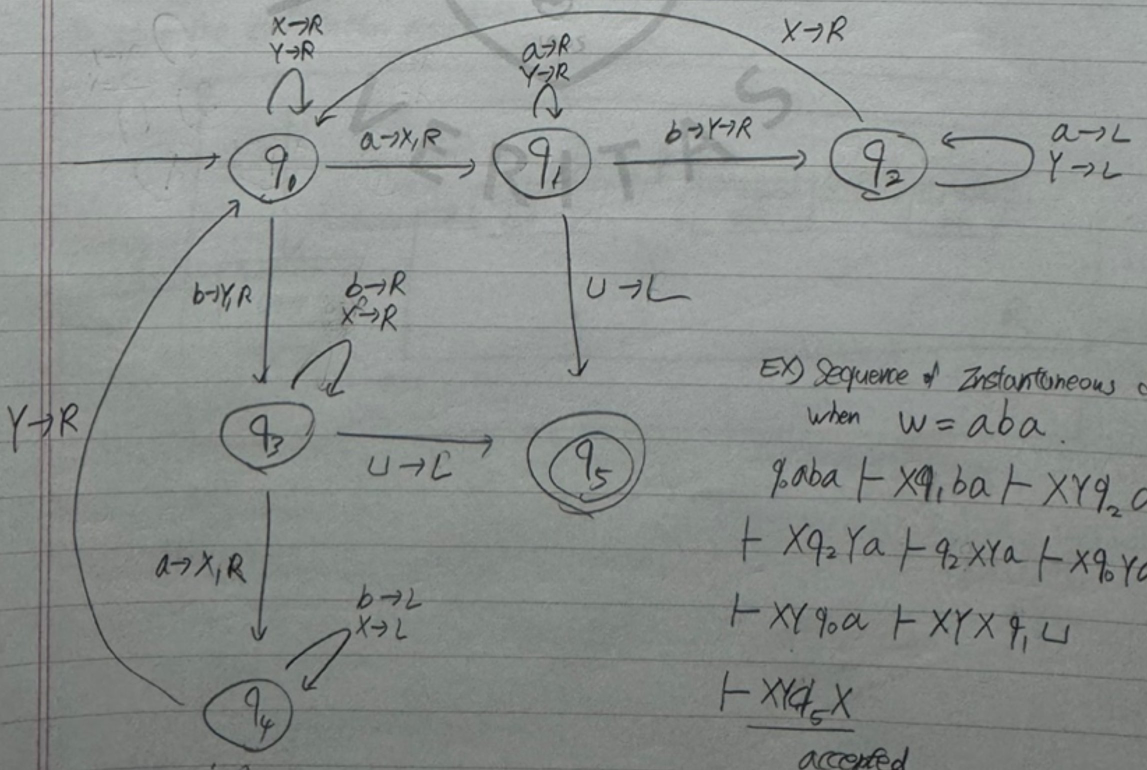
$\delta(q_0, a) = (q_1, X, R)$
$\delta(q_0, b) = (q_3, Y, R)$
$\delta(q_0, X) = (q_0, X, R)$
$\delta(q_0, Y) = (q_0, Y, R)$

$\delta(q_1, a) = (q_1, a, R)$
$\delta(q_1, b) = (q_2, Y, L)$
$\delta(q_1, Y) = (q_1, Y, R)$
$\delta(q_1, \sqcup) = (q_5, \sqcup, L)$

$\delta(q_2, a) = (q_2, a, L)$
$\delta(q_2, X) = (q_0, X, R)$
$\delta(q_2, Y) = (q_2, Y, L)$

$\delta(q_3, a) = (q_4, X, R)$
$\delta(q_3, b) = (q_3, b, R)$
$\delta(q_3, X) = (q_3, X, R)$
$\delta(q_3, \sqcup) = (q_5, \sqcup, L)$

$\delta(q_4, b) = (q_4, b, L)$
$\delta(q_4, X) = (q_0, X, L)$
$\delta(q_4, Y) = (q_0, Y, R)$



EX) Sequence of Instantaneous descriptions when $w = aba$.

$q_0aba \vdash Xq_1ba \vdash XYq_2a$
$\vdash Xq_2Ya \vdash q_2XYa \vdash Xq_0Ya$
$\vdash XYq_0a \vdash XYXq_1\sqcup$
$\vdash XYq_5X$

accepted

3. Using adders, subtracters, comparers, copiers, or multipliers, draw block diagram for Turing machines that compute the following functions for all positive integer n:

$$f(n) = n!$$

Ans Before drawing Block diagram for TM that computes $f(n) = n!$ $(\because n \geq 1)$, we need to consider the properties on $n!$

$$n! = n(n-1)(n-2)\cdots 1$$

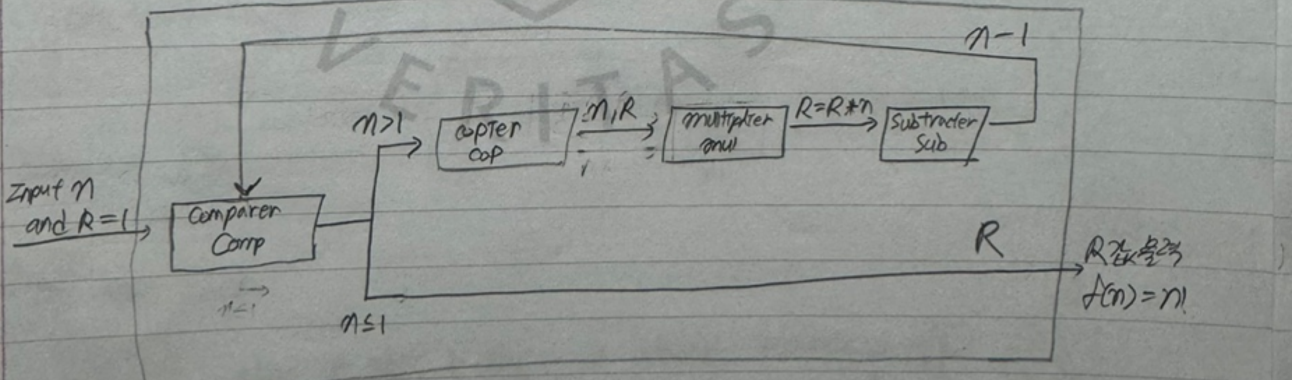
$$-1 \qquad -1$$

we will use a comparer to check if the value to be used in next operation is greater than or equal to 1. If the value is equal to 1, we need to move to the termination branch.

In the above comparison, if the value is greater than 1, copy the value, multiply it by the result value, subtract 1, and then pass the value back to the condition branch.

The subtracter, copier, and multiplier will be used in this process.

In each step of the process, we will also use an additional result variable R to store the calculation results.



4. Give a formal definition of a two-tape TM; then write programs that accept the languages below. Assume that : $\Sigma = \{a, b, c\}$ and that the input is initially all on tape 1.

$$L = \{a^n b^n c^m,\ m > n\}.$$

[Ans] Two-tape Turing machine can be defined as follow:

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, \sqcup, F\}$$

Q is finite, non-empty set of states.

Σ is the finite input alphabet not containing $\sqcup$.

Γ is the finite tape alphabet containing Σ.

$\delta : Q \times \Gamma^2 \rightarrow Q \times \Gamma^2 \times \{L, R\}^2$

q_0 : the start state

$\sqcup$: Blank symbol

F : $F \in Q$ is the set of accepting states.

formal definition

Let a two-tape TM $M = \{Q, \Sigma, \Gamma, \delta, q_0, \sqcup, F\}$, where

$Q = \{q_0, q_1, q_2, q_3, q_4\}$, $\{q_4\} \in F$, And δ as follows:

$\delta(q_0, (a, \sqcup)) = (q_1, (a, a), (R, R))$

$\delta(q_1, (a, \sqcup)) = (q_1, (a, a), (R, R))$

$\delta(q_1, (b, \sqcup)) = (q_2, (b, K), (R, L))$

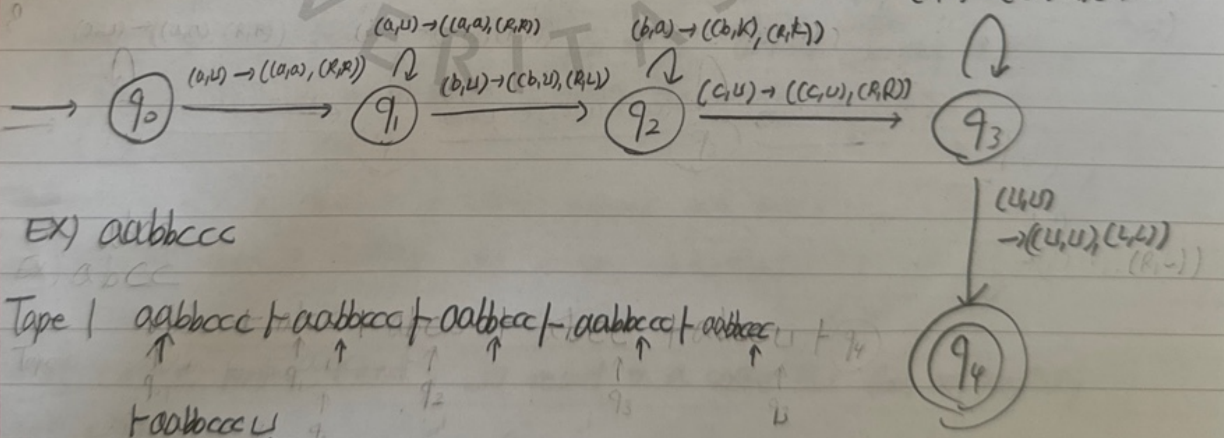
$\delta(q_2, (b, a)) = (q_2, (b, K), (R, L))$

$\delta(q_2, (c, \sqcup)) = (q_3, (c, \sqcup), (R, R))$

$\delta(q_3, (c, \sqcup)) = (q_3, (c, \sqcup), (R, R))$

$\delta(q_3, (c, K)) = (q_3, (c, \sqcup), (R, R))$

$\delta(q_3, (\sqcup, \sqcup)) = (q_4, (\sqcup, \sqcup), (L, L))$



EX) aabbccc

Tape 1 aabbccc ⊢ aabbccc ⊢ aabbccc ⊢ aabbccc ⊢ aabbccc

⊢ aabbccc⊔

Tape 2 ⊔⊔ ⊢ ⊔a⊔ ⊢ ⊔aK ⊢ ⊔KK ⊢ ⊔KK

⊢ ⊔⊔K ⊢ ⊔⊔⊔⊔