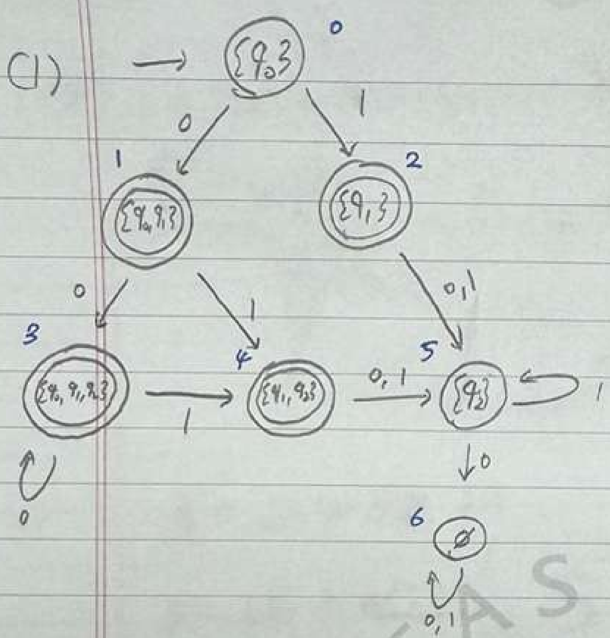
(1)

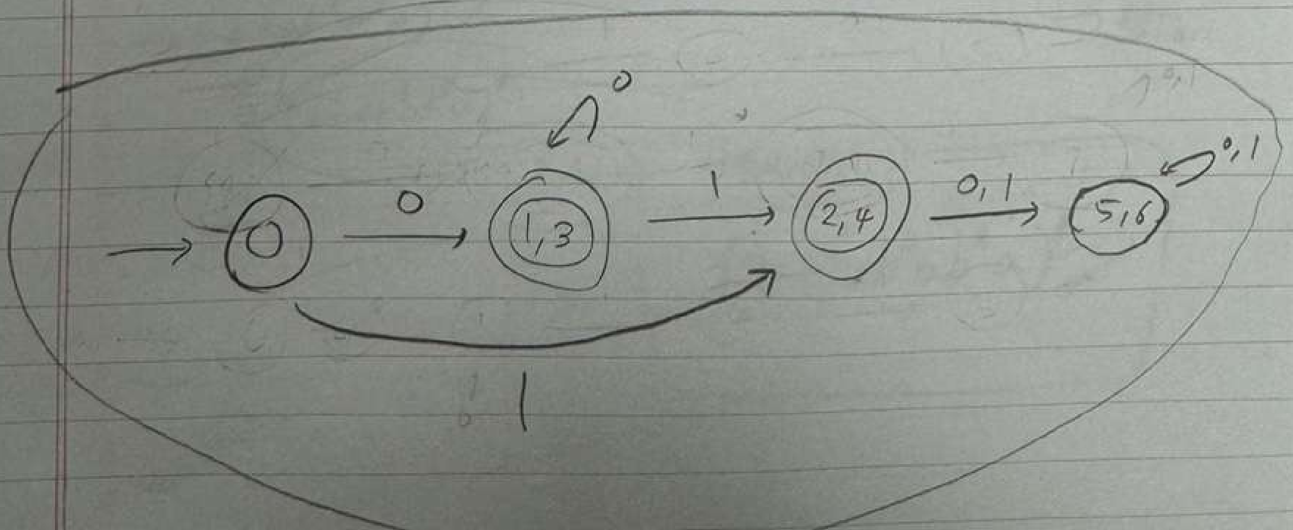


1	X					
2	X	X				
3	X	?	X			
4	X	X	?	X		
5	X	X	X	X	X	
6	X	X	X	X	X	?
	0	1	2	3	4	5

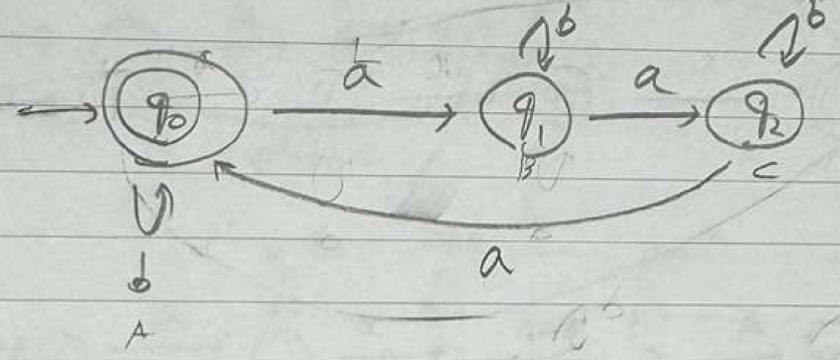
$(1,3) \xrightarrow{0} (3,3)$
$\quad\ \xrightarrow{1} (4,4)$ Indistinguishable

$(2,4) \xrightarrow{0} (5,5)$
$\quad\ \xrightarrow{1} (5,5)$ Indistinguishable

$(5,6) \xrightarrow{0} (6,6)$
$\quad\ \xrightarrow{1} (5,6)$ Indistinguishable



(2) $L = \{w : n_a(w) \bmod 3 = 0\}$. / $\Sigma = \{a, b\}$



$$A = bA + aB + \lambda$$

$$B = bB + aC$$

$$C = bC + aA$$

$C = b^*(aA)$ → Arden's rule

$B = bB + ab^*(aA)$

$= b^*(ab^*(aA))$ → Arden's rule

$A = bA + ab^*(ab^*(aA))) + \lambda$

$= bA + ab^*ab^*aA + \lambda$

$= (b + ab^*ab^*a)A + \lambda$

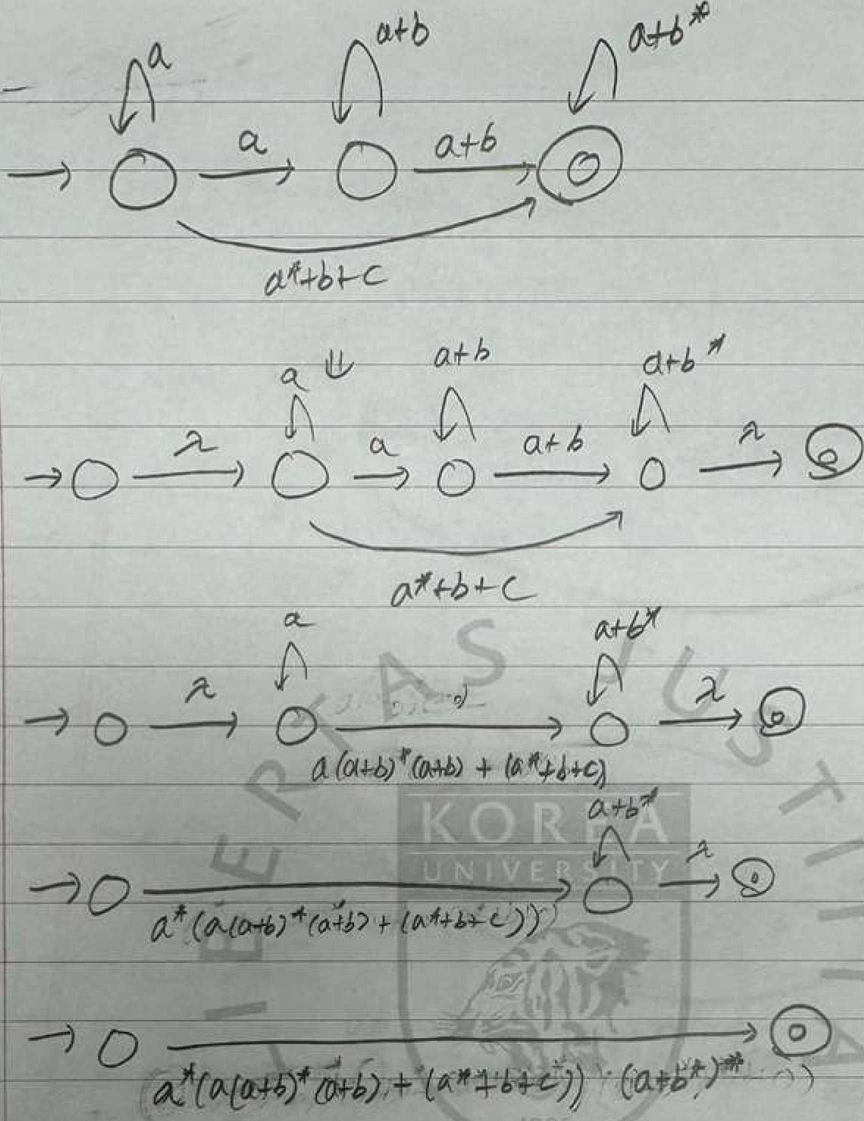
$\rightarrow (b + ab^*ab^*a)^*\lambda$

$= (b + ab^*ab^*a)^*$

Answer

$(b + ab^*ab^*a)^*$

(3)

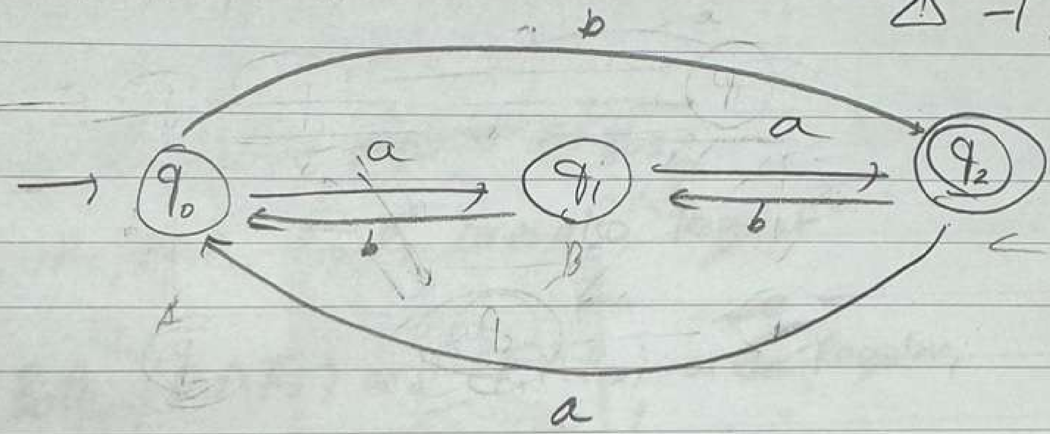


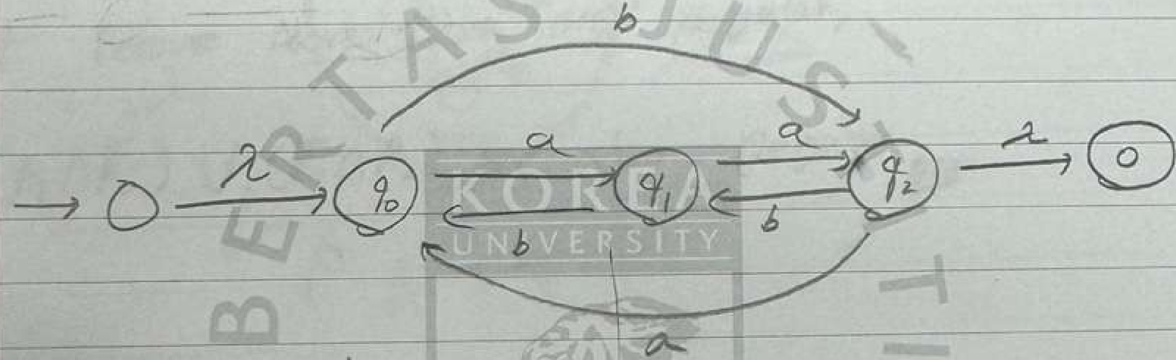
$$\boxed{r = a^*(a(a+b)^*(a+b) + (a^*+b+c))(a+b^*)^*}$$ //

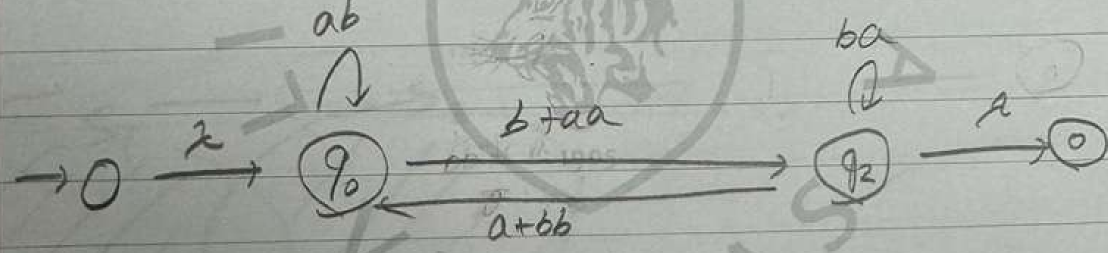
(4) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 2\}$, $\Sigma = \{a, b\}$

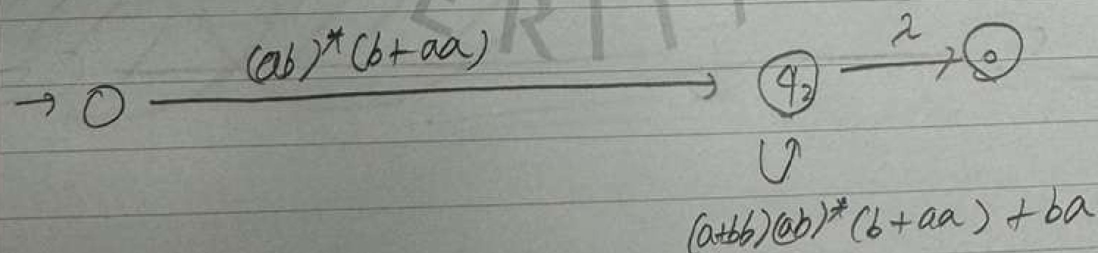
$\rightarrow \{aa, aaaabb, aaaaaabbbb, \dots\}$

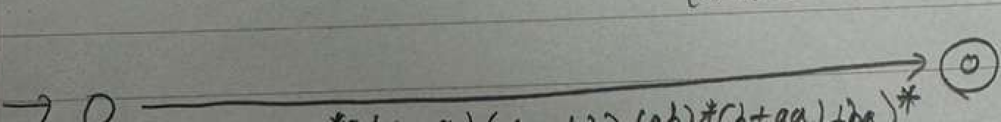
⚠ $-1 \% 3 = 2$











$r = (ab)^*(b+aa)((a+bb)(ab)^*(b+aa)+ba)^*$

(5) $S_1 \ominus S_2 = \{x : x \in S_1 \text{ or } x \in S_2, \text{ but } x \text{ is not in both } S_1 \text{ and } S_2\}$

let's suppose R_1, R_2 are regular.

Then, we will show $R_1 \ominus R_2$ is also regular.

$R_1 \ominus R_2 = (R_1 \cap \overline{R_2}) \cup (\overline{R_1} \cap R_2)$ → By definition of '$\ominus$'

If R_1 and R_2 are regular,

Because of closure properties of RLs,

$R_1 \cup R_2$, $R_1 \cap R_2$, $\overline{R_1}$, $\overline{R_2}$ are also Regular.

So, Both $(R_1 \cap \overline{R_2})$ and $(\overline{R_1} \cap R_2)$ are Regular,

and Because Above these are Regular,

$(R_1 \cap \overline{R_2}) \cup (\overline{R_1} \cap R_2)$ are also Regular ∎

+)