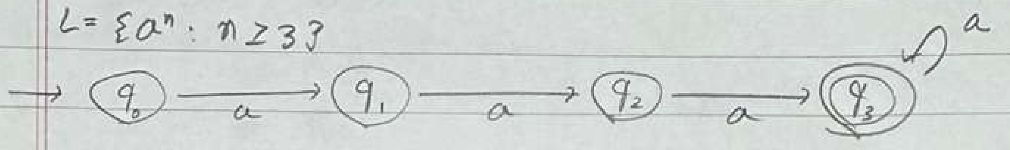
③ To show L is regular, we must show that L has a DFA for language L.

$L=\{a^n : n \geq 3\}$

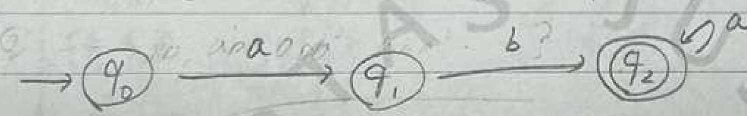


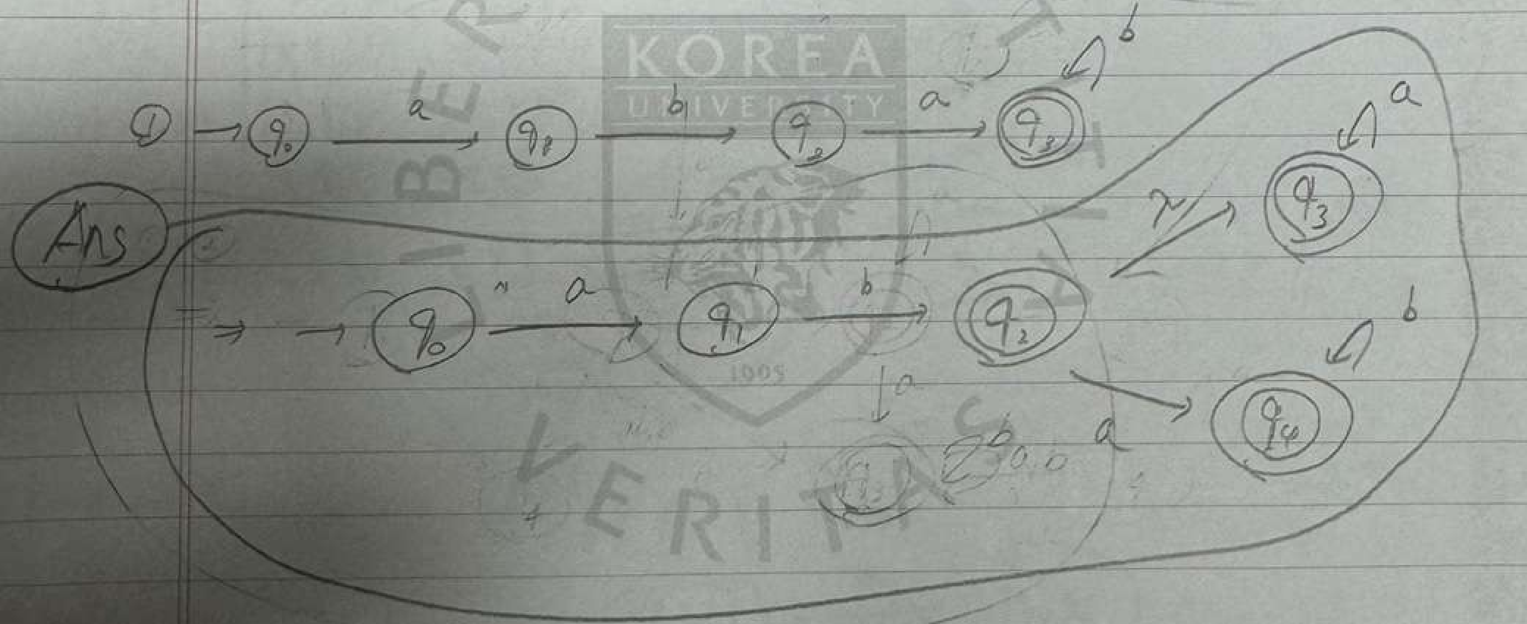
▲ DFA for L. // $L(M)$

(e) $M=(\{q_0, q_1, q_2, q_3\}, \{a\}, \delta, q_0, \{q_3\})$

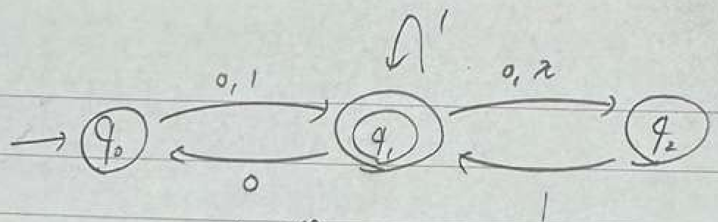
④ nfa// $\underbrace{\{abab^n : n \geq 0\}}_{①} \cup \underbrace{\{aba^n : n \geq 0\}}_{②}$

① ∪ ② = {ab, aba, aba, abab, abaa, ababb, ... }

② 

① 

⑤



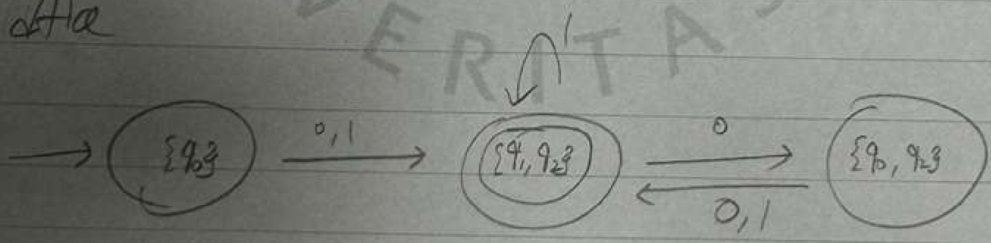
▲ nfa

$q_0' = E(\{q_0\}) = \{q_0\}$ // $M_D = (Q_D, \Sigma_D, \delta_D, \{q_0\}, F_D)$

○: Accept

2^Q \ Σ	0	1
$\emptyset$	$\emptyset$	$\emptyset$
$\{q_0\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_0, q_2\}$	$\{q_1, q_2\}$
$\{q_2\}$	$\emptyset$	$\{q_1, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$

⇒ dfa



$M_D = (\{\{q_0\}, \{q_1, q_2\}, \{q_0, q_2\}\}, \{0, 1\}, \delta_D, \{q_0\}, \{q_1, q_2\})$??

① $\overbrace{S_1 = S_2}^{p} \leftrightarrow \overbrace{(S_1 \wedge \overline{S_2}) \cup (\overline{S_1} \wedge S_2) = \emptyset}^{q}$

Proof

If $S_1 = S_2$, then $(S_1 \wedge \overline{S_2}) \cup (\overline{S_1} \wedge S_2) = \emptyset$ can be written,

$(S_1 \wedge \overline{S_1}) \cup (\overline{S_1} \wedge S_1)$

$S_1 \wedge \overline{S_1} = \emptyset$, $\overline{S_1} \wedge S_1 = \emptyset$

So, $(S_1 \wedge \overline{S_1}) \cup (\overline{S_1} \wedge S_1) = \emptyset$

We show $p \rightarrow q$ is true.

Above, we show $p \rightarrow q$ is True.

Then, we will show $q \rightarrow p$ is also True.

If, $(S_1 \cap \overline{S_2}) \cup (\overline{S_1} \cap S_2) \doteq \emptyset$

$(S_1 \cap \overline{S_2})$, $(\overline{S_1} \cap S_2)$ must be $\emptyset$.

$(S_1 \cap \overline{S_2}) = \emptyset$ means $S_1 = S_2$, and $(\overline{S_1} \cap S_2) = \emptyset$ means also $S_1 = S_2$.

$q \rightarrow p$

Because $p \rightarrow q$, $q \rightarrow p$ are True, $S_1 = S_2$ iff $(S_1 \cap \overline{S_2}) \cup (\overline{S_1} \cap S_2) = \emptyset$ is True.

② $\Sigma = \{a, b\}$ / $L = \{w \ (n_a(w)^{2} - n_b(w)) \bmod 3 \equiv 0\}$ / aaa aaaaaabbb ~~aaaaaa~~ b

P: $S \rightarrow aS$

$S \rightarrow bS$

$S \rightarrow \lambda$

$q_0 \Rightarrow (n_a(w) - n_b(w)) \bmod 3 = 0$

$q_1 \Rightarrow$ // // $= 1$

$q_2 \Rightarrow$ // // $= 2$

Ans