DELHI TECHNOLOGICAL UNIVERSITY

OPERATIONS RESEARCH PROJECT REPORT

Application of Queuing Theory in Automobile Assembly Line Problem

SUBMITTED BY:

KUMAR APURVA (DTU/2K18/MC/058)

NILAYA AGARWAL (DTU/2K18/MC/072)

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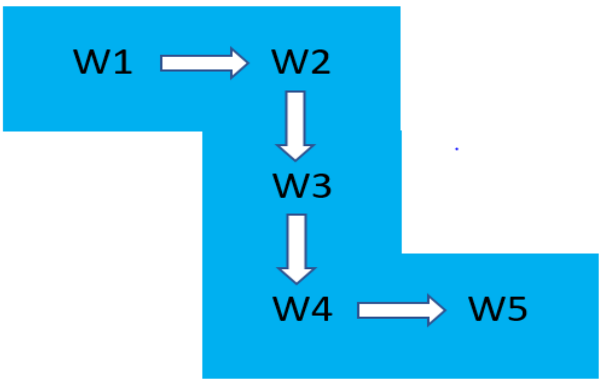
INTRODUCTION

Waiting is a phenomenon found in everyday life like in post offices, banks and filling stations. The waiting phenomenon is not an experience limited to human beings only; jobs wait to be processed on a machine and cars stop at traffic lights. In situations where facilities are limited and cannot satisfy the demand made upon them, bottlenecks occur which manifest as queue. Queuing theory refers to the mathematical study of the formation, function, and congestion of waiting lines, or queues.

An assembly line is a manufacturing process in which parts are added to a product in a sequential manner using optimally planned logistics to create a finished product in the fastest possible way. It is a flow-oriented production system where the productive units performing the operations, referred to as workstations, are aligned in a serial manner. Assembly lines are mostly designed for a sequential organization of workers, tools or machines and parts. The work pieces visit stations successively as they are moved along the line usually by some kind of transportation system, e.g. a conveyor belt. Assembly lines can be classified into single-model, batch- model and mixed-model lines. In a batch-model assembly system, a few product models are produced in batches, but one product at a time, on the same line and a change over time is allotted to make the line ready for production of another model. A procedure is needed to determine a particular configuration for the products to be produced on the line which will not only minimize the balance delay or number of workstations but also satisfy the other conflicting criteria like production rate, variety, minimum distance moved, division of labour and quality.

The main purpose of applying queuing theory in assembly plant is to model the assembly process in manufacturing plant by using an appropriate analytical model of queuing theory in order to increase the efficiency of each workstation and the overall production system.



An Assembly Line with 5 Workstations

BACKGROUND KNOWLEDGE

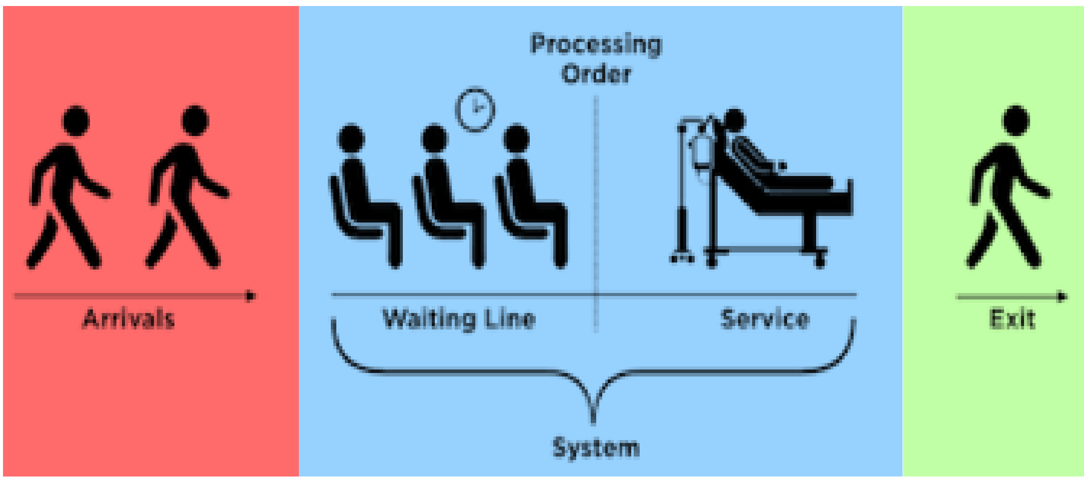
BASICS OF QUEUING THEORY

Queuing theory refers to the mathematical study of the formation, function, and congestion of waiting lines, or queues.

At its core, a queuing situation involves two parts.

1. Someone or something that requests a service—usually referred to as the customer, job, or request.
2. Someone or something that completes or delivers the services—usually referred to as the server.

Queuing theory examines every component of waiting in line to be served, including the arrival process, service process, number of servers, number of system places, and the number of customers—which might be people, data packets, cars, etc. As a branch of operations research, queuing theory can help users make informed business decisions on how to build efficient and cost effective workflow systems. Queues happen when resources are limited. In fact, queues make economic sense; no queues would equate to costly overcapacity. Queuing theory helps in the design of balanced systems that serve customers quickly and efficiently but do not cost too much to be sustainable. All queuing systems are broken down into the entities queuing for an activity. At its most elementary level, queuing theory involves the analysis of arrivals at a facility, such as a bank or fast food restaurant, then the service requirements of that facility, e.g., tellers or attendants.



A Queuing Process

Notation and Formulas for Queuing theory

Queuing theory uses the Kendall notation to classify the different types of queuing systems, or nodes. Queuing nodes are classified using the notation *A/S/c/K/N/D* where:

- **A** is the arrival process
- ***S*** is the mathematical distribution of the service time
- ***c*** is the number of servers
- ***K*** is the capacity of the queue, omitted if unlimited
- ***N*** is the number of possible customers, omitted if unlimited
- ***D*** is the queuing discipline, assumed first-in-first-out if omitted

Some important parameters in Queuing Theory are :

• Arrival rate in the system (**λ**)

• Service rate of the servers (**μ**)

• Expected Number of Busy Servers / Server Utilization (**ρ**) = **λ/μ**

• Expected Number of Customers in the System (**Ls**)

• Expected Number of Customers waiting in the Queue (**Lq**)

• Expected Waiting Time in the System (**Ws**)

• Expected Waiting Time in the Queue (**Wq**)

Little's Law states that the long-term average number of customers in a stable system L is equal to the long-term average effective arrival rate, λ, multiplied by the average time a customer spends in the system,W.

$$Ls = \lambda Ws$$

$$Lq = \lambda Wq$$

Single Server Queuing Model (M/M/1)

In queueing theory, a M/M/1 queuing model represents a system having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution.

Probability of having zero customers in the system (**P0**) = 1 – ρ

Expected Number of Customers in the System (**Ls**) = ρ/(1 – ρ)

Expected Number of Customers waiting in the Queue (**Lq**) = Ls - ρ

Expected Waiting Time in the System (**Ws**) = Ls/ λ

Expected Waiting Time in the Queue (**Wq**) = Lq/ λ

Expected Number of Busy Servers in the system ($\bar{c}$) = ρ

Multiple Server Queuing Model (M/M/c)

In queueing theory, a discipline within the mathematical theory of probability, the M/M/c queue (or Erlang–C model) is a multi-server queueing model.

Probability of having zero customers in the system (**P0**)

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!(1-\frac{\rho}{c})}}$$

Expected Number of Customers waiting in the Queue (**Lq**)

$$Lq = \frac{\rho}{(c-1)!(c-\rho)^2} P_0$$

Expected Number of Customers in the System (**Ls**) = Lq + ρ

Expected Waiting Time in the System (**Ws**) = Ls/ λ

Expected Waiting Time in the Queue (**Wq**) = Lq/ λ

Expected Number of Busy Servers in the system ($\bar{c}$) = ρ/c

CASE STUDY

The main assembly line problem is the queuing among stations during task achievement which is an obstacle to an effective and efficient assembly line.

The aim of this project is to carry out queuing analysis to examine an automobile assembly line performance to reduce queuing through harmonizing the tasks in each workstation. There are total 5 workstations in the automobile assembly line which is shown in the table below.

Workstation	Description of operation
1	Chassis assembly, assembly of suspension legs
2	Chassis painting
3	Drying station
4	Programming, filling of fuel, vehicle inspection and start up
5	Vehicle final quality inspection

The Kendall's notation for the queuing problem is M/M/C: FCFS/∞/∞. It is a multi-channel multi – server service with infinite system capacity and an infinite number of calling population.

The arriving and service distribution data for the system were determined. These data were employed to estimate the performance parameter of the system. The Analysis of the model was done on the basis of parameters like Utilization Factor, Average Length of the Queue, Average Waiting Time in the Queue and the Loss of Income due to Customers Waiting in the Queue.

The results obtained from the analysis were used to predict the efficiency and effectiveness of the system and make logical recommendations on how to improve the system.

A simple solution suggested was to hire additional workers in the workstations. The worker is allotted to that workstation where he can reduce the lost income as higher as possible. Based on this rule, as many workers were hired on the condition that they do not further increase the loss of income.

After the hiring, the performance parameters were again measured and Comparative Analysis was done with the old parameters.

Based on the results obtained, it can be concluded that if the allotment of workers is done strategically, the waiting no. of customers will be reduced thereby reducing the cost of waiting.

DATA

There are 5 workstations in a company. Customers arrive at a different rate at each station. Each station has its service rate and the no. of workers. The information is summarized in the given table.

Workstation	Arrival rate (per hour) (λ)	Service rate (per hour) (μ)	No. of workers (c)
1	20	18	2
2	27	20	3
3	12	9	2
4	14	11	2
5	17	10	3

APPROACH & IMPLEMENTATION WITH MATLAB CODE

M/M/1: ∞/∞/FCFS Queuing model Code

```
function[Ls, Lq, Ws, Wq, Workstation_Utilization] =
MM1_model(lamda, mu)
    rho = lamda/mu;
    p0 = 1 - rho;
    Ls = rho/(1 - rho);
    Lq = Ls - rho;
    Ws = Ls/lamda;
    Wq = Lq/lamda;
    Workstation_Utilization = rho;
```

M/M/c: ∞/∞/FCFS Queuing model Code

```
function[Ls, Lq, Ws, Wq, Workstation_Utilization] =
MMc_model(lamda, mu, c)
    rho = lamda/mu;
    val = 0;
    for i = 0:c-1
        val = val + (rho.^i)/factorial(i);
    end
    val = val + (rho.^c)/(factorial(c)*(1 - (rho/c)));
    p0 = 1./val;
    Lq = ((rho.^(c+1))*p0)/(factorial(c-1)*((c-rho).^2));
    Ls = Lq + rho;
    Ws = Ls/lamda;
    Wq = Lq/lamda;
    Workstation_Utilization = rho/c;
```

Command Window

```
>> [Ls, Lq, Ws, Wq, Workstation_Utilization] = MMc_model(20, 18, 2)
Ls =
    1.6071
Lq =
    0.4960
Ws =
    0.0804
Wq =
    0.0248
Workstation_Utilization =
    0.5556
```

Command Window

```
>> [Ls, Lq, Ws, Wq, Workstation_Utilization] = MMc_model(27, 20, 3)
Ls =
    1.5022
Lq =
    0.1522
Ws =
    0.0556
Wq =
    0.0056
Workstation_Utilization =
    0.4500
```

Command Window

```
>> [Ls, Lq, Ws, Wq, Workstation_Utilization] = MMc_model(20, 18, 3)
Ls =
    1.1802
Lq =
    0.0691
Ws =
    0.0590
Wq =
    0.0035
Workstation_Utilization =
    0.3704
```

Similarly, we computed the queue components of other workstations and summarized it in given table :

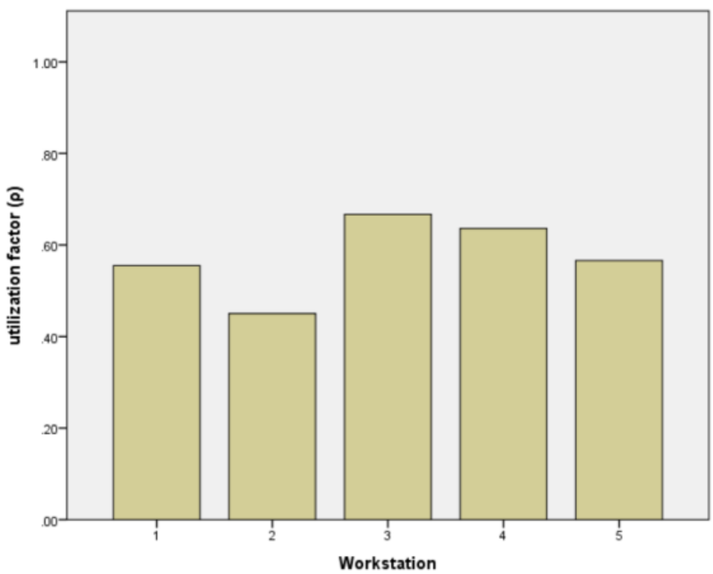
workstation	Arrival rate (per hr) (λ)	Service rate (per hr) (μ)	No. of workers (c)	Workstation utilization (λ /cμ)	Length in system (L_s)	Length in queue (L_q)	Waiting time in system (per hr) (W_s)	Waiting time in queue (per hr) (W_q)
1	20	18	2	0.555	1.607	0.496	0.080	0.024
2	27	20	3	0.45	1.502	0.152	0.055	0.005
3	12	9	2	0.667	2.4	1.066	0.2	0.088
4	14	11	2	0.636	2.138	0.866	0.152	0.061
5	17	10	3	0.566	2.109	0.409	0.124	0.024

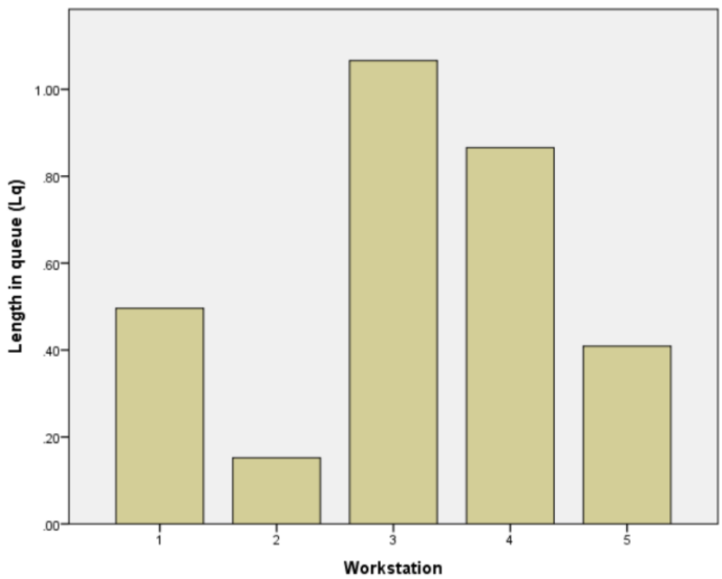
To address the concern of loss of income, the company uses the factor of delayed job (which is indicated by the L_q component).

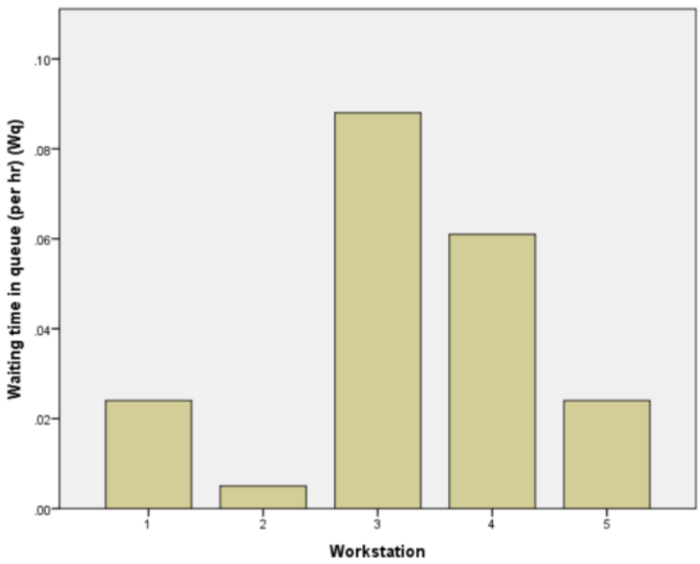
“A delayed job represents lost income” which is estimated by the company in the given table (each station has a different rate of loss of income)

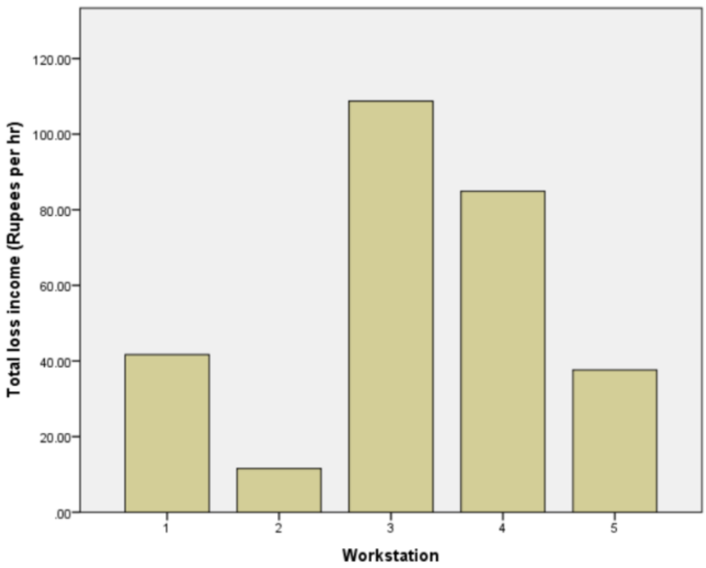
workstation	Rate of loss income (Rupees per waiting job per hr)	No. of waiting jobs (L_q)	Total loss income (Rupees per hr)
1	84	0.496	41.664
2	76	0.152	11.552
3	102	1.066	108.732
4	98	0.866	84.868
5	92	0.409	37.628

<u>ANALYSIS</u>









We conclude from the above graphs that **the workstation 3 is the most busy** among all the stations. This can be inferred from the highest utilization factor, length in queue, waiting time in queue and the loss of income due to customers waiting in the queue. Whereas, **the station 2 is the most idle station**.
We also observe that **a total of 284.444 Rs is lost per hour** due to customers waiting in the queue.

SOLUTION

To reduce the lost income due to the waiting job, the company plans on hiring additional workers. The worker is allotted to that workstation where he can reduce the lost income as higher as possible. The **hiring cost of a worker is 800 Rs per day.**

Eg :- initially workstation 1 has two workers (MM2 model). Let suppose, the new worker is allotted there, so the model becomes MM3 with reduced L_q . hence, the lost income will be reduced.

Therefore, we calculate the reduction of lost income in each case, and summarize the results in the following table:

1) Allotment location of the 1st worker

workstation	Final no. of workers	Lq_{Final}	New loss of income (Rupees per hr)	Initial loss of income (Rupees per hr)	Money saved (Rupees per hr)	Money saved (Rupees per day)
1	3	0.069	5.796	41.664	35.868	860.832
2	4	0.027	2.052	11.552	9.5	228
3	3	0.144	11.628	108.732	97.104	2330.496
4	3	0.119	11.662	84.868	73.206	1756.944
5	4	0.08	7.360	37.628	30.268	726.432

We observe that if the **worker is allotted to workstation 3**, an amount of 2330.496 Rs will be saved at the expense of 800 Rs. (Which is the maximum possible)

Hence **profit will be of 1530.496 Rs per day.**

Therefore **we allot the 1st worker to workstation 3** and update the initial loss of income in it.

We again apply this process for 2nd worker.

2) Allotment location of the 2nd worker

workstation	Final no. of workers	Lq $_{Final}$	New loss of income (Rupees per hr)	Initial loss of income (Rupees per hr)	Money saved (Rupees per hr)	Money saved (Rupees per day)
1	3	0.069	5.796	41.664	35.868	860.832
2	4	0.027	2.052	11.552	9.5	228
3	4	0.025	2.55	11.628	9.078	217.872
4	3	0.119	11.662	84.868	73.206	1756.944
5	4	0.08	7.360	37.628	30.268	726.432

We observe that if the **worker is allotted to workstation 4**, an amount of 1756.944 Rs will be saved at the expense of 800 Rs. (Which is the maximum possible)

Hence **profit will be of 956.944 Rs per day.**

Therefore **we allot the 2nd worker to workstation 4** and update the initial loss of income in it.

We continue the process for 3rd worker.

3) Allotment location of the 3rd worker

workstation	Final no. of workers	Lq $_{Final}$	New loss of income (Rupees per hr)	Initial loss of income (Rupees per hr)	Money saved (Rupees per hr)	Money saved (Rupees per day)
1	3	0.069	5.796	41.664	35.868	860.832
2	4	0.027	2.052	11.552	9.5	228
3	4	0.025	2.55	11.628	9.078	217.872
4	4	0.02	1.96	11.662	9.666	231.984
5	4	0.08	7.360	37.628	30.268	726.432

We observe that if the **worker is allotted to workstation 1**, an amount of 860.832 Rs will be saved at the expense of 800 Rs. (Which is the maximum possible)

Hence **profit will be of 30.832 Rs per day.**

Therefore **we allot the 3rd worker to workstation 1** and update the initial loss of income in it.

We continue the process for 4th worker.

4) Allotment location of the 4th worker

workstation	Final no. of workers	$L_{q\ Final}$	New loss of income (Rupees per hr)	Initial loss of income (Rupees per hr)	Money saved (Rupees per hr)	Money saved (Rupees per day)
1	4	0.011	0.924	5.796	4.872	116.928
2	4	0.027	2.052	11.552	9.5	228
3	4	0.025	2.55	11.628	9.078	217.872
4	4	0.02	1.96	11.662	9.666	231.984
5	4	0.08	7.360	37.628	30.268	726.432

We observe that if we allot a 4th worker to any of the station, the money saved would not catch up with the hiring cost of that worker. Hence, **addition of 4th worker will further increase the loss of company.**

Hence, we conclude that **addition of three workers at workstation 1,3 and 4 would maximize the profit of the company**.

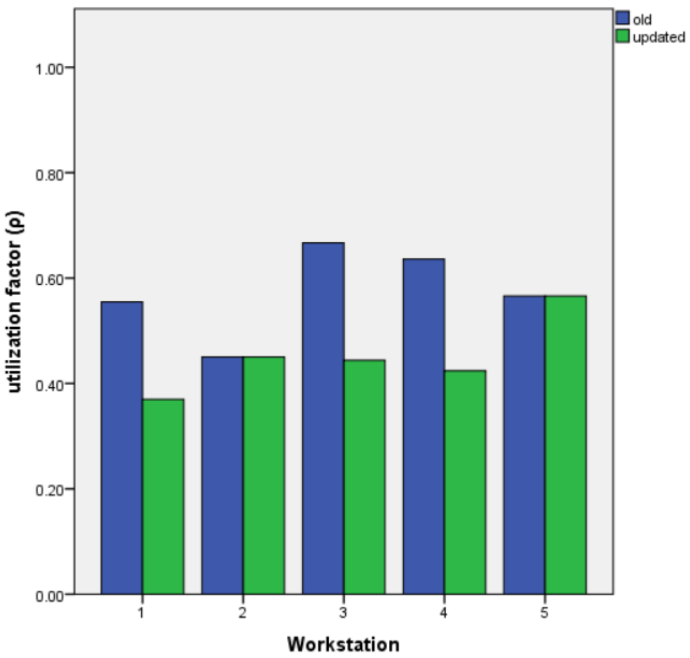
The **net profit will be** 1530.496 + 956.944 + 30.832 = **2518.272 Rs per day.**

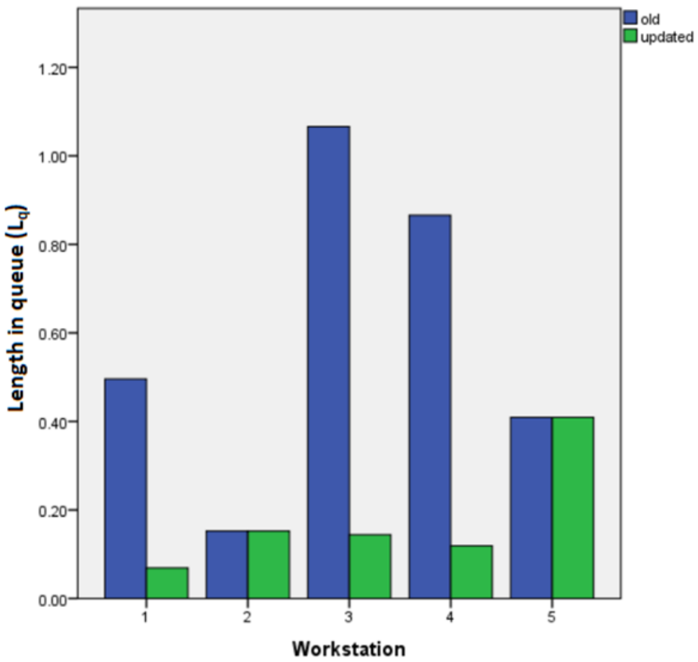
We **update all the queuing components** after the addition of 3 workers according to the above procedure and summarized it in the given table:

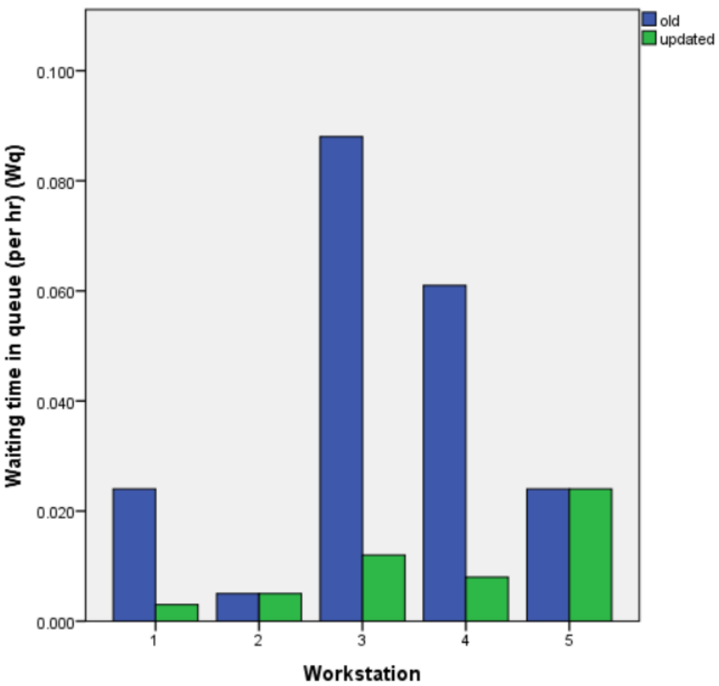
workstation	Arrival rate (per hr) (λ)	Service rate (per hr) (μ)	No. of workers (c)	Workstation utilization (λ /cμ)	Length in system (L_s)	Length in queue (L_q)	Waiting time in system (per hr) (W_s)	Waiting time in queue (per hr) (W_q)
1	20	18	3	0.37	1.18	0.069	0.059	0.003
2	27	20	3	0.45	1.502	0.152	0.055	0.005
3	12	9	3	0.444	1.477	0.144	0.123	0.012
4	14	11	3	0.424	1.392	0.119	0.099	0.008
5	17	10	3	0.566	2.109	0.409	0.124	0.024

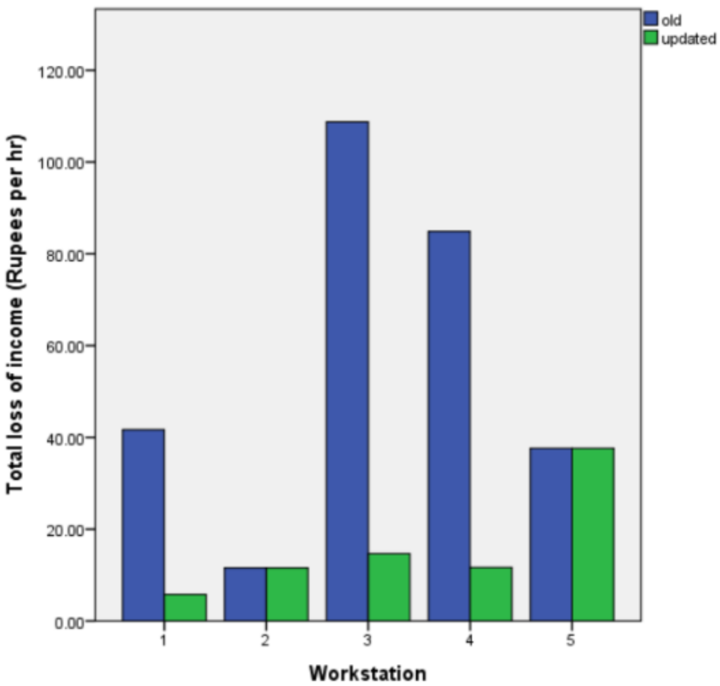
workstation	Rate of loss income (Rupees per waiting job per hr)	No. of waiting jobs (L_q)	Total loss income (Rupees per hr)
1	84	0.069	5.796
2	76	0.152	11.552
3	102	0.144	14.688
4	98	0.119	11.662
5	92	0.409	37.628

COMPARATIVE ANALYSIS









On comparing the queue elements of the workstations before and after hiring 3 extra workers, we observe that **where ever a worker is hired, the utilization factor, length in queue, waiting time in queue and the loss of income due to customers waiting in the queue, in that workstation has reduced drastically** which is a good thing for a company.

We also observe that **now a total of only 81.326 Rs is lost per hour** due to less customers waiting in the queue as compared to **284.444 Rs per hour before**.

Hence, **a profit of 203.118 Rs per hour (2518.272 Rs per day)** is made by the company.

CONCLUSION AND LEARNING

From the study, it was discovered that the problem of congestion in assembly line is not caused by only inadequate space but majorly by the operational managerial inefficiency. A lot of money and time is wasted because of customers waiting in the queue. Hence, increasing the number of workers and putting in mind the cost implications is recommended. The optimum number of workers should be selected for each station such that the overall production cost and the assembly time is minimized. This study is very valuable for the company, because by knowing all information related to the performance of its assembly line, it is more effective and easier for the management of company to plan their production in future.

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