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Q.1).

$$\left.\begin{aligned} 9x + 2y &= 0 \\ 2x + ay &= 0 \end{aligned}\right\} \text{given eq}^n$$

The value of a for which there will be whole lines of sol^n:

$$\frac{a}{2} = \frac{2}{a}$$

$$a^2 = 4$$

$$\boxed{a = \pm 2}$$ Ans

Q.2).

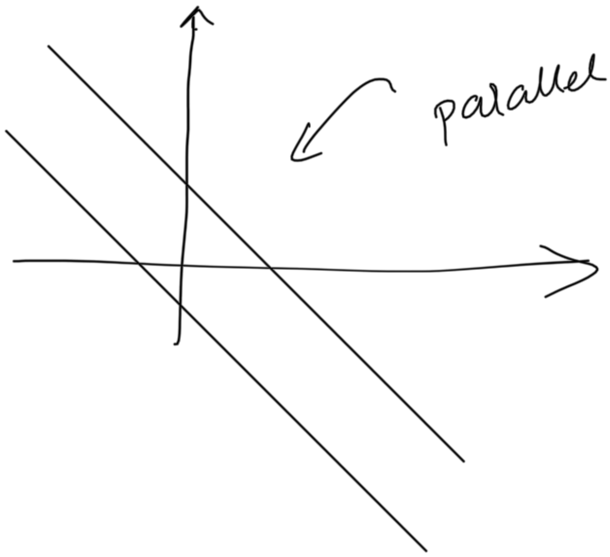
$$\left.\begin{aligned} 3x + 2y &= 10 \\ 6x + 4y &= a \end{aligned}\right\} \text{given eq}^n\text{'s.}$$

i) No sol^n:

Possible when both the lines are parallel

$$3x + 2y = 10 \quad - (1)$$

$$3x + 2y = \frac{a}{2} \quad - (2)$$



parallel lines

$\therefore$ Condition for no solⁿ

$$\frac{a}{2} \neq 10$$

$$\boxed{a \neq 20}$$

ii)

Infinitely Many solⁿ:

$$\frac{a}{2} = 10$$

$\boxed{a = 20}$ when both lines overlap.

Q.3).

$$A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \qquad A(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

To prove $\quad A(\theta_1)A(\theta_2) = A(\theta_1+\theta_2)$

$$A(\theta_1)A(\theta_2) = \begin{bmatrix} (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) & -(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2) \\ (\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2) & (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} = L.H.S$$

$$A(\theta_1+\theta_2) = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix}$$

$$L.H.S = R.H.S$$

$$A(\theta).A(-\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta + \cos^2\theta & \sin\theta\cos\theta - \cos\theta.\sin\theta \\ -\sin\theta\cos\theta + \sin\theta.\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Q.4).

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \begin{matrix} R_3' \to R_3 + 2R_1 \\ R_2' \to R_2 - 4R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{bmatrix} \quad R_3'' \to R_3' - 2R_2'$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix} = U$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2'} R_2 - 4R_1$$

$$E_{21}(-4) = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{31}(2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$E_{32}(-2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

These three matrices put A into upper triangular form U.

$$E_{32} E_{31} E_{21} A = U$$

$$A = LU$$

$$L^{-1} A = L^{-1} L U$$

$$L^{-1} A = I U$$

$$MA = U$$

$$\boxed{M = L^{-1}}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{M = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}}$$

$$L = E_{32}^{-1}(-2)\, E_{31}^{-1}(2)\, E_{21}^{-1}(4)$$

$$A = LU$$

$$\boxed{L = M^{-1}}$$

Q. 5):

on R^2

$$(x, y) + (x_1, y_1) = (x + x_1, 0) \quad — ①$$

$$c(x, y) = (cx, 0) \quad — ②$$

$\therefore \quad a(x, y) + b(x_1, y_1) = (ax + bx_1, 0)$

for any linear Combination of points in R^2

the Solution space is x axis which is on R.

The above set of operations maps from R^2 to R on x axis.

$\therefore$ The above set of operations are Vector Space from $R^2 \to R$.