ENPM673 HW1

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1 Problem 1

1.1 To calculate Field of View of the camera in the horizontal and vertical direction.

$$Focal\ length, f = 15mm \tag{1}$$

$$Width\ of\ camera\ sensor, d = 14mm \tag{2}$$

$$Field\ Of\ View = \tan^{-1}(\frac{d}{2f}) = \tan^{-1}(\frac{14}{2 \times 15}) = 25.02^\circ \tag{3}$$

1.2 Computing the minimum number of pixels that the object will occupy in the image.

$$Object\ width, w = 5cm \tag{4}$$

$$Distance\ of\ object\ from\ the\ camera = 20m \tag{5}$$

A 5 MP camera has a resolution of $3072 * 1728 = 5308416$ pixels.
So for a camera with square sensor, resolution = $\sqrt{5308416}$ = 2304*2304 pixels

$$Field\ of\ view\ occupied\ by\ one\ pixel = (\frac{50.04^\circ}{2304\ pixels}) = 0.02172^\circ \tag{6}$$

The figure 1 shows the object placed at 20m from the camera.We can calculate the angle formed by the object which should be equal to the angle formed between the reference line and image as both the angles are vertically opposite angles.

$$\theta = \frac{0.05}{20} = 0.1432^\circ \tag{7}$$

$$Number\ of\ pixels\ occupied\ by\ object\ in\ image = \frac{0.1432}{0.0217} \tag{8}$$

Minimum number of pixels occupied by object = 6*6 pixels

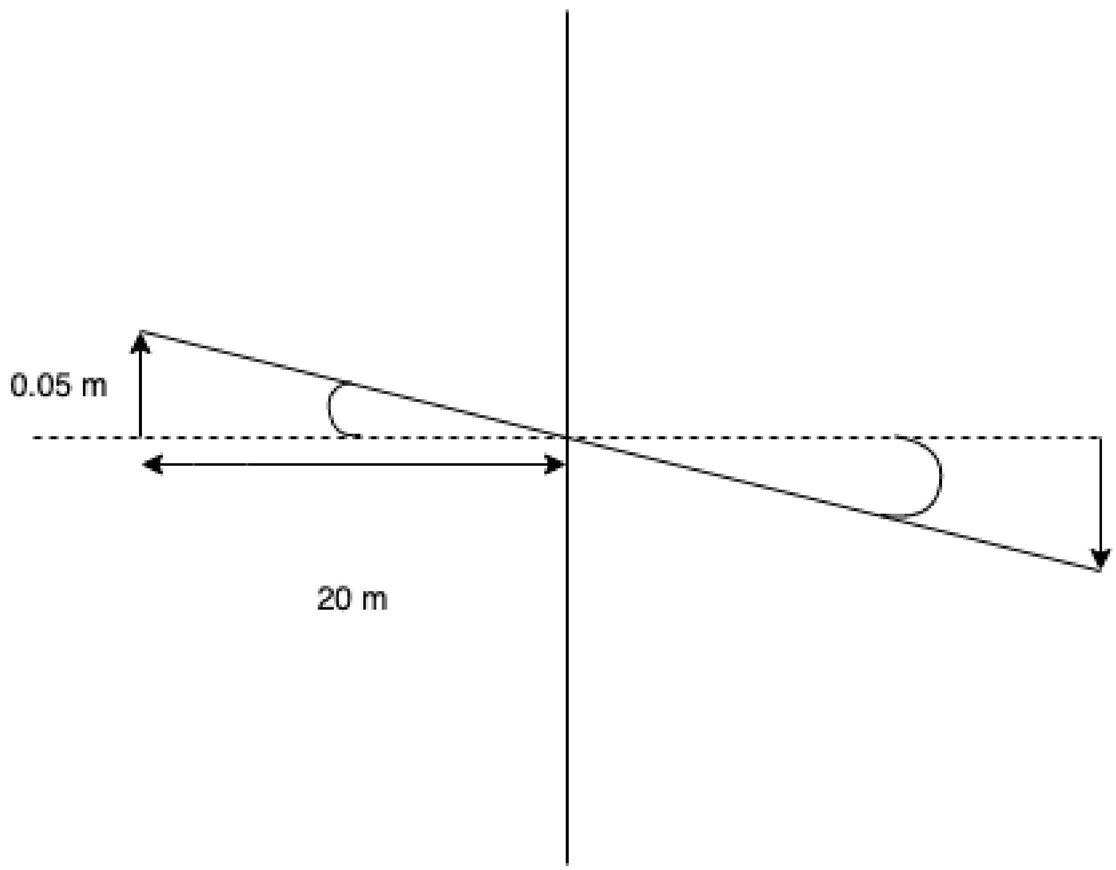

Figure 1: Ray Diagram

2 Problem 2

We are given a dataset of points in the cartesian cordinate system. We are required to fit this data to a parabola. There are four methods to fit a parabola from the given data set:

1. Least Squares Fitting Method (LS)
2. Total Least Squares Method (TLS)
3. TLS with Regularisation
4. Random Sample Consensus(RANSAC)

2.1 Least Squares Fitting Method

The method of least squares assumes that the best fit curve of a given type is the curve that minimizes a given error function. In this technique, it is assumed that there is no error in the x-coordintes. Hence, we define the error function as the deviation of the data point in the y-direction. For a parabola we define this error function as

$$E = \sum_{i=0}^{n} \left[y - \left(a x_i^2 + b x_i + c \right) \right]^2 = 0 \tag{9}$$

Hence, we find the parameters using the following approach

$$a x_i^2 + b x_i + c = y_i \tag{10}$$

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n^2 & x_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \tag{11}$$

$$AX = B \tag{12}$$

$$A^T A X = A^\top B \tag{13}$$

$$X = \left(A^\top A \right)^{-1} A^\top B \tag{14}$$

Algorithm 1 Least Squares

Require: $data$
  $x2_i^0 \leftarrow 0, i = 1, \ldots, n$
  $ones_i^0 \leftarrow 1, i = 1, \ldots, n$
  $i \leftarrow 0$
  while $i < lengthofdata$ **do**
    $x2[i] \leftarrow data[i][0]^2$
  end while
  $A \leftarrow [x2.Transpose, data[...][0].Transpose, ones.Transpose]$
  $B \leftarrow data[...][1].Transpose$
  return $(A^T A)^{-1} A^T B$

2.2 Random Sample Consensus (RANSAC)

. RANSAC is an iterative outlier detection method for data points in a dataset. outliers are defined as abnormal data points that disrupt the estimates made by optimization techniques like least squares. Hence it is required to filter out the outliers. Refer Algorithm 1 for RANSAC pseudo-code.

RASNAC randomly selects three points from that data set and fits a parabola to these points. Then RANSAC counts the number of outliers based on the offset provided on the new parabola. This process is repeated for a desired number of iterations or until the desired number of outliers are filtered out by the fitted parabola.

Then, the Least Squares Method is performed on the inliers to find the best fitted parabola.

Algorithm 2 RANSAC

Require: $data, dist, N, fit3points, pointsampler$

  $param \leftarrow [0, 0, 0]$
  $out \leftarrow 0$
  $iter \leftarrow 0$
  while $iter < N$ **do**
    $newpoints \leftarrow pointsampler(data)$
    $newparams \leftarrow fit3points(newpoints)$
    $newout \leftarrow 0$
    $i \leftarrow 0$
    while $i <= sizeofdata$ **do**
      $x \leftarrow data[i][0]$
      $y \leftarrow data[i][1]$
      $a \leftarrow newparams[0]$
      $b \leftarrow newparams[1]$
      $c \leftarrow newparams[2]$
      if $|ax^2 + bx + c - y| >= dist$ **then**
        $newout \leftarrow newout + 1$
      end if
      $i \leftarrow i + 1$
    end while
    if $newout > out$ **then**
      $out \leftarrow newout$
      $params \leftarrow newparams$
    end if
    $iter \leftarrow iter + 1$
  end while
  return $params$

2.3 Our Approach

Since the x coordinates were uniformly distributed for both data1 and data2, it is assumed that no error propagated through the x-direction. Hence, Least Squares can be used. However, upon inspection of data2, multiple outliers disrupt the estimate of least squares. Hence, we use RANSAC algorithm to filter out the outliers and pass the inliers through Least Squares algorithm to fit the data

3 Problem 3

To find the homography matrix:-

$$A = \begin{bmatrix} -5 & -5 & -1 & 0 & 0 & 0 & 500 & 500 & 100 \\ 0 & 0 & 0 & -5 & -5 & -1 & 500 & 500 & 100 \\ -150 & -5 & -1 & 0 & 0 & 0 & 30000 & 1000 & 200 \\ 0 & 0 & 0 & -150 & -5 & -1 & 12000 & 400 & 80 \\ -150 & -150 & -1 & 0 & 0 & 0 & 33000 & 33000 & 220 \\ 0 & 0 & 0 & -150 & -150 & -1 & 12000 & 12000 & 80 \\ -5 & -150 & -1 & 0 & 0 & 0 & 500 & 15000 & 100 \\ 0 & 0 & 0 & -5 & -150 & -1 & 1000 & 3000 & 200 \end{bmatrix} \tag{15}$$

Transforming the matrix to the reduced row Echelon form :-

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-216}{31} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{20}{31} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{-2500}{31} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{-72}{31} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16}{31} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{-3200}{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-24}{775} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{155} \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = 0 \tag{16}$$

3.1 Show mathematically how to compute SVD for an arbitrary matrix A

For an arbitrary $A \in \mathbb{R}^{M\times N}$ of rank R, Singular Value Decomposition is defined as follows

$$A = U\Sigma V^T \tag{17}$$

where:

$U \in \mathbb{R}^{M\times M}$
$V \in \mathbb{R}^{N\times N}$

$\Sigma \in \mathbb{R}^{M \times N}$
The columns of U are orthogonal eigen vectors of AA^T. The columns of V are orthogonal eigen vectors of $A^T A$ and the singular values in Σ are square roots of eigenvalues from AA^T or $A^T A$.

$$\sigma_i = \sqrt{\lambda_i} \tag{18}$$

$$diag(\Sigma) = \sigma_1 ... \sigma_M \tag{19}$$

The singular values are the diagonal entries of the S matrix and are arranged in descending order.