8/4/25 Name ____________________________ APID _____________

US25 MTH 124 Activity 5 (9.2/6.1/6.3/6.4/7.2) (ALL WORK REQUIRED)

Note: (1) If you think the answer doesn't exist, just demonstrate your work and write "DNE" or "doesn't exist".

(2) Each question is worth 5 points. And the final score will be rescaled to the total 20 points and then rounded to 2 decimal place.

$\left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2}$

1) (10points) Compute $f'(x)$ for the following function.

[i] $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$ $= \sec^2(x)$

$g = \sin x$ $\quad g' = \cos x$

$h = \cos x$ $\quad h' = -\sin x$

$f'(x) = \frac{\cos x \cdot \cos x - \sin(x) \cdot (-\sin x)}{\cos x^2} = \frac{1}{\cos^2 x}$

$= \sec^2 x$

[ii] $f(x) = \sin\left(e^{3x}\right)$

$g(x) = \sin x$ $\quad g' = \cos x$

$h(x) = e^x$ $\quad h' = e^x$

$I(x) = 3x$ $\quad I' = 3$

$\Downarrow$

$f(x) = g(h(I(x)))$

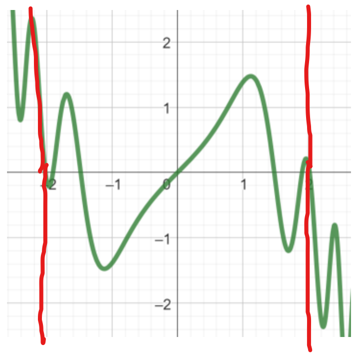
$f'(x) = g'(h(I(x))) \cdot (h(I(x)))'$

$= \underline{g'(h(I(x)))} \cdot \underline{h'(I(x))} \cdot \underline{I'(x)}$

$f'(x) = \underline{\cos(e^{3x})} \cdot \underline{e^{3x}} \cdot \underline{3}$

2) (15points) Compute the following indefinite/definite integrals.

[i] $\int_{-2}^{2} \sin\left(x^3\right) + x\cos(x)\,dx$ (The graph of $\sin\left(x^3\right) + x\cos(x)$ is given.)



(Hint: Interpretation of definite integral)

Area between function & x-axis

(w/ ± sign)

Ans = 0

indefinite integral

$2 \cdot x^{-1.3}$

[ii] $\int \frac{2}{x^{1.3}} - 3^x\,dx$ $= 2 \cdot \frac{1}{-0.3} x^{-0.3} - \frac{1}{\ln 3} 3^x + C$

$n \neq -1$

$\int x^n\,dx = \frac{1}{n+1} x^{n+1} + C$

$\int a^x\,dx = \frac{1}{\ln a} a^x + C$

[iii] $\int_{100}^{200} \frac{1}{x}\,dx$ $= \Big[\ln(x)\Big]_{100}^{200} = \ln(200) - \ln(100)$

$\int \frac{1}{x}\,dx = \ln x + C$

$= \ln\left(\frac{200}{100}\right) = \ln 2$ ✱

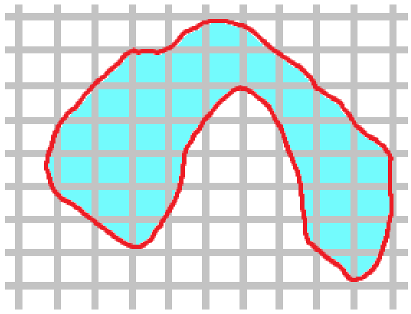
3) (10points) Environmental scientists are studying satellite images to estimate the size of freshwater lakes in a remote region. The image below has been simplified into a grid. Use the Riemann sum to determine the area of the lake.

[i] Use the left Riemann sum to estimate the area of the lake.

left endpoint × width.

$$1 \times (2+4+6+5+3+2+2+5+5)$$

$$= 34$$



2 4 6 5 3 2 2 5 5 2

[ii] Use the right Riemann sum to estimate the area of the lake.

right endpoint × width

$$1 \times (4+6+5+3+2+2+5+5+2) = 34$$

4) (10points) A clothing company manufactures expensive soccer jerseys for high school soccer athletes to be sold at all area local schools. Suppose the daily revenue from selling jerseys is

$R(t)-C(t)$
$= 50-2t$
putit per day at t day

$$R(t) = 150 + t$$

dollars per day, where t represents days from the beginning, while the daily costs are

$$C(t) = 100 + 3t$$

from 0 day to t day

[i] Find the profit $P(t)$ of selling jerseys for t days.

$$P(t) = \int_0^t 50-2s\, ds = \left[50s - s^2\right]_0^t = 50t - t^2$$

[ii] At which day the company can reach the biggest profit?

$$-t^2+50t = -(t^2-50t+25^2)+25^2$$
$$= -(t-25)^2+625$$

Ans: 25 day
Profit 625

5) (5points) Suppose a worker is tasked with painting a side of an arch bridge. He wants to calculate how much paint he needs to order to paint the bridge. The arch bridge is composed of two curves. The upper curve is

$$y = -\frac{x^2}{3} + \frac{1}{2},\ -1 \le x \le 1.$$

The lower curve is

$$y = -\frac{x^2}{2},\ -1 \le x \le 1.$$

What is the area the worker needs to paint?

$$\int_{-1}^{1} \text{upper} - \text{lower}\ dx = \text{Area}$$

$$\int_{-1}^{1} \left(-\frac{x^2}{3}+\frac{1}{2}\right) - \left(-\frac{x^2}{2}\right) dx = \int_{-1}^{1} \frac{1}{6}x^2 + \frac{1}{2}\, dx$$

$$= \left[\frac{1}{18}x^3 + \frac{1}{2}x\right]_{-1}^{1} = \frac{1}{18} + \frac{1}{2} - \left(-\frac{1}{18} - \frac{1}{2}\right) = \frac{1}{9} + 1 = \frac{10}{9}$$

6) (10points) On a straight road, the velocity of a car is described by the following equation:

$$V(t) = t^2 - 5t + 4, \quad 0 \le t \le 6,$$

from Shop A to Home. The unit is mile/hr.

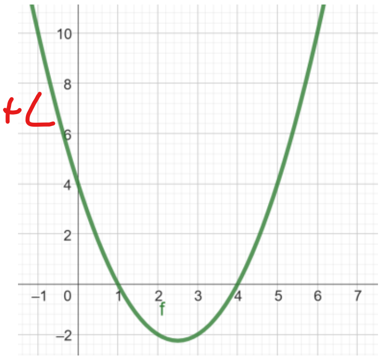
integral of $V(t)$

[i] What is the distance separating Shop A and Home?

$$\int_0^6 V(t)\,dt = \int_0^6 t^2 - 5t + 4\,dt = \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t\right]_0^6$$

$$= \frac{1}{3}6^3 - \frac{5}{2}6^2 + 24 - 0 = 72 - 90 + 24 = 6 \text{ miles}$$

[ii] What is the total distance traveled by the car during this journey?



$\int V(t)\,dt = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t + C$

A

$t: 0 \sim 1$ $\qquad \int_0^1 V(t)\,dt$

$t\ 1 \sim 4$ $\qquad -\int_1^4 V(t)\,dt$

$t: 4 \sim 6$ $\qquad +\int_4^6 V(t)\,dt$

$$= \left[\frac{1}{3} - \frac{5}{2} + 4\right] - 0 - \left(\frac{1}{3}4^3 - \frac{5}{2}4^2 + 16\right) + \left(\frac{1}{3} - \frac{5}{2} + 4\right) + 6 - \left(\frac{1}{3}4^3 - \frac{5}{2}4^2 + 16\right)$$

$$= 2\left(\frac{1}{3} + \frac{3}{2}\right) - 2\left(\underbrace{\frac{64}{3} \cancel{- 40 + 16}}_{-24}\right) + 6 \qquad -\frac{8}{3}$$

$$= \frac{2}{3} + 3 + \frac{16}{3} + 6 = 6 + 3 + 6 = 15 \quad \text{miles}$$

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