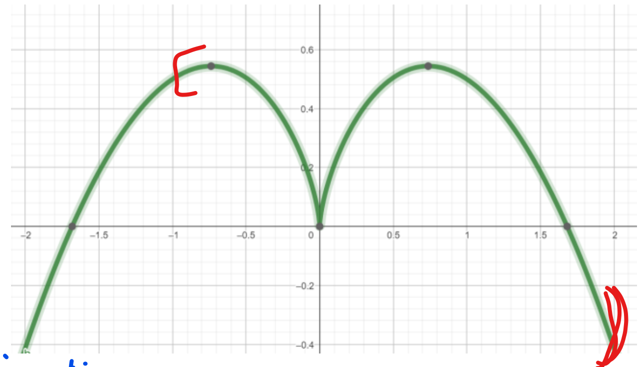
7/28/25 Name ____________________________ APID _____________

US25 MTH 124 Activity 4 (4.1/4.2/4.3/4.4/4.5/5.1/5.3) (ALL WORK REQUIRED)

Note: (1) If you think the answer doesn't exist, just demonstrate your work and write "DNE" or "doesn't exist".

(2) Each question is worth 5 points. And the final score will be rescaled to the total 20 points and then rounded to 2 decimal place.

1) (20points) The graph of the funciton $f(x)=x^{\frac{2}{3}}-\frac{x^2}{2}$ is given below. Follow the following steps to find out the extrema of the function over the interval [-1, 2) .



no deviative

[i] What are the singular points of $f(x)$?

$(x^n)' = nx^{n-1}$

$(0,0)$

$f'(x)=0$

[ii] What are the stationary points of $f(x)$?

$f'(x)=\frac{2}{3}x^{-\frac{1}{3}}-\frac{1}{2}\cdot 2x' = \frac{2}{3}x^{-\frac{1}{3}}-x=0$

$x^{-\frac{1}{3}}\left(\frac{2}{3}-x^{\frac{4}{3}}\right)=0$

$x^{\frac{4}{3}}=\frac{2}{3}$

$x=\pm\left(\frac{2}{3}\right)^{\frac{3}{4}}$

(1) $\left(+\left(\frac{2}{3}\right)^{\frac{3}{4}}, f(\cdots)\right)$

(2) $\left(-\left(\frac{2}{3}\right)^{\frac{3}{4}}, f(-(\cdots))\right)$

[iii] What are the endpoints of $f(x)$ over this interval [-1, 2)?

$(-1, f(-1))$, $f(-1)=(-1)^{\frac{2}{3}}-\frac{1}{2}=1-\frac{1}{2}=\frac{1}{2}$, $\left(-1,\frac{1}{2}\right)$ ✗

[iv] Find all extrema of the function f and determine they are relatively/absolutely minimum/maximum.

(1) $(0,0)$ rel. min.

(2) $\left(+\left(\frac{2}{3}\right)^{\frac{3}{4}}, f(\cdots)\right)$ abs. max

(3) $\left(-\left(\frac{2}{3}\right)^{\frac{3}{4}}, f(-\cdots)\right)$ abs max

(4) $\left(-1,\frac{1}{2}\right)$ rel. min

quotient rule

$$\left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2}$$

2) (10points) Compute $f'(x)$ for the following function.

[i] $f(x) = \dfrac{x^2-2}{x+1}$

$$f'(x) = \frac{2x(x+1) - (x^2-2)\cdot 1}{(x+1)^2} = \frac{x^2+2x+2}{(x+1)^2}$$ #

$g = x^2-2$, $g' = 2x$

$h = x+1$, $h' = 1$

Chain rule

[ii] $f(x) = e^{-|x|}$

$g(h(x))' = g'(h(x))\cdot h'(x)$

$$f'(x) = \underbrace{-e^{-|x|}}_{g'(h(x))} \cdot \underbrace{\frac{|x|}{x}}_{h'(x)}$$

$g(x) = e^{-x}$ $\quad g'(x) = -e^{-x}$

$h(x) = |x|$ $\quad h'(x) = \dfrac{|x|}{x}$

$g(h(x)) = e^{-|x|}$

3) (5points) Find the values of the constants a.

[i] $f(x) = \dfrac{2x+a}{x+1}$ with $f'(1) = 1$

$$f(x) = \frac{2x+a}{x+1} = \frac{2(x+1)-2+a}{x+1} = 2 + \frac{-2+a}{x+1}$$

$$f'(x) = 0 + (-2+a)\cdot\underbrace{\left(-\frac{1}{(x+1)^2}\right)}_{\left(\frac{1}{x+1}\right)'}$$

$$f'(1) = 1 = (-2+a)\cdot\left(-\frac{1}{4}\right)$$

$$-4 = -2+a$$

$$a = -2$$ #

$A(r) = \pi r^2 \qquad A(r(t)) = \pi (r(t))^2$

$r'(t) = 0.4$ cm/day

$\frac{d}{dt}$

$A'(r=5)$

4) (5points) A formation of mold is growing in a circular rate in a petri dish. The radius is measured to be growing at a rate of 0.4 cm/day. How fast is the area of the mold growing when the culture has reached a radius of 5cm?

$$\frac{d}{dt}A(t) = \frac{dA(r)}{dr} \cdot \frac{dr}{dt}$$

$$= A'(r(t)) \cdot r'(t)$$

$$= 2\pi r(t) \cdot r'(t)$$

$$= 2\pi \; 5 \cdot 0.4 = 4\pi \; \text{cm}^2/\text{day}$$

5) (10points) **Rounded the answers in this problem to 1 decimal places.**

On a road, the position of a car is described by the following equation:

$$P(t) = 6(t-1)^3 + 6t + 6, \quad 0 \le t \le 3$$

Where the unit of t is hour and the unit of P is mile.

[i] Does this car go over the speed limit? Assume the speed limit is 60 mile/hr.

$P'(t) = 18(t-1)^2 + 6 > 60$ $\qquad t > 1+\sqrt{3} = 2.73 \fallingdotseq 2.7$

$18(t-1)^2 > 54$

$(t-1)^2 > 3$

$\Rightarrow \boxed{t > 1+\sqrt{3}}$

$t < 1-\sqrt{3}$ (non sense)

Yes, when $t > 2.7$

[ii] When does this car have no acceleration?

$P''(t) = 0$

$P''(t) = 36(t-1) = 0$

$t = 1$ #

6) (15points) Given a circle centered at the origin with radius 2. There is a point $C(1, \sqrt{3})$ on the circle. Follow the following steps to find the equation of the tangent of C to the circle

$$x^2+y^2=r^2=4$$

[i] Formulate the y coordinate of the upper circle as a funciton of the x coordinate.

$g(x)=\sqrt{x}$

$h(x)=4-x^2$

$g'(x)=\frac{1}{2}x^{-\frac{1}{2}}$

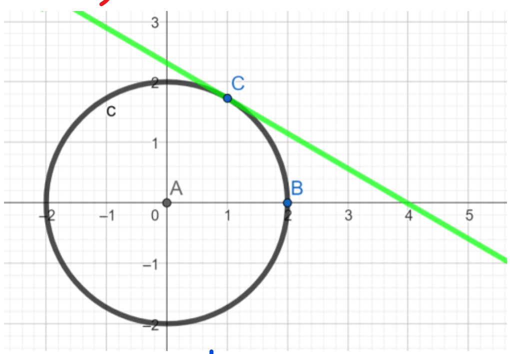
$h'(x)=-2x$

$y^2=4-x^2 \Rightarrow y=\sqrt{4-x^2}$ (upper circle)

$=(4-x^2)^{\frac{1}{2}}$

$y=\pm\sqrt{4-x^2}$

$-$ → lower circle



[ii]What's the slope of the tangent line at C?

$$y'=\underbrace{\frac{1}{2}(4-x^2)^{-\frac{1}{2}}}_{g'(h(x))}\cdot\underbrace{(4-x^2)'}_{h'(x)}=\frac{-x}{\sqrt{4-x^2}} \qquad y'(1)=\frac{-1}{\sqrt{3}} \#$$

[iii] What's the equation of the tangent line at C?

$$y=ax+b$$

$$\Rightarrow y=\frac{-1}{\sqrt{3}}x+b$$

plug in $c(1,\sqrt{3}) \Rightarrow \sqrt{3}=-\frac{1}{\sqrt{3}}\cdot 1+b$

$$b=\sqrt{3}+\frac{1}{\sqrt{3}}=\frac{4}{\sqrt{3}}$$

$$y=-\frac{1}{\sqrt{3}}x+\frac{4}{\sqrt{3}}$$