

6/17/24 Name _____ APID _____

US24 MTH 124 Activity 5 (9.2/6.1/6.3/6.4/7.2) (ALL WORK REQUIRED)

Note: (1) If you think the answer doesn't exist, just demonstrate your work and write "DNE" or "doesn't exist".

(2) Each question is worth 5 points. And the final score will be rescaled to the total 20 points and then rounded to 2 decimal place.

1) (10points) Compute $f'(x)$ for the following function.

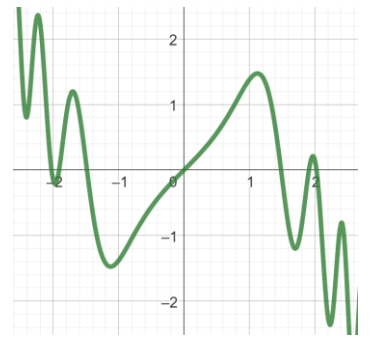
[i] $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

[ii] $f(x) = \sin(e^{3x})$

2) (15points) Compute the following indefinite/definite integrals.

[i] $\int_{-2}^2 \sin(x^3) + x \cos(x) dx$ (The graph of $\sin(x^3) + x \cos(x)$ is given.)

(Hint: Interpretation of definite integral)



[ii] $\int \frac{2}{x^{1.3}} - 3^x dx$

[iii] $\int_{100}^{200} \frac{1}{x} dx$

3) (20points) Follow the steps below to solve the ordinary differential equation.

$$-\frac{1}{2}f(x) + xf'(x) = x, \quad f(1) = 3.$$

[i] Find values a, b such that the equation $x^a \left(-\frac{1}{2}f(x) + xf'(x) \right) = (x^b f(x))'$ holds.

Thus, we know if we multiply x^a on the both sides of $-\frac{1}{2}f(x) + xf'(x) = x$, we get $x^a \left(-\frac{1}{2}f(x) + xf'(x) \right) = (x^b f(x))' = x^a \cdot x = x^{a+1}$.

[ii] Compute the indefinite integral $\int x^{a+1} dx$, and denote the answer by $F(x) + c$

So we get $x^b f(x) = F(x) + c$, and then $f(x) = x^{-b} F(x) + cx^{-b}$.
[iii] Using $f(1) = 3$ to solve the constant c .

[iv] Write down the solution of the ordinary differential equation and verify it does

satisfy $-\frac{1}{2}f(x) + xf'(x) = x$ and $f(1) = 3$

- 4) (10points) A clothing company manufactures expensive soccer jerseys for high school soccer athletes to be sold at all area local schools. Suppose the daily revenue from selling jerseys is

$$R(s) = 150 + s$$

dollars per day, where t represents days from the beginning, while the daily costs are

$$C(s) = 100 + 3s$$

[i] Find the profit $P(t)$ of selling jerseys for t days.

[ii] At which day the company can reach the biggest profit?



- 5) (5points) Suppose a worker is tasked with painting a side of an arch bridge. He wants to calculate how much paint he needs to order to paint the bridge. The arch bridge is composed of two curves. The upper curve is

$$y = -\frac{x^2}{3} + \frac{1}{2}, \quad -1 \leq x \leq 1.$$

The lower curve is

$$y = -\frac{x^2}{2}, \quad -1 \leq x \leq 1.$$

What is the area the worker needs to paint?



- 6) (10points) On a straight road, the velocity of a car is described by the following equation:

$$V(t) = t^2 - 5t + 4, \quad 0 \leq t \leq 6,$$

from Shop A to Home. The unit is mile/hr.

[i] What is the distance separating Shop A and Home?



[ii] What is the total distance traveled by the car during this journey?

