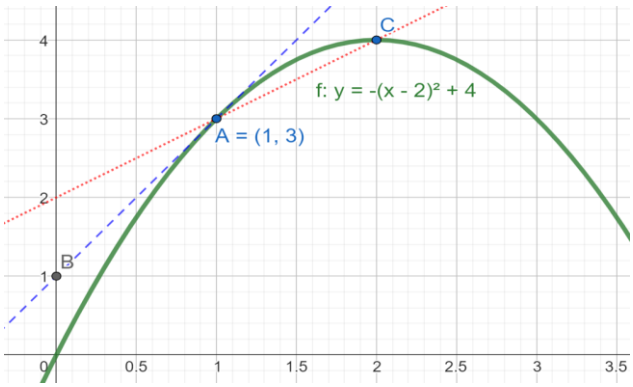
7/21/25 Name _____ APID _____

US25 MTH 124 Activity 3 (3.4/3.5/3.6) (ALL WORK REQUIRED)

Note: (1) If you think the answer doesn't exist, just demonstrate your work and write "DNE" or "doesn't exist".

(2) Each question is worth 5 points. And the final score will be rescaled to the total 20 points and then rounded to 2 decimal place.

1) (20points) Use the information in the graph to answer the following questions.



derivative

the slope of the tangent line

[i] What is the **instantaneous** rate of change of the function f at the point $A(1,3)$? What information do use and what is the geometric meaning of that?

B(0,1) (1) $\frac{3-1}{1-0}=\frac{2}{1}=2$ (2) the slope of the tangent line $\overleftrightarrow{AB}$

the slope of the secant line

[ii] What is the **avereage** rate of change of the function f from the point $A(1,3)$ to $C(2,4)$? What information do use and what is the geometric meaning of that?

(1) $\frac{4-3}{2-1}=\frac{1}{1}=1$ (2) the slope of the secant line $\overleftrightarrow{AC}$

$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

2) (15points) Compute $f'(a)$ algebraically for the given value of a.

[i] $f(x)=x^2-2x$, $a=3$ [ii] $f(x)=\frac{2}{x}$, $a=-1$ [iii] $f(x)=px+q$, $a=100$

(i)

$f(3+h)=(3+h)^2-2(3+h)$

$=9+6h+h^2-6-2h$

$=3+4h+h^2$

$f(3)=9-6=3$

$\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$=\lim_{h\to 0}\frac{4h+h^2}{h}=\lim_{h\to 0}4+h=4$

4

$f(-1+h)=\frac{2}{-1+h}$ -2

$\frac{2}{-1+h}-\frac{2}{-1}=\frac{2\cdot(-1)-2(-1+h)}{(-1+h)(-1)}$

$=\frac{0-2h}{1-h}=\frac{-2h}{1-h}$

$\lim_{h\to 0}\frac{\frac{-2h}{1-h}}{h}=\lim_{h\to 0}\frac{-2}{1-h}$

$=-2$

p

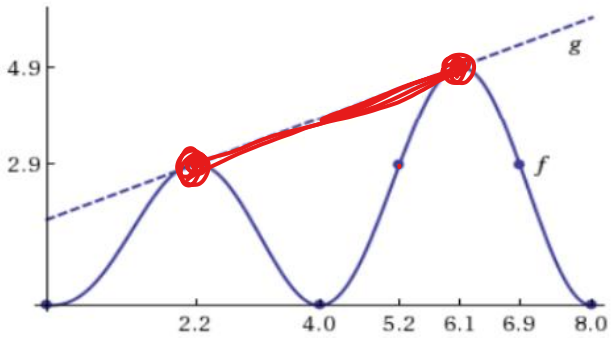
$f(100+h)=p(100+h)+q$

$f(100)=p\cdot 100+q$

$f(100+h)-f(100)=ph$

$\lim_{h\to 0}\frac{ph}{h}=\lim_{h\to 0}p=p$

3) (25points) Comparing average rate of change of two functions, f and g.



For each interval given below, decide whether the average rate of change of f(x) or g(x) is greater over that particular interval.

slope of secant line

Interval	Which function has GREATER average rate of change?
$0 \le x \le 8$	☐ f ☑ g ☐ both have an equal rate of change.
$5.2 \le x \le 8$	☐ f ☑ g ☐ both have an equal rate of change.
$5.2 \le x \le 6.1$	☑ f ☐ g ☐ both have an equal rate of change.
$5.2 \le x \le 6.9$	☐ f ☑ g ☐ both have an equal rate of change.
$2.2 \le x \le 6.1$	☐ f ☐ g ☑ both have an equal rate of change.

4) (10points) **Rounded the answers in this problem to 1 decimal places.**

On a road, the position of a car is described by the following equation:

$$P(t) = x^3 + x.$$

[i] Estimate $\dfrac{P(3)-P(2)}{3-2}$ and interpret the answer in this context.

$P(3) = 3^3 + 3 = 27 + 3 = 30$

$P(2) = 2^3 + 2 = 8 + 2 = 10$

$\dfrac{P(3)-P(2)}{3-2} = \dfrac{20}{1} = 20$

the average velocity between 2, 3

[ii] Estimate $\lim\limits_{h\to 0^+} \dfrac{P(2+h)-P(2)}{h}$ and interpret the answer in this context.

$(a+b)^3$

$a^3 + 3a^2b + 3ab^2 + b^3$

$P(2+h) = (2+h)^3 + 2 + h$

$= 8 + 12h + 6h^2 + h^3 + 2 + h$

$= 10 + 13h + 6h^2 + h^3$

$P(2) = 10$

$\lim\limits_{h\to 0^+} \dfrac{P(2+h)-P(2)}{h}$

$= \lim\limits_{h\to 0^+} \dfrac{13h + 6h^2 + h^3}{h}$

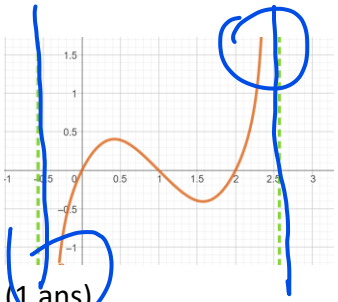
$= \lim\limits_{h\to 0^+} 13 + 6h + h^2 = 13$

the instantaneous velocity at $t = 2$.

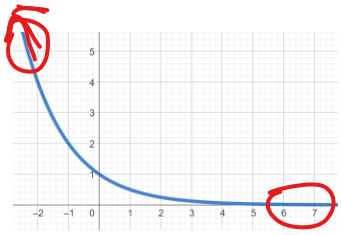
slope → +∞ slope → −∞ slope → −∞

5) (15points) Pair the following pictures with the correct statements:

slope → +∞

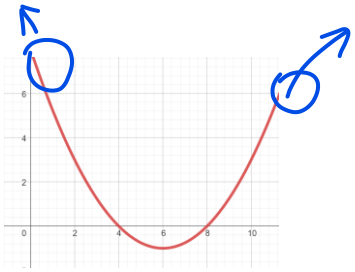


(1 ans)



(2 ans)

slope → 0



(3 ans)

slope → +∞ C

A B

A C D

(A) For any time t, the instantaneous rate of change at time t is less than every average rate of change over every period after time t.

(B) For any time t, the instantaneous rate of change at time t is less than every average rate of change over every period after time t. And the instantaneous rate of change is always negative.

(C)There is at least one time at which the instantaneous rate of change is zero. Plus, the instantaneous rate of change can be any positive number you want.

(D) There is at least one time at which the instantaneous rate of change is zero. Plus, the instantaneous rate of change can be any negative number you want.