

7/07/25 Name \_\_\_\_\_ APID \_\_\_\_\_

US25 MTH 124 Activity 1 (1.1/1.2/1.3/2.1/2.2) (ALL WORK REQUIRED)

Note: (1) If you think the answer doesn't exist, just demonstrate your work and write "DNE" or "doesn't exist".

**(2) Each question is worth 5 points. And the final score will be rescaled to the total 20 points and then rounded to 2 decimal place.**

E.g.: One gets 50 points originally. Then he/she will get 10 points in this activity.

1) (25points) Using the function graphed

here, provide the following:

(Use interval notation where appropriate.)

[i] Domain:

$$(-1, \infty)$$

[ii] Range:

$$\left[-\frac{1}{2}, \infty\right)$$

[iii] x-intercept(s):

$$(1, 0), (3, 0)$$

[iv] y-intercept(s)

$$(0, \frac{3}{2})$$

[v] The coordinate of vertex of the graph, point G:

$$x = -\frac{b}{2a} = \frac{2}{1} = 2 \quad (h, k) \quad (2, -\frac{1}{2})$$

$$y = a(x-h)^2 + k$$

$$\begin{aligned} x=2 \\ \Rightarrow y &= \frac{1}{2}(4-8+3) = -\frac{1}{2} \quad \frac{1}{2}(x^2-4x+3) = \frac{1}{2}(x^2-4x+4-4+3) \\ &= \frac{1}{2}(x-2)^2 + \frac{1}{2}(-4+3) = \frac{1}{2}(x-2)^2 - \frac{1}{2} \end{aligned}$$

2) (20points) Given the function shown, find

these values.

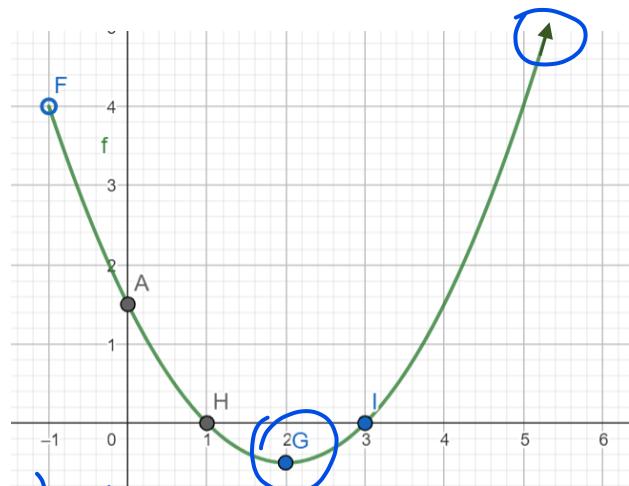
[i]  $V(-6)$  9

[ii]  $V(-2)$  1

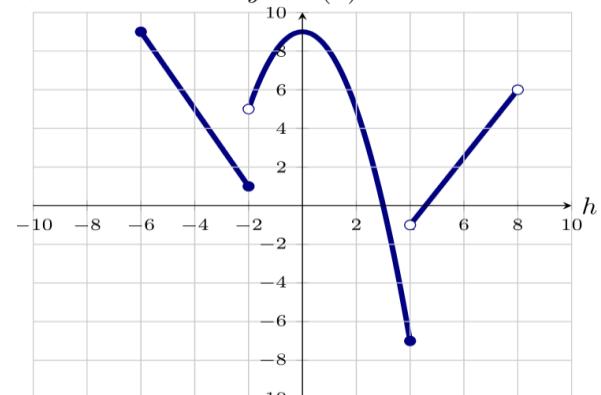
[iii]  $2^*V(4)$   
 $2 \cdot (-9) = -14$

[iv]  $V(8)$

DNE



$$\begin{aligned} y &= \frac{1}{2}(x^2 - 4x + 3) \\ &= \frac{1}{2}x^2 - 2x + \frac{3}{2} \end{aligned}$$



3) (10points) The U.S. Federal income tax is a function of taxable income. Write  $T$  for the tax owed on a taxable income of  $I$  dollars. For tax year 2023, the function  $T$  for a single taxpayer was specified by the table below.



→

If your taxable income ( $I$ ) was		Adjusted taxable Income ( $I'$ )	Your tax $T$ is
Over-	But not over-		
\$0	\$1000	$I$	$10\% \times I$
\$1000	\$2000	$I - 1000$	$100 + 15\% \times I'$
\$2000	...	$I - 2000$	$250 + 20\% \times I'$

$$y = ax + b$$

nref 2 points

[i] What was the owed by a single taxpayer on a taxable income of \$1500?

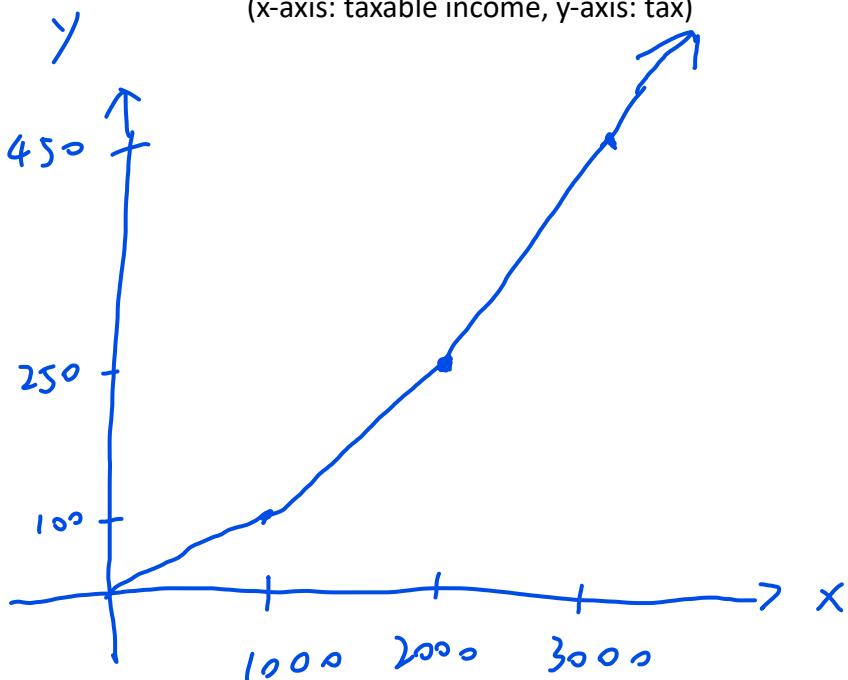
$$T = 100 + 15\% \cdot I'$$

$$I' = 1500 - 1000 = 500$$

$$T = 100 + 15\% \cdot 500 = 175$$

[ii] Draw the graph of tax versus the taxable income.

(x-axis: taxable income, y-axis: tax)



$$I = 1000 \Rightarrow T = 10\% \cdot 1000 \\ = 100$$

$$I = 2000 \Rightarrow I' = 1000$$

$$T = 100 + 15\% \cdot 1000 \\ = 250$$

$$I = 3000 \Rightarrow I' = 3000 - 2000 \\ = 1000$$

$$T = 250 + 20\% \cdot 1000 \\ = 450$$

4) (15points) A clothing company manufactures expensive soccer jerseys for high school soccer athletes to be sold at all area local schools. The company plans to produce  $x$  jerseys. The cost  $c(x)$  of each jersey depends on how many jerseys produced and follows the formula below.

$$\text{Profit} = \text{revenue} - \text{cost}$$

$$c(x) = 150 - \frac{x}{5}$$

But as they produced more jerseys, they need to lower the price of jerseys in order to sell out all jerseys. And the price of each jersey depends on how many jerseys produced and follows the formula below.

$$\text{revenue} = \text{price} \times \# \text{jersseys}$$

$$p(x) = 100 - \frac{x}{10}$$

[i] Determine a revenue function for this company.

$$(100 - \frac{x}{10})x = -\frac{1}{10}x^2 + 100x$$

[ii] Determine a Profit function for this company.

$$\text{Cost} = \text{cost} \times \# \text{jersseys} = (150 - \frac{x}{5}) \cdot x = -\frac{1}{5}x^2 + 150x$$

[iii] How many jerseys must they sell to break even?

$$\text{Profit} = 0 = \frac{1}{10}x^2 - 50x$$

$$= \frac{1}{10}x(x - 500)$$

$x = 0, \boxed{500} \times$



$$\begin{aligned} \text{Profit} &= -\frac{1}{10}x^2 + 100x - \left[ -\frac{1}{5}x^2 + 150x \right] \\ &= \frac{1}{10}x^2 - 50x \end{aligned}$$

$\times$

$$y = a \cdot r^t$$

5) (10points) After a large number of drinks, a person has a blood alcohol level of 200 mg/dL (milligrams per deciliter). If the amount of alcohol in the blood decays exponentially, and after 2 hours, 150 mg/dL remain.

[i] Find an exponential model for the person's blood alcohol level.



$$a = 200$$

$$b = \frac{1}{2}$$

$$r = \frac{150}{200} = \frac{3}{4}$$

$$\rightarrow y(t) = 200 \cdot \left(\frac{3}{4}\right)^{\frac{t}{2}} \text{ mg/dL}, t: \text{hours}$$

[ii] Use the exponential model to estimate the person's blood alcohol level after 4 hours.

$$t = 4 \quad \left(\frac{4}{2}\right) = 2$$

$$y(4) = 200 \cdot \left(\frac{3}{4}\right)^2 = 200 \cdot \frac{9}{16} = 112.5 \text{ mg/dL}$$

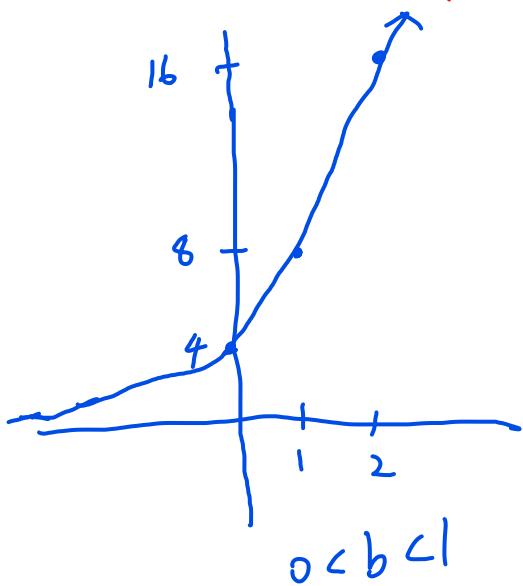
6) (20points) Graph the given function below. And find

(a) the x-intercept(s) and (b) y-intercept(s).

[i]  $y = f(x) = 4(2)^x$

x	0	1	2
y	4	8	16

y-int:  $(0, 4)$   
x-int: DNE



[ii]  $y = g(x) = \left(\frac{1}{3}\right)^x$

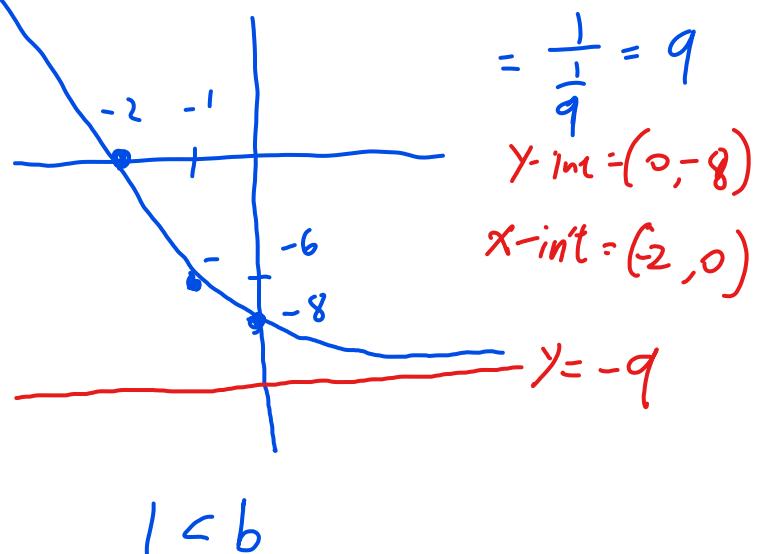
x	0	-1	-2
y	1	-3	-9

$$\left(\frac{1}{3}\right)^{-1} = 3$$

$$\left(\frac{1}{3}\right)^2 = \frac{1}{\left(\frac{1}{3}\right)^2}$$

$$= \frac{1}{\frac{1}{9}} = 9$$

y-int:  $(0, -8)$   
x-int:  $(-2, 0)$



$$1 < b$$

