# VLIW Processors

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# Limits on Deep Pipelining

- Ideally, CycleTime<sub>Pipelined</sub> = CycleTime<sub>SingleCycle</sub> / Number of Stages
   In theory, can indefinitely improve performance with more stages
- Limitation 1: **Cycle time** does not improve indefinitely
  - Latch delay + unbalanced stages (manufacturing variability)
- Limitation 2: CPI tends to increase with deep pipelines
  - Penalty due to branch misprediction increases
  - Stalls due to data hazards cause more bubbles
- Is there another way to improve performance?



#### What if we improve CPI?

Remember the three components of performance?

```
instructions x cycles x seconds cycle
```

- Pipelining focused on seconds / cycle, or cycle time
- Can we improve cycles / instruction, or CPI?
  - O But the best we can get is CPI = 1, right?
  - O How can an instruction be executed in less than a cycle?



# Wide Issue Processors



#### From CPI to IPC

- How about if we execute two instructions each cycle?
  - Maybe, fetch one ALU instruction and one load/store instruction

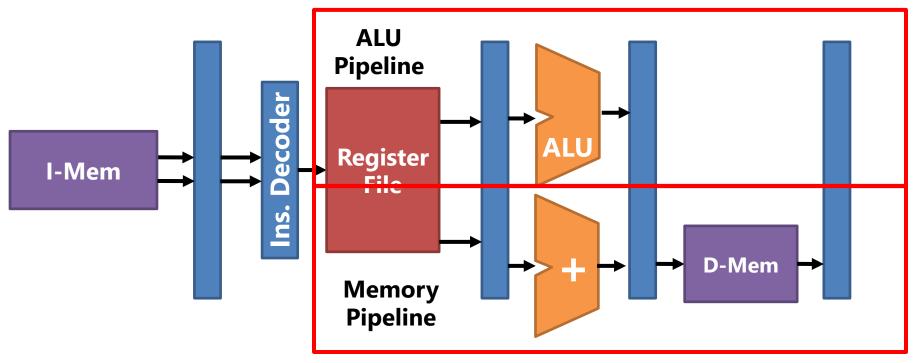
Instruction type				Pip	e stages			
ALU or branch instruction	IF	ID	EX	MEM	WB			
Load or store instruction	IF	ID	EX	MEM	WB			
ALU or branch instruction		IF	ID	EX	MEM	WB		
Load or store instruction		IF	ID	EX	MEM	WB		
ALU or branch instruction			IF	ID	EX	MEM	WB	
Load or store instruction			IF	ID	EX	MEM	WB	
ALU or branch instruction				IF	ID	EX	MEM	WB
Load or store instruction				IF	ID	EX	MEM	WB

- Then, IPC (Instructions per Cycle) = 2
  - And by extension, CPI = 1 / IPC = 0.5!
- Wide-issue processors can execute multiple instructions per cycle



#### 2X speedup with mere addition of one 📙 unit?

One pipeline for ALU/Branches and one for loads and stores

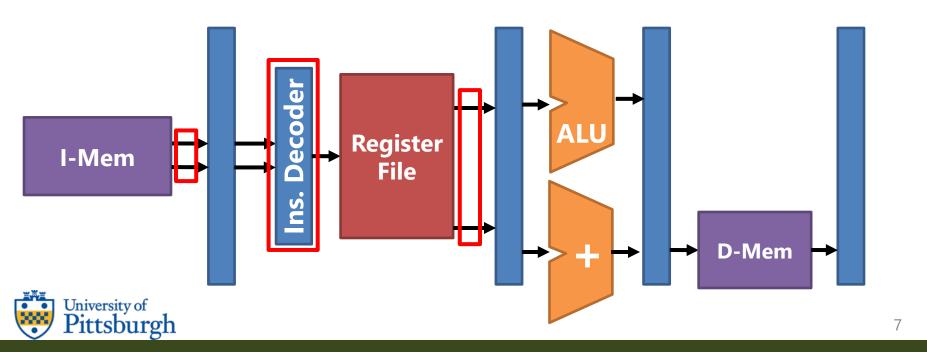


Causes contention on shared resources and structural hazards!



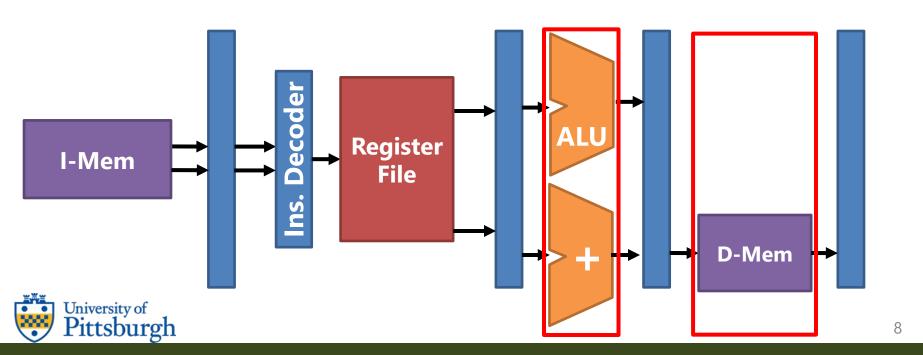
#### Extra investments needed to avoid structural hazard

- At max, every clock cycle in this 2-wide processor:
  - $\circ$  2 instructions fetched from I-MEM  $\rightarrow$  1 extra read port needed
  - 2 instructions decoded → Extra wide instruction decoder needed
  - $\circ$  3 reads from register file  $\rightarrow$  1 extra read port needed
  - $\circ$  2 writes to register file  $\rightarrow$  1 extra write port needed



#### And fundamental structural hazards still remain

- Structural hazard on EX units
  - Top ALU can handle all arithmetic (+, -, \*, /)
  - Bottom ALU can only handle +, needed for address calculation
- Structural hazard on MEM unit
  - ALU pipeline does not have a MEM unit to access memory



#### Structural Hazard in Functional Units: Example

Code on the left will result in a timeline on the right
If it were not for the bubbles, we could have finished in 4 cycles!

1w	\$t0,	0(\$s	1)
1w	\$t1,	-4(\$s	1)
addi	\$t2,	\$t2,	-8
add	\$t3,	\$t0,	<b>\$s1</b>
add	\$t4,	<b>\$</b> s1,	<b>\$s1</b>
SW	\$t5,	8(\$t3	)
SW	\$t6.	4(\$s1	)

CC	ALU Pipeline	Mem Pipeline
1		lw t0
2	addi t2	lw t1
3	add t3	
4	add t4	sw t5
5		sw t6



#### Structural Hazard Solution: Reordering

Of course we can come up with a better schedule
 While still adhering to the data dependencies

1w	\$t0,	0(\$	s <b>1</b> )
lw	\$t1,	-4(\$	s <b>1</b> )
addi	\$t2,	\$t2,	-8
add	\$t3,	\$t0,	<b>\$s1</b>
add	\$t4,	<b>\$</b> s1,	<b>\$s1</b>
SW	\$t5,	8(\$t3	3)
SW	\$t6.	4(\$s:	1)

CC	ALU Pipeline	Mem Pipeline
1		lw t0
2	addi t2	lw t1
3	add t3	
4	add t4	sw t5
5		sw t6



### Why not just duplicate all resources?

- Why not have a full ALU and a MEM unit in each pipeline?
  - That way, we can avoid those structural hazards in the first place
  - But that leads to low utilization
    - ALU/Branch type instructions will not use the MEM unit
    - Load/Store instructions will not need the full ALU
- Most processors have specialized pipelines for different instructions
  - o Integer ALU pipeline, FP ALU pipeline, Load/Store pipeline, ...
  - With scheduling, can achieve high utilization and performance
- Who does the scheduling? Well, we talked about this already:
  - Static scheduling → Compiler
  - Dynamic scheduling → Processor



#### VLIW vs. Superscalar

- There are two types of wide-issue processors
- If the compiler does static scheduling, the processor is called:
  - VLIW (Very Long Instruction Word) processor
  - This is what we will learn this chapter
- If the processor does **dynamic scheduling**, the processor is called:
  - Superscalar processor
  - This is what we will learn next chapter



# VLIW Processors



#### **VLIW Processor Overview**

- What does Very Long Instruction Word mean anyway?
  - o It means one instruction is very long!
  - Why? Because it contains multiple operations in one instruction
- A (64 bits long) VLIW instruction for our example architecture:

ALU/Branch Operation (32 bits) Load/Store Operation (32 bits)

An example instruction could be:

addi \$t2, \$t0, -8 lw \$t1, -4(\$s1)

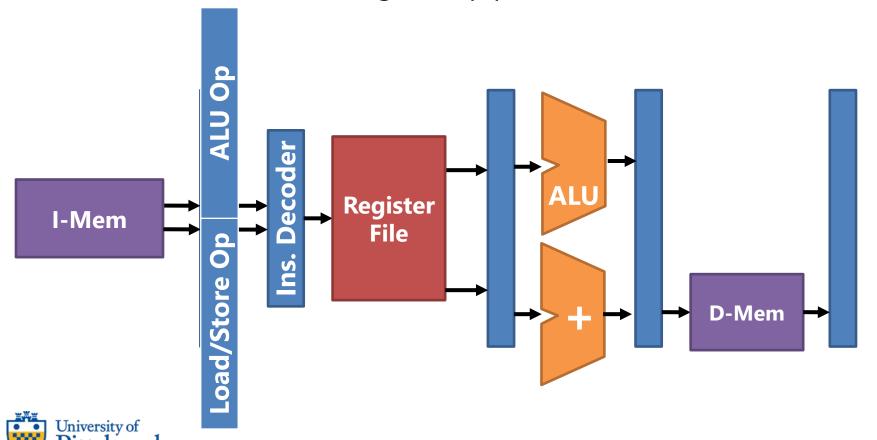
Or another example could be:

nop lw \$t1, -4(\$s1)



#### A VLIW instruction is one instruction

For all purposes, a VLIW instruction acts like one instruction
 It moves as a unit through the pipeline



# VLIW instruction encoding for example

```
nop
lw $t0, 0($s1)

addi $t2, $t2, -8
lw $t1, -4($s1)

add $t3, $t0, $s1
nop
```

add	\$t4,	\$s1, \$s1	
SW	\$t5,	8(\$t3)	

```
nop
sw $t6, 4($s1)
```

Inst	ALU Op	Load/Store Op
1	nop	lw t0
2	addi t2	lw t1
3	add t3	nop
4	add t4	sw t5
5	nop	sw t6

- Each square is an instruction. (There are 5 instructions.)
- Nops are inserted by the compiler.



# VLIW instruction encoding (after reordering)

```
add $t4, $s1, $s1
lw $t0, 0($s1)
```

```
add $t3, $t0, $s1
sw $t6, 4($s1)
```

```
nop
sw $t5, 8($t3)
```

Inst	ALU Op	Load/Store Op
1	add t4	lw t0
2	addi t2	lw t1
3	add t3	sw t6
4	nop	sw t5

Same program with 4 instructions!



# VLIW Architectures are (Very) Power Efficient

- All scheduling is done by the compiler offline
- No need for the Hazard Detection Unit
  - Nops are inserted by the compiler when necessary
- No need for a dynamic scheduler
  - Which can be even more power hungry than the HDU
- Even no need for the Forwarding Unit
  - If compiler is good enough and fill all bubbles with instructions
  - Or, may have cheap compiler-controlled forwarding in ISA



#### Challenges of VLIW

- All the challenges of static scheduling apply here X 2
- Review: what were the limitations?
  - Compiler must make assumptions about the pipeline
    - → ISA now becomes much more than instruction set + registers
    - → ISA restricts modification of pipeline in future generations
  - Compiler must do scheduling without runtime information
    - → Length of MEM stage is hard to predict (due to Memory Wall)
    - → Data dependencies are hard (must do pointer analysis)
- These limitations are exacerbated with VLIW



# Not Portable due to Assumptions About Pipeline

- VLIW ties ISA to a particular processor design
  - One that is 2-wide and has an ALU op and a Load/Store op
  - What if future processors are wider or contain different ops?
- Code must be recompiled repeatedly for future processors
  - Not suitable for releasing general purpose software
  - Reason VLIW is most often used for embedded software (Because embedded software is not expected to be portable)
- Is there any way to get around this problem?



#### Making VLIW Software Portable

- There are mainly two ways VLIW software can become portable
- 1. Allow CPU to exploit parallelism according to capability
  - o Analogy: multithreaded software does not specify number of cores
    - SW: Makes parallelism explicit by coding using threads
    - CPU: Exploits parallelism to the extent it has number of cores
  - o Portable VLIW: ISA does not specify number of ops in instruction
    - SW: Makes parallelism explicit by using bundles
      - Bundle: a group of ops that can execute together
      - Wider processors fetch several bundles to form one instruction
      - A "stop bit" tells processor to stop fetching the next bundle
    - Intel Itanium EPIC(Explicitly Parallel Instruction Computing)
      - A general-purpose ISA that uses bundles



#### Making VLIW Software Portable

There are mainly two ways VLIW software can become portable

#### 2. Binary translation

- Have firmware translate binary to new VLIW ISA on the fly
- Open't this go against the power efficiency of VLIWs?
  - Yes, but if SW runs for long time, one-time translation is nothing
  - Translation can be cached in file system for next run
- Transmeta processors convert x86 to an ultra low-power VLIW
- Other examples of binary translation
  - Apple Rosetta converts x86 to ARM ISA for M1 or M2 chips
  - NVIDIA GPUs convert PTX (Parallel Thread Execution) to SASS
    - SASS (Stream ASSembly) is different for every generation of GPU



### Scheduling without Runtime Information

- Up to the compiler to create schedule with minimal nops
  - Use reordering to fill nops with useful operations
- All the challenges of static scheduling remain
  - Length of MEM stage is hard to predict (due to Memory Wall)
  - Data dependencies are hard to figure out (due to pointer analysis)
- And these challenges become especially acute for VLIW
  - o For 4-wide VLIW, need to find 4 operations to fill "one" bubble!
  - Operations in one instruction must be data independent
    - Data forwarding will not work within one instruction (Obviously because they are executing on the same cycle)
  - Operations must also be control independent



### Predicates Help in Compiler Scheduling

- Predicates can enlarge "instruction window" for scheduling
  - Reordering cannot happen across control dependencies

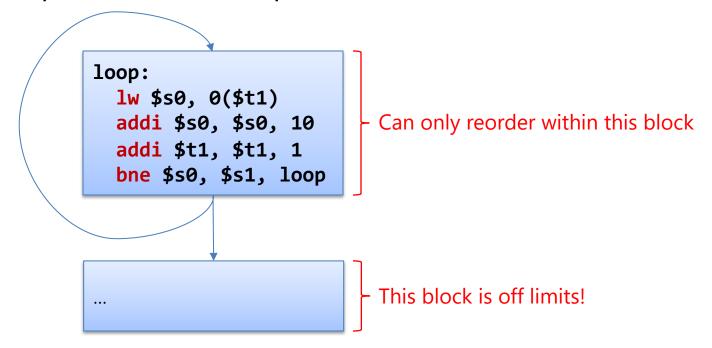
Predicates can convert if-then-else code into one big block

```
lw $t1, 0($s0)
pne $p1, $s1, $s2
lw $t2, 1($s0), !$p1
addi $t1, $t1, 1
lw $t2, 2($s0), $p1
- Can reorder within a larger window!
```



#### But Predicates Cannot Remove Loopback Branches

- **Loops** are particularly challenging to the compiler. Why?
  - Scheduling is limited to within the loop
  - o For tight loops, not much compiler can do with a handful of insts





### Let's use this loop as a running example

```
for (i = 0; i < 100; i++)
  a[i] = a[i] + x;
             Is equivalent to
for (p = &a[0]; p < &a[100]; p++)
  *p = *p + x;
          Translated to MIPS
// Map p \rightarrow $s1, x \rightarrow $s2, and &a[100] \rightarrow $s3
Loop:
  lw $t0, 0($s1)
                             // $t0 = *p
  add $t0, $t0, $s2   // $t0 = $t0 + x
  sw $t0, 0($s1)
                       // *p = $t0
  addi $s1, $s1, 4
                           // p++
  blt $s1, $s3, Loop
                      // loopback if p < &a[100]
```



#### This is the loop schedule on VLIW as-is

• This is the best the compiler can do with current order:

```
Loop:

lw $t0, 0($s1)

add $t0, $t0, $s2

sw $t0, 0($s1)

addi $s1, $s1, 4

blt $s1, $s3, Loop
```

	ALU/Branch Op	Load/Store Op
Loop:	nop	lw \$t0, 0(\$s1)
	nop	nop
	add \$t0, \$t0, \$s2	nop
	addi \$s1, \$s1, 4	SW \$t0, 0(\$s1)
	blt \$s1, \$s3, Loop	X nop

- lw and add separated by 2 cycles  $\rightarrow$  due to use-after-load on \$t0
- add and sw separated by 1 cycle → due to dependence on \$t0
- addi and sw on same cycle → sw uses old value of \$s1
- IPC = 5 / 5 = 1. On a 2-wide VLIW. We need to better!
  - But chain of dependencies prevents any reordering... or does it?



### Dependencies can be broken (with proper patch-up)

We broke the WAR (Write-After-Read) dependence on \$s1!

Loop:			
<u>lw</u>	\$t0,	0(\$s:	1)
addi	<b>\$s1</b> ,	<b>\$</b> s <b>1</b> ,	4
add	\$t0,	\$t0,	<b>\$</b> s2
SW	\$t0,	-4(\$	51)
blt	<b>\$s1</b> ,	\$s3,	Loop

	ALU/Branch Op	Load/Store Op
Loop:	addi \$s1, \$s1, 4	lw \$t0, 0(\$s1)
	nop	nop
	add \$t0, \$t0, \$s2	nop
	<b>blt</b> \$s1, \$s3, Loop	sw \$t0, -4(\$s1)

- Now, sw uses the new value of \$s1 produced by addi
- The compiler compensates by changing the sw offset by -4:
  - $\circ$  sw \$t0, 0(\$s1)  $\to$  sw \$t0, -4(\$s1)
- Now, **IPC** = **5** / **4** = **1.25**. Can we do even better?



Expand scheduling window using multiple iterations of the loop!



# Loop unrolling



#### What is Loop Unrolling?

- Loop unrolling: a compiler technique to enlarge loop body
  - By duplicating loop body for an X number of iterations

- What does this buy us?
  - And expanded scheduling window to reorder and hide bubbles
  - And less instructions to execute as a whole
    - Less frequent loop branches
    - Two i++ are merged into one i+= 2



#### 1. Unroll the loop

```
Loop:
    lw $t0, 0($s1)
    addi $s1, $s1, 4
    add $t0, $t0, $s2
    sw $t0, -4($s1)
    blt $s1, $s3, Loop
```

```
Unroll 2X
```

```
Loop:
 lw $t0, 0($s1)
 addi $s1, $s1, 4
 add $t0, $t0, $s2
      $t0, -4($s1)
 SW
 lw $t1, 0($s1)
 addi $s1, $s1, 4
 add $t1, $t1, $s2
      $t1, -4($s1)
 SW
```

```
blt $s1, $s3, Loop
```

- Instructions are duplicated but using \$t1 instead of \$t0
- This is intentional to minimize false dependencies during reordering



#### 2. Interleave iterations to space out dependencies

```
Loop:
                                   Loop:
     $t0, 0($s1)
                                         $t0, 0($s1)
 lw
                                     lw
                          Reorder!
      $t1, 4($s1)
                                    addi $s1, $s1, 4
 lw
                                    add $t0, $t0, $s2
 addi $s1\\$s1, 4
                                         $t0, -4($s1)
                                     SW
 addi $s1,\$s1, 4
                                         $t1, 0($s1)
                                    lw
                                    addi $s1, $s1, 4
      $t0, $t0, $s2
 add
      $t1, $t1, $s2
                                         $t1, $t1, $s2
 add
                                     add
                                         $t1, -4($s1)
                                     SW
     $t0, -8($s1)
 SW
      $t1, -4($s1)
                                     blt
                                         $s1, $s3, Loop
  SW
 blt $s1, $s3, Loop
```

Interleaving iterations spaces out dependencies by 2X (unroll factor)



#### 3. Merge induction variable increment

```
Loop:
                                 Loop:
 lw $t0, 0($s1)
                                   lw $t0, 0($s1)
                         Merge
 lw $t1, 4($s1)
                                   lw $t1, 4($s1)
                                   addi $s1, $s1, 8
 addi $s1, $s1, 4
 addi $s1, $s1, 4
                                   add $t0, $t0, $s2
                                   add $t1, $t1, $s2
 add $t0, $t0, $s2
 add $t1, $t1, $s2
                                   sw $t0, -8($s1)
                                   sw $t1, -4($s1)
 sw $t0, -8($s1)
    $t1, -4($s1)
 SW
                                   blt $s1, $s3, Loop
 blt $s1, $s3, Loop
```

Two addi \$s1, \$s1, 4 are merged into addi \$s1, \$s1, 8

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#### 4. Schedule unrolled loop onto VLIW

#### Loop:

```
lw $t0, 0($s1)
lw $t1, 4($s1)
addi $s1, $s1, 8
add $t0, $t0, $s2
add $t1, $t1, $s2
sw $t0, -8($s1)
sw $t1, -4($s1)
blt $s1, $s3, Loop
```

	ALU/Branch Op	Load/Store Op
Loop:	nop	lw \$t0, 0(\$s1)
	addi \$s1, \$s1, 8	lw \$t1, 4(\$s1)
	add \$t0, \$t0, \$s2	nop
	add \$t1, \$t1, \$s2	sw \$t0, -8(\$s1)
	blt \$s1, \$s3, Loop	sw \$t1, -4(\$s1)

- Now we spend 5 cycles for 2 iterations of the loop
  - $\circ$  So, 5 / 2 = **2.5** cycles per iteration
  - Much better than the previous 4 cycles for 1 iteration!



#### Unrolling loop 4X buys us even more speedup!

4X Unrolled loop converted to VLIW:

	ALU/Branch Op	Load/Store Op	Inst
Loop:	addi \$s1, \$s1, 16	lw \$t0, 0(\$s1)	1
	nop	<pre>Iw \$t1, -12(\$s1)</pre>	2
	add \$t0, \$t0, \$s2	lw \$t2, -8(\$s1)	3
	add \$t1, \$t1, \$s2	lw \$t3, -4(\$s1)	4
	add \$t2, \$t1, \$s2	sw \$t0, -16(\$s1)	5
	add \$t3, \$t1, \$s2	sw \$t1, -12(\$s1)	6
	nop	sw \$t2, -8(\$s1)	7
	blt \$s1, \$s3, Loop	sw \$t3, -4(\$s1)	8

- Now we spend 8 cycles for 4 iterations of the loop
  - $\circ$  So, 8 / 4 = 2 cycles per iteration
  - Even better than 2.5 cycles per iteration for 2X unrolling



# Unrolling beyond that won't buys us anything

8X Unrolled loop converted to VLIW:

	ALU/Branch Op	Load/Store Op	Inst
Loop:	addi \$s1, \$s1, 32	lw \$t0, 0(\$s1)	1
	nop	lw \$t1, -28(\$s1)	2
	•••	•••	•••
	add \$t1, \$t1, \$s2	lw \$t7, -4(\$s1)	8
	add \$t2, \$t1, \$s2	sw \$t0, -32(\$s1)	9
	add \$t3, \$t1, \$s2	sw \$t1, -28(\$s1)	10
	•••	•••	•••
	blt \$s1, \$s3, Loop	sw \$t7, -4(\$s1)	16

- Now we spend 16 cycles for 8 iterations of the loop
  - $\circ$  So, 16 / 8 = 2 cycles per iteration (no improvement over 4X)
  - 2 is minimum because you need one lw and one sw per iteration



# When should the compiler stop unrolling?

- When dependencies are already sufficiently spaced far apart
- There are constraints that can prevent unrolling even before that
- 1. Limitation in number of registers
  - o More unrolling uses more registers \$t0, \$t1, \$t2, ...
  - o For this reason, VLIW ISAs have many more registers than MIPS
    - Intel Itanium has 256 registers!
- 2. Limitation in code space
  - More unrolling means more code bloat
  - o Embedded processors don't have lots of code memory
  - Matters even for general purpose processors because of caching (Code that overflows i-cache can lead to lots of cache misses)



# List Scheduling



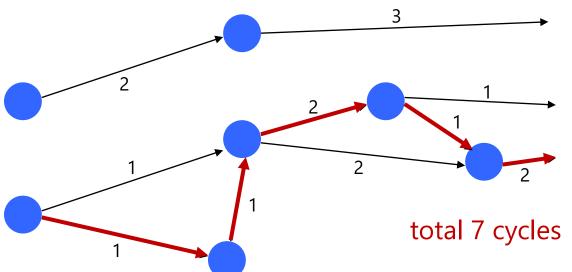
#### How does the compiler schedule instructions?

- Compiler will first expand the **instruction window** that it looks at
  - Instruction window: block of code without branches
  - Compiler uses predication and loop unrolling
- Once compiler has a sizable window, it will construct the schedule
- A popular scheduling algorithm is *list scheduling* 
  - o Idea: list instructions in some order of priority and schedule
  - o Instructions on the critical path should be prioritized
- List scheduling can be used with any statically scheduled processor
  - Simple single-issue statically scheduled processor (not just VLIW)
  - o GPUs are also statically scheduled using list scheduling



#### Critical Path in Code

- At below is a data dependence graph for a code with 7 instructions
  - Nodes are instructions
  - Arrows are data dependencies annotated with required delay
- Q: How long is the critical path in this code?

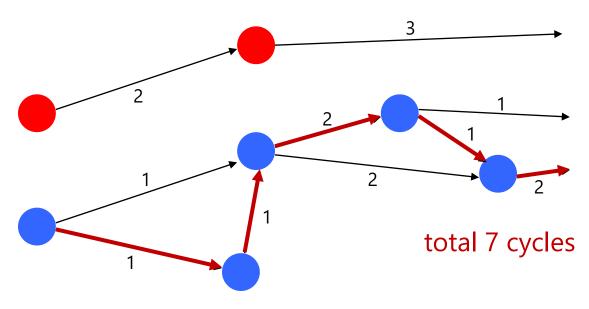


That means, at minimum, this code will take 7 cycles, period. Regardless of how wide your processor is or how well you do your scheduling.



#### Instruction Level Parallelism (ILP)

- The 7 cycles is achievable thanks to **instruction level parallelism** 
  - Property of graph that allows parallel execution of instructions
  - o The nodes marked in red can execute in parallel with blue nodes
- This tell us that this code is where a VLIW processor can shine





#### Maximizing Instruction Level Parallelism (ILP)

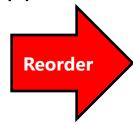
- The more ILP code has, the more VLIW will shine
- So before list scheduling, compiler maximizes ILP in code
  - O What constrains ILP? Data dependencies!
  - Some data dependencies can be removed by the compiler
- There are 3 types of data dependencies actually:
  - o RAW (Read-After-Write): cannot be removed
  - WAR (Write-After-Read): can be removed
  - o WAW (Write-After-Write): can also be removed
- How about Read-After-Read? Not a data dependency.



#### Read-After-Write (RAW) Dependency

- RAW dependencies are also called *true dependencies* 
  - o In the sense that other dependencies are not "real" dependencies
- Suppose we reorder this snippet of code:

```
RAW! (lw t0, 0(s0) addi t1, t0, 4
```



```
addi t1, t0, 4 lw t0, 0(s0)
```

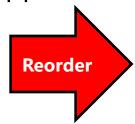
- The code is incorrect because now **t1** has a wrong value
  - Value in to can be read only after written
  - No amount of compiler tinkering will allow this reordering



#### Write-After-Read (WAR) Dependency

- WAR dependencies are also called anti-dependencies
  - In the sense that they are the opposite of true dependencies
- Suppose we reorder this snippet of code:

```
lw t0, 0(s0)
addi t1, t0, 4
lw t0, 0(s1)
```



```
lw t0, 0(s0)
lw t0, 0(s1)
addi t1, t0, 4
```

- The code is again incorrect because **t1** has the wrong value
  - o addi should not read to produced by lw to, O(s1)
- Q: Is there a way for addi to not use that value?
  - o to and to contain different values. Why use the same register?
  - o Just rename register to some other register!



# Removing WAR with SSA

- Static Single Assignment:
  - o Renaming registers when a different value is stored into it
  - A register is assigned a value only a single time (never reused)
- Reordering after converting to SSA form:

```
lw t0, 0(s0)
addi t1, t0, 4
lw t2, 0(s1)

Reorder
lw t0, 0(s0)
lw t2, 0(s1)
addi t1, t0, 4
```

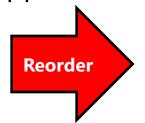
- Note how destination registers always use a new register
  - Yes, if you do this, you will need lots of registers
  - o But, no more WAR dependencies!



# Write-After-Write (WAW) Dependency

- WAW dependencies are also called *false dependencies* 
  - In the sense that they are not real dependencies
- Suppose we reorder this snippet of code:

```
waw! lw t0, 0(s0)
lw t0, 0(s1)
addi t1, t0, 4
```



```
lw t0, 0(s1)
lw t0, 0(s0)
addi t1, t0, 4
```

- The code is again incorrect because t1 has the wrong value
  addi should not read t0 produced by lw t0, 0(s0)
- Q: Is there a way for **addi** to *not use* that value?
  - O Again, rename register to some other register!



#### Removing WAW with SSA

- Again, Static Single Assignment (SSA) to the rescue!
  - SSA removes both WAR and WAW dependencies
- Reordering after converting to SSA form:

```
lw t0, 0(s0)
lw t1, 0(s1)
addi t2, t1, 4

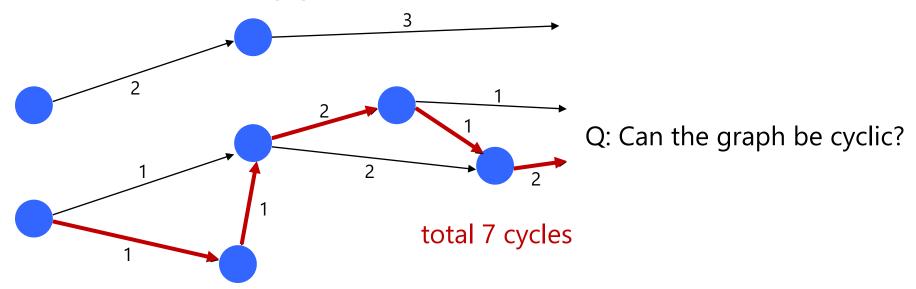
Reorder lw t1, 0(s1)
lw t0, 0(s0)
addi t2, t1, 4
```

- SSA form is now the norm in all mature compilers
  - Clang / LLVM ("Apple" Compiler)
  - GCC (GNU C Compiler)
  - Java Hotspot / OpenJDK Compiler
  - Chrome JavaScript Compiler



# Back to List Scheduling

- With SSA, the only data dependences that remain are RAW ones.
- The critical path length is 7 cycles but that is not always achievable
  - o If processor is not wide enough for the available parallelism
  - o If compiler does a bad job at scheduling instructions
    - → List scheduling guarantees compiler is within 2X of optimal





# List Scheduling is a Greedy Algorithm

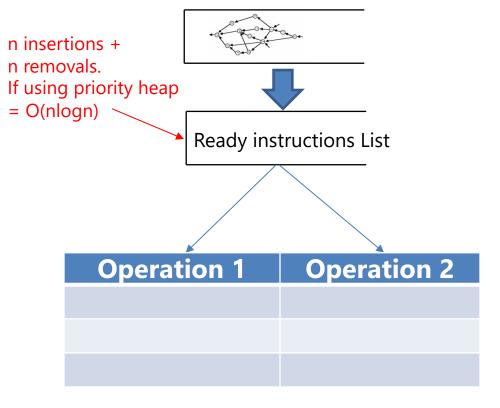
- Idea: Greedily prioritize instructions on the critical path
- Steps:
  - 1. Create a data dependence graph
  - 2. Assign a **priority** to each node (instruction)
    - Priority = critical path length starting from that node
  - 3. Schedule nodes one by one starting from **ready** instructions
    - Ready = all dependencies have been fulfilled (Initially, only roots of dependency chains are ready)
    - When there are multiple nodes that are ready
      - → Choose the node with the highest priority

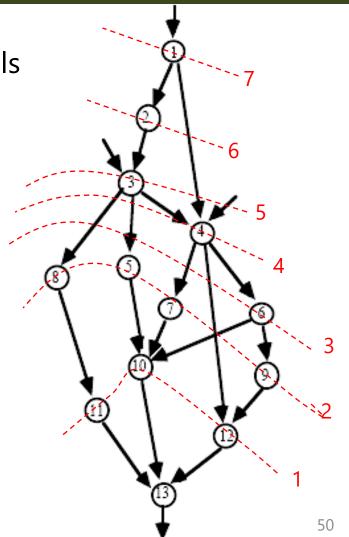


# List Scheduling Example

Assume all edges have a delay of 1

Red dashed lines indicate priority levels







## List Scheduling Example

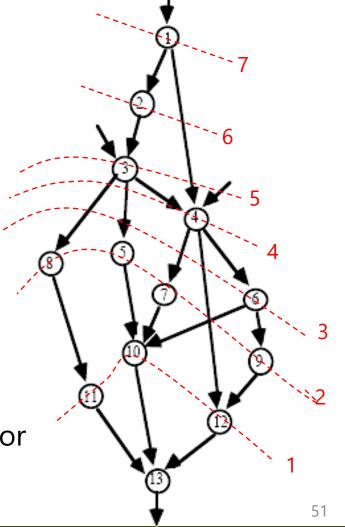
• This will result in the following schedule:

Operation 1	Operation 2
1	
2	
3	
4	5
6	7
8	9
10	11
12	
13	

• 9 cycles. We couldn't achieve 7 cycles!

o But could've if we had a wider processor





#### List Scheduling is not optimal

- Optimal scheduling is an NP-Complete problem:
  - o John L. Hennessy and Thomas Gross. "Postpass Code Optimization of Pipeline Constraints", ACM Trans. Program. Lang. Syst., July 1983.
- List scheduling achieves worst-case **2 1/n** of optimal schedule
  - Where **n** is the width of the processor
  - R. L. Graham. "Bounds on multiprocessing timing anomalies", SIAM Journal of Applied Mathematics, 1969.
- More elaborate algorithms exist to approach optimal
  - Using constraint solvers or machine learning
- Still, list scheduling using critical path lengths is most widely used
- GCC uses other heuristics in addition to critical path length:
  - https://github.com/gcc-mirror/gcc/blob/master/gcc/haifa-sched.cc

