Matrix Theory - Assignment 1

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Problem 1: Write down a unit vector in the xy-plane, making an angle of 30° with the positive direction of the x-axis ?

Solution: Let us consider a unit vector $\vec{a}$ in the xy-plane. Since the vector lies in xy-plane, any vector in this plane is formed by linear combination of $\hat{i}$ (unit vector in direction of x-axis) and $\hat{j}$ (unit vector in direction of y-axis).
This can be written in the form of equation as below:

$$\vec{a} = x\hat{i} + y\hat{j}$$

We know from question that $\vec{a}$ makes an angle of 30° with the positive direction of the x-axis. Similarly, as x and y axis are perpendicular to each other, we can also infer that $\vec{a}$ makes an angle of 60° with the positive direction of the y-axis.

From the definition of dot product we know that,

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

(a) As $\vec{a}$ makes an angle of 30° with the positive direction of the x-axis, let us substitute $\vec{a} = \vec{a}$, $\vec{b} = \hat{i}$ (unit vector along x axis) and $\theta = 30^\circ$.

$$\vec{a} \cdot \hat{i} = |\vec{a}||\hat{i}| \cos 30^\circ$$

As $\vec{a}$ and $\vec{i}$ are unit vectors, their magnitude is 1. Therefore,

$$\vec{a} \cdot \hat{i} = 1 \times 1 \times \frac{\sqrt{3}}{2}$$
$$\therefore \vec{a} \cdot \hat{i} = \frac{\sqrt{3}}{2}$$

Substituting, $\vec{a}$ from above,

$$\left(x\hat{i} + y\hat{j}\right) \cdot \hat{i} = \frac{\sqrt{3}}{2}$$
$$\implies \boxed{x = \frac{\sqrt{3}}{2}}$$

(b) Similarly, as $\vec{a}$ makes an angle of 60° with the positive direction of the y-axis, let us substitute $\vec{a} = \vec{a}$, $\vec{b} = \hat{j}$ (unit vector along y axis) and $\theta = 60^\circ$.

$$\vec{a} \cdot \hat{j} = |\vec{a}||\hat{j}| \cos 60^\circ$$

As $\vec{a}$ and $\vec{j}$ are unit vectors, their magnitude is 1. Therefore,

$$\vec{a} \cdot \hat{j} = 1 \times 1 \times \frac{1}{2}$$
$$\therefore \vec{a} \cdot \hat{j} = \frac{1}{2}$$

Substituting, $\vec{a}$ from above,

$$\left(x\hat{i} + y\hat{j}\right) \cdot \hat{j} = \frac{1}{2}$$
$$\implies \boxed{y = \frac{1}{2}}$$

As, $\vec{a} = \mathrm{x}\hat{i} + \mathrm{y}\hat{j}$
Substituting x and y from above, we get

$$\implies \boxed{\vec{a} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}}$$

This $\vec{a}$ is the unit vector that makes an angle of 30° with the positive x axis.