QR decomposition

KUSUMA PRIYA
EE20MTECH11007

Download latex codes from

https://github.com/KUSUMAPRIYAPULAVARTY/QRdecomposition

1 Question

Perform the QR decomposition of matrix A

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \tag{1.0.1}$$

2 Explanation

If α and β are the columns of a (2×2) matrix $\mathbf{A}$, then A can be decomposed as

$$\mathbf{A} = \mathbf{QR} \tag{2.0.1}$$

$$\text{where, } \mathbf{U} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix}, \tag{2.0.2}$$

$$\text{uppertriangular matrix } \mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.3}$$

$$k_1 = \|\alpha\|, \mathbf{u_1} = \frac{\alpha}{k_1} \tag{2.0.4}$$

$$r_1 = \frac{\mathbf{u_1}^T \beta}{\|\mathbf{u_1}\|^2} \tag{2.0.5}$$

$$\mathbf{u_2} = \frac{\beta - r_1 \mathbf{u_1}}{\|\beta - r_1 \mathbf{u_1}\|}, k_2 = \mathbf{u_2}^T \beta \tag{2.0.6}$$

3 Solution

$$\alpha = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \beta = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{3.0.1}$$

$$\text{From, (2.0.4), } k_1 = \|\alpha\| = \sqrt{10} \tag{3.0.2}$$

$$\text{and } \mathbf{u_1} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{3.0.3}$$

$$\text{From (2.0.5), } r_1 = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{5}{\sqrt{10}} \tag{3.0.4}$$

$$\beta - r_1 \mathbf{u_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \frac{5}{\sqrt{10}} \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{3.0.5}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{-1}{2} \end{pmatrix} \tag{3.0.6}$$

$$\text{From (2.0.6), } \mathbf{u_2} = \frac{\begin{pmatrix} \frac{3}{2} \\ \frac{-1}{2} \end{pmatrix}}{\sqrt{\frac{9}{4} + \frac{1}{4}}} \tag{3.0.7}$$

$$\implies \mathbf{u_2} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}, \tag{3.0.8}$$

$$k_2 = \begin{pmatrix} \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{5}{\sqrt{10}} \tag{3.0.9}$$

Note that,

$$\mathbf{Q}^T \mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{10}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I} \tag{3.0.10}$$

The matrix A can now be rewritten using (2.0.1) as

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \sqrt{10} & \frac{5}{\sqrt{10}} \\ 0 & \frac{5}{\sqrt{10}} \end{pmatrix} \tag{3.0.11}$$