Assignment 1

Kusuma Priya

Download all python codes from

```
https://github.com/KUSUMAPRIYAPULAVARTY/
    assignment1/tree/master/codes
```

and latex-tikz codes from

```
https://github.com/KUSUMAPRIYAPULAVARTY/
    assignment1
```

1 Question No. 40

Two lines passing through the point $\begin{pmatrix}2\\3\end{pmatrix}$ intersect each other at an angle of 60°. If one line has slope 2, find equation of the other line.

2 Explanation

Directional vector of a line having slope 2 is $\begin{pmatrix}1\\2\end{pmatrix}$ Hence normal vector n_1 is given as

$$n_1 = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix}-2\\1\end{pmatrix} \tag{2.0.2}$$

Similarly normal vector for line 2

$$n_2 = \begin{pmatrix}-m_2\\1\end{pmatrix} \tag{2.0.3}$$

Angle between two lines θ can be given by

$$\cos\theta = \frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{2.0.4}$$

$$\implies \cos 60^\circ = \frac{1}{2} \tag{2.0.5}$$

$$= \frac{2m_2+1}{\sqrt{5}\times\sqrt{1+m_2}} \tag{2.0.6}$$

$$\implies 11m_2^2 + 16m_2 - 1 = 0 \tag{2.0.7}$$

Solving, m_2 yields values $\frac{-8+5\sqrt{3}}{11}$ and $\frac{-8-5\sqrt{3}}{11}$ Equation of line with normal vectorn and passing through point A is given by

$$\mathbf{n}^T(\mathbf{X} - A) = 0 \tag{2.0.8}$$

Hence,equation of line with slope $\frac{-8+5\sqrt{3}}{11}$ passing through $\begin{pmatrix}2\\3\end{pmatrix}$ is

$$\begin{pmatrix}\frac{8-5\sqrt{3}}{11} & 1\end{pmatrix}\left(\mathbf{X} - \begin{pmatrix}2\\3\end{pmatrix}\right) = 0 \tag{2.0.9}$$

$$\implies \begin{pmatrix}\frac{8-5\sqrt{3}}{11} & 1\end{pmatrix}\mathbf{X} = \frac{49-10\sqrt{3}}{11} \tag{2.0.10}$$

Similarly,equation of line with slope $\frac{-8-5\sqrt{3}}{11}$ passing through $\begin{pmatrix}2\\3\end{pmatrix}$ is

$$\begin{pmatrix}\frac{8+5\sqrt{3}}{11} & 1\end{pmatrix}\left(\mathbf{X} - \begin{pmatrix}2\\3\end{pmatrix}\right) = 0 \tag{2.0.11}$$

$$\implies \begin{pmatrix}\frac{8+5\sqrt{3}}{11} & 1\end{pmatrix}\mathbf{X} = \frac{49+10\sqrt{3}}{11} \tag{2.0.12}$$

Thus, the required line equations are

$$\begin{pmatrix}\frac{8-5\sqrt{3}}{11} & 1\end{pmatrix}\mathbf{X} = \frac{49-10\sqrt{3}}{11} \tag{2.0.13}$$

$$\text{and}\quad \begin{pmatrix}\frac{8+5\sqrt{3}}{11} & 1\end{pmatrix}\mathbf{X} = \frac{49+10\sqrt{3}}{11} \tag{2.0.14}$$

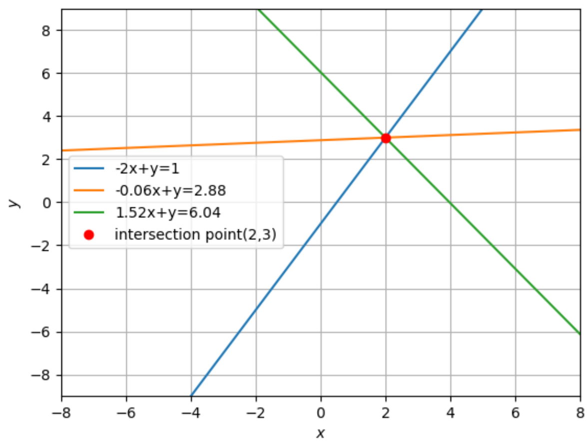

Fig. 0: plot showing intersection of lines