#### 1

# Assignment 1

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Download all python codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment1/tree/master/codes

and latex-tikz codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment1

### 1 Question No. 40

Two lines passing through the point  $\binom{2}{3}$  intersect each other at an angle of  $60^{\circ}$ . If one line has slope 2, find equation of the other line.

#### 2 Explanation

Directional vector of a line having slope 2 is  $\binom{1}{2}$ Hence normal vector  $\mathbf{n}_1$  is given as

$$n_{1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
 (2.0.1)  
=  $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$  (2.0.2)

Similarly normal vector for line 2

$$n_2 = \begin{pmatrix} -m_2 \\ 1 \end{pmatrix} \tag{2.0.3}$$

Angle between two lines  $\theta$  can be given by

$$\cos \theta = \frac{{\mathbf{n_1}}^T \mathbf{n_2}}{\|\mathbf{n_1}\| \|\mathbf{n_2}\|}$$
 (2.0.4)

$$\implies \cos 60^\circ = \frac{1}{2} \tag{2.0.5}$$

$$=\frac{2m_2+1}{\sqrt{5}\times\sqrt{1+m_2}}\tag{2.0.6}$$

$$\implies 11m_2^2 + 16m_2 - 1 = 0 \tag{2.0.7}$$

Solving,  $m_2$  yields values  $\frac{-8+5\sqrt{3}}{11}$  and  $\frac{-8-5\sqrt{3}}{11}$  Equation of line with normal vectorn and passing through point A is given by

$$\mathbf{n}^T(\mathbf{X} - A) = 0 \tag{2.0.8}$$

Hence, equation of line with slope  $\frac{-8+5\sqrt{3}}{11}$  passing through  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$  is

$$\left(\frac{8-5\sqrt{3}}{11} \quad 1\right) \left(\mathbf{X} - \begin{pmatrix} 2\\3 \end{pmatrix}\right) = 0 \tag{2.0.9}$$

$$\implies \left(\frac{8-5\sqrt{3}}{11} \quad 1\right)\mathbf{X} = \frac{49-10\sqrt{3}}{11} \qquad (2.0.10)$$

Similarly, equation of line with slope  $\frac{-8-5\sqrt{3}}{11}$  passing through  $\binom{2}{3}$  is

$$\left(\frac{8+5\sqrt{3}}{11} \quad 1\right)\left(\mathbf{X} - \begin{pmatrix} 2\\3 \end{pmatrix}\right) = 0 \qquad (2.0.11)$$

$$\implies \left(\frac{8+5\sqrt{3}}{11} \quad 1\right)\mathbf{X} = \frac{49+10\sqrt{3}}{11} \tag{2.0.12}$$

(2.0.1) Thus, the required line equations are

$$\left(\frac{8-5\sqrt{3}}{11} \quad 1\right)\mathbf{X} = \frac{49-10\sqrt{3}}{11}$$
 (2.0.13)

and 
$$(2.0.14)$$

$$\left(\frac{8+5\sqrt{3}}{11} \quad 1\right)\mathbf{X} = \frac{49+10\sqrt{3}}{11}$$
 (2.0.15)

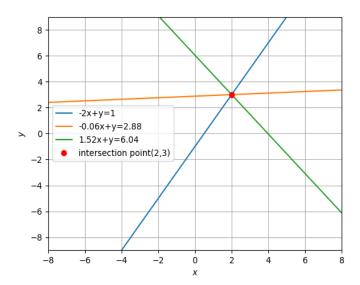


Fig. 0: plot showing intersection of lines