Assignment 1

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Download all python codes from

https://github.com/KUSUMAPRIYAPULAVARTY/
assignment1/tree/master/codes

and latex-tikz codes from

https://github.com/KUSUMAPRIYAPULAVARTY/
assignment1

1 QUESTION No. 40

Two lines passing through the point $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ intersect each other at an angle of 60°. If one line has slope 2, find equation of the other line.

2 EXPLANATION

Directional vector of a line1 having slope 2 is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Hence normal vector of line1 is given as

$$\mathbf{n_1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} -2 \\ 1 \end{pmatrix} \tag{2.0.2}$$

Similarly, normal vector for line 2

$$\mathbf{n_2} = \begin{pmatrix} -m_2 \\ 1 \end{pmatrix} \tag{2.0.3}$$

Angle between two lines θ can be given by

$$\cos\theta = \frac{\mathbf{n_1}^T \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{2.0.4}$$

$$\implies \cos 60° = \frac{1}{2} \tag{2.0.5}$$

$$= \frac{2m_2 + 1}{\sqrt{5} \times \sqrt{1 + m_2}} \tag{2.0.6}$$

$$\implies 11m_2^2 + 16m_2 - 1 = 0 \tag{2.0.7}$$

Solving, m_2 yields values $\frac{-8+5\sqrt{3}}{11}$ and $\frac{-8-5\sqrt{3}}{11}$

Equation of line with normal vectorn and passing through point A is given by

$$\mathbf{n}^T(\mathbf{x} - A) = 0 \tag{2.0.8}$$

Hence,equation of line with slope $\frac{-8+5\sqrt{3}}{11}$ passing through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is

$$\left(\frac{8-5\sqrt{3}}{11} \quad 1\right)\left(\mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = 0 \tag{2.0.9}$$

$$\implies \left(\frac{8-5\sqrt{3}}{11} \quad 1\right)\mathbf{x} = \frac{49 - 10\sqrt{3}}{11} \tag{2.0.10}$$

Similarly,equation of line with slope $\frac{-8-5\sqrt{3}}{11}$ passing through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is

$$\left(\frac{8+5\sqrt{3}}{11} \quad 1\right)\left(\mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = 0 \tag{2.0.11}$$

$$\implies \left(\frac{8+5\sqrt{3}}{11} \quad 1\right)\mathbf{x} = \frac{49 + 10\sqrt{3}}{11} \tag{2.0.12}$$

Thus, the required line equations are

$$\left(\frac{8-5\sqrt{3}}{11} \quad 1\right)\mathbf{x} = \frac{49 - 10\sqrt{3}}{11} \tag{2.0.13}$$

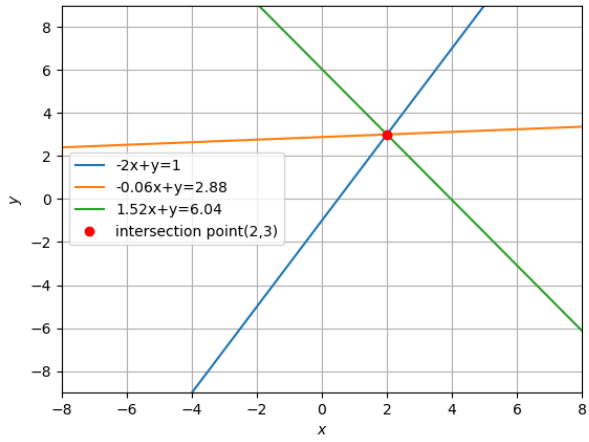$$\text{and} \quad \left(\frac{8+5\sqrt{3}}{11} \quad 1\right)\mathbf{x} = \frac{49 + 10\sqrt{3}}{11} \tag{2.0.14}$$

Fig. 0: plot showing intersection of lines