Assignment 10

KUSUMA PRIYA
EE20MTECH11007

Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment10

1 QUESTION

Let $\mathbf{A}$ be an $m \times n$ matrix and $\mathbf{B}$ be an $n \times k$ matrix.Show that the columns of $\mathbf{C} = \mathbf{AB}$ are linear combinations of columns of $\mathbf{A}$.If $\alpha_1, \alpha_2, \dots, \alpha_n$ are the columns of $\mathbf{A}$ and $\gamma_1, \gamma_2, \dots, \gamma_k$ are the columns of $\mathbf{C}$ then,

$$\gamma_{\mathbf{j}} = \sum_{r=1}^{n} B_{rj}\alpha_{\mathbf{r}} \tag{1.0.1}$$

2 Solution

$$\mathbf{C} = \mathbf{AB} \tag{2.0.1}$$

$$\mathbf{C} = \begin{pmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_k \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{A} = \begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{B} = \begin{pmatrix} \beta_1 & \beta_2 & \dots & \beta_k \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} B_{11} & B_{12} & \dots & B_{1k} \\ B_{21} & B_{22} & \dots & B_{2k} \\ \vdots & \vdots & \dots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nk} \end{pmatrix} \tag{2.0.5}$$

By matrix multiplication, we can write

$$\begin{pmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_k \end{pmatrix} = \begin{pmatrix} \mathbf{A}\beta_1 & \mathbf{A}\beta_2 & \dots & \mathbf{A}\beta_k \end{pmatrix} \tag{2.0.6}$$

Consider γ_1

$$\gamma_1 = \mathbf{A}\beta_1 \tag{2.0.7}$$

$$= \begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \end{pmatrix} \begin{pmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{n1} \end{pmatrix} \tag{2.0.8}$$

$$= B_{11}\alpha_1 + B_{21}\alpha_2 + \dots + B_{n1}\alpha_n \tag{2.0.9}$$

Similarly, considering j^{th} column of $\mathbf{C}$

$$\gamma_{\mathbf{j}} = \begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \end{pmatrix} \begin{pmatrix} B_{1j} \\ B_{2j} \\ \vdots \\ B_{nj} \end{pmatrix} \tag{2.0.10}$$

$$= B_{1j}\alpha_1 + B_{2j}\alpha_2 + \dots + B_{nj}\alpha_n \tag{2.0.11}$$

$$\implies \gamma_{\mathbf{j}} = \sum_{r=1}^{n} B_{rj}\alpha_{\mathbf{r}} \tag{2.0.12}$$

which proves that columns of $\mathbf{C}$ are linear combinations of columns of $\mathbf{A}$