Assignment 11

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment11

1 Question

Let $\mathbf{W_1}$ and $\mathbf{W_2}$ be subspaces of a vector space $\mathbf{V}$ such that

$$\mathbf{W_1} + \mathbf{W_2} = \mathbf{V} \tag{1.0.1}$$

$$\text{and } \mathbf{W_1} \cap \mathbf{W_2} = \mathbf{0} \tag{1.0.2}$$

Prove that for each vector α in $\mathbf{V}$ there are unique vectors α_1 in $\mathbf{W_1}$ and α_2 in $\mathbf{W_2}$ such that

$$\alpha = \alpha_1 + \alpha_2 \tag{1.0.3}$$

2 Solution

Suppose, vectors α_1 and α_2 are not unique. Consider

$$\alpha_1' \in \mathbf{W_1}, \tag{2.0.1}$$

$$\alpha_2' \in \mathbf{W_2} \tag{2.0.2}$$

$$\text{such that } \alpha = \alpha_1' + \alpha_2' \tag{2.0.3}$$

(1.0.3) and (2.0.3) indicate

$$\alpha_1 + \alpha_2 = \alpha_1' + \alpha_2' \tag{2.0.4}$$

$$\implies \alpha_1 - \alpha_1' = \alpha_2' - \alpha_2 \tag{2.0.5}$$

For α_1 and α_1' lying in subspace $\mathbf{W_1}$, defined on field F, the following holds

$$\alpha_1 + c\alpha_1' \in \mathbf{W_1}, c \in F \tag{2.0.6}$$

$$c = -1 \implies \alpha_1 - \alpha_1' \in \mathbf{W_1} \tag{2.0.7}$$

$$\text{Similarly, } \alpha_2' - \alpha_2 \in \mathbf{W_2} \tag{2.0.8}$$

$$(2.0.5) \implies \alpha_1 - \alpha_1' \in \mathbf{W_2} \tag{2.0.9}$$

(1.0.2),(2.0.7),(2.0.9) indicate

$$\alpha_1 - \alpha_1' = \alpha_2' - \alpha_2 = \mathbf{0} \tag{2.0.10}$$

$$\implies \alpha_1 = \alpha_1' \tag{2.0.11}$$

$$\alpha_2 = \alpha_2' \tag{2.0.12}$$

So, there exists a unique $\alpha_1 \in \mathbf{W_1}$ and $\alpha_2 \in \mathbf{W_2}$ such that

$$\alpha = \alpha_1 + \alpha_2 \tag{2.0.13}$$

where $\alpha \in \mathbf{V}$