

Assignment 12

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Download codes from

<https://github.com/KUSUMAPRIYAPULAVARTY/assignment12>

1 QUESTION

Let \mathbf{V} be the set of real numbers. Regard \mathbf{V} as a vector space over the field of rational numbers, with usual operations. Prove that this vector space is not finite-dimensional.

2 SOLUTION

Given \mathbf{V} is a vector space over field Q (rational numbers)

It is finite dimensional with dimensionality n if every vector \mathbf{v} in \mathbf{V} can be written as

$$\mathbf{v} = \sum_{i=0}^{n-1} c_i \alpha_i \quad (2.0.1)$$

$$\text{where } c_i \in Q \quad (2.0.2)$$

$$\text{and } \mathbf{B} = \{\alpha_0, \alpha_1, \dots, \alpha_{n-1}\} \quad (2.0.3)$$

is the basis with linearly independent α_i

that is, basis is the largest set with linearly independent vectors from \mathbf{V}

Consider the space of polynomials in x of degree less than equal to one where x is irrational.

Proving that $\{1, x\}$ forms its basis,

Assume there exists non zero $\beta_0, \beta_1 \in Q$ such that

$$\beta_0 + \beta_1 x = 0 \quad (2.0.4)$$

$$\implies x = -\frac{\beta_0}{\beta_1} \quad (2.0.5)$$

But x is irrational and $-\frac{\beta_0}{\beta_1}$ is rational so (2.0.5) can't be possible so $\beta_0, \beta_1 = 0$

Hence $\{1, x\}$ are independent.

Similarly for the set $\{1, x, x^2\}$

for $\beta_0, \beta_1, \beta_2 \in Q$

$$\beta_0 + \beta_1 x + \beta_2 x^2 = 0 \quad (2.0.6)$$

$\beta_1 x + \beta_2 x^2$ is irrational and β_0 is rational. Therefore

$$\beta_0 = 0 \quad (2.0.7)$$

$$\text{and } \beta_1 x + \beta_2 x^2 = 0, (x \neq 0) \quad (2.0.8)$$

$$\implies \beta_1 + \beta_2 x = 0 \quad (2.0.9)$$

$$\implies \beta_1, \beta_2 = 0 \quad (2.0.10)$$

$$\therefore \beta_0 + \beta_1 x + \beta_2 x^2 = 0 \quad (2.0.11)$$

$$\iff \beta_0, \beta_1, \beta_2 = 0 \quad (2.0.12)$$

Hence $\{1, x, x^2\}$ are independent and form basis for space of polynomials of degree less than equal to two.