Assignment 12

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let $\mathbf{V}$ be the set of real numbers.Regard $\mathbf{V}$ as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 Solution

Given $\mathbf{V}$ is a vector space over field Q (rational numbers)
It is finite dimensional with dimensionality n if every vector $\mathbf{v}$ in $\mathbf{V}$ can be written as

$$\mathbf{v} = \sum_{i=0}^{n-1} c_i \alpha_i \tag{2.0.1}$$

$$\text{where } c_i \in Q \tag{2.0.2}$$

$$\text{and } \mathbf{B} = \{\alpha_0, \alpha_1, \ldots, \alpha_{n-1}\} \tag{2.0.3}$$

is the basis with linearly independent α_i
We know that e^k is a vector that lies in $\mathbf{V}$

$$e^k = \sum_{i=0}^{\infty} \frac{k^i}{i!} \tag{2.0.4}$$

$$= 1 + \frac{k}{1!} + \frac{k^2}{2!} + \frac{k^3}{3!} + \ldots \tag{2.0.5}$$

Representing e^k as a linear combination of basis vectors

$$e^k = \sum_{i=0}^{n-1} c_i \alpha_i \tag{2.0.6}$$

$$\text{where } c_i = \begin{cases} \frac{1}{i!} & \text{if } \alpha_i = k^i \\ 0 & \text{otherwise} \end{cases} \tag{2.0.7}$$

Clearly the set of vectors $\left\{1, k, k^2, k^3, \ldots \infty\right\}$ forms a linearly independent set since
for $\beta_0, \beta_1, \beta_2, \ldots \in Q$

$$\beta_0(1) + \beta_1(k) + \beta_2(k^2) + \ldots = 0 \tag{2.0.8}$$

$$\iff \beta_0, \beta_1, \beta_2, \ldots = 0 \tag{2.0.9}$$

Since, (2.0.4) refers to an infinite summation, the representation in (2.0.6) is achieved only if n is ∞
So, the vector space $\mathbf{V}$ of real numbers is not finite dimensional over the field of rational numbers Q