Assignment 12

KUSUMA PRIYA
EE20MTECH11007

Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let **V** be the set of real numbers.Regard **V** as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 SOLUTION

Given **V** is a vector space over field Q (rational numbers)
It is finite dimensional if every vector **v** in **V** can be written as

$$\mathbf{v} = \sum_{i=1}^{n} c_i \alpha_i \tag{2.0.1}$$

$$\text{where } c_i \in Q \tag{2.0.2}$$

$$\text{and } \mathbf{B} = \{\alpha_1, \alpha_2, \ldots, \alpha_n\} \tag{2.0.3}$$

is the basis with linearly independent α_i
To denote the vectors $\left\{e, e^2, e^3, \ldots\right\}$ lying in the vector space **V**,the basis vectors have to contain e raised to some value.
Let e is one of the basis vectors and no other higher powers of e are in the basis

$$e^2 = e.e \tag{2.0.4}$$

$$\text{So,for rational } c_i,\ e^2 \neq \sum c_i \alpha_i \tag{2.0.5}$$

Since e^2 does not lie in span of **B**

$$\mathbf{B}' = \mathbf{B} \cup \left\{e^2\right\} \tag{2.0.6}$$

also has linearly independent vectors and can be the basis of **V**. But **B′** also cannot span the whole **V** since it cannot generate e^3 by any combinations of **B′** and this continues for any e^n present in the basis.
From this we can conclude that the vector space **V** of real numbers is not finite dimensional over the field of rational numbers Q