Assignment 12

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let $\mathbf{V}$ be the set of real numbers.Regard $\mathbf{V}$ as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 Solution

Given $\mathbf{V}$ is a vector space over field Q (rational numbers)
It is finite dimensional if every vector $\mathbf{v}$ in $\mathbf{V}$ can be written as

$$\mathbf{v} = \sum_{i=1}^{n} c_i \alpha_i \tag{2.0.1}$$

$$\text{where } c_i \in Q \tag{2.0.2}$$

$$\text{and } \mathbf{B} = \left(\alpha_1, \alpha_2, \ldots, \alpha_n\right) \tag{2.0.3}$$

is the basis with linearly independent α_i
Let e^n is one of the basis vectors and no other higher powers of e are in the basis

$$e^{n+1} = e.e^n \tag{2.0.4}$$

$$= e^2.e^{n-1} \tag{2.0.5}$$

$$\vdots \tag{2.0.6}$$

$$\text{So,for rational } c_i, \tag{2.0.7}$$

$$e^{n+1} \neq \sum c_i \alpha_i \tag{2.0.8}$$

Since e^{n+1} does not lie in span of $\mathbf{B}$

$$\mathbf{B}' = \mathbf{B} \cup \left(e^{n+1}\right) \tag{2.0.9}$$

also has linearly independent vectors and can be the basis of $\mathbf{V}$. But $\mathbf{B}'$ also cannot span the whole $\mathbf{V}$ since it cannot generate e^{n+2} by any combinations of $\mathbf{B}'$
From this we can conclude that the vector space $\mathbf{V}$ of real numbers is not finite dimensional over the field of rational numbers Q