Assignment 12

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let **V** be the set of real numbers.Regard **V** as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 SOLUTION

Given **V** is a vector space over field Q (rational numbers)
It is finite dimensional with dimensionality n if every vector **v** in **V** can be written as

$$\mathbf{v} = \sum_{i=0}^{n-1} c_i \alpha_i \tag{2.0.1}$$

$$\text{where } c_i \in Q \tag{2.0.2}$$

$$\text{and } \mathbf{B} = \{\alpha_0, \alpha_1, \ldots, \alpha_{n-1}\} \tag{2.0.3}$$

is the basis with linearly independent α_i
that is, basis is the largest set with linearly independent vectors from **V**
The set of infinite vectors $\mathbf{S} = \left\{ \sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt{7}, \ldots \right\}$ indicating the square root of prime numbers are irrational and lie in **V**
Consider the set of vectors $\mathbf{S_1} = \left\{ \sqrt{2},\ \sqrt{3} \right\} \in \mathbf{V}$
We can write

$$\text{for } \beta_0, \beta_1 \in Q \tag{2.0.4}$$

$$\beta_0(\sqrt{2}) + \beta_1(\sqrt{3}) = 0 \iff \beta_0, \beta_1 = 0 \tag{2.0.5}$$

Hence they are linearly independent.
Now considering

$$\mathbf{S_2} = \mathbf{S_1} \cup \left\{ \sqrt{5} \right\} \tag{2.0.6}$$

$$\text{for } \beta_0, \beta_1, \beta_2 \in Q \tag{2.0.7}$$

$$\beta_0(\sqrt{2}) + \beta_1(\sqrt{3}) + \beta_3(\sqrt{5}) = 0 \tag{2.0.8}$$

$$\iff \beta_0, \beta_1, \beta_2 = 0 \tag{2.0.9}$$

Following this, the set $\mathbf{S} = \left\{ \sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt{7}, \ldots \infty \right\}$ is a set of independent vectors that lie in the vector space **V** which indicates that the largest set with linearly independent vectors in **V** has infinite elements in it.
Hence, the vector space of real numbers is not finite dimensional over the field of rational numbers.