Assignment 12

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let **V** be the set of real numbers.Regard **V** as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 Solution

Given **V** is a vector space over field Q (rational numbers)
It is finite dimensional if every vector **v** in **V** can be written as

$$\mathbf{v} = \sum_{i=1}^{n} c_i \alpha_i \tag{2.0.1}$$

$$\text{where } c_i \in Q \tag{2.0.2}$$

$$\text{and } \mathbf{B} = \left(\alpha_1, \alpha_2, \ldots, \alpha_n\right) \tag{2.0.3}$$

is the basis with linearly independent α_i
Let e^n is one of the basis vectors and no other higher powers of e are in the basis

$$e^{n+1} = e.e^n \tag{2.0.4}$$

$$= e^2.e^{n-1} \ldots \tag{2.0.5}$$

$$\text{So,for rational } c_i,\ e^{n+1} \neq \sum c_i \alpha_i \tag{2.0.6}$$

Since e^{n+1} does not lie in span of **B**

$$\mathbf{B}' = \mathbf{B} \cup \left(e^{n+1}\right) \tag{2.0.7}$$

also has linearly independent vectors and can be the basis of **V**. But **B′** also cannot span the whole **V** since it cannot generate e^{n+2} by any combinations of **B′**
From this we can conclude that the vector space **V** of real numbers is not finite dimensional over the field of rational numbers Q