Assignment 12

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let **V** be the set of real numbers.Regard **V** as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 Solution

Given **V** is a vector space over field Q (rational numbers)
It is finite dimensional with dimensionality n if every vector $\mathbf{v}$ in **V** can be written as

$$\mathbf{v} = \sum_{i=1}^{n} c_i \alpha_i \tag{2.0.1}$$

$$\text{where } c_i \in Q \tag{2.0.2}$$

$$\text{and } \mathbf{B} = \{\alpha_1, \alpha_2, \ldots, \alpha_n\} \tag{2.0.3}$$

is the basis with linearly independent α_i
To denote the subspace of vectors

$$\mathbf{V}' = \left\{e, e^2, e^3, \ldots\right\} \tag{2.0.4}$$

lying in the vector space **V**,we consider the infinite series

$$e^k = \sum_{n=0}^{\infty} \frac{k^n}{n!} \tag{2.0.5}$$

$$= 1 + \frac{k}{1!} + \frac{k^2}{2!} + \frac{k^3}{3!} + \ldots \tag{2.0.6}$$

$$\tag{2.0.7}$$

where the scalars $\left\{\frac{1}{0!}, \frac{1}{1!}, \frac{1}{2!}, \ldots\right\} \in Q$
and the basis is $\left\{1, k, k^2, \ldots\right\}$
Since this is an infinite series,we have the dimensionality of $\mathbf{V}'$ and hence of **V** as ∞

So, the vector space **V** of real numbers is not finite dimensional over the field of rational numbers Q