

Assignment 12

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Download codes from

<https://github.com/KUSUMAPRIYAPULAVARTY/assignment12>

1 QUESTION

Let \mathbf{V} be the set of real numbers. Regard \mathbf{V} as a vector space over the field of rational numbers, with usual operations. Prove that this vector space is not finite-dimensional.

2 SOLUTION

Given \mathbf{V} is a vector space over field Q (rational numbers)

It is finite dimensional with dimensionality n if every vector \mathbf{v} in \mathbf{V} can be written as

$$\mathbf{v} = \sum_{i=0}^{n-1} c_i \alpha_i \quad (2.0.1)$$

$$\text{where } c_i \in Q \quad (2.0.2)$$

$$\text{and } \mathbf{B} = \{\alpha_0, \alpha_1, \dots, \alpha_{n-1}\} \quad (2.0.3)$$

is the basis with linearly independent α_i

that is, basis is the largest set with linearly independent vectors.

Consider the set of vectors $\{1, k\} \in \mathbf{V}$ is linearly independent since,

$$\text{for } \beta_0, \beta_1 \in Q \quad (2.0.4)$$

$$\beta_0(1) + \beta_1(k) = 0 \iff \beta_0, \beta_1 = 0 \quad (2.0.5)$$