Assignment 12

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment12

1 QUESTION

Let **V** be the set of real numbers.Regard **V** as a vector space over the field of rational numbers,with usual operations.Prove that this vector space is not finite-dimensional.

2 SOLUTION

Given **V** is a vector space over field Q (rational numbers)
It is finite dimensional with dimensionality n if every vector **v** in **V** can be written as

$$\mathbf{v} = \sum_{i=0}^{n-1} c_i \alpha_i \quad (2.0.1)$$

$$\text{where } c_i \in Q \quad (2.0.2)$$

$$\text{and } \mathbf{B} = \{\alpha_0, \alpha_1, \ldots, \alpha_{n-1}\} \quad (2.0.3)$$

is the basis with linearly independent α_i
that is, basis is the largest set with linearly independent vectors from **V**
Consider the space of polynomials in x of degree less than equal to one where x is irrational.
Proving that $\{1, x\}$ forms its basis,
Assume there exists non zero $\beta_0, \beta_1 \in Q$ such that

$$\beta_0 + \beta_1 x = 0 \quad (2.0.4)$$

$$\implies x = -\frac{\beta_0}{\beta_1} \quad (2.0.5)$$

But x is irrational and $-\frac{\beta_0}{\beta_1}$ is rational so (2.0.5) can't be possible so $\beta_0, \beta_1 = 0$
Hence $\{1, x\}$ are independent