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Assignment 13

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/ assignment13

1 QUESTION

Let F be a subfield of the complex numbers and let T be the function from F^3 into F^3 defined by

$$T(x_1, x_2, x_3) = (1.0.1)$$

$$(x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$$
 (1.0.2)

- (a) Verify that T is a linear transformation.
- (b) If (a,b,c) is a vector in F^3 , what are the conditions on a, b, c that the vector be in the range of T? What is the rank of T?
- (c) What are the conditions on a, b, c that (a, b, c)be in the null space of T? What is the nullity of T?

2 Solution

Representing the transformation in matrix form

$$T(x_1, x_2, x_3) = \mathbf{T}\mathbf{x}$$
 (2.0.1)

$$\mathbf{T} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ -1 & -2 & 2 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{2.0.3}$$

2.1 Part (a)

Consider the matrices $\mathbf{x}, \mathbf{y} \in F^3$ and the scalar

By the associativity of matrix multiplications, we can write

$$\mathbf{T}(c\mathbf{x} + \mathbf{y}) = \mathbf{T}(c\mathbf{x}) + \mathbf{T}\mathbf{y}$$
 (2.1.1)

$$= c\mathbf{T}\mathbf{x} + \mathbf{T}\mathbf{y} \tag{2.1.2}$$

So, T is a linear transformation.

2.2 Part (b)

range(**T**)= $\{$ **y** : **Tx** = **y** where **x**, **y** \in $F^3\}$

$$\mathbf{y} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{2.2.1}$$

$$\mathbf{T}\mathbf{x} = \mathbf{y} \qquad (2.2.2)$$

$$\implies$$
 BTx = By (2.2.3)

$$\implies \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0\\ \frac{-2}{3} & \frac{1}{3} & 0\\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2\\ 2 & 1 & 0\\ -1 & -2 & 2 \end{pmatrix} \mathbf{x} = (2.2.4)$$

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0\\ \frac{-2}{3} & \frac{1}{3} & 0\\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a\\b\\c \end{pmatrix} \qquad (2.2.5)$$

$$\begin{pmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{a+b}{3} \\ \frac{-2a+b}{3} \\ -a+b+c \end{pmatrix}$$
 (2.2.6)

So, rank(T)=2 and

$$-a + b + c = 0 (2.2.7)$$

 $I(x_1, x_2, x_3) = \mathbf{Tx}$ $\mathbf{T} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ -1 & -2 & 2 \end{pmatrix}$ (2.0.1) $All vectors \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in F^3 \text{ that satisfy } (2.2.7) \text{ lie in the}$ $(2.0.2) \text{ range of } \mathbf{T}$

2.3 Part (c)

nullspace(T)= $\{ \mathbf{x} : \mathbf{T}\mathbf{x} = \mathbf{0} \text{ where } \mathbf{x} \in F^3 \}$

$$\mathbf{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{T}\mathbf{x} = \mathbf{0} \tag{2.3.2}$$

$$\mathbf{BTx} = \mathbf{0} \tag{2.3.3}$$

where BT is in reduced row echelon form

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0\\ \frac{-2}{3} & \frac{1}{3} & 0\\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2\\ 2 & 1 & 0\\ -1 & -2 & 2 \end{pmatrix} \mathbf{x} = \mathbf{0}$$
 (2.3.4)

$$\implies \begin{pmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
 (2.3.5)

$$\implies a + \frac{2}{3}c = 0 \qquad (2.3.6)$$

$$b - \frac{4}{3}c = 0 \qquad (2.3.7)$$

$$b - \frac{4}{3}c = 0 \tag{2.3.7}$$

The number of free variables in the reduced row echelon form of T is 1 hence nullity(T) =1

So, the null space of **T** is set of all vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \in F^3$

that satisfy (2.3.6) and (2.3.7)

Note

 $rank(\mathbf{T}) + nullity(\mathbf{T}) = 2 + 1 = dim(F^3)$