Assignment 15

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment15

1 QUESTION

Let $\mathbf{V}$ be the set of complex numbers regarded as a vector space over the field of real numbers.We define a function T from $\mathbf{V}$ into the space of 2×2 real matrices,as follows.If $z = x + iy$ with x and y real numbers,then

$$T(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{1.0.1}$$

How would you describe the range of T?

2 SOLUTION

$$T : \mathbf{V} \to R^{2\times 2} \tag{2.0.1}$$

where $R^{2\times 2}$,is the space of all 2×2 real matrices

$$T(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.3}$$

$$T(\mathbf{x}) = \begin{pmatrix} \begin{pmatrix} 1 & 7 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 0 & 5 \end{pmatrix}\mathbf{x} \\ \begin{pmatrix} 0 & -10 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 1 & -7 \end{pmatrix}\mathbf{x} \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix}\mathbf{x} \end{pmatrix} \tag{2.0.5}$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \tag{2.0.7}$$

$$\implies T(\mathbf{x}) = \begin{pmatrix} \mathbf{A}\mathbf{x} & \mathbf{B}\mathbf{x} \end{pmatrix} \tag{2.0.8}$$

$$T(\mathbf{x}) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{x} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{2\times 1} & \mathbf{x} \end{pmatrix} \tag{2.0.9}$$

The kronecker product of $\mathbf{I}, \mathbf{x}$ gives the block matrix

$$\mathbf{I}_{2\times 2} \otimes \mathbf{x}_{2\times 1} = \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix}_{4\times 2} \tag{2.0.10}$$

$$(2.0.9) \implies T(\mathbf{x}) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \mathbf{x} \tag{2.0.11}$$

Verifying if the matrices $\mathbf{A}, \mathbf{B}$ are independent.

$$c_1\mathbf{A} + c_2\mathbf{B} = \mathbf{0} \tag{2.0.12}$$

$$\begin{pmatrix} 1 & 0 \\ 7 & 5 \\ 0 & 1 \\ -10 & -7 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{0} \tag{2.0.13}$$

$$\begin{pmatrix} 1 & 0 \\ 7 & 5 \\ 0 & 1 \\ -10 & -7 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - 7R_1]{R4 \leftarrow R_4 + 10R_1} \begin{pmatrix} 1 & 0 \\ 0 & 5 \\ 0 & 1 \\ 0 & -7 \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow[R_2 \leftarrow \frac{R_2}{5}]{R_4 \leftarrow \frac{R_4}{-7}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \tag{2.0.15}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_2]{R_4 \leftarrow R_4 - R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.16}$$

$$\implies c_1, c_2 = 0 \tag{2.0.17}$$

Hence $\mathbf{A}, \mathbf{B}$ are independent and

$$\text{range}(T) = \tag{2.0.18}$$

$$\text{span of } \left\{ \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix}, \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \right\} \tag{2.0.19}$$