# Assignment 15

KUSUMA PRIYA
EE20MTECH11007

Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment15

## 1 QUESTION

Let $\mathbf{V}$ be the set of complex numbers regarded as a vector space over the field of real numbers.We define a function $T$ from $\mathbf{V}$ into the space of $2 \times 2$ real matrices,as follows.If $z = x + iy$ with $x$ and $y$ real numbers,then

$$T(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{1.0.1}$$

How would you describe the range of $T$?

## 2 Solution

$$T : \mathbf{V} \to R^{2\times 2} \tag{2.0.1}$$

where $R^{2\times 2}$,is the space of all $2 \times 2$ real matrices

$$T(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.3}$$

$$T(\mathbf{x}) = \begin{pmatrix} \begin{pmatrix} 1 & 7 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 0 & 5 \end{pmatrix}\mathbf{x} \\ \begin{pmatrix} 0 & -10 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 1 & -7 \end{pmatrix}\mathbf{x} \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix}\mathbf{x} & \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix}\mathbf{x} \end{pmatrix} \tag{2.0.5}$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \tag{2.0.7}$$

$$\implies T(\mathbf{x}) = \begin{pmatrix} \mathbf{Ax} & \mathbf{Bx} \end{pmatrix} \tag{2.0.8}$$

$$T(\mathbf{x}) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{x} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{2\times 1} & \mathbf{x} \end{pmatrix} \tag{2.0.9}$$

The kronecker product of $\mathbf{I}, \mathbf{x}$ gives the block matrix

$$\mathbf{I}_{2\times 2} \otimes \mathbf{x}_{2\times 1} = \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix}_{4\times 2} \tag{2.0.10}$$

$$(2.0.9) \implies T(\mathbf{x}) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \mathbf{x} \tag{2.0.11}$$

$$= \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \left[ \begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \end{pmatrix} \right] \tag{2.0.12}$$

$$= x \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.13}$$

$$\mathbf{I} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{I} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.15}$$

Kronecker product in the first term of (2.0.13) picks out 1st columns of $\mathbf{A}, \mathbf{B}$ and in the second term picks out 2nd columns of $\mathbf{A}, \mathbf{B}$ so basis for range($T$) is

$$\left\{ \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \mathbf{I} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} = \tag{2.0.16}$$

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 7 & 5 \\ -10 & -7 \end{pmatrix} \right\} \tag{2.0.17}$$

$$\text{range}(T) = \tag{2.0.18}$$

$$\text{span of } \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 7 & 5 \\ -10 & -7 \end{pmatrix} \right\} \tag{2.0.19}$$