

Assignment 15

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Download codes from

<https://github.com/KUSUMAPRIYAPULAVARTY/assignment15>

The kronecker product of \mathbf{I}, \mathbf{x} gives the block matrix

$$\mathbf{I}_{2 \times 2} \otimes \mathbf{x}_{2 \times 1} = \begin{pmatrix} \mathbf{x} & \mathbf{0} \\ \mathbf{0} & \mathbf{x} \end{pmatrix}_{4 \times 2} \quad (2.0.10)$$

$$(2.0.9) \implies T(\mathbf{x}) = (\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \mathbf{x} \quad (2.0.11)$$

$$= (\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \left[\begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \end{pmatrix} \right] \quad (2.0.12)$$

$$= (\mathbf{A} \ \mathbf{B}) \left[\mathbf{I} \otimes \begin{pmatrix} x \\ 0 \end{pmatrix} + \mathbf{I} \otimes \begin{pmatrix} 0 \\ y \end{pmatrix} \right] \quad (2.0.13)$$

$$= (\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \begin{pmatrix} x \\ 0 \end{pmatrix} + (\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \begin{pmatrix} 0 \\ y \end{pmatrix} \quad (2.0.14)$$

$$= x(\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y(\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{x} = x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.16)$$

$$T(\mathbf{x}) = x(\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y(\mathbf{A} \ \mathbf{B}) \mathbf{I} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.17)$$

1 QUESTION

Let \mathbf{V} be the set of complex numbers regarded as a vector space over the field of real numbers. We define a function T from \mathbf{V} into the space of 2×2 real matrices, as follows. If $z = x + iy$ with x and y real numbers, then

$$T(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (1.0.1)$$

How would you describe the range of T ?

2 SOLUTION

$$T : \mathbf{V} \rightarrow R^{2 \times 2} \quad (2.0.1)$$

where $R^{2 \times 2}$, is the space of all 2×2 real matrices

$$T(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.3)$$

$$T(\mathbf{x}) = \begin{pmatrix} (1 \ 7)\mathbf{x} & (0 \ 5)\mathbf{x} \\ (0 \ -10)\mathbf{x} & (1 \ -7)\mathbf{x} \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} (1 \ 7)\mathbf{x} & (0 \ 5)\mathbf{x} \\ (0 \ -10)\mathbf{x} & (1 \ -7)\mathbf{x} \end{pmatrix} \quad (2.0.5)$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \quad (2.0.7)$$

$$\implies T(\mathbf{x}) = (\mathbf{A}\mathbf{x} \ \mathbf{B}\mathbf{x}) \quad (2.0.8)$$

$$T(\mathbf{x}) = (\mathbf{A} \ \mathbf{B}) \begin{pmatrix} \mathbf{x} & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{2 \times 1} & \mathbf{x} \end{pmatrix} \quad (2.0.9)$$