Assignment 15

KUSUMA PRIYA
EE20MTECH11007

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1 QUESTION

Let **V** be the set of complex numbers regarded as a vector space over the field of real numbers.We define a function T from **V** into the space of 2×2 real matrices,as follows.If $z = x + iy$ with x and y real numbers,then

$$T(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (1.0.1)$$

How would you decribe the range of T?

2 SOLUTION

$$T : \mathbf{V} \to R^{2 \times 2} \quad (2.0.1)$$

where $R^{2 \times 2}$,is the space of all 2×2 real matrices
If R^4 is the space of all 4-tuple vectors,then $R^{2 \times 2}$ and R^4 are isomorphic with

$$T_2 \left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} \right] = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \quad (2.0.2)$$

$$T_2 : R^{2 \times 2} \to R^4 \quad (2.0.3)$$

Rewriting (1.0.1)

$$T_2 T(z) = \begin{pmatrix} x + 7y \\ 5y \\ -10y \\ x - 7y \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} 1 & 7 \\ 0 & 5 \\ 0 & -10 \\ 1 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.5)$$

$$T_2 T : \mathbf{V} \to R^4 \quad (2.0.6)$$

$$rref \begin{pmatrix} 1 & 0 & 0 & 1 \\ 7 & 5 & -10 & -7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -\frac{14}{5} \end{pmatrix} \quad (2.0.7)$$

So,the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ 5 \\ -10 \\ -7 \end{pmatrix}$ are linearly independent.

From, (2.0.5)

range($T_2 T$)=columnspace of $\begin{pmatrix} 1 & 7 \\ 0 & 5 \\ 0 & -10 \\ 1 & -7 \end{pmatrix}$

range(T) is span of $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 7 & 5 \\ -10 & -7 \end{pmatrix} \right\}$