

Assignment 15

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Download codes from

<https://github.com/KUSUMAPRIYAPULAVARTY/assignment15>

Verifying if the matrices **A**, **B** are independent.

$$c_1 \mathbf{A} + c_2 \mathbf{B} = \mathbf{0} \quad (2.0.10)$$

$$\begin{pmatrix} 1 & 0 \\ 7 & 5 \\ 0 & 1 \\ -10 & -7 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{0} \quad (2.0.11)$$

$$\begin{pmatrix} 1 & 0 \\ 7 & 5 \\ 0 & 1 \\ -10 & -7 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - 7R_1]{R_4 \leftarrow R_4 + 10R_1} \begin{pmatrix} 1 & 0 \\ 0 & 5 \\ 0 & 1 \\ 0 & -7 \end{pmatrix} \quad (2.0.12)$$

$$\xleftrightarrow[R_2 \leftarrow \frac{R_2}{5}]{R_4 \leftarrow \frac{R_4}{-7}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_2]{R_4 \leftarrow R_4 - R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.0.14)$$

$$\implies c_1, c_2 = 0 \quad (2.0.15)$$

1 QUESTION

Let **V** be the set of complex numbers regarded as a vector space over the field of real numbers. We define a function T from **V** into the space of 2×2 real matrices, as follows. If $z = x + iy$ with x and y real numbers, then

$$T(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (1.0.1)$$

How would you describe the range of T ?

2 SOLUTION

$$T : \mathbf{V} \rightarrow R^{2 \times 2} \quad (2.0.1)$$

where $R^{2 \times 2}$, is the space of all 2×2 real matrices

Hence **A**, **B** are independent and

$$T(z) = \begin{pmatrix} x + 7y & 5y \\ -10y & x - 7y \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.3)$$

$$T(\mathbf{x}) = \begin{pmatrix} (1 \ 7)\mathbf{x} & (0 \ 5)\mathbf{x} \\ (0 \ -10)\mathbf{x} & (1 \ -7)\mathbf{x} \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} (1 \ 7)\mathbf{x} & (0 \ 5)\mathbf{x} \\ (0 \ -10)\mathbf{x} & (1 \ -7)\mathbf{x} \end{pmatrix} \quad (2.0.5)$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \quad (2.0.7)$$

$$\implies T(\mathbf{x}) = (\mathbf{A}\mathbf{x} \ \mathbf{B}\mathbf{x}) \quad (2.0.8)$$

$$T(\mathbf{x}) = (\mathbf{A} \ \mathbf{B}) \begin{pmatrix} \mathbf{x} & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{2 \times 1} & \mathbf{x} \end{pmatrix} \quad (2.0.9)$$

$$\text{range}(T) = \quad (2.0.16)$$

$$\text{span of } \left\{ \begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix}, \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \right\} \quad (2.0.17)$$