Assignment 17

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment17

1 QUESTION

Let $\mathbb{F}$ be a field of characteristic zero and let $\mathbf{V}$ be a finite dimensional vector space over $\mathbb{F}$.If $\alpha_1, \alpha_2, \ldots, \alpha_m$ are finitely many vectors in $\mathbf{V}$, each different from the zero vector, prove that there is a linear functional f on $\mathbf{V}$ such that

$$f(\alpha_i) \neq 0, i = 1, 2, \ldots, m \tag{1.0.1}$$

2 Theorem

We will make use of the following theorem
Let $\mathbf{V}$ be a finite dimensional vector space over the field $\mathbb{F}$ and let $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}\}$ be a basis for $\mathbf{V}$.Then there exists a unique dual basis $\{f_1, f_2, \ldots, f_n\}$ for $\mathbf{V}^*$ such that

$$f_i(\mathbf{v}_j) = \delta_{ij} \tag{2.0.1}$$

and $\mathbf{V}^*$ is the space of all linear functionals on $\mathbf{V}$

3 Solution

PARAMETERS	DESCRIPTION
F	Field
$\mathbf{V}$	Finite dimensional vector space over $\mathbb{F}$
$\alpha_1, \alpha_2, \ldots, \alpha_m$	non zero vectors in $\mathbf{V}$
$f : \mathbf{V} \to \mathbb{F}$	Linear functional on $\mathbf{V}$

TABLE 0:Input Parameters

PARAMETERS	MATRIX REPRESENTATION
Basis for $\mathbf{V}$	$\mathbf{B} = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_n \end{pmatrix}$
Basis for $\mathbf{V}^*$	$\mathbf{B}^* = \begin{pmatrix} \mathbf{f}_1 & \mathbf{f}_2 & \ldots & \mathbf{f}_n \end{pmatrix}$
$f_i(\mathbf{v}_j) = \delta_{ij}$	$(\mathbf{f}_i)^T \mathbf{v}_j = \delta_{ij}$
$\mathbf{B}, \mathbf{B}^*$ are dual basis	$(\mathbf{B}^*)^T \mathbf{B} = \mathbf{I}$
$\alpha_i = \sum_{k=1}^{n} a_k \mathbf{v}_k, i \in [1, m]$	$\alpha_i = \mathbf{Ba}$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, a_l \in \mathbb{F}, l \in [1, n]$
$\alpha_i \neq \mathbf{0}$	$\mathbf{a} \neq \mathbf{0}$
Any linear functional f over $\mathbf{V}$	$\mathbf{f} = \mathbf{B}^* \mathbf{c}$, where $\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}, c_l \in \mathbb{F}, l \in [1, n]$
$f(\alpha_i)$	$\mathbf{f}^T \alpha_i$ $= (\mathbf{B}^* \mathbf{c})^T \alpha_i$ $= \mathbf{c}^T (\mathbf{B}^*)^T \mathbf{Ba}$ $= \mathbf{c}^T \mathbf{a}$
f with $c_l \neq 0 \forall l \in [1, n]$ lies in $\mathbf{V}^*, \mathbf{a} \neq \mathbf{0}$	$f(\alpha_i) = \mathbf{c}^T \mathbf{a} \neq 0, i = 1, 2, \ldots, m$

TABLE 1:Proof