Assignment 18

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment18

1 QUESTION

Use theorem 20 to prove the following.If $\mathbf{W}$ is a subspace of a finite dimensional vector space $\mathbf{V}$ and if $\{g_1, g_2, \ldots, g_r\}$ is any basis for $\mathbf{W}^0$ then

$$\mathbf{W} = \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{1.0.1}$$

2 Theorem 20

Let $g, f_1, f_2, \ldots, f_r$ be linear functionals on vector space $\mathbf{V}$ with respective null spaces $\mathbf{N}, \mathbf{N}_1, \ldots, \mathbf{N}_r$. Then g is a linear combination of $f_1, \ldots, f_r$ if and only if $\mathbf{N}$ contains the intersection $\mathbf{N}_1 \cap \mathbf{N}_2 \cap \ldots \cap \mathbf{N}_r$.

3 Solution

PARAMETERS	**DESCRIPTION**
$\mathbf{V}$	Finite dimensional vector space with $dim(\mathbf{V}) = n$
$\mathbf{W}$	Subspace of $\mathbf{V}$
$\mathbf{W}^0$	Annihilator of $\mathbf{W}$
$g \in \mathbf{W}^0$	$g(\mathbf{w}) = 0 \forall \mathbf{w} \in \mathbf{W}$
$\{g_1, g_2, \ldots, g_r\}$	basis for $\mathbf{W}^0$
$\mathbf{N}_{g_i}$	Null space of g_i

TABLE 0: Input Parameters

STATEMENT	DESCRIPTION
$g \in \mathbf{W}^0$	$$g = \sum_{i=1}^{r} c_i g_i \tag{3.0.1}$$
If $\mathbf{N}$ is nullspace of g	From the theorem, $$\bigcap_{i=1}^{r} \mathbf{N}_{g_i} \subseteq \mathbf{N} \tag{3.0.2}$$
$\forall \mathbf{w} \in \mathbf{W}$	$$g(\mathbf{w}) = 0 \tag{3.0.3}$$
$(3.0.2), (3.0.3) \implies$	$$\mathbf{W} \subseteq \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{3.0.4}$$
Starting with method of contradiction	$$\mathbf{W} \neq \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{3.0.5}$$
From (3.0.4),(3.0.5) there exists a vector $\mathbf{e}$ such that	$$\mathbf{e} \in \bigcap_{i=1}^{r} \mathbf{N}_{g_i}, \mathbf{e} \notin \mathbf{W} \tag{3.0.6}$$ $$\text{So } \forall g \in \mathbf{W}^0, g(\mathbf{e}) = 0 \tag{3.0.7}$$
Since g is a linear functional on $\mathbf{V}$	$$(3.0.7) \implies \mathbf{e} \in \mathbf{V} \tag{3.0.8}$$
f	A linear functional on $\mathbf{V}$ defined as $$f(\alpha) \begin{cases} = 0 & \text{for } \alpha \in \mathbf{W} \\ \neq 0 & \text{for } \alpha \in \mathbf{V}, \alpha \notin \mathbf{W} \end{cases} \tag{3.0.9}$$ $$\therefore f \in \mathbf{W}^0 \text{ and } f(e) \neq 0 \tag{3.0.10}$$
(3.0.7),(3.0.10) contradict each other	$$\mathbf{W} = \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{3.0.11}$$

TABLE1: Proof