# Assignment 18

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment18

## 1 Question

Use theorem 20 to prove the following.If $\mathbf{W}$ is a subspace of a finite dimensional vector space $\mathbf{V}$ and if $\{g_1, g_2, \dots, g_r\}$ is any basis for $\mathbf{W}^0$ then

$$\mathbf{W} = \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{1.0.1}$$

## 2 Theorem 20

Let $g, f_1, f_2, \dots, f_r$ be linear functionals on vector space $\mathbf{V}$ with respective null spaces $\mathbf{N}, \mathbf{N}_1, \dots, \mathbf{N}_r$. Then $g$ is a linear combination of $f_1, \dots, f_r$ if and only if $\mathbf{N}$ contains the intersection $\mathbf{N}_1 \cap \mathbf{N}_2 \cap \dots \cap \mathbf{N}_r$.

## 3 Solution

| **PARAMETERS** | **DESCRIPTION** |
|---|---|
| $\mathbf{V}$ | Finite dimensional vector space with $dim(\mathbf{V}) = n$ |
| $\mathbf{W}$ | Subspace of $\mathbf{V}$ |
| $\mathbf{W}^0$ | Annihilator of $\mathbf{W}$ |
| $g \in \mathbf{W}^0$ | $g(\mathbf{w}) = 0 \forall \mathbf{w} \in \mathbf{W}$ |
| $\{g_1, g_2, \dots, g_r\}$ | basis for $\mathbf{W}^0$ |
| $\mathbf{N}_{g_i}$ | Null space of $g_i$ |

TABLE 0: Input Parameters

Any $g \in \mathbf{W}^0$ can be written as

$$g = \sum_{i=1}^{r} c_i g_i \tag{3.0.1}$$

So, from the theorem, if $\mathbf{N}$ is nullspace of $g$

$$\bigcap_{i=1}^{r} \mathbf{N}_{g_i} \subseteq \mathbf{N} \tag{3.0.2}$$

$$\forall \mathbf{w} \in \mathbf{W}, g(\mathbf{w}) = 0 \tag{3.0.3}$$

$$(3.0.2), (3.0.3) \implies \mathbf{W} \subseteq \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{3.0.4}$$

To prove using method of contradiction, assume

$$\mathbf{W} \neq \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{3.0.5}$$

From (3.0.4),(3.0.5) there exists a vector

$$\mathbf{e} \in \bigcap_{i=1}^{r} \mathbf{N}_{g_i}, \mathbf{e} \notin \mathbf{W} \tag{3.0.6}$$

$$\text{So } \forall g \in \mathbf{W}^0, g(\mathbf{e}) = 0 \tag{3.0.7}$$

Since $g$ is a linear functional on $\mathbf{V}$,

$$(3.0.7) \implies \mathbf{e} \in \mathbf{V} \tag{3.0.8}$$

Let us define a functional $f$ on $\mathbf{V}$ such that

$$f(\alpha) = \begin{cases} 0 & \text{for } \alpha \in \mathbf{W} \\ \neq 0 & \text{for } \alpha \in \mathbf{V}, \alpha \notin \mathbf{W} \end{cases} \tag{3.0.9}$$

$$\therefore f \in \mathbf{W}^0 \text{ and } f(e) \neq 0 \tag{3.0.10}$$

which contradicts with (3.0.7)
Hence

$$\mathbf{W} = \bigcap_{i=1}^{r} \mathbf{N}_{g_i} \tag{3.0.11}$$