Assignment 19

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment19

1 QUESTION

Let $\mathbf{V}$ be the vector space of all polynomial functions over the field of real numbers.Let a and b be fixed real numbers and let f be the linear functional on $\mathbf{V}$ defined by

$$f(p) = \int_a^b p(x)\,dx \tag{1.0.1}$$

If D is the differentiator operator on $\mathbf{V}$, what is $D^t f$?

2 Solution

PARAMETERS	DESCRIPTION
$\mathbb{R}$	Field of real numbers
$\mathbf{V}$	Vector space of all polynomials over $\mathbb{R}$
a, b	Fixed real numbers
f	Linear functional on $\mathbf{V}$
D	Differentiator operator on $\mathbf{V}$
D^t	Transpose of D

TABLE 0: Input Parameters

STATEMENTS	DERIVATIONS
D^t is transpose of D	$$(D^t f)(p) = f[D(p)] \tag{2.0.1}$$
A polynomial of degree n $p(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n$	$$p = \mathbf{c}^T \mathbf{x} \tag{2.0.2}$$ $$\mathbf{c} = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 1 \\ x \\ x^2 \\ \vdots \\ x^n \end{pmatrix} \tag{2.0.3}$$
$f(p) = \int_a^b p(x)\,dx$	$$f(p) = \mathbf{c}^T \mathbf{F} \tag{2.0.4}$$ $$\mathbf{F} = \begin{pmatrix} b-a \\ \frac{b^2-a^2}{2} \\ \frac{b^3-a^3}{3} \\ \vdots \\ \frac{b^{n+1}-a^{n+1}}{n+1} \end{pmatrix} \tag{2.0.5}$$
$D(p) = c_1 + 2c_2 x + 3c_3 x^2 + \ldots + nc_n x^{n-1}$	$$D(p) = \mathbf{c}^T \mathbf{D} \mathbf{x} \tag{2.0.6}$$ $$\mathbf{D} = \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 2 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 3 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & n & 0 \end{pmatrix} \tag{2.0.7}$$
$f[D(p)] = c_1(b-a) + c_2(b^2-a^2) + \ldots + c_n(b^n - a^n)$	$$\text{Let } D(p) = p' \tag{2.0.8}$$ $$\implies p' = \mathbf{c}'^T \mathbf{x} \tag{2.0.9}$$ $$\text{where } \mathbf{c}'^T = \begin{pmatrix} c_1 \\ 2c_2 \\ 3c_3 \\ \vdots \\ nc_n \\ 0 \end{pmatrix} \tag{2.0.10}$$ $$f[D(p)] = \mathbf{c}'^T \mathbf{F} \tag{2.0.11}$$
$(D^t f)(p) = p(b) - p(a)$	$$(D^t f)(p) = c_1(b-a) + c_2(b^2-a^2) + \tag{2.0.12}$$ $$\ldots + c_n(b^n - a^n) + c_0 - c_0 \tag{2.0.13}$$ $$= (c_0 + c_1 b + c_2 b^2 + \ldots + c_n b^n) - \tag{2.0.14}$$ $$(c_0 + c_1 a + c_2 a^2 + \ldots + c_n a^n) \tag{2.0.15}$$ $$= p(b) - p(a) \tag{2.0.16}$$

TABLE1: Proof