Assignment 2

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1 Question No. 15

Find the real numbers x, y such that $\begin{pmatrix} x \\ -y \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ is

conjugate of $\begin{pmatrix} -6 \\ -24 \end{pmatrix}$

2 Explanation

The matrix representation of a complex number

$$\begin{pmatrix} a \\ b \end{pmatrix} \; is \; \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2.0.1)$$

Conjugate of a complex number

$$\overline{\begin{pmatrix} a \\ b \end{pmatrix}} = \begin{pmatrix} a \\ -b \end{pmatrix} \qquad (2.0.2)$$

3 Solution

The conjugate of $\begin{pmatrix} -6 \\ -24 \end{pmatrix}$ is $\begin{pmatrix} -6 \\ 24 \end{pmatrix}$

$$\implies \begin{pmatrix} x \\ -y \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 24 \end{pmatrix} \qquad (3.0.1)$$

Using equivalent matrices for complex numbers, we have

$$\begin{pmatrix} x & y \\ -y & x \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 & -24 \\ 24 & -6 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3.0.2)$$

Multiplying both sides of Equation (3.0.1) by $\begin{pmatrix} 3 & -5 \\ 5 & 3 \end{pmatrix}^{-1}$,we have

$$\begin{pmatrix} x & y \\ -y & x \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 & -24 \\ 24 & -6 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 5 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3.0.3)$$

$$= \frac{1}{34}\begin{pmatrix} -6 & -24 \\ 24 & -6 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3.0.4)$$

$$= \frac{1}{34}\begin{pmatrix} 102 & -102 \\ 102 & 102 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3.0.5)$$

$$= \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (3.0.6)$$

$$\implies \begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \qquad (3.0.7)$$

$$\text{Therefore,} \quad x = 3, \qquad (3.0.8)$$

$$y = -3 \qquad (3.0.9)$$