Assignment 2

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Download all python codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment2/tree/master/codes

and latex-tikz codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment2

1 Question No. 15

Find the real numbers x, y such that $\begin{pmatrix} x \\ -y \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ is conjugate of $\begin{pmatrix} -6 \\ -24 \end{pmatrix}$

2 Explanation

The matrix representation of a complex number

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.1}$$

Conjugate of a complex number

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ -b \end{pmatrix} \tag{2.0.2}$$

3 Solution

The conjugate of $\begin{pmatrix} -6 \\ -24 \end{pmatrix}$ is $\begin{pmatrix} -6 \\ 24 \end{pmatrix}$

$$\implies \begin{pmatrix} x \\ -y \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 24 \end{pmatrix} \tag{3.0.1}$$

$$\implies \begin{pmatrix} x \\ -y \end{pmatrix} = \frac{\begin{pmatrix} -6 \\ 24 \end{pmatrix}}{\begin{pmatrix} 3 \\ 5 \end{pmatrix}} \tag{3.0.2}$$

Using equivalent matrices for complex numbers, we have

$$\begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} -6 & -24 \\ 24 & -6 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 5 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.3}$$

$$= \frac{1}{34}\begin{pmatrix} -6 & -24 \\ 24 & -6 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.4}$$

$$= \frac{1}{34}\begin{pmatrix} 102 & -102 \\ 102 & 102 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.5}$$

$$= \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.6}$$

$$\implies \begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{3.0.7}$$

$$\text{Therefore,} x = 3, \tag{3.0.8}$$

$$y = -3 \tag{3.0.9}$$