Assignment 3

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Download all python codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment3/tree/master/codes

and latex-tikz codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment3

1 Question No. 61

Examine the consistency of the system of given Equations.

$$5x + 2y = 3 \tag{1.0.1}$$
$$3x + 2y = 5 \tag{1.0.2}$$

2 Solution

The given set of equations can be represented in the matrix equation form as

$$\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{2.0.1}$$

The augmented matrix for this system becomes

$$\begin{pmatrix} 5 & 2 & 3 \\ 3 & 2 & 5 \end{pmatrix} \tag{2.0.2}$$

Row reducing the matrix

$$\begin{pmatrix} 5 & 2 & 3 \\ 3 & 2 & 5 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 \times \frac{5}{3} - R_1} \begin{pmatrix} 5 & 2 & 3 \\ 0 & \frac{4}{3} & \frac{16}{3} \end{pmatrix} \tag{2.0.3}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 \times \frac{3}{4}} \begin{pmatrix} 5 & 2 & 3 \\ 0 & 1 & 4 \end{pmatrix} \tag{2.0.4}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - 2 \times R_2} \begin{pmatrix} 5 & 0 & -5 \\ 0 & 1 & 4 \end{pmatrix} \tag{2.0.5}$$
$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{5}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \end{pmatrix} \tag{2.0.6}$$
$$\implies \text{Rank}\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix} = \text{Rank}\begin{pmatrix} 5 & 2 & 3 \\ 3 & 2 & 5 \end{pmatrix} = 2 \tag{2.0.7}$$
$$= dim\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix} \tag{2.0.8}$$

So, the given system of equations are consistent with a unique solution of

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{2.0.9}$$

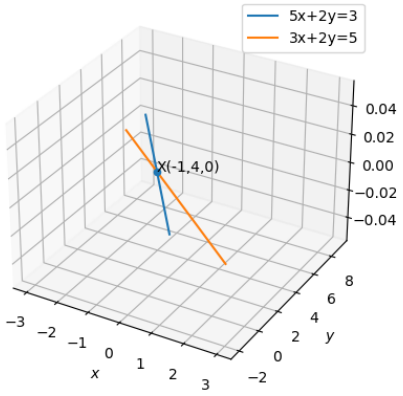

Fig. 0: plot showing intersection of lines