Assignment 4

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Download all python codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment4/tree/master/codes

and latex-tikz codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment4

1 QUESTION

Prove that the following equations represent two straight lines.Also find their point of intersection and the angle between them

$$3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0 \tag{1.0.1}$$

2 Explanation

The general form of equation representing a pair of straight lines is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \tag{2.0.1}$$

These represent a pair of lines if the discriminant of the equation is zero, that is,

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \tag{2.0.2}$$

3 Solution

The discriminant of (1.0.1) becomes

$$\begin{vmatrix} -3 & -4 & -\frac{29}{2} \\ -4 & 3 & \frac{3}{2} \\ -\frac{29}{2} & \frac{3}{2} & -18 \end{vmatrix} \tag{3.0.1}$$

Expanding equation (3.0.1), we have discriminant equal to zero.
Hence given equation represents a pair of straight lines.

4 Finding the individual lines

Rewriting equation (1.0.1) and solving for y,

$$y^2 - y\left(\frac{8x-3}{3}\right) = \frac{3x^2 + 29x + 18}{3} \tag{4.0.1}$$

$$\implies y^2 - y\left(\frac{8x-3}{3}\right) + \left(\frac{8x-3}{6}\right)^2 = \tag{4.0.2}$$

$$\frac{3x^2 + 29x + 18}{3} + \left(\frac{8x-3}{6}\right)^2 \tag{4.0.3}$$

$$\left[y - \left(\frac{8x-3}{6}\right)\right]^2 = \left[\frac{5}{6}(2x+3)\right]^2 \tag{4.0.4}$$

$$\implies y = \frac{8x-3}{6} - \frac{5}{6}(2x+3), \tag{4.0.5}$$

$$y = \frac{8x-3}{6} + \frac{5}{6}(2x+3) \tag{4.0.6}$$

The individual line equations therefore become

$$x + 3y + 9 = 0 \tag{4.0.7}$$

$$-3x + y - 2 = 0 \tag{4.0.8}$$

5 Intersection Point

The augmented matrix for the set of equations represented in (4.0.7), (4.0.8) is

$$\begin{pmatrix} 1 & 3 & -9 \\ -3 & 1 & 2 \end{pmatrix} \tag{5.0.1}$$

Row reducing the matrix

$$\begin{pmatrix} 1 & 3 & -9 \\ -3 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + 3\times R_1} \begin{pmatrix} 1 & 3 & -9 \\ 0 & 10 & -25 \end{pmatrix} \tag{5.0.2}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{3}{10}\times R_2} \begin{pmatrix} 1 & 0 & -\frac{3}{2} \\ 0 & 10 & -25 \end{pmatrix} \tag{5.0.3}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{10}} \begin{pmatrix} 1 & 0 & -\frac{3}{2} \\ 0 & 1 & -\frac{5}{2} \end{pmatrix} \tag{5.0.4}$$

$$\text{Hence, the intersection point is} \begin{pmatrix} -\frac{3}{2} \\ -\frac{5}{2} \end{pmatrix} \tag{5.0.5}$$

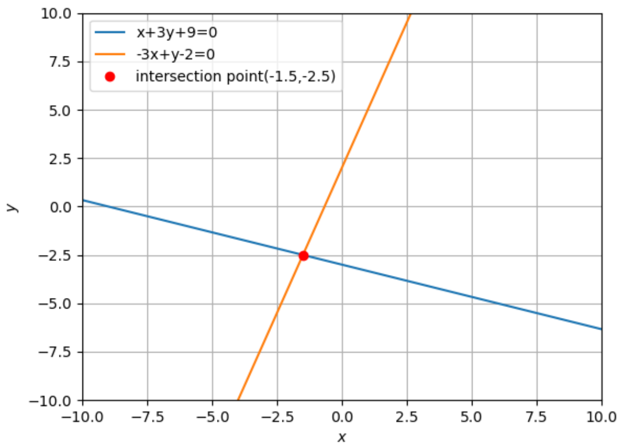

Fig. 0: plot showing intersection of lines

6 Angle Between The Lines

The direction vectors of lines in (4.0.7), (4.0.8) are

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix} \tag{6.0.1}$$

$$\mathbf{m_2} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{6.0.2}$$

Their corresponding normal vectors are

$$\mathbf{n_1} = \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix} \tag{6.0.3}$$

$$\mathbf{n_2} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{6.0.4}$$

Angle between two lines θ can be given by

$$\cos\theta = \frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{6.0.5}$$

$$= \frac{\begin{pmatrix} \frac{1}{3} & 1 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\sqrt{\left(\frac{1}{3}\right)^2 + 1} \times \sqrt{(-3)^2 + 1}} = 0 \tag{6.0.6}$$

$$\implies \theta = 90^\circ \tag{6.0.7}$$