Assignment 5

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1 QUESTION

Find equation of circle with centre $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and passes through the point $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

2 EXPLANATION

The general equation of a circle is

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \qquad (2.0.1)$$

$$\text{If } r \text{ is radius, } f = \mathbf{u}^T\mathbf{u} - r^2 \qquad (2.0.2)$$

$$\text{center } \mathbf{c} = -\mathbf{u} \qquad (2.0.3)$$

3 SOLUTION

Given centre is $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$$\implies \mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \qquad (3.0.1)$$

$$\implies \mathbf{u} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \qquad (3.0.2)$$

Equation (2.0.1) becomes

$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix} -4 & -4 \end{pmatrix}\mathbf{x} + f = 0 \qquad (3.0.3)$$

This passes through point $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Substituting $\mathbf{x} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ in (3.0.3)

$$\begin{pmatrix} 4 & 5 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -4 & -4 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \end{pmatrix} + f = 0 \qquad (3.0.4)$$

$$\implies f = -5 \qquad (3.0.5)$$

Also, radius can be determined as follows

$$f = \mathbf{u}^T\mathbf{u} - r^2 \qquad (3.0.6)$$

$$\implies -5 = \begin{pmatrix} -2 & -2 \end{pmatrix}\begin{pmatrix} -2 \\ -2 \end{pmatrix} - r^2 \qquad (3.0.7)$$

$$\implies -5 = 8 - r^2 \qquad (3.0.8)$$

$$\implies r = \sqrt{13} \qquad (3.0.9)$$

The equation of required circle is
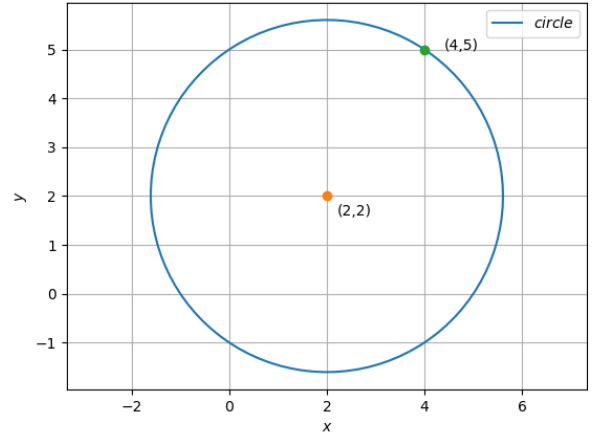
$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix} -4 & -4 \end{pmatrix}\mathbf{x} - 5 = 0 \qquad (3.0.10)$$

Fig. 0: plot showing the circle