Assignment 9

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Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment9

1 QUESTION

Prove that if two homogenous systems of linear equations in two unknowns have the same solutions, then they are equivalent.

2 SOLUTION

Let the two systems of homogenous equations be

$$\mathbf{A}\mathbf{x} = \mathbf{0} \tag{2.0.1}$$

$$\mathbf{B}\mathbf{x} = \mathbf{0} \tag{2.0.2}$$

$$\implies \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ \vdots & \vdots \\ A_{n1} & A_{n2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\text{and} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ \vdots & \vdots \\ B_{n1} & B_{n2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.4}$$

Writing the coefficient matrices in their row echelon forms

$$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22}A_{11} - A_{12}A21 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\begin{pmatrix} B_{11} & B_{12} \\ 0 & B_{22}B_{11} - B_{12}B21 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.6}$$

Since the two sets have the same solution, they must also satisfy

$$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22}A_{11} - A_{12}A21 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ B_{11} & B_{12} \\ 0 & B_{22}B_{11} - B_{12}B21 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.7}$$

For the same solution to exist for (2.0.5), (2.0.6), (2.0.7), all the coefficient matrices must have the same rank.
It is possible only if the row echelon form for coefficient matrix of (2.0.7) becomes

$$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22}A_{11} - A_{12}A21 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \tag{2.0.8}$$

This happens only if equations in (2.0.4) are linear combinations of equations in (2.0.3)
Hence, they are equivalent