Assignment 9

KUSUMA PRIYA
EE20MTECH11007

Download codes from

https://github.com/KUSUMAPRIYAPULAVARTY/assignment9

1 QUESTION

Prove that if two homogenous systems of linear equations in two unknowns have the same solutions, then they are equivalent.

2 Solution

Let the two systems of homogenous equations be

$$\mathbf{Ax} = \mathbf{0} \tag{2.0.1}$$

$$\mathbf{Bx} = \mathbf{0} \tag{2.0.2}$$

$$\implies \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ \vdots & \vdots \\ A_{n1} & A_{n2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\text{and } \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ \vdots & \vdots \\ B_{n1} & B_{n2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.0.4}$$

Consider i^{th} equation in (2.0.3) and j^{th} equation in (2.0.4)

$$\begin{pmatrix} A_{i1} & A_{i2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \tag{2.0.5}$$

$$\begin{pmatrix} B_{j1} & B_{j2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \tag{2.0.6}$$

Since they have the same solution, we can deduce

$$\begin{pmatrix} A_{i1} & A_{i2} \end{pmatrix} = k \begin{pmatrix} B_{j1} & B_{j2} \end{pmatrix} \tag{2.0.7}$$

where k is a scalar.
This is true for all the equations in (2.0.3) and (2.0.4) proving that they are equivalent.