ProblemSet3_Ex2_Cyan

March 13, 2024

[ ]:
```
import pandas as pd
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np
import matplotlib.ticker as ticker
import sys
from iminuit import Minuit
from matplotlib.colors import ListedColormap
from tqdm import tqdm
from sklearn.neighbors import KernelDensity
```

[ ]:
```
sys.path.append('../External_Functions')
from ExternalFunctions import Chi2Regression, BinnedLH, UnbinnedLH, simpson38
from ExternalFunctions import nice_string_output, add_text_to_ax
```

[ ]:
```
COLOUR = ['#1E90FF', # 0 # Dodgerblue
          '#FFBF00', # 1 # Amber
          '#FF6347', # 2 # Tomato
          '#00A86B', # 3 # Jade
          '#8A2BE2', # 4 # Blueviolet
          '#FF6FFF', # 5 # Ultra Pink
          '#00CCFF', # 6 # Vivid Sky Blue
          '#00ff40', # 7 # Erin
          '#FF004F', # 8 # Folly
          '#0063A6', # 9 # Lapis Lazuli
        ]
def setMplParam(classNum):
    # Define effective colors, line styles, and markers based on the class␣
 ↪number

    LINE = ['-', '-.', '--', '-.', ':','--','-.','-', ':', '--']
    MARKER = ['.','*', '^', 's', '.', 'p', 'o', 's', '.', 'd']
    COLOUR_EFF = COLOUR[:classNum]
    LINE_EFF = LINE[:classNum]
    MARKER_EFF = MARKER[:classNum]

    # Set the color cycle for lines including color, line style, and marker
    plt.rcParams['axes.prop_cycle'] = (plt.cycler(color=COLOUR_EFF) +
```

```
                                        plt.cycler(linestyle=LINE_EFF)+
                                        plt.cycler(marker=MARKER_EFF))

    # Set default line and marker sizes
    plt.rcParams['lines.markersize'] = 3  # Example size
    plt.rcParams['lines.linewidth'] = 2   # Example width for lines

    # Set label and title sizes
    plt.rcParams['axes.labelsize'] = 20
    plt.rcParams['axes.titlesize'] = 20

    # Set tick properties
    plt.rcParams['xtick.direction'] = 'in'
    plt.rcParams['xtick.labelsize'] = 20
    plt.rcParams['ytick.direction'] = 'in'
    plt.rcParams['ytick.labelsize'] = 20

    # Set legend font size
    plt.rcParams['legend.fontsize'] = 12

    # Enable and configure grid
    plt.rcParams['axes.grid'] = True
    plt.rcParams['grid.alpha'] = 0.8
    plt.rcParams['grid.linestyle'] = '--'
    plt.rcParams['grid.linewidth'] = 1

    # Set axes line width
    plt.rcParams['axes.linewidth'] = 2

    # Set tick sizes and widths
    plt.rcParams['xtick.major.size'] = 7
    plt.rcParams['xtick.major.width'] = 3
    plt.rcParams['xtick.minor.size'] = 2
    plt.rcParams['xtick.minor.width'] = 2

    plt.rcParams['ytick.major.size'] = 7
    plt.rcParams['ytick.major.width'] = 3
    plt.rcParams['ytick.minor.size'] = 2
    plt.rcParams['ytick.minor.width'] = 2

setMplParam(10)
```

[ ]:
```
def readData():
    path = 'resources/'
    fileName = 'cpd-crash-incidents.csv'
    data = pd.read_csv(path + fileName, sep=';')
    data.dropna(subset=['lon', 'lat', 'crash_date'], inplace=True)
```

```
    return data
data = readData()
# print(data['lon'][:10])
# print(data['lat'][:10])
# print(data['crash_date'][:10])
```

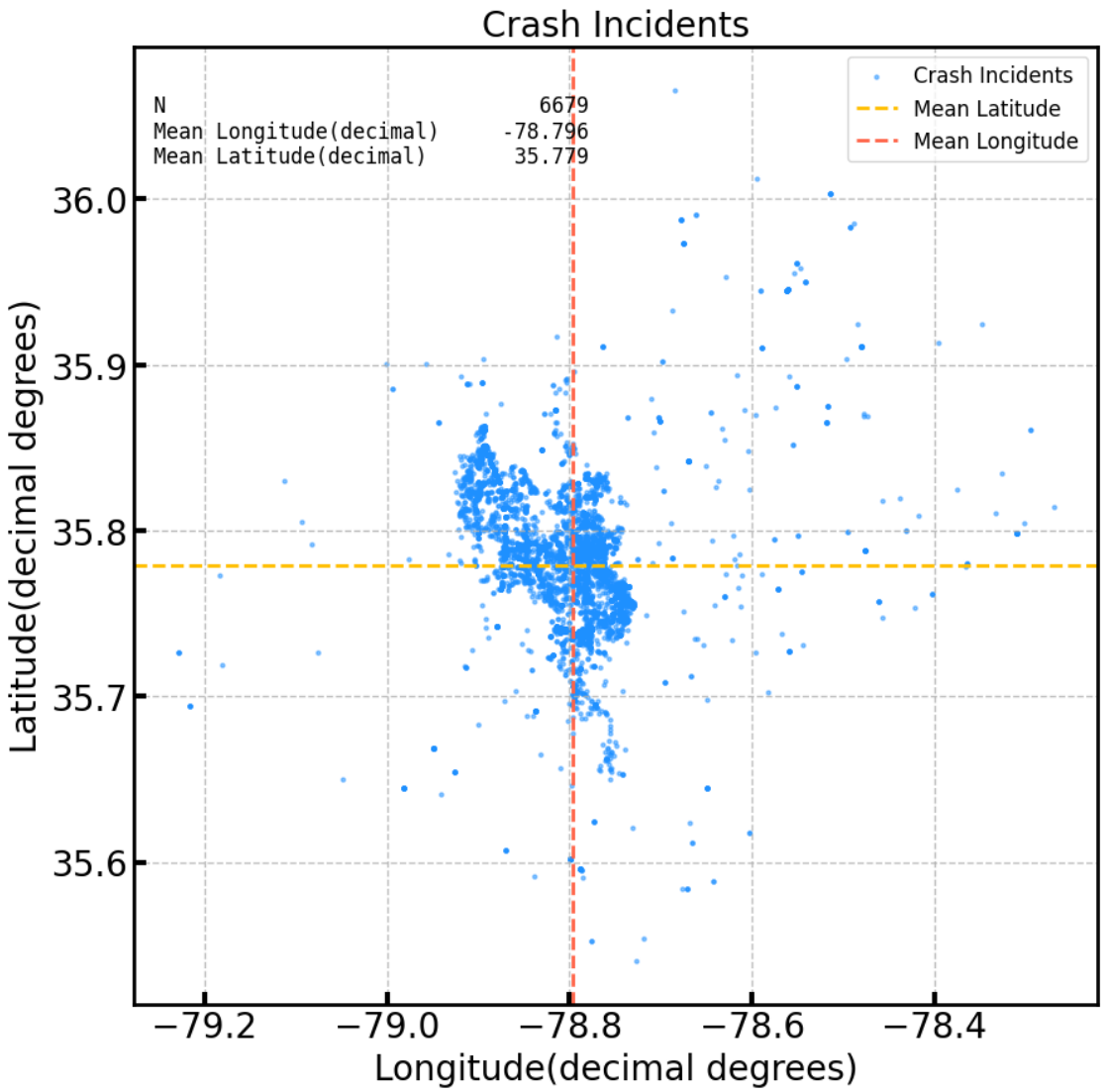
[ ]:
```
def makeTimeColumn(data):
    crash_date = pd.to_datetime(data['crash_date'])
    hours = crash_date.dt.hour
    minutes = crash_date.dt.minute
    seconds = crash_date.dt.second
    time = hours + minutes/60 + seconds/3600
    data['time'] = time

makeTimeColumn(data)
time = data['time']
```

[ ]:
```
def getShiftedTime_9(crash_date):
    crash_date = pd.to_datetime(data['crash_date'])
    hours = crash_date.dt.hour
    minutes = crash_date.dt.minute
    seconds = crash_date.dt.second
    time = hours + minutes/60 + seconds/3600
    shifted_time = time.apply(lambda x: x + 24 if x < 9 else x)
    shifted_time = shifted_time.apply(lambda x: x - 9)

    return shifted_time

time_shift_9 = getShiftedTime_9(data)
```

1 prob2a

[ ]:
```
def scatterLatLong(data):
    fig, ax = plt.subplots(figsize=(10, 10))
    ax.scatter(data['lon'], data['lat'], s=5, alpha=0.5, label='Crash␣
 ↪Incidents')
    ax.axhline(np.mean(data['lat']), c = COLOUR[1], linestyle='--', label='Mean␣
 ↪Latitude')
    ax.axvline(np.mean(data['lon']), c = COLOUR[2], linestyle='--', label='Mean␣
 ↪Longitude')
    ax.set_xlabel('Longitude(decimal degrees)')
    ax.set_ylabel('Latitude(decimal degrees)')
    ax.set_title('Crash Incidents')
    d = {'N' : len(data),
         'Mean Longitude(decimal)' : np.mean(data['lon']),
         'Mean Latitude(decimal)' : np.mean(data['lat'])}
```

```
    add_text_to_ax(0.02, 0.95, nice_string_output(d), ax, fontsize=12)
    ax.legend()
scatterLatLong(data)
```



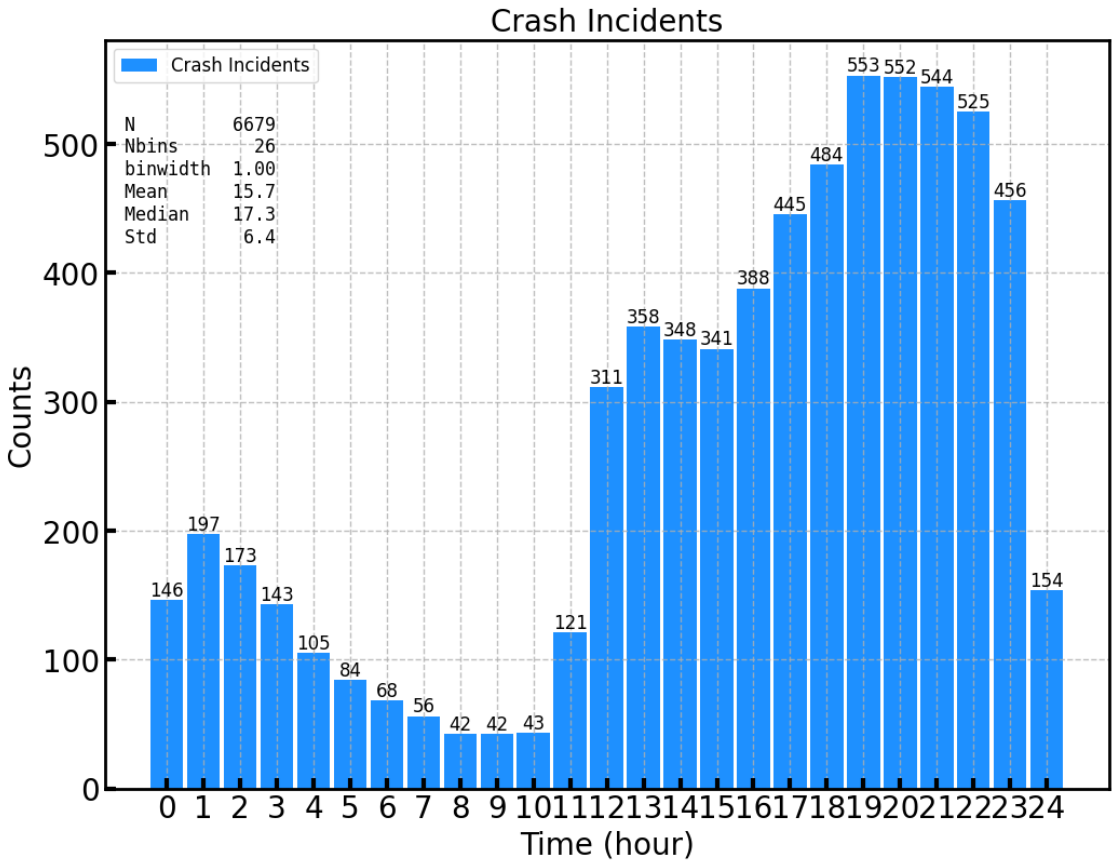
[ ]:
```
def getHistoParamWithBinWidth(data, binwidth, isDensity = False):
    start_bin = np.floor(data.min()) - 0.5 * binwidth
    end_bin = np.ceil(data.max()) + 0.5 * binwidth
    bins = np.arange(start_bin, end_bin + binwidth, binwidth)
    bin_centers = (bins[:-1] + bins[1:]) / 2
    counts, x_edges = np.histogram(data, bins=bins, density=isDensity)
    x_centres = x_edges[:-1] + binwidth/2
    return bins, counts, bin_centers, x_centres
```

[ ]:
```
def buildHistoTexts(title, label, binwidth,
                    d_text_x,
                    d_text_y,
                    xlabel = 'X',
                    ylabel = 'Y',
                    rotation = 0,
                    density = False,
                    additional_d = {}):
    texts = {}
    texts['title'] = title
    texts['label'] = label
    texts['binwidth'] = binwidth
    texts['d_x'] = d_text_x
    texts['d_y'] = d_text_y
    texts['xlabel'] = xlabel
    texts['ylabel'] = ylabel
    texts['rotation'] = rotation
    texts['density'] = density
    texts['add_d'] = additional_d
    return texts
```

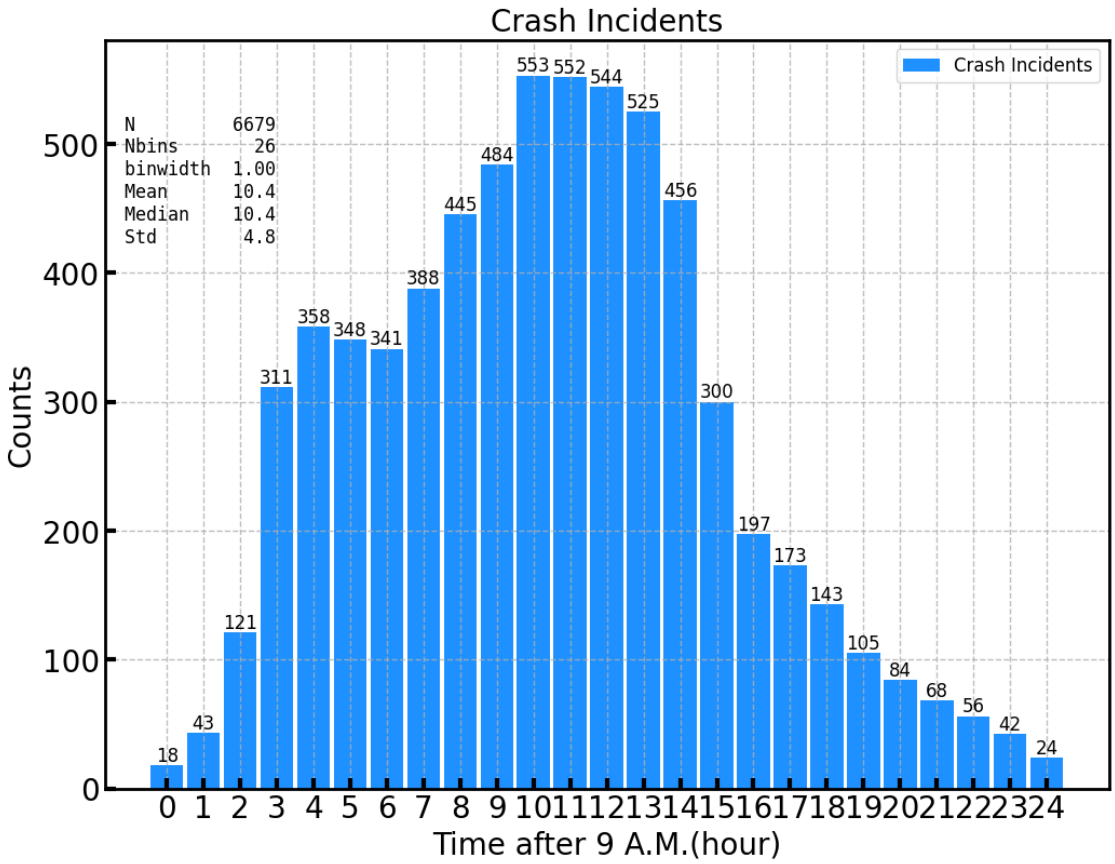
[ ]:
```
def plotHistoCore(data, texts):
    title = texts['title']
    label = texts['label']
    binwidth = texts['binwidth']
    xlabel = texts['xlabel']
    ylabel = texts['ylabel']
    d_text_x = texts['d_x']
    d_text_y = texts['d_y']
    rotation = texts['rotation']
    density = texts['density']

    fig, ax = plt.subplots(figsize=(12, 9))
    bins, counts, bin_centers, x_centres = getHistoParamWithBinWidth(data,␣
 ↪binwidth)
    Nbins = len(bins)
    _, _, patches = ax.hist(data, bins=bins, rwidth=0.9, label=label)
    ax.set_title(title)
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

    ax.set_xticks(bin_centers)
    ax.set_xticklabels([f'{int(label)}' for label in bin_centers],␣
 ↪rotation=rotation)
    ax.legend()

    # display the count of each bar on the top
```

```
    for patch, count in zip(patches, counts):
         height = patch.get_height()
         ax.text(patch.get_x() + patch.get_width() / 2.,
                 height, f'{count:.0f}',
                 ha='center', va='bottom',
                 fontsize=12)
    add_d = texts['add_d']
    d = {'N': len(data),
         'Nbins': Nbins,
         'binwidth': f'{binwidth:.2f}',
         'Mean': f'{np.mean(data):.1f}',
         'Median': f'{np.median(data):.1f}',
         'Std': f'{np.std(data):.1f}',

         }
    text = nice_string_output(d, extra_spacing=2, decimals=3)
    add_text_to_ax(d_text_x, d_text_y, text, ax, fontsize=12, color='k')
```

[ ]:
```
def plotHistogram():
    histoText_alive = buildHistoTexts('Crash Incidents',
                                      'Crash Incidents',
                                      1.0,
                                      0.02,
                                      0.90,
                                      'Time (hour)',
                                      'Counts')
    plotHistoCore(time, histoText_alive)
plotHistogram()
```



[ ]:
```
def plotHistogram_shifted():
    histoText_alive = buildHistoTexts('Crash Incidents',
                                      'Crash Incidents',
                                      1.0,
                                      0.02,
                                      0.90,
                                      'Time after 9 A.M.(hour)',
                                      'Counts')
    plotHistoCore(time_shift_9, histoText_alive)
plotHistogram_shifted()
```



1.1 prob2a Gaussian KDE

- the data is in time of clock which can be a continuous variable if I can think about the previous or the following day.
- So I may shift the histogram to locate its summit to be around centre of my horizontal dimension, and then I will have a giant Gaussian-like peak with some hints of several minor Gaussian-like peaks may also contribute to the profile
- I shall choose to cut the data before 9 A.M. and put after 24:00.
- Applying normal Gaussian formula for each time data point, it will give Gaussian KDE PDF
-

2 prob2b

2.1 Prob2b Epanechnikov

[ ]:
```
def Epanechnikov(x, x0):
    width = 0.8
    u = (x - x0) / width
    return np.where(np.abs(u) <= 1, 3/4 * (1 - u**2), 0)
```

[ ]:
```
def KDEcore(data):
    DAY_STEP = 0.01
    DAY_START = 0
    DAY_END = 24
    DAY = np.arange(DAY_START, DAY_END, DAY_STEP)
    Epanechnikovs = []
    for t in tqdm(DAY):
        Epanechnikovs.append(Epanechnikov(data, t).sum())
    dt = DAY[1] - DAY[0]
    area = np.trapz(Epanechnikovs, dx=dt)
    kde = Epanechnikovs / area
    normalisation = np.trapz(kde, dx=dt)

    return DAY, kde, normalisation
```

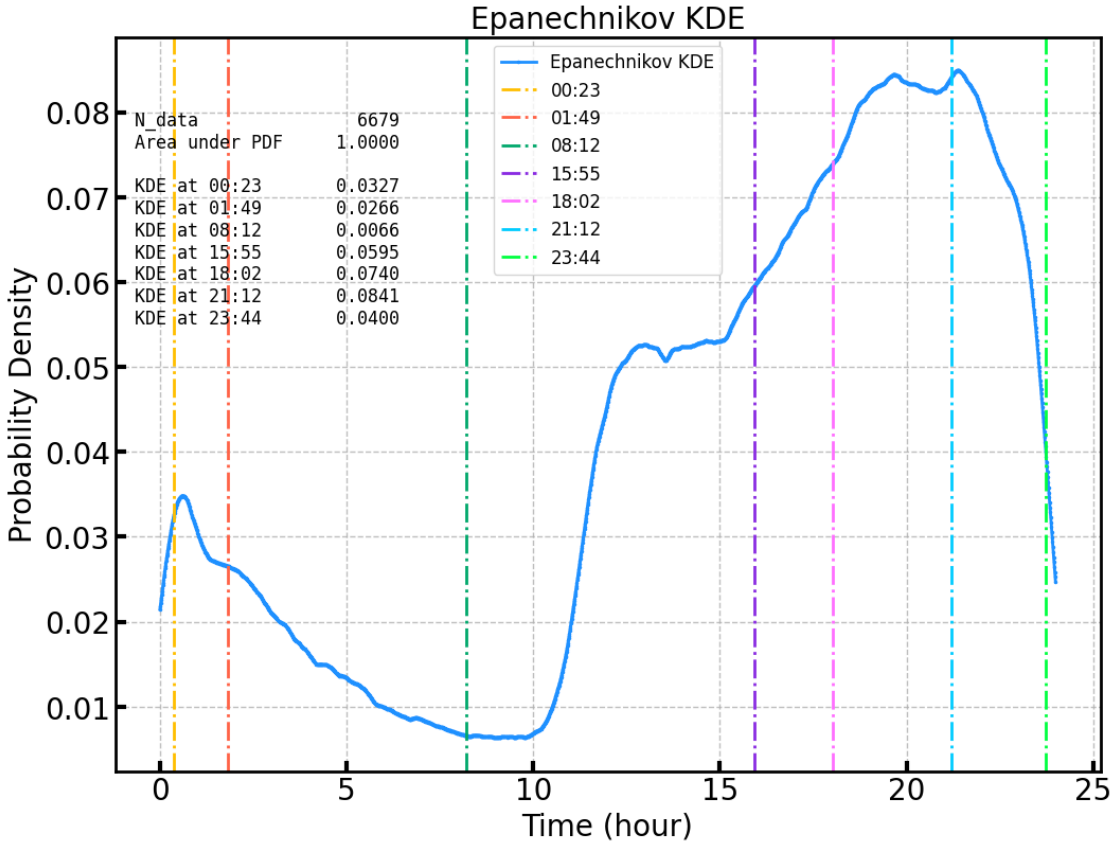
[ ]:
```
def getInterestedTime(isShifted = False):
    str = '00:23, 01:49, 08:12, 15:55, 18:02, 21:12, 23:44'
    arr_str = str.split(', ')
    arr_num = [int(x.split(':')[0]) + int(x.split(':')[1])/60 for x in arr_str]
    if isShifted:
        arr_num = [x + 24 if x < 9 else x for x in arr_num]
        arr_num = [x - 9 for x in arr_num]
    return arr_num, arr_str
```

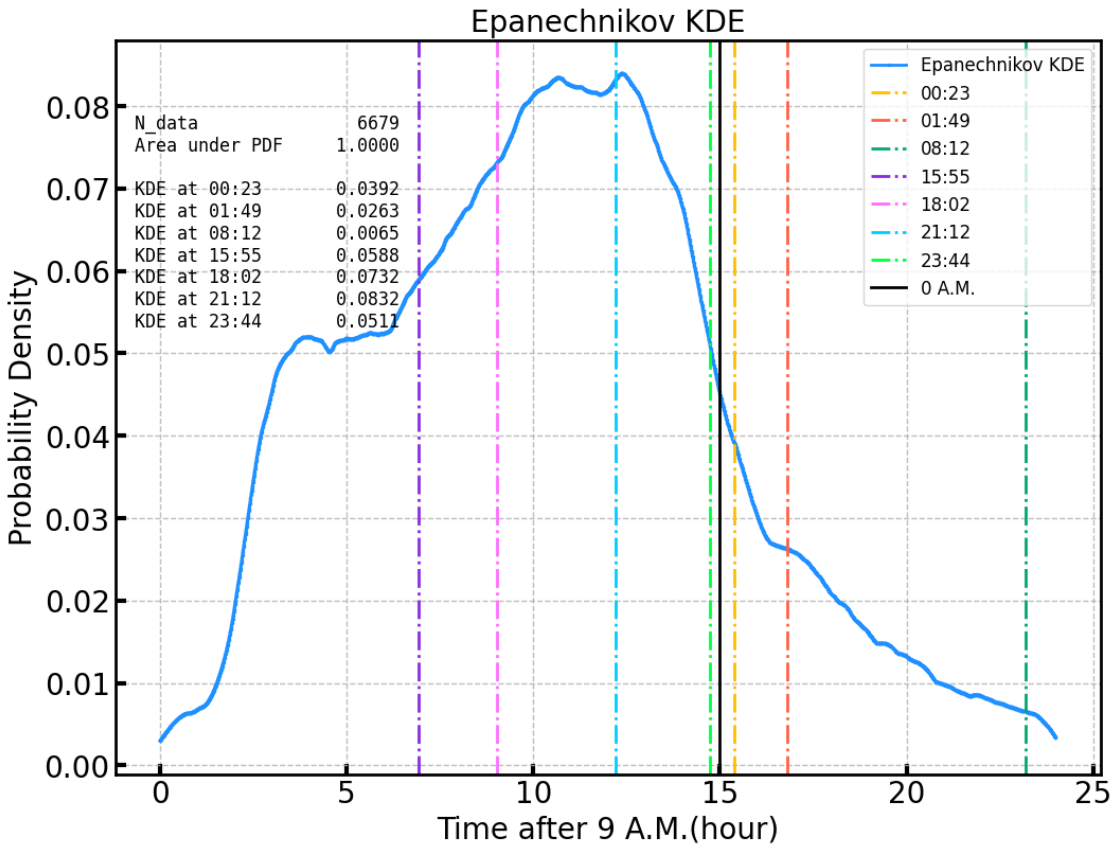
[ ]:
```
def getEvaluation(DAY, kde, isShifted = False):
    times, _ = getInterestedTime(isShifted)
    evaluation = np.zeros(len(times))
    dt = DAY[1] - DAY[0]
    for i, t in enumerate(times):
        evaluation[i] = kde[int(t/dt)]
    return evaluation
```

[ ]:
```
def plotEpanechnikovKDE(data, isShifted = False):
    DAY, kde, normalisation = KDEcore(data)
    evaluation = getEvaluation(DAY, kde, isShifted)
    t_intersted_num, t_intersted_str  = getInterestedTime(isShifted)

    fig, ax = plt.subplots(figsize=(12, 9))
    ax.plot(DAY, kde, label='Epanechnikov KDE')
    for i, t in enumerate(t_intersted_num):
        ax.axvline(x=t, color=COLOUR[i+1], linestyle='-.',␣
 ↪label=t_intersted_str[i])

    if isShifted:
        ax.axvline(x=15, color='k', linestyle='-', label='0 A.M.')
        ax.set_xlabel('Time after 9 A.M.(hour)')
        ax.legend(loc = 'upper right')
```

```
    else:
        ax.set_xlabel('Time (hour)')
        ax.legend(loc = 'upper center')
    ax.set_ylabel('Probability Density')
    ax.set_title('Epanechnikov KDE')
    d = {'N_data' : len(data),
         'Area under PDF' : f'{normalisation:.4f}',
         '': '',
         }
    for i, t in enumerate(t_intersted_num):
        d[f'KDE at {t_intersted_str[i]}'] = f'{evaluation[i]:.4f}'
    add_text_to_ax(0.02, 0.90, nice_string_output(d), ax, fontsize=12)
plotEpanechnikovKDE(time)
plotEpanechnikovKDE(time_shift_9, True)
```

```
100%|      | 2400/2400 [00:04<00:00, 571.53it/s]
100%|      | 2400/2400 [00:02<00:00, 954.25it/s]
```





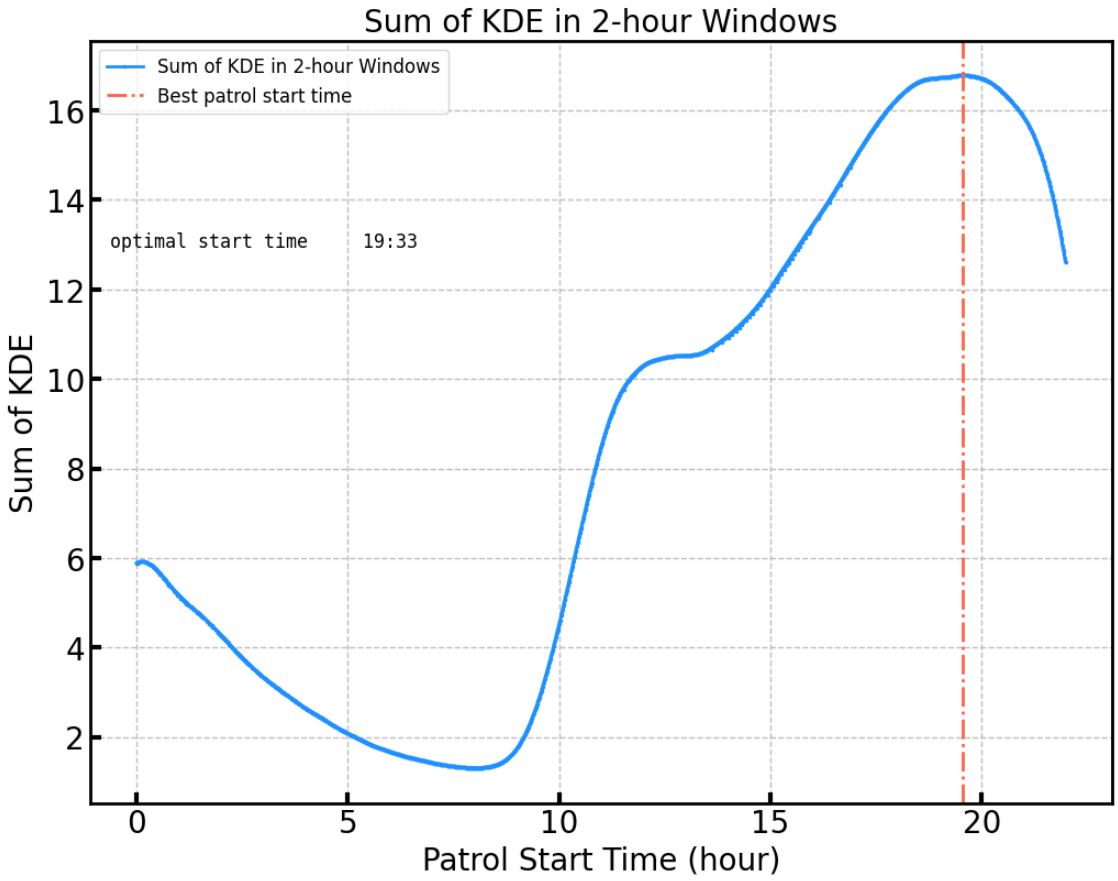
2.2 Prob2b : window

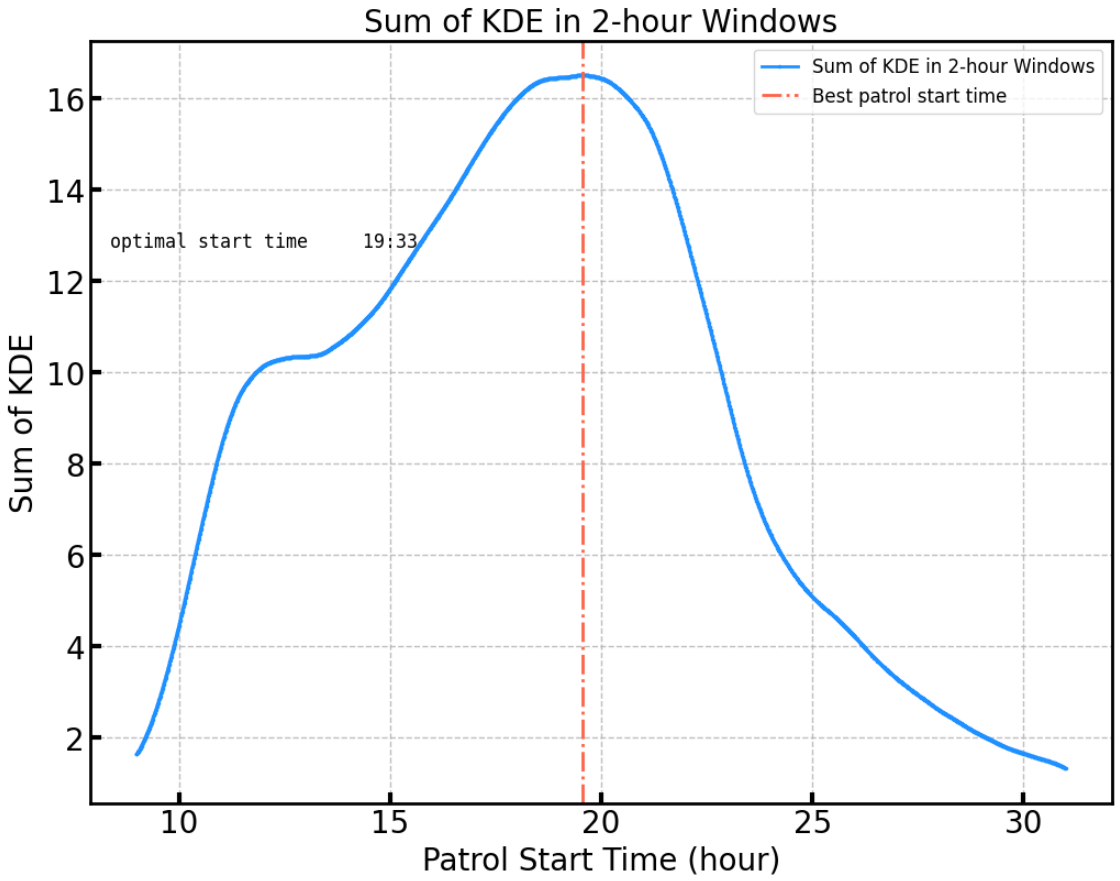
[ ]:
```
def getWindows(DAY, min, max, dt):
    WINDOW_SIZE = 2
    starts = np.arange(min, max, dt)
    windows = []
    for i, start in enumerate(starts):
        end = start + WINDOW_SIZE
        window = (DAY >= start) & (DAY <= end)
        windows.append(window)
    return windows
```

[ ]:
```
def best2hrWindow(data, isShifted = False):
    DAY, kde, _ = KDEcore(data)
    if isShifted:
        DAY = DAY + 9

    WINDOW_SIZE = 2
    w_min = DAY[0]
    w_max = DAY[-1] - WINDOW_SIZE
```

```
    dt = DAY[1] - DAY[0]

    window_starts = np.arange(w_min, w_max, dt)
    windows = getWindows(DAY, w_min, w_max, dt)
    window_sums = [kde[window].sum() for window in windows]

    best_start_time = window_starts[np.argmax(window_sums)]
    h = int(best_start_time)
    m = int((best_start_time - h) * 60)
    print(f'optimal choice: {h:02d}:{m:02d}')

    fig, ax = plt.subplots(figsize=(12, 9))
    ax.plot(window_starts, window_sums, label='Sum of KDE in 2-hour Windows')
    ax.axvline(x=best_start_time, color=COLOUR[2], linestyle='-.', label='Best␣
 ↪patrol start time')
    ax.set_xlabel('Patrol Start Time (hour)')
    ax.set_ylabel('Sum of KDE')
    ax.set_title('Sum of KDE in 2-hour Windows')
    ax.legend()
    d = {'optimal start time' : f'{h:02d}:{m:02d}',}
    add_text_to_ax(0.02, 0.75, nice_string_output(d), ax, fontsize=12)

best2hrWindow(time)
best2hrWindow(time_shift_9, True)
```

```
  0%|          | 0/2400 [00:00<?, ?it/s]100%|      | 2400/2400 [00:01<00:00,
1548.01it/s]

optimal choice: 19:33

100%|      | 2400/2400 [00:03<00:00, 739.64it/s]

optimal choice: 19:33
```


Sum of KDE in 2-hour Windows
Sum of KDE in 2-hour Windows
Best patrol start time
optimal start time 19:33
Sum of KDE
Patrol Start Time (hour)
0
5
10
15
20
2
4
6
8
10
12
14
16



3 prob2c

[ ]:
```
def normalisation(PDF_values, longitudes, latitudes):
    dx = np.diff(longitudes).mean()
    dy = np.diff(latitudes).mean()
    unit_area = dx * dy
    return PDF_values.sum() * unit_area
```

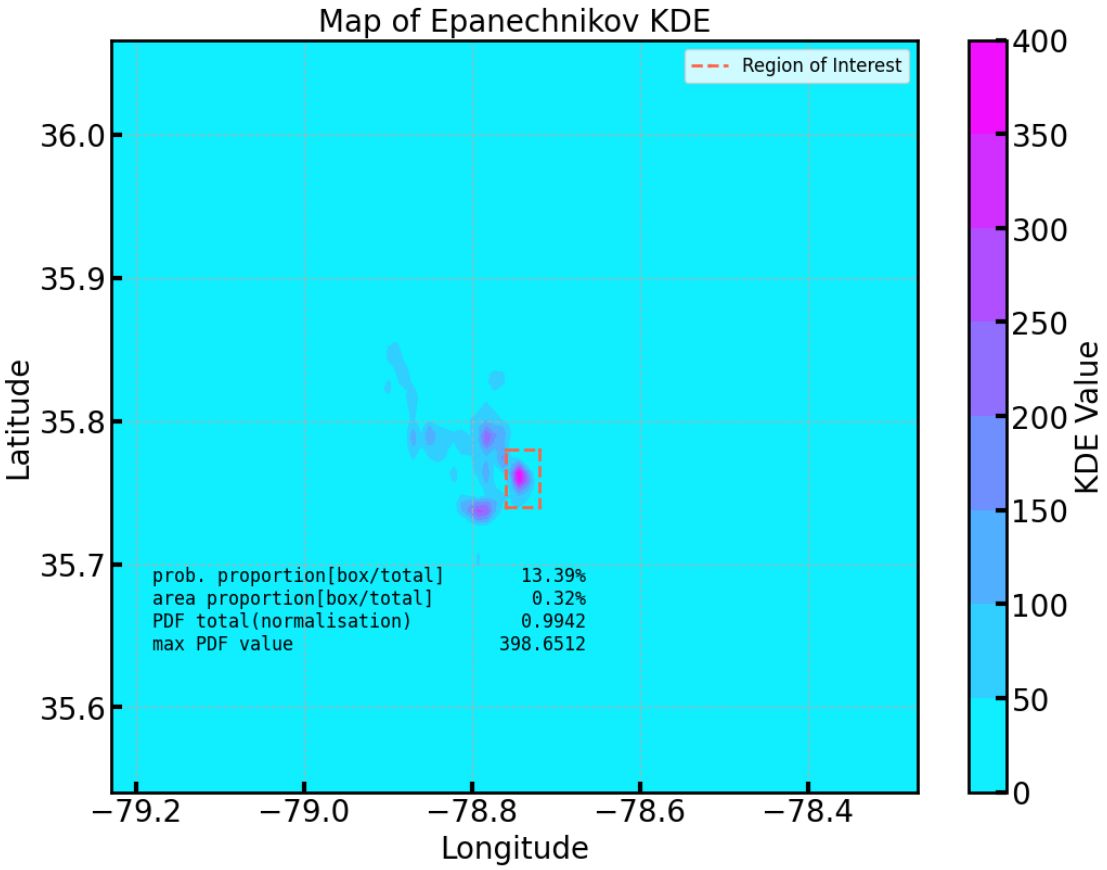
[ ]:
```
def getProportion(PDF_values, longitudes, latitudes, box_longitude,␣
↪box_latitude):
    lon_min, lon_max = box_longitude
    lat_min, lat_max = box_latitude

    lon_indices = np.where((longitudes >= lon_min) & (longitudes <= lon_max))[0]
    lat_indices = np.where((latitudes >= lat_min) & (latitudes <= lat_max))[0]

    box = PDF_values[min(lat_indices):max(lat_indices)+1, min(lon_indices):
↪max(lon_indices)+1]
```

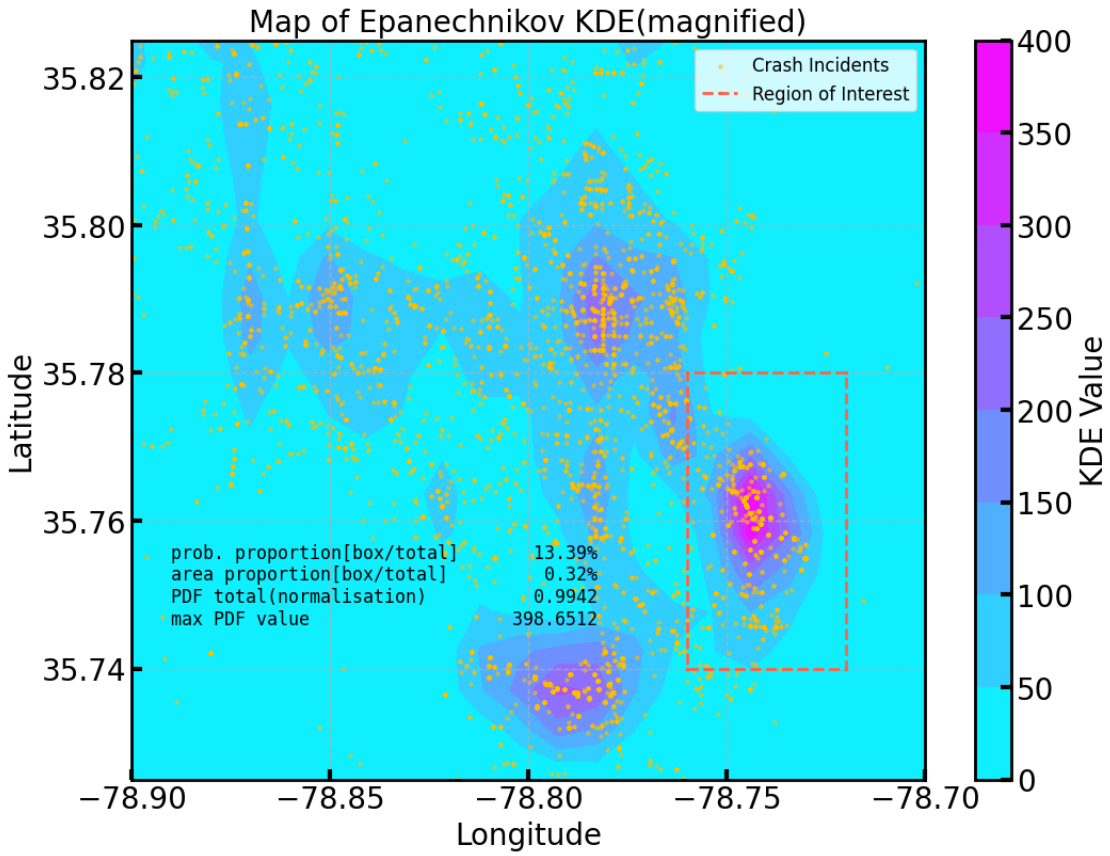
```
    volume_box = box.sum()
    volume_total = PDF_values.sum()

    return volume_box / volume_total
```

[ ]:
```
def twoDimensionalEpanechnikovKDE(data):
    X = data[['lon', 'lat']].values
    kde = KernelDensity(bandwidth=0.01, kernel='epanechnikov')
    kde.fit(X)
    longitudes = np.linspace(data['lon'].min(), data['lon'].max(), 100)
    latitudes = np.linspace(data['lat'].min(), data['lat'].max(), 100)
    total_area = (longitudes.max() - longitudes.min()) * (latitudes.max() -␣
 ↪latitudes.min())

    X_grid, Y_grid = np.meshgrid(longitudes, latitudes)
    grid_points = np.vstack([X_grid.ravel(), Y_grid.ravel()]).T

    # Calculate the log density (log PDF) values for the grid
    log_pdf_values = kde.score_samples(grid_points)
    PDF_values = np.exp(log_pdf_values).reshape(X_grid.shape)

    fig, ax = plt.subplots(figsize=(12, 9))
    contourf_plot = ax.contourf(X_grid, Y_grid, PDF_values, cmap = 'cool')
#     ax.scatter(data['lon'], data['lat'], s=5, color = COLOUR[2], alpha=0.5,␣
 ↪label='Crash Incidents')
    fig.colorbar(contourf_plot, ax=ax, label='KDE Value')
    ax.set_xlabel('Longitude')
    ax.set_ylabel('Latitude')
    ax.set_title('Map of Epanechnikov KDE')

    ## box volume proportion
    box_longitude = np.array([-78.76, -78.72])
    box_latitude = np.array([35.74, 35.78])
    box_area = (box_longitude[1] - box_longitude[0]) * (box_latitude[1] -␣
 ↪box_latitude[0])

    # plot
    ax.plot([box_longitude[0], box_longitude[1], box_longitude[1],␣
 ↪box_longitude[0], box_longitude[0]],
            [box_latitude[0], box_latitude[0], box_latitude[1],␣
 ↪box_latitude[1], box_latitude[0]],
            color=COLOUR[2], linestyle='--', label='Region of Interest')
    proportion = getProportion(PDF_values, longitudes, latitudes,␣
 ↪box_longitude, box_latitude)
    print(f"Proportion of the region within the total area: {proportion:.4f}")
    ax.legend()
```

```
    # Add text to the plot
    d = {'prob. proportion[box/total]': f'{proportion:.2%}',
         'area proportion[box/total]': f'{box_area / total_area:.2%}',
         'PDF total(normalisation)' : f'{normalisation(PDF_values, longitudes,␣
 ↪latitudes):.4f}',
         'max PDF value' : f'{PDF_values.max():.4f}',
         }
    add_text_to_ax(0.05, 0.30, nice_string_output(d), ax, fontsize=12)


    fig_mag, ax_mag = plt.subplots(figsize=(12, 9))
    contourf_plot_mag = ax_mag.contourf(X_grid, Y_grid, PDF_values, cmap =␣
 ↪'cool')
    fig_mag.colorbar(contourf_plot_mag, ax=ax_mag, label='KDE Value')
    ax_mag.scatter(data['lon'], data['lat'], s=5, color = COLOUR[1], alpha=0.5,␣
 ↪label='Crash Incidents')
    ax_mag.set_xlabel('Longitude')
    ax_mag.set_ylabel('Latitude')
    ax_mag.set_title('Map of Epanechnikov KDE(magnified)')
    ax_mag.set_xlim(-78.9, -78.7)
    ax_mag.set_ylim(35.725, 35.825)
    ax_mag.plot([box_longitude[0], box_longitude[1], box_longitude[1],␣
 ↪box_longitude[0], box_longitude[0]],
             [box_latitude[0], box_latitude[0], box_latitude[1],␣
 ↪box_latitude[1], box_latitude[0]],
             color=COLOUR[2], linestyle='--', label='Region of Interest')
    ax_mag.legend(loc = 'upper right')
    add_text_to_ax(0.05, 0.32, nice_string_output(d), ax_mag, fontsize=12)

twoDimensionalEpanechnikovKDE(data)
```

```
Proportion of the region within the total area: 0.1339
```


Map of Epanechnikov KDE
Region of Interest
prob. proportion[box/total] 13.39%
area proportion[box/total] 0.32%
PDF total(normalisation) 0.9942
max PDF value 398.6512
Latitude
Longitude
KDE Value
36.0
35.9
35.8
35.7
35.6
−79.2
−79.0
−78.8
−78.6
−78.4
400
350
300
250
200
150
100
50
0


Map of Epanechnikov KDE(magnified)
Crash Incidents
Region of Interest
prob. proportion[box/total] 13.39%
area proportion[box/total] 0.32%
PDF total(normalisation) 0.9942
max PDF value 398.6512
Latitude
Longitude
KDE Value
35.82
35.80
35.78
35.76
35.74
−78.90
−78.85
−78.80
−78.75
−78.70
400
350
300
250
200
150
100
50
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