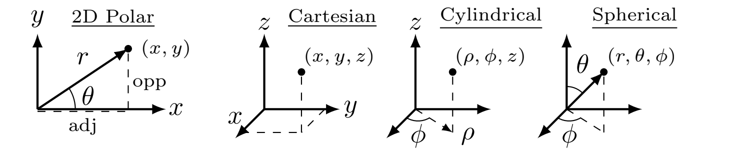
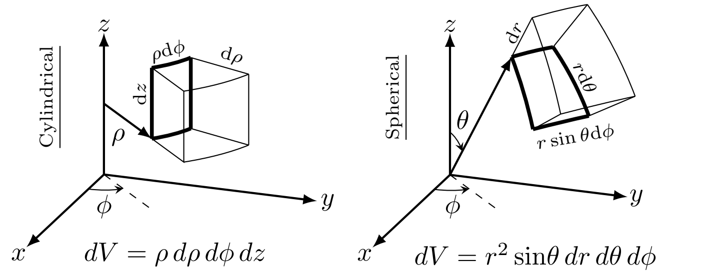
Coordinate Systems





$dV = \rho\, d\rho\, d\phi\, dz$ $\qquad$ $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$d\vec{\boldsymbol{\ell}} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$ $\qquad$ $d\vec{\boldsymbol{\ell}} = dr\hat{\boldsymbol{r}} + r d\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$

$\vec{\nabla} = \hat{\boldsymbol{\rho}}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{z}}\frac{\partial}{\partial z}$ $\qquad$ $\vec{\nabla} = \hat{\boldsymbol{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$

Trigonometry

$\sin(-x) = -\sin x$ (odd) $\qquad$ $\sin(\pi/2 - x) = \cos x$

$\cos(-x) = \cos x$ (even) $\qquad$ $\cos(\pi/2 - x) = \sin x$

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $1 + \tan^2 x = \sec^2 x$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$\sin(2x) = 2\sin x \cos x$ $\qquad$ $\sin(x/2) = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos(2x) = 1 - 2\sin^2 x$ $\qquad$ $\cos(x/2) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$ $\qquad$ $\tan(x/2) = \frac{1-\cos x}{\sin x}$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

⇐ Derivatives and Integrals ⇒

df/dx	$\boldsymbol{f(x)}$	$\int f dx$	integration strat
nx^{n-1}	x^n	$\left(\frac{1}{n+1}\right)x^{n+1}$	$n \neq -1$
ae^{ax}	e^{ax}	$(1/a)\, e^{ax}$	$u = ax$
$e^{ax}(ax+1)$	xe^{ax}	$e^{ax}\,\frac{(ax-1)}{a^2}$	$u = x \quad dv = e^{ax}dx$
$1/x$	$\ln(ax)$	$x\ln(ax) - x$	$u = \ln(ax) \quad dv = dx$
$\ln(a)a^x$	a^x	$\frac{a^x}{\ln(a)}$	$e^{\ln a^x}; u = x\ln(a)$
$\cos(x)$	$\sin(x)$	$-\cos(x)$	
$-\sin(x)$	$\cos(x)$	$\sin(x)$	
$\sec^2(x)$	$\tan(x)$	$-\ln\lvert\cos x\rvert$	$u = \cos(x)$
$2a\sin(ax)\cos(ax)$	$\sin^2(ax)$	$\frac{x}{2} - \frac{\sin(2ax)}{4a}$	trig identity
$-2a\sin(ax)\cos(ax)$	$\cos^2(ax)$	$\frac{x}{2} + \frac{\sin(2ax)}{4a}$	trig identity

$\frac{d}{dx}\sin^{-1}(x) = 1/\sqrt{1-x^2}$ $\qquad$ $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right|$

$\frac{d}{dx}\cos^{-1}(x) = -1/\sqrt{1-x^2}$ $\qquad$ $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$

$\frac{d}{dx}\tan^{-1}(x) = 1/(1+x^2)$ $\qquad$ $\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$

Vector Calculus

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \quad \text{Cartesian}$$

$\vec{\nabla} f$ — Gradient — $\int_{\vec{a}}^{\vec{b}}(\vec{\nabla} f)\cdot d\vec{\ell} = f(\vec{b}) - f(\vec{a})$

$\vec{\nabla}\cdot\vec{F}$ — Divergence — $\iiint(\vec{\nabla}\cdot\vec{F})dV = \oint_S \vec{F}\cdot d\vec{A}$

$\vec{\nabla}\times\vec{F}$ — Curl — $\iint(\vec{\nabla}\times\vec{F})\cdot d\vec{A} = \oint_C \vec{F}\cdot d\vec{\ell}$

Vector Multiplication

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\times\vec{B} = \left|\begin{array}{ccc|cc} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ A_x & A_y & A_z & A_x & A_y \\ B_x & B_y & B_z & B_x & B_y \end{array}\right. \quad = AB\sin\theta$$

← *Cross Product*
down right(+)
down left(−)

Electric Field $[N/C]$ $[V/m]$

$\vec{F} = q\vec{E}$ $\qquad$ $\left|\vec{F}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_1\, q_2}{r^2}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$ $\qquad$ $\vec{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i$

Gauss' Law $\quad \varepsilon_0 = 8.85\times 10^{-12}[F/m]$ permittivity

$\Phi_E = \iint_S \vec{E}\cdot\hat{n}\, dA$ $\qquad$ Flux $\Phi_E = EA\cos\theta$

$\oiint_S \vec{E}\cdot\hat{n}\, dA = \frac{q_{\text{in}}}{\varepsilon_0}$ $\qquad$ $E\cdot dA = \frac{q_{\text{in}}}{\varepsilon_0}$

Electric Potential [Volts] $[V]$ $[J/C]$

$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{\ell}$ $\qquad$ $\vec{E} = -\vec{\nabla}V$ $\qquad$ $\vec{E}_x = -\frac{dV}{dx}\hat{x}$

$\Delta V_{\text{point}} = \frac{kq}{r}\Big|_A^B$ $\qquad$ $W_{nc} = \Delta K + \Delta U$

$\Delta V_{\text{line}} = \frac{-\lambda}{2\pi\varepsilon_0}\ln(r)\Big|_A^B$ $\qquad$ $K = \frac{1}{2}mv^2$ $\qquad$ $U_g = mgh$

$\Delta V_{\text{plane}} = -E\Delta x\Big|_{x_i}^{x_f}$ $\qquad$ $U_E = qV$ $\qquad$ $U_s = \frac{1}{2}kx^2$

Magnetic Field [Tesla] $[T]$ $[10^4$ Gauss$]$

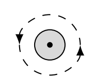
$\vec{F} = q\,\vec{v}\times\vec{B} = I\vec{\ell}\times\vec{B}$ $\qquad$ $\vec{\mu} = I\,\vec{A} = \frac{q}{2m}\vec{L}$

$\vec{\tau} = \vec{\mu}\times\vec{B}$

$r = \frac{mv}{qB}$ $\qquad$ $T = \frac{2\pi m}{qB}$ $\qquad$ $U_B = -\vec{\mu}\cdot\vec{B}$

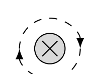
Ampere's Law $\quad \mu_0 = 4\pi\times 10^{-7}[N/A^2]$ permeability

$\Phi_B = \iint_S \vec{B}\cdot\hat{n}\, dA$ $\qquad$ Flux $\Phi_B = BA\cos\theta$



$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I$ $\qquad$ $\oiint_S \vec{B}\cdot d\vec{A} = 0$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$ $\qquad$ $\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$



$B = \frac{\mu_0 I}{2\pi r}$ $\qquad$ $B = \mu_0\frac{N}{\ell}I$

Faraday's Law of Induction

$\mathcal{E} = \oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$ $\qquad$ nature abhors change in magnetic flux

$\mathcal{E} = NBA\omega\sin(\omega t)$ $\qquad$ $\mathcal{E} = B\ell v$

Maxwells Equations

$$\vec{\boldsymbol{F}} = q\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}\right) \quad \text{Lorrentz}$$

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ — Gauss — $\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$

$\vec{\nabla}\cdot\vec{B} = 0$ — NoName — $\oint_S \vec{B}\cdot d\vec{A} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ — Faraday — $\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{\partial\Phi_B}{dt}$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{I} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$ — Ampere — $\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I + \varepsilon_0\mu_0\frac{\partial\Phi_E}{dt}$

$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ — EM-Wave — $\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0\mu_0\frac{\partial^2 E_y}{\partial t^2}$

Conversions

$1\,in = 2.54\,cm$	$1\,ft\text{-}lb = 1.356\,N{\cdot}m$	tera	T	10^{12}
$1\,ft = 0.3048\,m$	$1\,cal = 4.186\;J$	giga	G	10^{9}
$1\,yard = 3\,ft$	$1\,kWh = 3.60\,MJ$	mega	M	10^{6}
$1\,mi = 5280\,ft$	$1\,hp = 746\,J/s$	kilo	k	10^{3}
$1\,\frac{mi}{hr} = 0.447\,\frac{m}{s}$	$1\;C = 6.25\cdot10^{18}\;e$	centi	c	10^{-2}
$1\,L = 1000\,cm^3$	$1eV = 1.60\cdot10^{-19}\;J$	milli	m	10^{-3}
$1\,gal = 3.786\,L$	$T_C = 5/9(T_F - 32)$	micro	μ	10^{-6}
$1\,ft^3 = 28.32\,L$	$T = T_C + 273$	nano	n	10^{-9}
$1\,lb = 4.448\,N$	$1\,atm = 101325\,Pa$	pico	p	10^{-12}

Greek Symbols

alpha	α		iota	ι		rho	ρ	
beta	β		kappa	κ		sigma	σ	Σ
gamma	γ	Γ	lambda	λ	Λ	tau	τ	
delta	δ	Δ	mu	μ		upsilon	υ	Υ
epsilon	ϵ		nu	ν		phi	ϕ	Φ
zeta	ζ		xi	ξ	Ξ	chi	χ	
eta	η		omicron	o		psi	ψ	Ψ
theta	θ	Θ	pi	π	Π	omega	ω	Ω

Physical Constants

Name	Symbol	Value
Gravitation Constant	G	$6.67\cdot10^{-11}\,N{\cdot}m^2/kg^2$
Earth Surface Accel	g	$9.8\;m/s^2$
Boltzmann Constant	k_B	$1.38\cdot10^{-23}\;J/K$
Avogadros Number	N_A	$6.02\cdot10^{23}\;\text{mol}^{-1}$
Ideal Gas Constant	R	$8.314\;J/(\text{mol}{\cdot}K)$
Stefan-Boltzmann	σ	$5.67\cdot10^{-8}\;W/(m^2{\cdot}K^4)$
Speed of Light	c	$2.99792458\cdot10^{8}\;m/s$
Electrostatic Constant	k_e	$8.99\cdot10^{9}\;N{\cdot}m^2/C^2$
Electric Permittivity	ε_0	$8.854\cdot10^{-12}\;C^2/(N{\cdot}m^2)$
Magnetic Permeability	μ_0	$4\pi\cdot10^{-7}\;T{\cdot}m/A$
Elementary Charge	e	$1.60\cdot10^{-19}\;C$
Planck Constant	h	$6.63\cdot10^{-34}\;J\cdot s$
Rydberg Constant	R_H	$1.09737\cdot10^{-7}\;m^{-1}$
Bohr Magneton	μ_B	$9.274\cdot10^{-24}\;J/T$
Bohr Radius	a_0	$5.292\cdot10^{-11}\;m$
Rest mass of Electron	m_e	$9.1093\cdot10^{-31}\;kg$
Rest mass of Proton	m_p	$1.6726\cdot10^{-27}\;kg$
Rest mass of Neutron	m_n	$1.6749\cdot10^{-27}\;kg$
Atomic Mass Unit	u	$931.49\;MeV/c^2$

Capacitance C [Farad] [F]

$$\Delta Q = C\Delta V \qquad C = \epsilon_0\frac{A}{d} \qquad C = \kappa C_0 \qquad \kappa = \frac{\varepsilon_0}{\varepsilon}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \leftarrow\text{series} \quad \text{parallel}\rightarrow \quad C_{eq} = C_1 + C_2$$

$$U_C = \frac{1}{2}V^2C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Current I [Amp] [A] [C/s] Resistance R [Ohm] [Ω]

$$I = \frac{dQ}{dt} \qquad I = \iint \vec{J}\cdot d\vec{A}$$

$$I = nqAv_d \qquad E = \rho J$$

$$R = \rho\frac{\ell}{A} \qquad \rho = \rho_0\big[1 + \alpha(T - T_0)\big]$$

Direct-Current Circuits

$$\Delta V = IR \qquad \mathbb{P} = IV = I^2R = \frac{V^2}{R} \qquad \text{junction} \rightarrow I_{in} = I_{out}$$

$$\text{loop} \rightarrow \Sigma\Delta V = 0$$

$$R_{eq} = R_1 + R_2 \quad \leftarrow\text{series} \quad \text{parallel}\rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$q_\uparrow(t) = Q\left(1 - e^{-t/\tau}\right) \qquad q_\downarrow(t) = Qe^{-t/\tau} \qquad \tau = RC$$

Inductance L

$$\mathcal{E} = -L\frac{dI}{dt} \qquad L = \frac{N\Phi_B}{I} = \mu_0\frac{N^2}{\ell}A$$

$$I_\uparrow(t) = \frac{\mathcal{E}}{R}\left(1 - e^{\frac{-t}{\tau}}\right) \qquad I_\downarrow(t) = \frac{\mathcal{E}}{R}e^{\frac{-t}{\tau}} \qquad \tau = \frac{L}{R}$$

Alternating-Current Circuits

$$X_L = \omega L \qquad X_C = \frac{1}{\omega C} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{\text{RMS}} = \frac{I_{\max}}{2} \qquad \Delta V_{\text{RMS}} = I_{\text{RMS}}Z \qquad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\mathbb{P}_{avg} = I_{\text{RMS}}\Delta V_{\text{RMS}}\cos(\phi) = \frac{1}{2}I_{\max}V_{\max}\cos\phi$$

$$q(t) = Q_{\text{Max}}e^{\frac{-RT}{2L}}\cos\omega_d t \qquad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \Delta V_{\text{sec}} = \frac{N_{\text{sec}}}{N_{\text{pri}}}\Delta V_{\text{pri}}$$