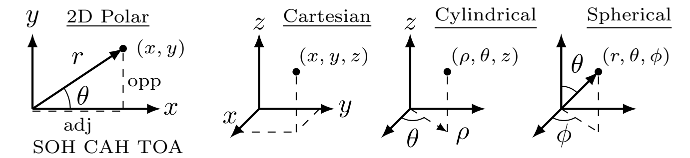
Physics - Mechanics

Coordinate Systems



Kinematics - Constant Acceleration

$$\vec{v}_f = \vec{v}_i + \vec{a}\,t \qquad \vec{x}_f = \vec{x}_i + \vec{v}_i\,t + \frac{1}{2}\vec{a}\,t^2$$

$v_f^2 = v_i^2 + 2\,a(x_f - x_i)$	$\vec{a}$: acceleration $[m/s^2]$
$\vec{a} = \frac{d\vec{v}}{dt} \qquad \vec{v} = \frac{d\vec{x}}{dt}$	$\vec{v}$: velocity $[m/s]$
$v = \frac{\Delta x}{\Delta t}$ (only if $a = 0$)	$\vec{x}$: position $[m]$ $\quad t$: time $[s]$

Force $\mathbf{F}$ [Newton] $[N]$ $[kg \cdot m/s^2]$

$$\sum \vec{F} = m\vec{a} \qquad \vec{F}_{12} = -\vec{F}_{21} \ \ \text{(by,on)}$$

$a_c = v^2/r$	Weight	mg	*down,center of Earth*
$a = \sqrt{a_t^2 + a_c^2}$	Tension	T	*only pulls*
	Normal	N	*perpendicular,reacts*
$f = \mu N$	Friction	f	*opposes motion*
$F_s = -kx$	Spring	F_s	*spring const* k $[N/m]$

Energy $\mathbf{E}$ [Joule] $[J]$

$$W_{nc} = \Delta K + \Delta U \qquad F_x = -\frac{dU}{dx} \qquad W = \int \vec{F}(r)\cdot d\vec{r}$$

$K = \frac{1}{2}mv^2$	$\leftarrow$ Kinetic E	$\mathbb{P} = \frac{dE}{dt} = \vec{F}\cdot\vec{v} = \vec{\tau}\cdot\vec{\omega}$
$K_{\text{Rot}} = \frac{1}{2}I\omega^2$	Power $\rightarrow$	$[Watt = J/s]$
$U_g = m\,g\,h$	$\leftarrow$ Potential E	$\vec{A}\cdot\vec{B} = A\,B\,\cos(\theta)$
$U_s = \frac{1}{2}kx^2$	*Dot Product* $\rightarrow$	$= A_x\,B_x + A_y\,B_y + A_z\,B_z$

Momentum $\mathbf{p}$ $[kg \cdot m/s]$

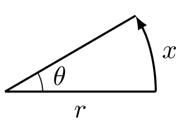
$$\vec{p} = m\vec{v} \qquad \vec{F} = \frac{d\vec{p}}{dt} \qquad \Delta\vec{p}_{\text{sys}} = \int \vec{F}_{\text{ext}}(t)dt$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \qquad (\leftarrow \Delta p = 0)$$

$$\frac{1}{2}m_1|\vec{v}_{1i}|^2 + \frac{1}{2}m_2|\vec{v}_{2i}|^2 = \frac{1}{2}m_1|\vec{v}_{1f}|^2 + \frac{1}{2}m_2|\vec{v}_{2f}|^2 \qquad (\leftarrow \Delta K = 0)$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f} \qquad (\leftarrow \text{1D elastic})$$

Angular Variables

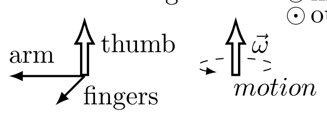


$x = r\theta$

$v = r\omega$

$a = r\alpha$

thumb=arm×fingers



$$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\,t \qquad \vec{\theta} = \vec{\theta}_i + \vec{\omega}_i\,t + 1/2\,\vec{\alpha}\,t^2$$

θ	ang pos $[rad]$	x	A	amplitude	
ω	ang vel $[rad/s]$	v	f	frequency $[Hz]$	$\omega = 2\pi f$
α	ang acc $[rad/s^2]$	a	λ	wavelength $[m]$	
τ	torque $\leftrightarrow$ force	F	k_λ	wave number	$k_\lambda = 2\pi/\lambda$
I	MoI $\leftrightarrow$ mass	m	ϕ	phase angle	
L	ang momentum	p	T	period $[s]$	$T = 1/f$

Torque $\boldsymbol{\tau}$ $[N \cdot m]$ Angular Momentum $\mathbf{L}$ $[kg \cdot m^2/s]$

$$\sum \vec{\tau} = I\vec{\alpha} \qquad \vec{\tau} = \vec{r}\times\vec{F} = \frac{d\vec{L}}{dt} \qquad |\vec{\tau}| = rF\sin\theta$$

$$\vec{L} = I\vec{\omega} \qquad \vec{L} = \vec{r}\times\vec{p} \qquad \left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

$$\vec{A}\times\vec{B} = \left|\begin{array}{ccc|cc} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ A_x & A_y & A_z & A_x & A_y \\ B_x & B_y & B_z & B_x & B_y \end{array}\right.$$

$\leftarrow$ *Cross Product*

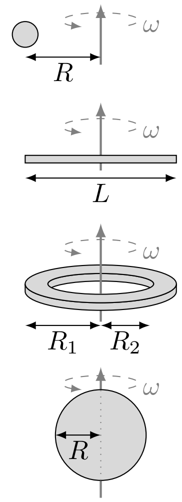
down right$(+)$

down left$(-)$

Center of Mass $[m]$

$$x_{\text{CM}} = \frac{\sum x_i \cdot m_i}{\sum m_i} \qquad \vec{r}_{\text{CM}} = \frac{1}{M}\int \vec{r}\,dm = \frac{1}{M}\iiint \rho(\vec{r})\,\vec{r}\,dV$$

Moment of Inertia $[kg \cdot m^2]$



$$I = \int r^2 dm \qquad I_\parallel = I_{\text{CM}} + MD^2$$

$$\rho = m/V \qquad \sigma = m/A \qquad \lambda = m/\ell$$

Shape	MoI
Point Mass	MR^2
Thin Rod (center)	$1/12 \cdot ML^2$
Thin Rod (end)	$1/3 \cdot ML^2$
Rectangular Plate	$1/12 \cdot M(L_1^2 + L_2^2)$
Cylindrical Shell	MR^2
Solid Cylinder	$1/2 \cdot MR^2$
Hollow Cylinder	$1/2 \cdot M(R_1^2 + R_2^2)$
Spherical Shell	$2/3 \cdot MR^2$
Solid Sphere	$2/5 \cdot MR^2$
Hollow Sphere	$2/5 \cdot \frac{R_1^5 - R_2^5}{R_1^3 - R_2^3}$

Universal Gravitation

$$F_G = G\frac{m_1 m_2}{r^2} \qquad U_G = -G\frac{m_1 m_2}{r} \qquad v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

Fluid Dynamics - Pressure $\mathbf{P}$ [Pascal] $[Pa]$ $[N/m^2]$

$$F = \int P\,dA \qquad P = P_0 + \rho g h \qquad F_B = \rho g V_{\text{disp}}$$

$$A_1 v_1 = A_2 v_2 = \text{const} \qquad P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{const}$$

Oscillations - 2^{nd} order ODE

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \Rightarrow \quad x(t) = A\cos(\omega t + \phi)$$

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx \quad \Rightarrow \quad x(t) = Ae^{-\omega_d t}\cos(\omega t + \phi)$$

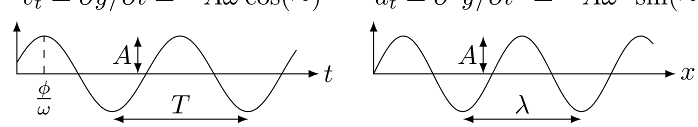
Simple	AngFreq	Damped	Behavior
mass-spring	$\omega = \sqrt{k/m}$	$\omega_0 = \sqrt{k/m}$	$\omega_d < \omega_0$ under
pendulum	$\omega = \sqrt{g/\ell}$	$\omega_d = b/2m$	$\omega_d = \omega_0$ critical
rigid pend	$\omega = \sqrt{\frac{mgd}{I}}$	$\omega^2 = \omega_0^2 - \omega_d^2$	$\omega_d > \omega_0$ over

Travelling Waves - 2^{nd} order PDE

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \Rightarrow \quad y(x,t) = f(x \mp vt)$$

$$y(x,t) = A\sin(k_\lambda x - \omega t + \phi) \qquad v = \lambda f = \sqrt{\frac{F_T}{m/\ell}}$$

$$v_t = \partial y/\partial t = -A\omega\cos(\sim) \qquad a_t = \partial^2 y/\partial t^2 = -A\omega^2\sin(\sim)$$



Mechanical Waves

$$f' = \left(\frac{v + v_o}{v - v_s}\right) f \qquad f_{\text{beat}} = |f_1 - f_2|$$

$$f_n = \frac{(2n)v}{4L} \quad \leftarrow \text{fixed/fixed} \qquad \text{fixed/free} \rightarrow \quad f_n = \frac{(2n+1)v}{4L}$$

Conversions

$1\,in = 2.54\,cm$	$1\,ft\text{-}lb = 1.356\,N{\cdot}m$	tera	T	10^{12}
$1\,ft = 0.3048\,m$	$1\,cal = 4.186\,J$	giga	G	10^{9}
$1\,yard = 3\,ft$	$1\,kWh = 3.60\,MJ$	mega	M	10^{6}
$1\,mi = 5280\,ft$	$1\,hp = 746\,J/s$	kilo	k	10^{3}
$1\,\frac{mi}{hr} = 0.447\,\frac{m}{s}$	$1\,C = 6.25\cdot 10^{18}\,e$	centi	c	10^{-2}
$1\,L = 1000\,cm^3$	$1eV = 1.60\cdot 10^{-19}\,J$	milli	m	10^{-3}
$1\,gal = 3.786\,L$	$T_C = 5/9(T_F - 32)$	micro	μ	10^{-6}
$1\,ft^3 = 28.32\,L$	$T = T_C + 273$	nano	n	10^{-9}
$1\,lb = 4.448\,N$	$1\,atm = 101325\,Pa$	pico	p	10^{-12}

Greek Symbols

alpha	α		iota	ι		rho	ρ	
beta	β		kappa	κ		sigma	σ	Σ
gamma	γ	Γ	lambda	λ	Λ	tau	τ	
delta	δ	Δ	mu	μ		upsilon	υ	Υ
epsilon	ϵ		nu	ν		phi	ϕ	Φ
zeta	ζ		xi	ξ	Ξ	chi	χ	
eta	η		omicron	o		psi	ψ	Ψ
theta	θ	Θ	pi	π	Π	omega	ω	Ω

Physical Constants

Name	Symbol	Value
Gravitation Constant	G	$6.67\cdot 10^{-11} N{\cdot}m^2/kg^2$
Earth Surface Accel	g	$9.8\ m/s^2$
Boltzmann Constant	k_B	$1.38\cdot 10^{-23}\ J/K$
Avogadros Number	N_A	$6.02\cdot 10^{23}\ \text{mol}^{-1}$
Ideal Gas Constant	R	$8.314\ J/(\text{mol}{\cdot}K)$
Stefan-Boltzmann	σ	$5.67\cdot 10^{-8}\ W/(m^2{\cdot}K^4)$
Speed of Light	c	$2.99792458\cdot 10^{8}\ m/s$
Electrostatic Constant	k_e	$8.99\cdot 10^{9}\ N{\cdot}m^2/C^2$
Electric Permittivity	ε_0	$8.854\cdot 10^{-12}\ C^2/(N{\cdot}m^2)$
Magnetic Permeability	μ_0	$4\pi\cdot 10^{-7}\ T{\cdot}m/A$
Elementary Charge	e	$1.60\cdot 10^{-19}\ C$
Planck Constant	h	$6.63\cdot 10^{-34}\ J\cdot s$
Rydberg Constant	R_H	$1.09737\cdot 10^{-7}\ m^{-1}$
Bohr Magneton	μ_B	$9.274\cdot 10^{-24}\ J/T$
Bohr Radius	a_0	$5.292\cdot 10^{-11}\ m$
Rest mass of Electron	m_e	$9.1093\cdot 10^{-31}\ kg$
Rest mass of Proton	m_p	$1.6726\cdot 10^{-27}\ kg$
Rest mass of Neutron	m_n	$1.6749\cdot 10^{-27}\ kg$
Atomic Mass Unit	u	$931.49\ MeV/c^2$
Mass Earth	M_E	$5.97\cdot 10^{24}\ kg$
Mass Sun	$M_\odot$	$1.99\cdot 10^{30}\ kg$
Radius Earth	R_E	$6.37\cdot 10^{6}\ m$
Radius Sun	$R_\odot$	$6.96\cdot 10^{8}\ m$
Distance Earth/Sun	AU	$1.496\cdot 10^{11}\ m$

Speed of Sound $[m/s]$

Material	v	Material	v	Material	v
Air 25℃	343	Ice	3845	Copper	5010
Air 0℃	331	Glass	5640	Aluminum	6420
Steam	478	P-wave	4k-7k	Iron	5950
Water	1492	S-wave	2k-5k	Lead	1960

Friction Coefficients $f = \mu N$

Material	μ_k	μ_s	Material	μ_k	μ_s
Tire-DryConc	0.68	0.90	Glass-Glass	0.4	0.94
Tire-WetConc	0.58		Wood-Wood	0.30	0.42
Tire-Ice	0.15		Ski-Snow	0.05	0.14
Steel-Steel	0.57	0.74	Teflon-Teflon	0.04	0.04
Copper-Steel	0.36	0.53	Synovial Fluid	0.01	0.01

Equation of State - P $[Pa]$ V $[m^3]$ T $[K]$ n $[mol]$

$PV = nRT$ $\quad PV = Nk_BT$ $\quad$ Ideal Gas

$$P = \frac{nRT}{V-b} - \frac{a}{V^2}$$ Van der Waals $\quad a\to$strength $b\to$volume

Statistical Thermodynamics *Ideal Gas*

$$\Delta U = n\,C_V dT \qquad P = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}m\bar{v}^2\right)$$

$$C_V = \frac{\text{DoF}}{2}R \qquad C_P = \frac{\text{DoF}+2}{2}R \qquad \gamma = C_P/C_V$$

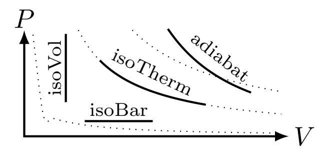
$$S = k_B\ln(\mathbb{P}) \qquad \mathbb{P}_{\text{Boltz}} \propto exp\,(-E_i/k_bT)$$

1^{st} Law of Thermodynamics $[J]$ [Joule]

$$\Delta U = \Delta Q + \Delta W \qquad dU = TdS + (\text{-}PdV)$$

$$\Delta W = \int_{V_i}^{V_f} -P\,dV \quad \Rightarrow$$

$$\Delta Q = \text{“Heat”} = \int TdS$$



Heat Engine

Q_H → Engine → $W_{\text{eng}} = |Q_H| - |Q_C|$, Q_C

$$e = \frac{W_{\text{eng}}}{Q_H} = 1 - \frac{|Q_C|}{|Q_H|}$$

$$e_{\text{carnot}} = 1 - \frac{T_C}{T_H}$$

PV^m	Process	ΔU_{int}	ΔQ	ΔW
	General	nC_VdT	$dU - dW$	$-\int P\,dV$
∞	Isovolume	$nC_V\Delta T$	$nC_V\Delta T$	0
0	Isobaric	$nC_V\Delta T$	$nC_P\Delta T$	$-P\Delta V$
1	Isothermal	0	$-\Delta W$	$-nRT\ln(V_f/V_i)$
γ	Adiabatic	$nC_V\Delta T$	0	$-\frac{P_fV_f - P_iV_i}{(1-\gamma)}$

$P_1V_1^\gamma = P_2V_2^\gamma = \text{constant}$ $\quad$ ($\leftarrow$ Adiabatic $dQ = 0$)

Thermal Material Properties

Material	Density ρ (kg/m^3)	Specific Heat c $(\frac{J}{kg℃})$	Thermal Cond k $(\frac{W}{m℃})$	Volume Expans β $(\frac{1}{℃})\cdot 10^{-6}$
Steam	0.590	2010	0.027	3400
Air	1.29	1004	0.023	3670
Water	1000	4186	0.6	214
Ethanol	785.1	2440	0.17	109
Ice	920	2090	~ 2	150
Glass	2500	840	1.05	27
Aluminum	2700	900	238	72
Copper	8600	387	397	51
Iron	7800	448	79.5	33
Lead	11300	128	34.7	87

Latent Heat of Materials

$$\Sigma Q_{\text{iso}} = 0 \qquad \Delta Q = mc\Delta T \qquad \Delta Q = L\Delta m$$

Material	Melting (℃)	L_{fusion} (J/kg)	Boiling (℃)	L_{vapor} (J/kg)
Nitrogen	-209.97	2.55×10^4	-195.81	2.01×10^5
Water	0.00	3.33×10^5	100.0	2.26×10^6
Ethanol	-114	1.04×10^5	78	8.54×10^5

Physics - Thermodynamics