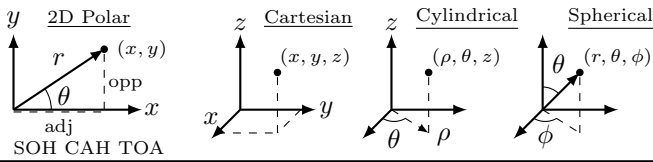


Coordinate Systems



Kinematics - Constant Acceleration

$$\vec{v}_f = \vec{v}_i + \vec{a}t \quad \vec{x}_f = \vec{x}_i + \vec{v}_i t + \frac{1}{2} \vec{a}t^2$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \quad \vec{a} : \text{acceleration [m/s}^2\text{]}$$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \vec{v} = \frac{d\vec{x}}{dt} \quad \vec{v} : \text{velocity [m/s]}$$

$$v = \frac{\Delta x}{\Delta t} \quad (\text{only if } a = 0) \quad \vec{x} : \text{position [m]} \quad t : \text{time [s]}$$

Force **F** [Newton] [N] [kg · m/s²]

$$\sum \vec{F} = m\vec{a} \quad \vec{F}_{12} = -\vec{F}_{21} \quad (\text{by, on})$$

$$a_c = v^2/r \quad \text{Weight } mg \quad \text{down, center of Earth}$$

$$a = \sqrt{a_t^2 + a_c^2} \quad \text{Tension } T \quad \text{only pulls}$$

$$f = \mu N \quad \text{Normal } N \quad \text{perpendicular, reacts}$$

$$F_s = -kx \quad \text{Friction } f \quad \text{opposes motion}$$

$$\quad \quad \quad \text{Spring } F_s \quad \text{spring const } k \text{ [N/m]}$$

Energy **E** [Joule] [J]

$$W_{nc} = \Delta K + \Delta U \quad F_x = -\frac{dU}{dx} \quad W = \int \vec{F}(r) \cdot d\vec{r}$$

$$\begin{array}{|l|l|l|} \hline K = \frac{1}{2}mv^2 & \leftarrow \text{Kinetic E} & \mathbb{P} = \frac{dE}{dt} = \vec{F} \cdot \vec{v} = \vec{\tau} \cdot \vec{\omega} \\ K_{\text{Rot}} = \frac{1}{2}I\omega^2 & \text{Power } \rightarrow & [\text{Watt} = \text{J/s}] \\ \hline U_g = mgh & \leftarrow \text{Potential E} & \vec{A} \cdot \vec{B} = AB \cos(\theta) \\ U_s = \frac{1}{2}kx^2 & \text{Dot Product } \rightarrow & = A_x B_x + A_y B_y + A_z B_z \end{array}$$

Momentum **p** [kg · m/s]

$$\vec{p} = m\vec{v} \quad \vec{F} = \frac{d\vec{p}}{dt} \quad \Delta \vec{p}_{\text{sys}} = \int \vec{F}_{\text{ext}}(t) dt$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (\leftarrow \Delta p = 0)$$

$$\frac{1}{2}m_1 |\vec{v}_{1i}|^2 + \frac{1}{2}m_2 |\vec{v}_{2i}|^2 = \frac{1}{2}m_1 |\vec{v}_{1f}|^2 + \frac{1}{2}m_2 |\vec{v}_{2f}|^2 \quad (\leftarrow \Delta K = 0)$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f} \quad (\leftarrow \text{1D elastic})$$

Angular Variables

thumb=arm × fingers ⊗ in
⊙ out

arm ↑ thumb
fingers ↓ fingers

$\vec{\omega}$ motion

$$x = r\theta \quad v = r\omega \quad a = r\alpha$$

$$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}t \quad \vec{\theta} = \vec{\theta}_i + \vec{\omega}_i t + \frac{1}{2} \vec{\alpha}t^2$$

θ	ang pos [rad]	x	A	amplitude	
ω	ang vel [rad/s]	v	f	frequency [Hz]	$\omega = 2\pi f$
α	ang acc [rad/s ²]	a	λ	wavelength [m]	
τ	torque ↔ force	F	k_λ	wave number	$k_\lambda = 2\pi/\lambda$
I	MoI ↔ mass	m	ϕ	phase angle	
L	ang momentum	p	T	period [s]	$T = 1/f$

Torque **τ** [N · m] Angular Momentum **L** [kg · m²/s]

$$\sum \vec{\tau} = I\vec{\alpha} \quad \vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} \quad |\vec{\tau}| = rF \sin \theta$$

$$\vec{L} = I\vec{\omega} \quad \vec{L} = \vec{r} \times \vec{p} \quad |\vec{A} \times \vec{B}| = AB \sin \theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \leftarrow \text{Cross Product}$$

down right(+)
down left(-)

Center of Mass [m]

$$x_{\text{CM}} = \frac{\sum x_i \cdot m_i}{\sum m_i} \quad \vec{r}_{\text{CM}} = \frac{1}{M} \int \vec{r} dm = \frac{1}{M} \iiint \rho(\vec{r}) \vec{r} dV$$

Moment of Inertia [kg · m²]

$$I = \int r^2 dm \quad I_{\parallel} = I_{\text{CM}} + MD^2$$

$$\rho = m/V \quad \sigma = m/A \quad \lambda = m/\ell$$

Shape	MoI
Point Mass	MR^2
Thin Rod (center)	$1/12 \cdot ML^2$
Thin Rod (end)	$1/3 \cdot ML^2$
Rectangular Plate	$1/12 \cdot M(L_1^2 + L_2^2)$
Cylindrical Shell	MR^2
Solid Cylinder	$1/2 \cdot MR^2$
Hollow Cylinder	$1/2 \cdot M(R_1^2 + R_2^2)$
Spherical Shell	$2/3 \cdot MR^2$
Solid Sphere	$2/5 \cdot MR^2$
Hollow Sphere	$2/5 \cdot \frac{R_1^5 - R_2^5}{R_1^3 - R_2^3}$

Universal Gravitation

$$F_G = G \frac{m_1 m_2}{r^2} \quad U_G = -G \frac{m_1 m_2}{r} \quad v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

Fluid Dynamics - Pressure **P** [Pascal] [Pa] [N/m²]

$$F = \int P dA \quad P = P_0 + \rho gh \quad F_B = \rho g V_{\text{displ}}$$

$$A_1 v_1 = A_2 v_2 = \text{const} \quad P + \frac{1}{2} \rho v^2 + \rho gy = \text{const}$$

Oscillations - 2nd order ODE

$$\frac{d^2 x}{dt^2} = -\omega^2 x \Rightarrow x(t) = A \cos(\omega t + \phi)$$

$$m \frac{d^2 x}{dt^2} = -b \frac{dx}{dt} - kx \Rightarrow x(t) = A e^{-\omega_d t} \cos(\omega t + \phi)$$

Simple	AngFreq	Damped	Behavior
mass-spring	$\omega = \sqrt{k/m}$	$\omega_0 = \sqrt{k/m}$	$\omega_d < \omega_0$ under
pendulum	$\omega = \sqrt{g/\ell}$	$\omega_d = b/2m$	$\omega_d = \omega_0$ critical
rigid pend	$\omega = \sqrt{\frac{mgd}{I}}$	$\omega^2 = \omega_0^2 - \omega_d^2$	$\omega_d > \omega_0$ over

Travelling Waves - 2nd order PDE

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \Rightarrow y(x, t) = f(x \mp vt)$$

$$y(x, t) = A \sin(k_\lambda x - \omega t + \phi) \quad v = \lambda f = \sqrt{\frac{F_T}{m/\ell}}$$

$$v_t = \partial y / \partial t = -A\omega \cos(\sim) \quad a_t = \partial^2 y / \partial t^2 = -A\omega^2 \sin(\sim)$$

Mechanical Waves

$$f' = \left(\frac{v + v_o}{v - v_s} \right) f \quad f_{\text{beat}} = |f_1 - f_2|$$

$$f_n = \frac{(2n)v}{4L} \quad \leftarrow \begin{array}{l} \text{fixed/fixed} \\ \text{fixed/free } \rightarrow \end{array} \quad f_n = \frac{(2n+1)v}{4L}$$

Physics - Mechanics

Conversions

$1\text{ in} = 2.54\text{ cm}$	$1\text{ ft}\cdot\text{lb} = 1.356\text{ N}\cdot\text{m}$	tera	T	10^{12}
$1\text{ ft} = 0.3048\text{ m}$	$1\text{ cal} = 4.186\text{ J}$	giga	G	10^9
$1\text{ yard} = 3\text{ ft}$	$1\text{ kWh} = 3.60\text{ MJ}$	mega	M	10^6
$1\text{ mi} = 5280\text{ ft}$	$1\text{ hp} = 746\text{ J/s}$	kilo	k	10^3
$1\frac{\text{mi}}{\text{hr}} = 0.447\frac{\text{m}}{\text{s}}$	$1\text{ C} = 6.25 \cdot 10^{18}\text{ e}$	centi	c	10^{-2}
$1\text{ L} = 1000\text{ cm}^3$	$1\text{ eV} = 1.60 \cdot 10^{-19}\text{ J}$	milli	m	10^{-3}
$1\text{ gal} = 3.786\text{ L}$	$T_C = 5/9(T_F - 32)$	micro	μ	10^{-6}
$1\text{ ft}^3 = 28.32\text{ L}$	$T = T_C + 273$	nano	n	10^{-9}
$1\text{ lb} = 4.448\text{ N}$	$1\text{ atm} = 101325\text{ Pa}$	pico	p	10^{-12}

Greek Symbols

alpha	α	iota	ι	rho	ρ
beta	β	kappa	κ	sigma	σ Σ
gamma	γ Γ	lambda	λ Λ	tau	τ
delta	δ Δ	mu	μ	upsilon	ν Υ
epsilon	ϵ	nu	ν	phi	ϕ Φ
zeta	ζ	xi	ξ Ξ	chi	χ
eta	η	omicron	\omicron	psi	ψ Ψ
theta	θ Θ	pi	π Π	omega	ω Ω

Physical Constants

Name	Symbol	Value
Gravitation Constant	G	$6.67 \cdot 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$
Earth Surface Accel	g	9.8 m/s^2
Boltzmann Constant	k_B	$1.38 \cdot 10^{-23}\text{ J/K}$
Avogadros Number	N_A	$6.02 \cdot 10^{23}\text{ mol}^{-1}$
Ideal Gas Constant	R	$8.314\text{ J}/(\text{mol}\cdot\text{K})$
Stefan-Boltzmann	σ	$5.67 \cdot 10^{-8}\text{ W}/(\text{m}^2\cdot\text{K}^4)$
Speed of Light	c	$2.99792458 \cdot 10^8\text{ m/s}$
Electrostatic Constant	k_e	$8.99 \cdot 10^9\text{ N}\cdot\text{m}^2/\text{C}^2$
Electric Permittivity	ϵ_0	$8.854 \cdot 10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)$
Magnetic Permeability	μ_0	$4\pi \cdot 10^{-7}\text{ T}\cdot\text{m/A}$
Elementary Charge	e	$1.60 \cdot 10^{-19}\text{ C}$
Planck Constant	h	$6.63 \cdot 10^{-34}\text{ J}\cdot\text{s}$
Rydberg Constant	R_H	$1.09737 \cdot 10^{-7}\text{ m}^{-1}$
Bohr Magneton	μ_B	$9.274 \cdot 10^{-24}\text{ J/T}$
Bohr Radius	a_0	$5.292 \cdot 10^{-11}\text{ m}$
Rest mass of Electron	m_e	$9.1093 \cdot 10^{-31}\text{ kg}$
Rest mass of Proton	m_p	$1.6726 \cdot 10^{-27}\text{ kg}$
Rest mass of Neutron	m_n	$1.6749 \cdot 10^{-27}\text{ kg}$
Atomic Mass Unit	u	$931.49\text{ MeV}/c^2$
Mass Earth	M_E	$5.97 \cdot 10^{24}\text{ kg}$
Mass Sun	M_\odot	$1.99 \cdot 10^{30}\text{ kg}$
Radius Earth	R_E	$6.37 \cdot 10^6\text{ m}$
Radius Sun	R_\odot	$6.96 \cdot 10^8\text{ m}$
Distance Earth/Sun	AU	$1.496 \cdot 10^{11}\text{ m}$

Speed of Sound [m/s]

Material	v	Material	v	Material	v
Air 25°C	343	Ice	3845	Copper	5010
Air 0°C	331	Glass	5640	Aluminum	6420
Steam	478	P-wave	4k-7k	Iron	5950
Water	1492	S-wave	2k-5k	Lead	1960

Friction Coefficients $f = \mu N$

Material	μ_k	μ_s	Material	μ_k	μ_s
Tire-DryConc	0.68	0.90	Glass-Glass	0.4	0.94
Tire-WetConc	0.58		Wood-Wood	0.30	0.42
Tire-Ice	0.15		Ski-Snow	0.05	0.14
Steel-Steel	0.57	0.74	Teflon-Teflon	0.04	0.04
Copper-Steel	0.36	0.53	Synovial Fluid	0.01	0.01

Equation of State - P [Pa] V [m³] T [K] n [mol]

$$PV = nRT \quad PV = Nk_B T \quad \text{Ideal Gas}$$

$$P = \frac{nRT}{V-b} - \frac{a}{V^2} \quad \text{Van der Waals} \quad \begin{matrix} a \rightarrow \text{strength} \\ b \rightarrow \text{volume} \end{matrix}$$

Statistical Thermodynamics Ideal Gas

$$\Delta U = nC_V dT \quad P = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m \bar{v}^2 \right)$$

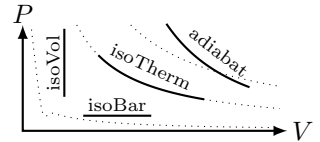
$$C_V = \frac{\text{DoF}}{2} R \quad C_P = \frac{\text{DoF} + 2}{2} R \quad \gamma = C_P/C_V$$
$$S = k_B \ln(\mathbb{P}) \quad \mathbb{P}_{\text{Boltz}} \propto \exp(-E_i/k_B T)$$

1st Law of Thermodynamics [J] [Joule]

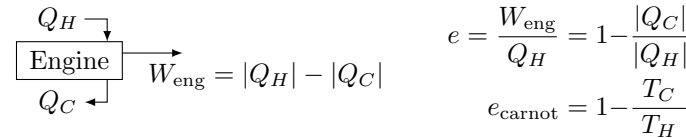
$$\Delta U = \Delta Q + \Delta W \quad dU = TdS + (-PdV)$$

$$\Delta W = \int_{V_i}^{V_f} -P dV \Rightarrow$$

$$\Delta Q = \text{“Heat”} = \int TdS$$



Heat Engine



PV^m	Process	ΔU_{int}	ΔQ	ΔW
	General	$nC_V dT$	$dU - dW$	$-\int P dV$
∞	Isovolume	$nC_V \Delta T$	$nC_V \Delta T$	0
0	Isobaric	$nC_V \Delta T$	$nC_P \Delta T$	$-P \Delta V$
1	Isothermal	0	$-\Delta W$	$-nRT \ln(V_f/V_i)$
γ	Adiabatic	$nC_V \Delta T$	0	$-\frac{P_f V_f - P_i V_i}{(1-\gamma)}$
$P_1 V_1^\gamma = P_2 V_2^\gamma = \text{constant}$				(\leftarrow Adiabatic $dQ = 0$)

Thermal Material Properties

Material	Density ρ (kg/m^3)	Specific Heat c ($\frac{\text{J}}{\text{kg}\cdot^\circ\text{C}}$)	Thermal Cond k ($\frac{\text{W}}{\text{m}\cdot^\circ\text{C}}$)	Volume Expans β ($\frac{1}{^\circ\text{C}}$) $\cdot 10^{-6}$
Steam	0.590	2010	0.027	3400
Air	1.29	1004	0.023	3670
Water	1000	4186	0.6	214
Ethanol	785.1	2440	0.17	109
Ice	920	2090	~ 2	150
Glass	2500	840	1.05	27
Aluminum	2700	900	238	72
Copper	8600	387	397	51
Iron	7800	448	79.5	33
Lead	11300	128	34.7	87

Latent Heat of Materials

Material	Melting ($^\circ\text{C}$)	L_{fusion} (J/kg)	Boiling ($^\circ\text{C}$)	L_{vapor} (J/kg)
Nitrogen	-209.97	2.55×10^4	-195.81	2.01×10^5
Water	0.00	3.33×10^5	100.0	2.26×10^6
Ethanol	-114	1.04×10^5	78	8.54×10^5

Physics - Thermodynamics