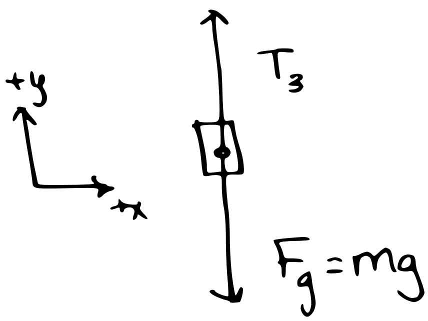
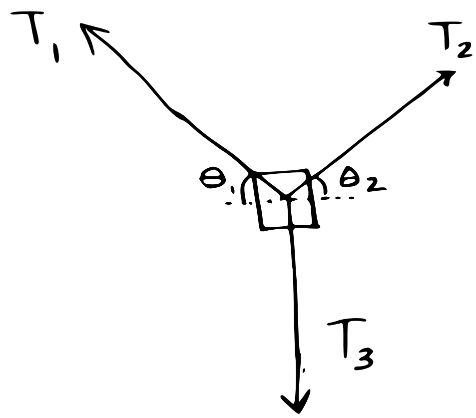
2.1

If the bag is stationary, the overall force must be $\underline{\text{zero}}$! First make a free body diagram, then sum all forces:



FBD for bag



FBD for cable connection

$$\sum F_x = 0 = 0$$

$$\sum F_y = -mg + T_3 = 0$$

$$\boxed{T_3 = mg}$$

$$\sum F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

$$\sum F_y = -T_3 + T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0$$

$$\boxed{\begin{array}{c} mg = T_1 \sin\theta_1 + T_2 \sin\theta_2 \\ T_2 \cos\theta_2 = T_1 \cos\theta_1 \end{array}}$$

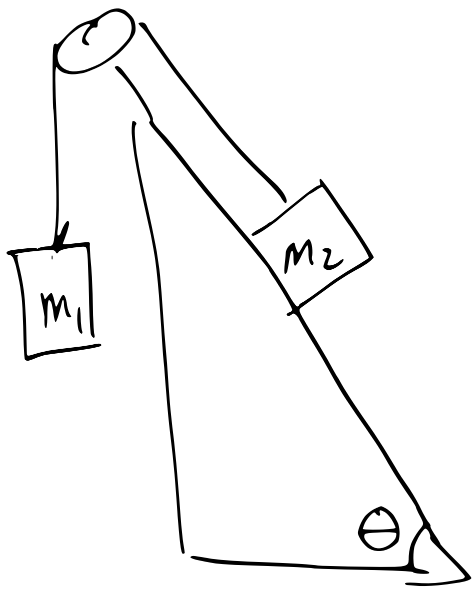
2 equs, 2 unknowns

$$mg = T_1 \sin\theta_1 + \frac{T_1 \cos\theta_1}{\cos\theta_2} \sin\theta_2$$

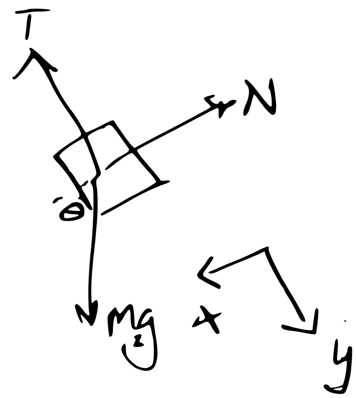
$$mg\cos\theta_2 = T_1 \sin\theta_1 \cos\theta_2 + T_1 \cos\theta_1 \sin\theta_2$$

$$= T_1 \sin(\theta_1 + \theta_2) \Rightarrow \boxed{T_1 = \frac{mg\cos\theta_2}{[illegible]}}$$

2.2

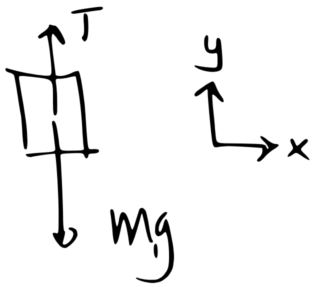


FBD 2



a)

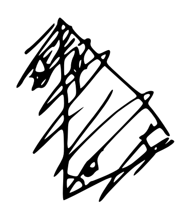
FBD 1



$$\Sigma F_{y_1} = T - m_1 g = m_1 a$$

$$\Sigma F_{y_2} = -T + m_2 g \sin\theta = m_2 a$$

add together



$$-m_1 g + m_2 g \sin\theta = (m_1 + m_2) a$$

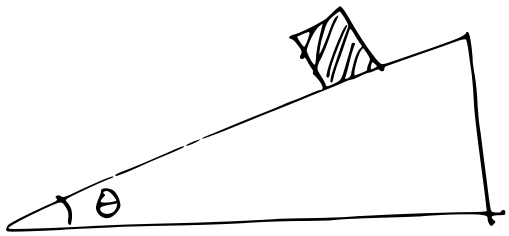
b) $\boxed{a = \dfrac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}}$

c) $\boxed{T = m_1 a + m_1 g}$

d) use kinematics!

$$v = \cancel{v_i}^{\,0} + at = \boxed{at}$$

2.3



how far up does the block travel?

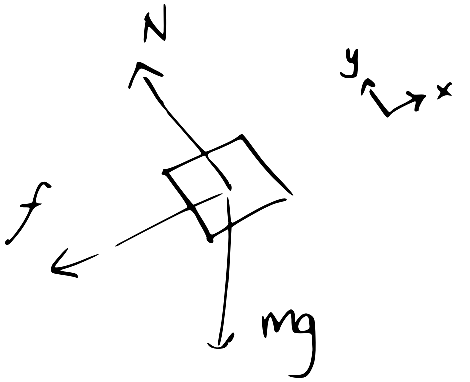
use kinematics!

$x_i = 0$ $\quad v_i = 5\,m/s$ $\quad a = ?$

$x_f = ?$ $\quad v_f = 0$ $\quad t = ?$

when you get 3 unknowns in kinematics, you cannot solve the eqns! Instead use Newton's Laws to find acceleration

FBD



$$\sum F_x = -f - mg\sin\theta = ma$$

$$\sum F_y = N - mg\cos\theta = 0$$

$$\boxed{a = -\mu g\cos\theta - g\sin\theta}$$

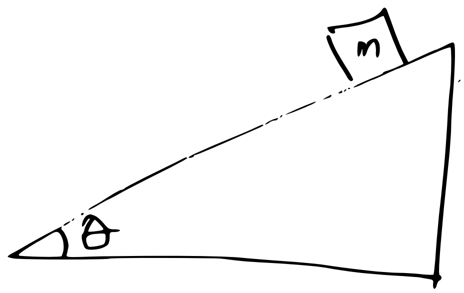
then use kinematics

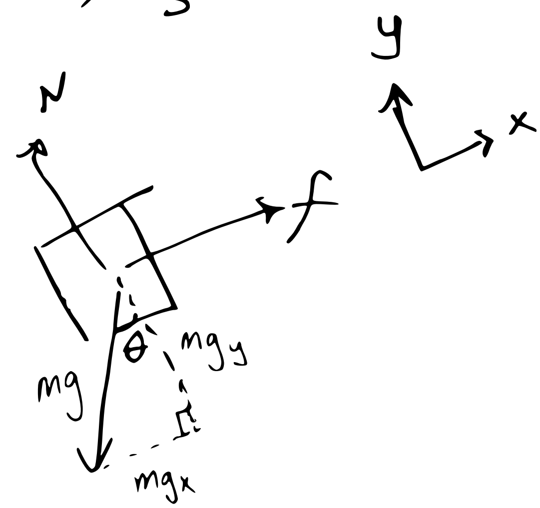
$$\cancel{v_f^2}^{\,0} = v_i^2 + 2a(x - \cancel{x_i}^{\,0})$$

$$\boxed{x = \frac{-v_i^2}{2a}}$$

2.4

find μ_k
μ_s





$$\sum F_x = f - mg\sin\theta = 0$$

$$\sum F_y = N - mg\cos\theta = 0$$

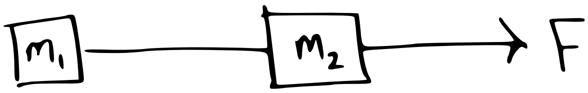
$$\mu mg\cos\theta - mg\sin\theta = 0$$

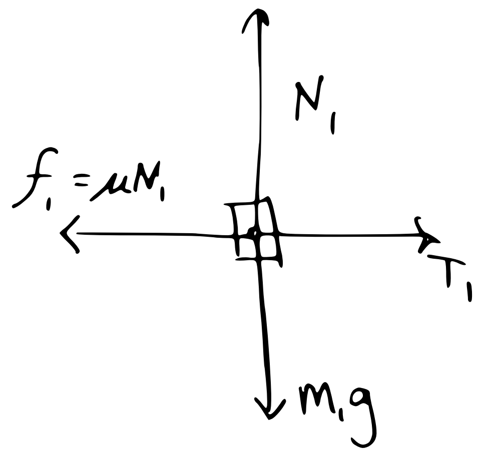
$$\mu = \tan\theta$$

$$\mu_s = \tan 36 = \boxed{0.727}$$

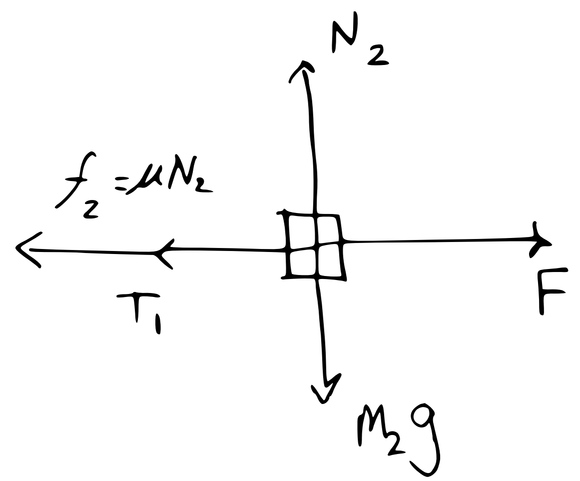
$$\mu_k = \tan 30 = \boxed{0.577}$$

2.5





FBD1



FBD2

$$\Sigma F_x = \mu N_1 + T_1 = m_1 a \qquad \Sigma F_x = -\mu N_2 - T_1 + F = m_2 a$$

$$\Sigma F_y = N_1 - m_1 g = 0 \qquad \Sigma F_y = N_2 - m_2 g = 0$$

Find the Normal forces using the y-direction

$$N_1 = m_1 g \qquad N_2 = m_2 g$$

Substitute into x eqns, add eqns together

$$-\mu m_1 g + T_1 = m_1 a$$

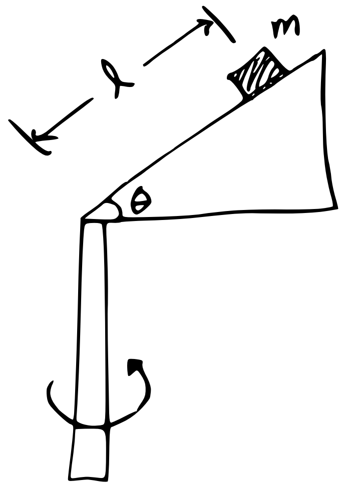
$$+ \quad -\mu m_2 g - T_1 + F = m_2 a$$

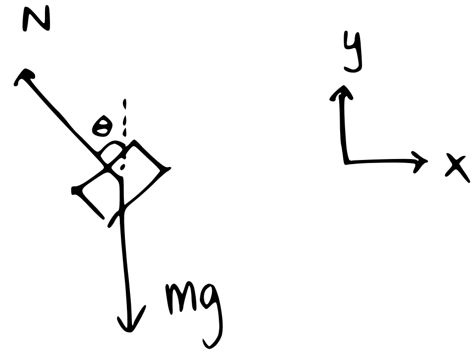
$$-\mu m_1 g - \mu m_2 g + T_1 - T_1 + F = (m_1 + m_2) a$$

Solution:

$$a = \frac{F - \mu g (m_1 + m_2)}{m_1 + m_2} = 1.28 \frac{m}{s^2} \quad \Big| \quad T = m_1 a + \mu m_1 g = 27.2 N$$

2.b





eqn #1 $$\sum F_x = -N\sin\theta = -\frac{mv^2}{r} = -\frac{mv^2}{l\cos\theta}$$

eqn #2 $$\sum F_y = -mg + N\cos\theta = 0$$

$$N = \frac{mg}{\cos\theta}$$

substitute back into eqn #1

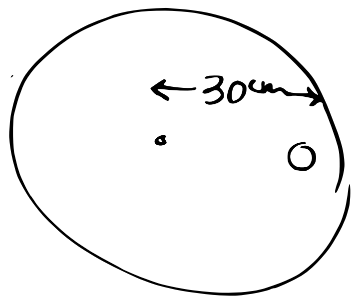
$$-\frac{\cancel{m}g}{\cancel{\cos\theta}}\sin\theta = -\frac{\cancel{m}v^2}{l\cancel{\cos\theta}}$$

$$v^2 = gl\sin\theta$$

$$\boxed{v = \sqrt{gl\sin\theta}}$$

2.7

top view

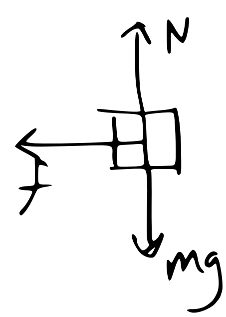


when $v = 50\,\frac{cm}{s}$

the coin slips

side view

FBD



$$\sum F_y = N - mg = 0$$

$$\sum F_x = f = ma = \frac{mv^2}{r}$$

a) what force? $\Rightarrow$ Friction!

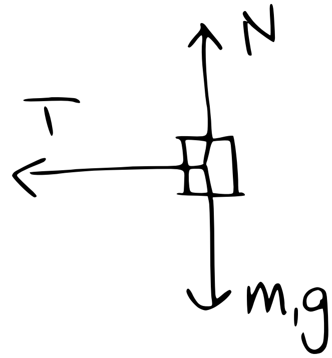
b)

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r} \Rightarrow \mu = \frac{v^2}{gr}$$

$$= \frac{(0.5)^2}{(9.8)(0.3)} = \boxed{0.085}$$

2.8

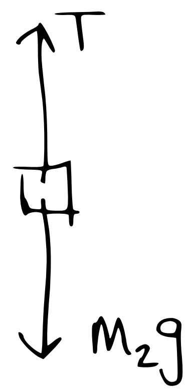
FBD 1



$\Sigma F_x = -T = -m_1 v^2 / r$

$\Sigma F_y = N - m_1 g = 0$

FBD 2



$\Sigma F_y = T - m_2 g = 0$

a) $T = m_2 g = \boxed{9.8\,N}$

b) $T = m_2 g = \boxed{9.8\,N}$

c) $v = \sqrt{\frac{Tr}{m_1}} = \sqrt{\frac{m_2 g r}{m_1}}$

$\boxed{= 6.26\ m/s}$

2.9

There are two ways to calculate a dot product

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}| \cos\theta = \sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2} \cos\theta$$

set these equal to each other and solve for θ

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)$$

I will show this for vectors in part (b)

$$\vec{A} = -2\hat{i} + 4\hat{j}$$
$$\vec{B} = 3\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\vec{A}\cdot\vec{B} = (-2)(3) + (4)(-4) + (0)(2) = -6 - 16 = -22$$

$$|\vec{A}| = \sqrt{(-2)^2 + (4)^2} = \sqrt{20} = 4.47$$

$$|B| = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{29} = 5.38$$

$$\theta = \cos^{-1}\left(\frac{-22}{4.47 \cdot 5.38}\right) = \boxed{156^\circ}$$

Repeat this calculation to find answers for a) & c)

2.10

$$F = -(k_1 x + k_2 x^2)$$

Find the work done by a force $(-F)$

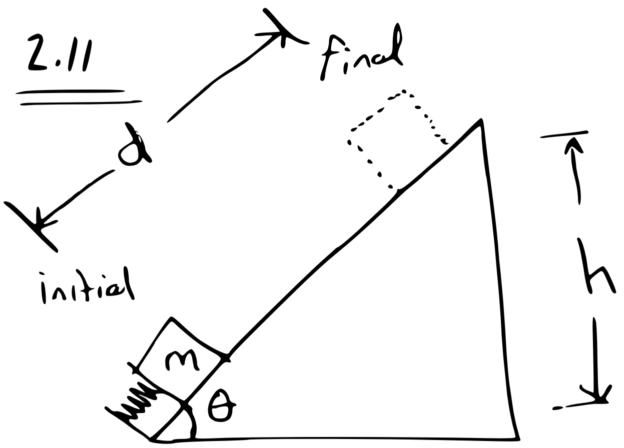
$$W = \int \vec{F} \cdot d\vec{r} = \int_0^{x_{max}} F(x)\, dx$$

$$= \int_0^{x_{max}} (k_1 x + k_2 x^2)\, dx$$

$$= \frac{1}{2} k_1 x^2 \Big|_0^{x_{max}} + \frac{k_2}{3} x^3 \Big|_0^{x_{max}}$$

$$\boxed{= \frac{1}{2} k_1 x_{max}^2 + \frac{1}{3} k_2 x_{max}^3}$$

2.11



$M = 0.2\,kg$

From geometry: $h = d\sin\theta$

Initial state:

* spring compressed $x = 0.10\,m$, $k = 1400\,N$
* initial height $h = 0$
* initial speed $v = 0$

Final state:

* spring uncompressed $x = 0$
* final height (unknown) $h = ?$
* final speed $v = 0$

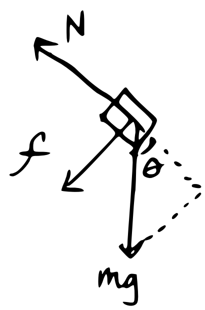
a)

$$W_{nc} = \Delta K + \Delta U$$

$$0 = mgh_f - \frac{1}{2}kx_i^2$$

$$h = \frac{kx^2}{2mg} = 3.57\,m$$

$$d = \frac{h}{\sin\theta} = \boxed{4.12\,m}$$



$N = mg\cos\theta$

$f = \mu N$
$= \mu mg\cos\theta$

b) add friction! draw a free body diagram!

$$W_{nc} = Fd\cos\underbrace{\phi}_{\text{angle between } F \,\&\, d} = (\mu mg\cos\theta)(d)\cos(180)$$

$$= -\mu mgd\cos\theta$$

$$-\mu mgd\cos\theta = mgh_f - \frac{1}{2}kx_i^2 = mgd\sin\theta - \frac{1}{2}kx^2$$

$$d = \frac{kx^2}{2mg(\mu\cos\theta + \sin\theta)} = \boxed{3.35\,m}$$

2.12 This problem has tricky wording and tricky units:

First recall power $\Rightarrow$ $\mathbb{P} = \frac{dW}{dt}$ units: Watts $= \frac{J}{s}$

$$120 \text{ Watts} \cdot hr = 120 \frac{J}{\cancel{s}} \cdot \cancel{hr} \left(\frac{3600 \cancel{s}}{\cancel{hr}} \right) = 432 kJ$$

* this is the energy available from the battery

60% of this energy is lost to ~~friction~~

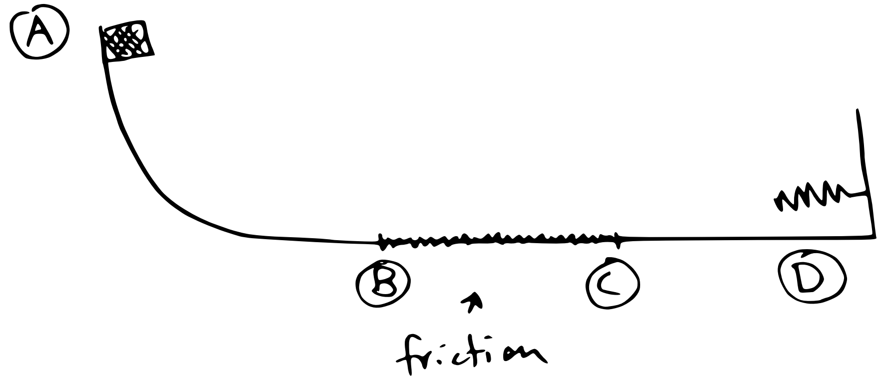
$$432 kJ \cdot (0.60) = \underline{\underline{259,200 J}}$$

* this is the energy which will transform to gravitational potential

$$259200 J = mgh$$

$$h = \frac{259,200 J}{890 N} = \boxed{291.2 m}$$

2.13



Break the problem into 3 parts

A → B

B → C

C → D

solve A → B and C → D. then solve B → C

A → B: No friction

$$W_{nc} = \frac{1}{2}mV_B^2 - mgh_A = 0$$

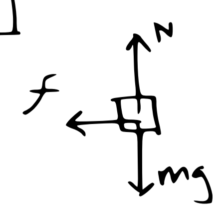
$$V_B = \sqrt{2gh} = \boxed{7.66 \, m/s}$$

C → D: No friction

$$W_{nc} = \frac{1}{2}kX_f^2 - \frac{1}{2}mV_c^2$$

$$V_c = \sqrt{k/m}\, x = \boxed{4.5 \, m/s}$$

B → C: Include friction : FBD ⇒

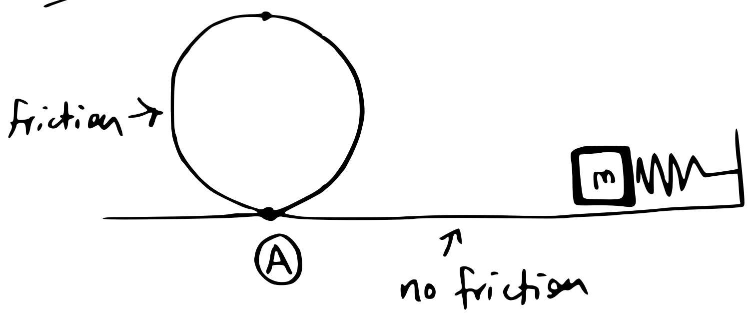


$$W_{nc} = Fd\cos\theta = -\mu mgd$$

$$-\mu mgd = \frac{1}{2}mV_c^2 - \frac{1}{2}mV_B^2$$

$$\mu = \frac{V_B^2 - V_c^2}{2gd} = \boxed{0.327}$$

2.14



$V_A = 12\,\frac{m}{s}$

a) From start to point A

$$W_{nc} = 0 = \frac{1}{2} m V_A^2 - \frac{1}{2} k x^2$$

$$x = \sqrt{m/k}\; V_A = \boxed{0.4\,m}$$

b) If the block reaches the top, it must be going in a circle! centripital acceleration!

$$\sum F = -mg = -ma_c = \frac{mv^2}{R}$$

$$V = \sqrt{gR} = \boxed{3.13\,\frac{m}{s}}$$

c) From pt A to top

$W_{nc} \neq 0$ there is a friction force $f = 7N$

$$W_{nc} = Fd\cos\theta = (f)(\pi R)(\cos 180) = -22J$$

$$-22J = \frac{1}{2} m v_{top}^2 - \frac{1}{2} m V_A^2 + mg(2R)$$

$$V_{top} = \sqrt{\left(-22 - mg(2R) + \frac{1}{2} m V_A^2\right)\frac{2}{m}} = \boxed{4.09\,\frac{m}{s}}$$

$4.09\,\frac{m}{s} > 3.13\,\frac{m}{s} \Rightarrow$ It reaches the top