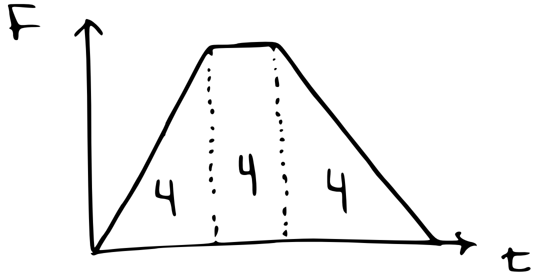
Impulse is the area under F vs t curve

$$I = \int \Sigma F(t)\, dt$$

From geometry: count the boxes



$$I = 4 + 4 + 4 = \boxed{12\ \frac{kg\,m}{s}}$$

From calculus:

$$I = \int_0^2 2t\,dt + \int_2^3 4\,dt + \int_3^5 (-2t+10)\,dt$$

$$= t^2\Big|_0^2 + 4t\Big|_2^3 + (-t^2+10t)\Big|_3^5$$

$$= (4-0) + (12-8) + (-25+50+9-30)$$

$$= 4 + 4 + 4$$

$$= \boxed{12\ \frac{kg\,m}{s}}$$

b) $\Delta p = I = m(V_f - V_i) = 12$

$$V_f = \frac{12}{m} + V_i = \boxed{4.8\ m/s}$$

c) $V_f = \frac{12}{m} + V_i = \boxed{2.8\ m/s}$

d) $\overline{F} = 2\cdot\left(\frac{2}{5}\right) + 4\left(\frac{1}{5}\right) + 2\left(\frac{2}{5}\right) = \boxed{2.4\ N}$

car:

$m_c = 1200\,kg$

$V_{ci} = 25\,m/s$

$V_{cf} = 18\,m/s$

truck:

$m_T = 9000\,kg$

$V_{Ti} = 20\,m/s$

$V_{Tf} = ?$

* The problem does not state "elastic", so please do not assume $\Delta K = 0$

* You can use conservation of momentum

a)

$$\vec{P}_i = \vec{P}_f$$

$$m_c V_{ci} + m_T V_{Ti} = m_c V_{cf} + m_T V_{Tf}$$

$$V_{Tf} = \frac{m_c V_{ci} + m_T V_{Ti} - m_c V_{cf}}{m_T}$$

$$\boxed{= 20.93\,m/s}$$

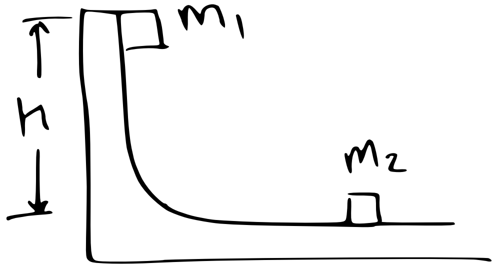
b)

$$\Delta K = \frac{1}{2} m_c V_{cf}^2 + \frac{1}{2} m_T V_{Tf}^2 - \frac{1}{2} m_c V_{ci}^2 - \frac{1}{2} m_T V_{Ti}^2$$

$$\boxed{= -9307\,J}$$

c) Energy was transformed into:

- crunching the car hood and truck bumper
- heat from deformation of metal
- sound from the crash
- etc...



$m_1 = 5\,kg$
$m_2 = 10\,kg$

Step 1: m_1 falls down ramp. Use conservation of energy!

$$W_{nc} = \Delta K + \Delta U$$

$$0 = \frac{1}{2} m_1 V_{1f}^2 - m_1 g h_i \Rightarrow V_{f1} = \sqrt{2gh_i} = \boxed{9.89\,\frac{m}{s}}$$

Step 2: elastic collision. Use conservation of momentum and energy

$$\Delta p = 0 \Rightarrow m_1 V_{1i} + \cancel{m_2 V_{2i}}^{0} = m_1 V_{1f} + m_2 V_{2f}$$

$$\Delta K = 0 \Rightarrow V_{1f} + V_{1i} = V_{2f} + \cancel{V_{2i}}^{0}$$

$$V_{1i} = 9.89\,\frac{m}{s} \qquad V_{2i} = 0$$

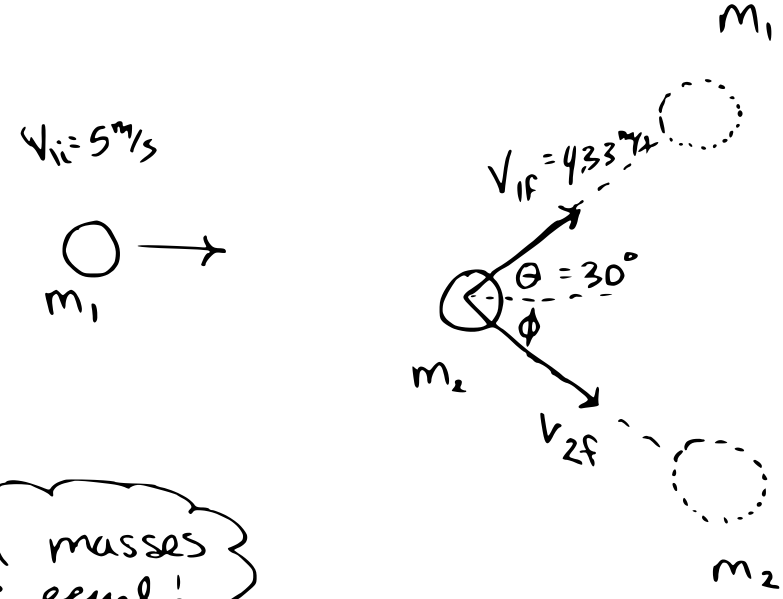
solve the system of equations

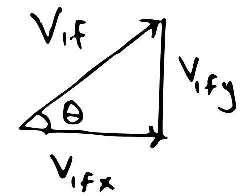
$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} * V_{1i} = \boxed{-3.299\,\frac{m}{s}}$$

Step 3: m_1 returns up ramp. Use conservation of energy

$$0 = -\frac{1}{2} m V_i^2 + m g h_f$$

$$h = \frac{V^2}{2g} = \boxed{0.556\,m}$$





$V_{1fx} = V_{1f}\cos\theta$

$V_{1fy} = V_{1f}\sin\theta$

* all masses are equal!

Conservation of momentum

$$\cancel{m}V_{1ix} + \cancel{m}\cancel{V_{2ix}}^{\,0} = \cancel{m}V_{1fx} + \cancel{m}V_{2fx}$$

$$\cancel{m}\cancel{V_{1iy}}^{\,0} + \cancel{m}\cancel{V_{2iy}}^{\,0} = \cancel{m}V_{1fy} + \cancel{m}V_{2fy}$$

$$V_{1ix} = V_{1fx} + V_{2fx}$$

$$0 = V_{1fy} + V_{2fy}$$

$$V_{2fx} = V_{1ix} - V_{1fx} = 5 - 3.75 = \boxed{1.25\,m/s}$$

$$V_{2fy} = -V_{1fy} = \boxed{-2.165\,m/s}$$

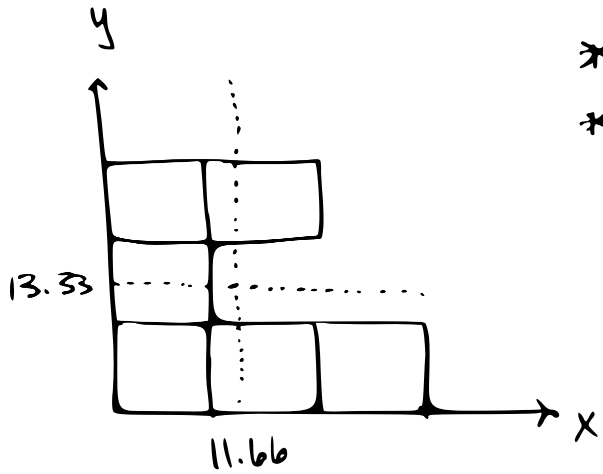
---

in polar coords:

$$V_{2f} = \sqrt{V_{2fx}^2 + V_{2fy}^2} = \boxed{2.5\,m/s}$$

at $\phi = \tan^{-1}\left(\frac{V_{2fy}}{V_{2fx}}\right)$

$$\boxed{\phi = -60°}$$



* each square is 10 cm x 10 cm
* all masses are equivalent

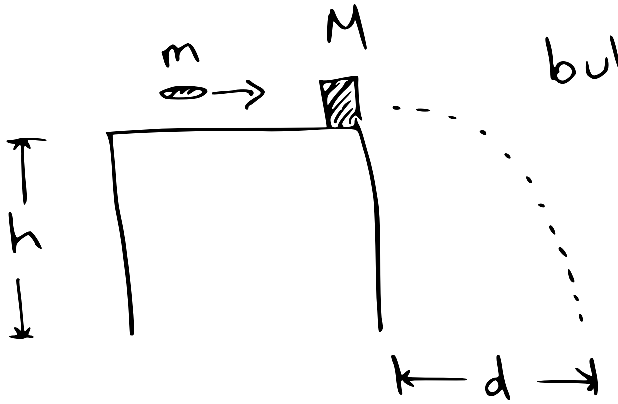
$$\vec{r}_{cm} = \frac{\sum \vec{r}_i m_i}{\sum m_i}$$

$$x_{cm} = \frac{(3)(5)m + (2)(15)m + (1)(25)m}{6m}$$

$$\boxed{= 11.66\text{ cm}}$$

$$y_{cm} = \frac{(3)(5)m + (1)(15)m + (2)(25)m}{6m}$$

$$\boxed{= 13.33\text{ cm}}$$



bullet sticks to block

perfectly inelastic

use kinematics to find the required velocity of the fall

y-dir: $y = y_i + V_{iy} t + \frac{1}{2} a_y t^2$

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

x-dir: $x = x_i + V_{ix} t + \cancel{\frac{1}{2} a_x t^2}^{\,0}$

$$V_{ix} = \frac{d}{t} = \sqrt{\frac{g}{2h}}\, d = \boxed{4.42 \text{ m/s}}$$

Then move on to the collision: $\Delta \vec{P} = 0$

$$m_1 V_{1i} + \cancel{m_2 V_{2i}}^{\,0} = (m_1 + m_2) V_f$$

$$V_{1i} = \frac{m_1 + m_2}{m_1} V_f = \boxed{142 \text{ m/s}}$$

$\theta = 2.5t^2 - 0.6t^3$

diameter is 1m
$r = 0.5m$

$\omega = \frac{d\theta}{dt} = 5t - 1.8t^2$

$\alpha = \frac{d\omega}{dt} = 5 - 3.6t$

a) max speed → "max" means take derivative and find the zero*

$\frac{d\omega}{dt} = \alpha = 5 - 3.6t = 0$

$t = 1.38 \text{ sec}$ ← time of max speed

$\omega(1.38) = 5(1.38) - 1.8(1.38)^2 = \boxed{3.47 \frac{rad}{sec}}$

b) tangential speed → find velocity of the rim of the roller

$V = r\omega = \boxed{1.736 \frac{m}{s}}$

c) reverse direction → this happens when $\omega = 0$

$5t - 1.8t^2 = 0$ solutions $t = 0s$

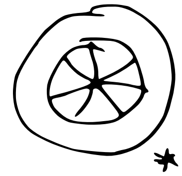
$\boxed{t = 2.77s}$

d) $\theta(2.77) = 6.43$ radians

$6.43 \text{ rad} \left(\frac{1 \text{ rot}}{2\pi \text{ rad}}\right) = \boxed{1.023 \text{ rotations}}$

$\sum \tau = I\alpha$

wheel



$\sum \tau = 36\,N\cdot m$

*applied for 6 seconds

there are two torques:

* Applied force $\rightarrow \tau_A$
* Friction force $\rightarrow \tau_f$

$$\tau_A - \tau_f = 36\,N\cdot m = I\alpha$$

a) use kinematics

$$w_f = w_i + \alpha t \Rightarrow \alpha = \frac{w_f - w_i}{t}$$

$w_f = 10\,\frac{rad}{sec}$ $\quad w_i = 0$ $\quad t = 6\,sec$ $\quad = \frac{10-0}{6} = 1.66\,\frac{rad}{s^2}$

Now back to torque:

$$\sum \tau = I\alpha \Rightarrow 36\,N\cdot m = I\left(1.66\,\frac{rad}{s^2}\right)$$

$$\boxed{I = 21.68\,kgm^2}$$

b) Now remove the Applied force

$w_f = 0$ $\quad w_i = 10\,\frac{rad}{sec}$ $\quad t = 60\,sec$ $\quad \alpha = -0.167\,\frac{rad}{sec^2}$

$$\tau_f = I\alpha = \boxed{-3.61\,N\cdot m}$$

first part: spinning the wheel up

$$\theta = \theta_i + w_i t + \tfrac{1}{2}\alpha t^2$$

$$= 0 + 0 + \tfrac{1}{2}(1.66)(6)^2 = 29.88\,rad$$

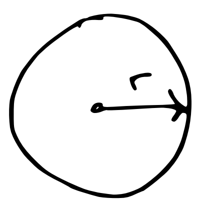
second part: spinning the wheel down

$$\theta = \theta_i + w_i t + \tfrac{1}{2}\alpha t^2$$

$$= 29.88 + 10(60) + \tfrac{1}{2}(-0.1667)(60)^2$$

$$= 329.88\,rad \quad \boxed{= 52.5\,rev}$$

m



$r = 7cm = 0.07m$

$m = 2kg$

$\tau = 0.6 \ N \cdot m$

uniform solid disk $\Rightarrow$ $I = \frac{1}{2} MR^2$

$$\tau = I\alpha = \left(\frac{1}{2} mr^2\right) \alpha$$

$$\alpha = \frac{2\tau}{mr^2} = 122.45 \ \frac{rad}{sec^2}$$

Before you start, do unit conversion! ω in $\frac{rad}{sec}$

$$1200 \ \frac{\cancel{rev}}{\cancel{min}} \left(\frac{2\pi \ rad}{1 \ \cancel{rev}}\right)\left(\frac{1 \ \cancel{min}}{60 \ sec}\right) = 125.7 \ \frac{rad}{sec}$$

then use kinematics

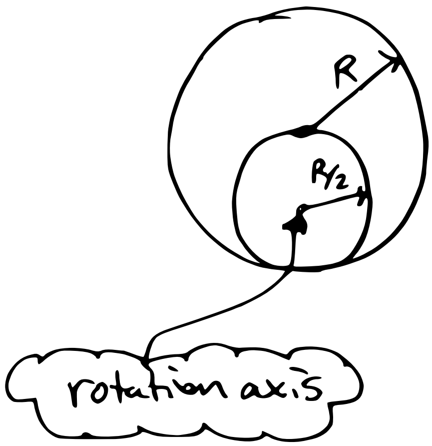
a) $\omega_f = \omega_i + \alpha t \Rightarrow t = \frac{\omega_f - \omega_i}{\alpha} = \frac{125.7 - 0}{122.45}$

$$\boxed{= 1.026 s}$$

b) $\theta = \cancel{\theta_i}^{\,0} + \cancel{\omega_i t}^{\,0} + \frac{1}{2} \alpha t^2$

$$= \frac{1}{2}(122.45)(1.026)^2 = 63.69 \ rad$$

$$\boxed{= 10.13 \ rev}$$



* both objects are solid disks

$$I_{cm} = \frac{1}{2}MR^2$$

* both disks have mass M

<u>small disk:</u>

$$I = I_{cm} + MD^2$$
$$= \frac{1}{2}MR^2 + M(0)^2$$
$$= \frac{1}{2}MR^2$$

<u>large disk:</u>

$$I = I_{cm} + MD^2$$
$$= \frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2$$
$$= \frac{3}{4}MR^2$$

$$I_{total} = \frac{1}{2}MR^2 + \frac{3}{4}MR^2 = \boxed{\frac{5}{4}MR^2}$$

$$K_{ROT} = \frac{1}{2}I\omega^2 =$$

$$\boxed{= \frac{5}{8}MR^2\omega^2}$$

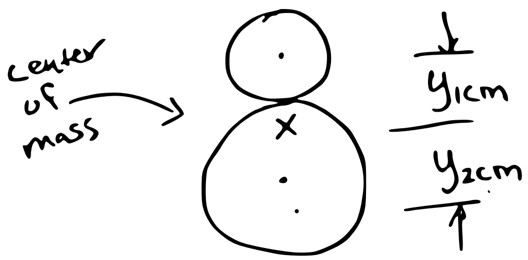
$$|\vec{A}\times\vec{B}| = AB\sin\theta$$
$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$\Rightarrow$

$$AB\sin\theta = AB\cos\theta$$
$$\tan\theta = 1$$
$$\boxed{\theta = \pi/4 \text{ OR } 45^\circ}$$

8.2

First calculate the center of mass

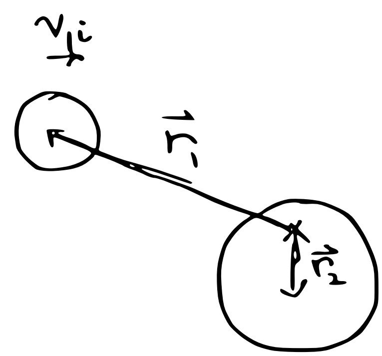


$$y_{1cm} = \frac{(r_1+r_2)\,m_2}{m_1+m_2}$$

$$y_{2cm} = \frac{(r_1+r_2)\,m_1}{m_1+m_2}$$

Now write down the radial vectors and momentum vectors

$$\vec{r}_1 = (x_i + v_{ix}t,\ y_{1cm},\ 0)$$
$$\vec{p}_1 = (m_1 v_{1i},\ 0,\ 0)$$
$$\vec{r}_2 = (0,\ -y_{2cm},\ 0)$$
$$\vec{p}_2 = (0,\ 0,\ 0)$$



Then calculate the angular momentum w/ the cross product

$$\vec{L} = \vec{r}\times\vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p} = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ x_i + v_{ix}t & y_{1cm} & 0 \\ m_1 v_{ix} & 0 & 0 \end{matrix} \right| \begin{matrix} \hat{i} & \hat{j} \\ x_i + v_{ix}t & y_{1cm} \\ m_1 v_{ix} & 0 \end{matrix}$$

$$= \left[\cancel{(y_{1cm})(0)}^{\,0} - \cancel{(0)(0)}^{\,0} \right] \hat{i}$$

$$+ \left[\cancel{(0)(m_1 v_{ix})}^{\,0} - \cancel{(x_i + v_{ix}t)(0)}^{\,0} \right] \hat{j}$$

$$+ \left[\cancel{(x_i + v_{ix}t)(0)}^{\,0} - y_{1cm} m_1 v_{ix} \right] \hat{k}$$

$$\vec{L} = -m_1 y_{1cm} v_{ix} \hat{k} = \boxed{-0.0157 \hat{k} \ \frac{kg m^2}{s}}$$

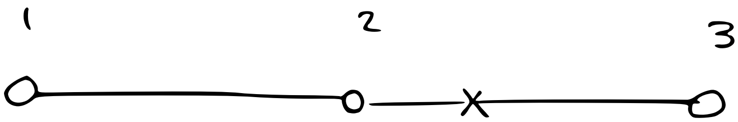
b) $\vec{L} = I \vec{\omega} \Rightarrow \vec{\omega} = \dfrac{\vec{L}}{I}$

use parallel axis theorem for both disks

$$I = \frac{1}{2} m_1 r_1^2 + m_1 y_{1cm}^2 + \frac{1}{2} m_2 r_2^2 + m_2 y_{2cm}^2$$

$$\vec{\omega} = \frac{\vec{L}}{I} = \boxed{-8.24 \hat{k} \ \frac{rad}{sec}}$$

8.4



a) $I = I_1 + I_2 + I_3$

$= m\left(\frac{4}{3}d\right)^2 + m\left(\frac{1}{3}d\right)^2 + m\left(\frac{2}{3}d\right)^2$

$= md^2\left(\frac{16}{9} + \frac{1}{9} + \frac{4}{9}\right) = \boxed{\frac{7}{3}md^2}$

b) $\tau_z = \frac{4}{3}dmg + \frac{1}{3}dmg - \frac{2}{3}dmg$

$\boxed{= dmg\,\hat{z}}$

c) $\Sigma\tau = I\alpha \Rightarrow \alpha = \frac{\Sigma\tau}{I} = \frac{dmg}{\frac{7}{3}md^2}\hat{z} = \boxed{\frac{3}{7}\frac{g}{d}\hat{z}}$

d) $a = r\alpha = \left(\frac{2}{3}d\right)\left(\frac{3}{7}\frac{g}{d}\hat{z}\right) = \boxed{\frac{2}{7}g\,\hat{z}}$

e) $K = \frac{4}{3}mgd + \frac{1}{3}mgd - \frac{2}{3}mgd = \boxed{mgd}$

f) $\frac{1}{2}I\omega^2 = K \Rightarrow \omega = \sqrt{\frac{2K}{I}} = \boxed{\sqrt{\frac{6}{7}\frac{g}{d}}\;\hat{z}}$

g) $L = I\omega = \frac{7}{3}md^2\sqrt{\frac{6}{7}\frac{g}{d}}\;\hat{z} = \boxed{\sqrt{\frac{14}{3}m^2d^3g}\;\hat{z}}$

h) $v = r\omega = \boxed{\frac{1}{3}d\sqrt{\frac{6}{7}\frac{g}{d}}}$

$L_i = r \times p = r_i m v_i$

Angular momentum is conserved

$L_i = L_f = r_i m v_i = r_f m v_f$

a) $v_f = v_i \frac{r_i}{r_f} = \boxed{4.5 \text{ m/s}}$

b) $\sum F = m a_c = T = \frac{m v_f^2}{r_f} = \boxed{10.125 N}$

c) Energy is <u>not</u> conserved (Inelastic colision)

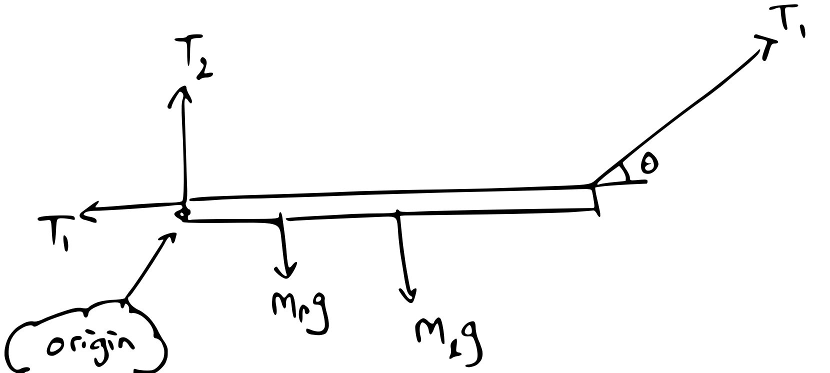
$$W = \int F \cdot dr = \int \frac{m v^2}{r} dr = \int_{0.3}^{0.1} \frac{m v_i^2 r_i^2}{r^3} dr$$

$$= m v_i^2 r_i^2 \int r^{-3} dr$$

$$= \frac{1}{-2} m v_i^2 r_i^2 \left(\frac{1}{r^2} \Big|_{0.3}^{0.1} \right)$$

$$\boxed{= .45 J}$$

FBD



choose the origin to be at left end so that the torque due to T_1 & T_2 are zero!

$$\tau_z = 0 = -d\, m_p g - \frac{\ell}{2} m_\ell g + \ell T_1 \sin\theta$$

$$T_1 = \frac{d\, m_p g + \frac{\ell}{2} m_\ell g}{\ell \sin\theta} = \boxed{501\text{ N}}$$

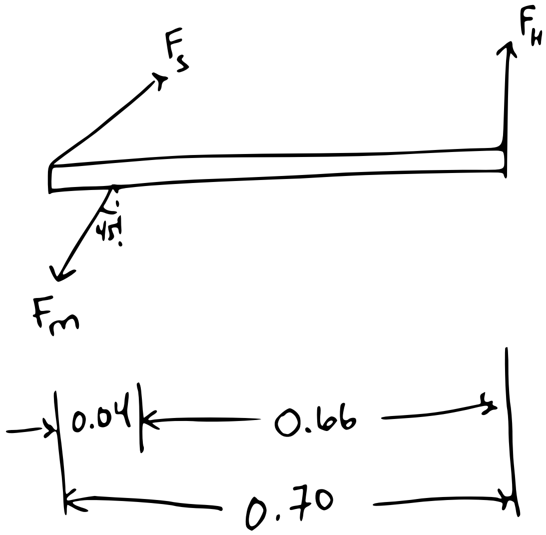
$$\sum F_x = -T_3 + T_1 \cos\theta = 0$$

$$T_3 = T_1 \cos\theta = \boxed{384\text{ N}}$$

$$\sum F_y = T_2 + T_1 \sin\theta - m_p g - m_\ell g = 0$$

$$= T_2 + \frac{d}{\ell} m_p g + \frac{1}{2} m_\ell g - m_p g - m_\ell g$$

$$T_2 = \frac{1}{2} m_\ell g + \left(1 - \frac{d}{2}\right) m_p g = \boxed{672\text{ N}}$$

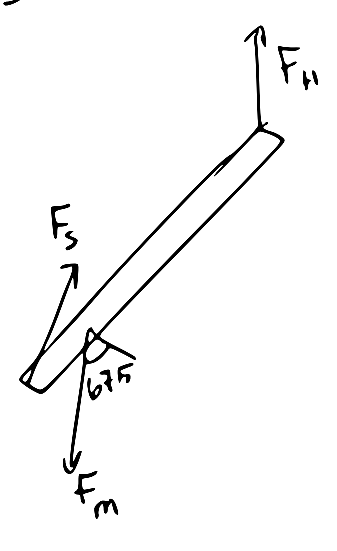


$$F_H = \frac{1}{2} F_{weight} = 375N$$

$$\Sigma \tau = -(0.04)\,F_m \sin 45 + (0.7)\,F_H = 0$$

$$F_m = \frac{0.7\,F_H}{0.04 \sin 45} = \boxed{9280N}$$

As your arm moves closer to vertical, the angle which your muscle pulls at gets closer and closer to opposite the direction of your shoulder's Normal force



$$\Sigma \tau = -(0.04)F_m \sin 67.5 + (0.7)F_H \sin 45 = 0$$

$$F_m = \frac{0.7\,F_H \sin 45}{0.04 \sin 67.5} = \underline{5022\,N}$$