2.1

Volume is $\frac{1}{3}Bh$

First convert units:

$$B = 13\,\text{acres}\left(\frac{43560\ ft^2}{1\ acre}\right)\left(\frac{12\ in}{1\ ft}\right)^2\left(\frac{2.54\ cm}{1\ in}\right)^2\left(\frac{1\ m}{100\ cm}\right)^2$$

$$\boxed{B = 52{,}609\ m^2}$$

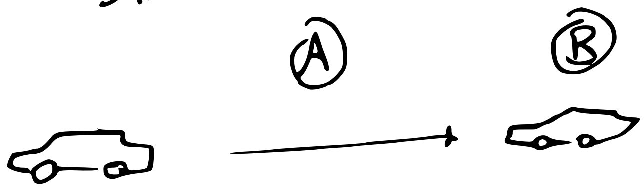
$$h = 481\ ft\left(\frac{12\ in}{1\ ft}\right)\left(\frac{2.54\ cm}{1\ in}\right)\left(\frac{1\ m}{100\ cm}\right)$$

$$h = 146\ m$$

$$V = \frac{1}{3}Bh = \boxed{2.56\times10^6\ m^3}$$

2.2

Start with a sketch



Break the problem into 2 parts:

A → driving @ $89.5 \frac{km}{hr}$

B → stopped for 22 mins

<u>knowns</u>:

$\Delta X_A = ?$

$t_A = ?$

$v_A = 89.5 \frac{km}{hr}$

$\Delta X_B = 0\ m$

$t_B = 0.3\bar{6}\ hr$

$v_B = 0\ m/s$

$\Delta X_{total} = \Delta X_A = ?$

$t_{total} = t_A + t_B$

$v_{total} = 77.8 \frac{km}{hr}$

$$V_A = \frac{\Delta X_A}{t_A} \Rightarrow 89.5 = \frac{X_A}{t_A}$$

$$V_{total} = \frac{\Delta X_{total}}{t_{total}} \Rightarrow 77.8 = \frac{X_A}{t_A + 0.3\bar{6}}$$

⇐ two equations and two unknowns

solve top eqn for X_A

$$89.5\, t_A = X_A$$

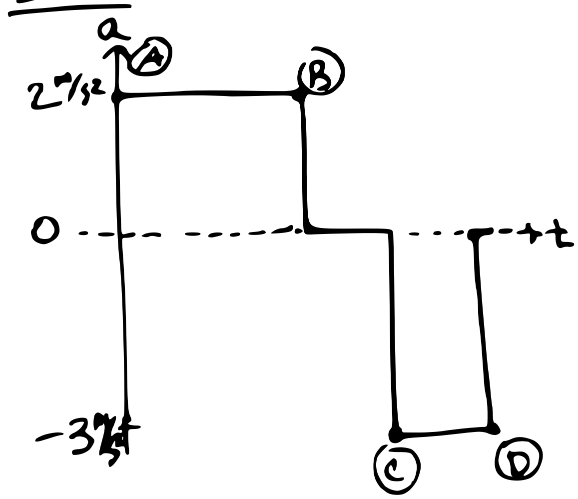
put this solution into second eqn

$$77.8 = \frac{89.5\, t_A}{t_A + 0.366}$$

solve for t_A and X_A:

$\boxed{t_A = 2.43\ hr}$ $\boxed{X_A = 218\ km}$

2.3



the acceleration is constant between the points

$a_{AB} = 2\,m/s^2$, $t_{AB} = 10s$

$a_{BC} = 0\,m/s^2$, $t_{BC} = 5s$

$a_{CD} = -3\,m/s^2$, $t_{CD} = 5s$

Model: 1D motion w/ constant acceleration (Kinematics)

* $v_f = v_i + at$
* $x_f = x_i + v_i t + \frac{1}{2}at^2$

Find the velocity & position @ each critical point:

$v_B = v_A + a_{AB}t_{AB} \Rightarrow v_B = 0 + 2(10)$

$\boxed{v_B = 20\,m/s}$

$v_C = v_B + a_{BC}t_{BC} \Rightarrow \boxed{v_C = 20\,m/s}$

$v_D = v_C + a_{CD}t_{CD} \Rightarrow v_D = 20 - 3(5) = \boxed{5\,m/s}$

$x_B = x_A + v_A t_{AB} + \frac{1}{2}a_{AB}t_{AB}^2 = 0 + 0(10) + \frac{1}{2}(2)(10)^2 = \boxed{100m}$

$x_C = x_B + v_B t_{BC} + \frac{1}{2}a_{BC}t_{BC}^2 = 100 + 20(5) + 0 = \boxed{200\,m}$

$x_D = x_C + v_C t_{CD} + \frac{1}{2}a_{CD}t_{CD}^2 = 200 + 20(5) + \frac{1}{2}(-3)5^2 = \boxed{262.5m}$

2.4

accel $a = -5.60 \frac{m}{s^2}$ for $t = 4.2$ sec

62.4 m

model: constant accel in 1D

* Dont know initial or final velocity:

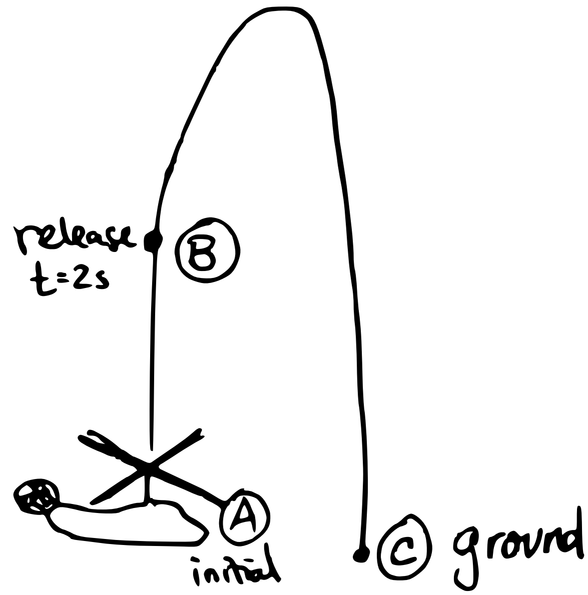
$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_i = \frac{1}{t}\left(x_f - x_i - \frac{1}{2} a t^2\right)$$

$$\boxed{v_i = 26.6 \frac{m}{s}}$$

$$v_f = v_i + at = \boxed{3.09 \frac{m}{s}}$$

2.5 Use calculus!



$x(t) = 3t^3$

$v(t) = \frac{dx}{dt} = 9t^2$

$a(t) = \frac{dv}{dt} = 18t$

Note: acceleration is NOT constant!

cannot use kinematic equations

First part (helocopter accelerating)

$v_B = 9t^2 = \boxed{36 \frac{m}{s}}$

$x_B = 3t^2 = \boxed{24\ m}$

After release $\Rightarrow$ accel is constant (Free fall)

$$x_c = x_B + v_B t + \frac{1}{2} a t^2$$

$$0 = 24 + 36t + \frac{1}{2}(-9.8)t^2$$

use quadratic equation:

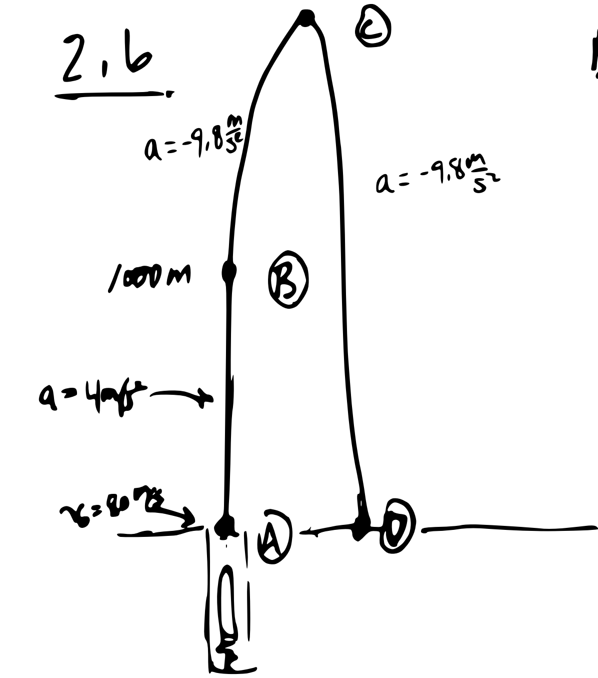
Solutions

$t = -0.615\,s$

OR

$\boxed{+7.96\,s}$

2.6



Model: const accel in 1D

Kinematic eqns:

#1 $x_f = x_i + v_i t + \frac{1}{2} a t^2$

#2 $v_f = v_i + at$

#3 $v_f^2 = v_i^2 + 2a(x_f - x_i)$

$v_A = 80 \frac{m}{s}$

$x_A = 0$

$x_B = 1000 m$

$x_D = 0 m$

$a_{AB} = 4 m/s^2$

$a_{BC} = -9.8 m/s^2$

$a_{CD} = -9.8 m/s^2$

Answers:

a) $t = t_{AB} + t_{BC} + t_{CD}$
$= 41.0 \text{ sec}$

b) $x_{max} = x_C = 1734 m$

c) $v_D = -184 m/s$

Motion from A → B ($a_{AB} = 4 m/s^2$)

eqn #3 $\quad v_B = \sqrt{v_A^2 + 2a_{AB}(x_B - x_A)} = \boxed{120 \frac{m}{s^2}}$

eqn #2 $\quad t_{AB} = \frac{v_B - v_A}{a_{AB}} = \boxed{10 s}$

Motion from B → C (max height → $v_f = 0 \frac{m}{s}$)

eqn #2 $\quad t_{BC} = \frac{v_C - v_B}{a_{BC}} = \boxed{12.24 s}$

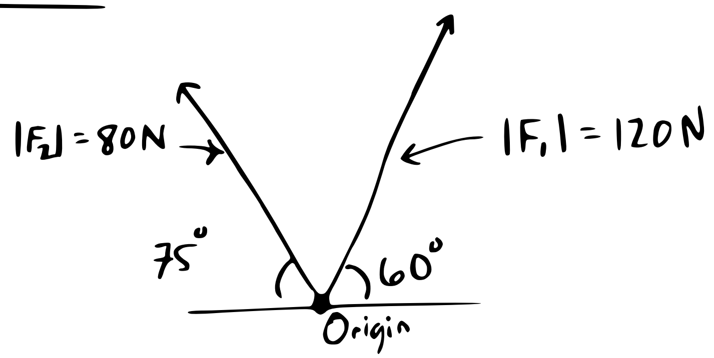
eqn #1 $\quad x_C = x_B + v_B t_{BC} + \frac{1}{2} a_{BC} t_{BC}^2 = \boxed{1734 m}$

Motion from C → D

eqn #1 $\quad t_{CD} = \sqrt{\frac{2(x_D - x_C)}{a_{CD}}} = \boxed{18.8 s}$

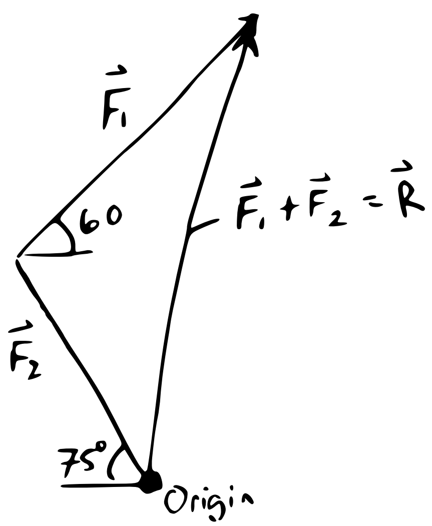
eqn #2 $\quad v_D = v_C + a_{CD} t_{CD} = \boxed{-184 m/s}$

3.1



These vectors can be moved! adding (tip to tail)

a) $F_1 + F_2$ gives resulting vector



Lets add with components of x & y

$F_{1x} = F_1 \cos 60$

$F_{1y} = F_1 \sin 60$

$F_{2x} = -F_2 \cos 75$ dont forget the negative!

$F_{2y} = F_2 \sin 75$

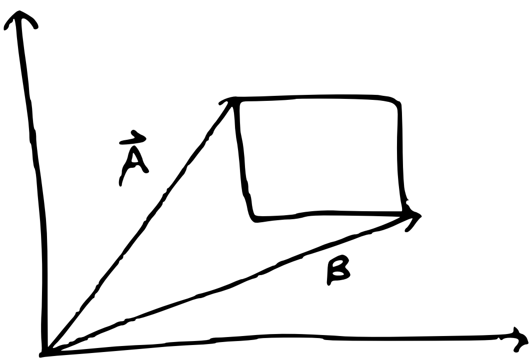
$\vec{R} = (R_x, R_y) = (F_{1x} + F_{2x}, F_{1y} + F_{2y})$

$\boxed{= (+39.29\text{ N}, +181.19\text{ N})}$

b) to reverse this, reverse both x & y

$\vec{R}_{opp} = \boxed{(-39.29\text{ N}, -181.19\text{ N})}$

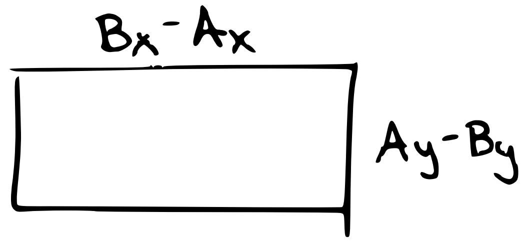
3.2



First convert from polar to cartesian coords

$$\vec{A} = (10\cos 50,\ 10\sin 50)$$
$$= (6.42,\ 7.66)$$

$$\vec{B} = (12\cos 30,\ 12\sin 30)$$
$$= (10.39,\ 6.00)$$



a) perimeter : $2(|B_x - A_x| + |B_y - A_y|) = \boxed{11.25\,m}$

b) Vector to the upper right corner

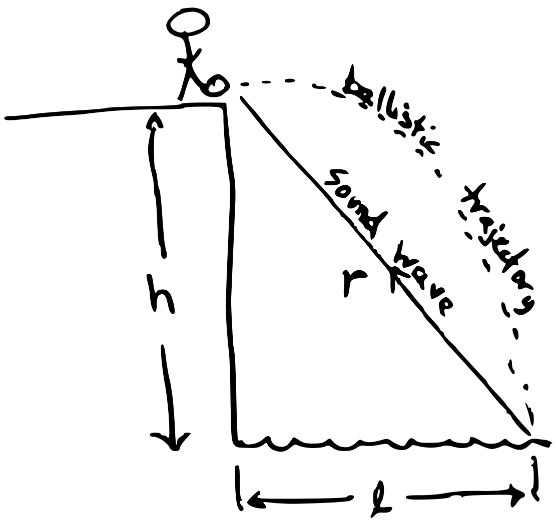
$$(B_x, A_y) = (10.39,\ 7.66)$$

convert back to polar coords

$$\theta = \tan^{-1}(A_y/B_x) = \boxed{36.39^\circ}$$

$$|\vec{R}| = \sqrt{A_y^2 + B_x^2} = \boxed{12.91\,m}$$

3.3



time taken for ball to reach the water: t_w

$$\cancel{y_f}^{0} = y_i + \cancel{v_{iy}}^{0} t_w + \frac{1}{2} a_y t_w^2$$

$$t_w = \sqrt{\frac{2y_i}{g}} = 2.85\,s$$

time taken for sound to return to your ear: t_s

$$t_s = 3 - \sqrt{\frac{2y_i}{g}} = 0.15\,s$$

the ball will travel a length l given by (use kinematics)

$$l = v_{ix} t_w$$

solving for the initial velocity:

$$v_{ix} = \frac{l}{t_w} = \frac{r\cos\theta}{t_w}$$

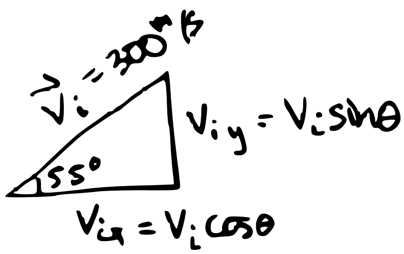
the sound travel distance (r) is $\quad (v_s = 343\,\tfrac{m}{s})$

$$r = v_s t_s$$

giving the initial velocity

$$v_{ix} = \frac{v_s t_s \cos\theta}{t_w}$$

3.4



$$X_f = \cancelto{0}{X_i} + V_{ix}t + \cancelto{0}{\tfrac{1}{2}a_x t^2}$$

$$X_f = V_i \cos\theta\, t = \boxed{7227\,m}$$

$$y_f = y_i + V_{iy}t + \tfrac{1}{2}a_y t^2$$

$$= V_i \sin\theta\, t + \tfrac{1}{2}(-9.8)t^2$$

$$\boxed{= 1668\,m}$$

3.5

$$\vec{a} = \sqrt{a_t^2 + a_c^2}$$

particle can have centripital (a_c) and tangential (a_t) acceleration
the overall accel is given above

The question is: given $\vec{a}$ and $\vec{a}_c$, is $\vec{a}_t$ real or imaginary

a)

$$6 = \sqrt{a_t^2 + \left(\frac{3^2}{2}\right)^2}$$

$$a_t = \sqrt{6^2 - \left(\frac{3^2}{2}\right)^2} = \boxed{3.96 \frac{m}{s^2}}$$

yes, this is possible

b)

$$a_t = \sqrt{a^2 - a_c^2} = \sqrt{4^2 - \left(\frac{3^2}{2}\right)^2} = \sqrt{-4.25}$$

this is an imaginary number

$\boxed{\text{No, this is not possible}}$

3.6 First convert units!

$$8 \frac{rev}{s} = 8 \frac{\cancel{rev}}{sec} 2\pi \frac{rad}{\cancel{rev}} = 16\pi \frac{rad}{sec}$$

$$6 \frac{rev}{s} = 12\pi \frac{rad}{sec}$$

the linear velocity v is related to the angular velocity ω

$$v = \omega r$$

a) $v_1 = \omega_1 r_1 = (16\pi)(0.6) = 31.1 \frac{m}{s}$

$v_2 = \omega_2 r_2 = (12\pi)(0.9) = 33.9 \frac{m}{s}$

$\boxed{\text{shorter chain moves faster}}$

b) $a_c = \frac{v_1^2}{r_1} = \omega_1^2 r_1 = \boxed{1515 \frac{m}{s^2}}$

c) $a_c = \frac{v_2^2}{r_2} = \omega_2^2 r_2 = \boxed{1279 \frac{m}{s^2}}$