Problem 4.1:

"potential" → Electric Potential (not potential energy)

why? • unit is Volts (rather than Joules)

• only a single charge (energy requires two q's)

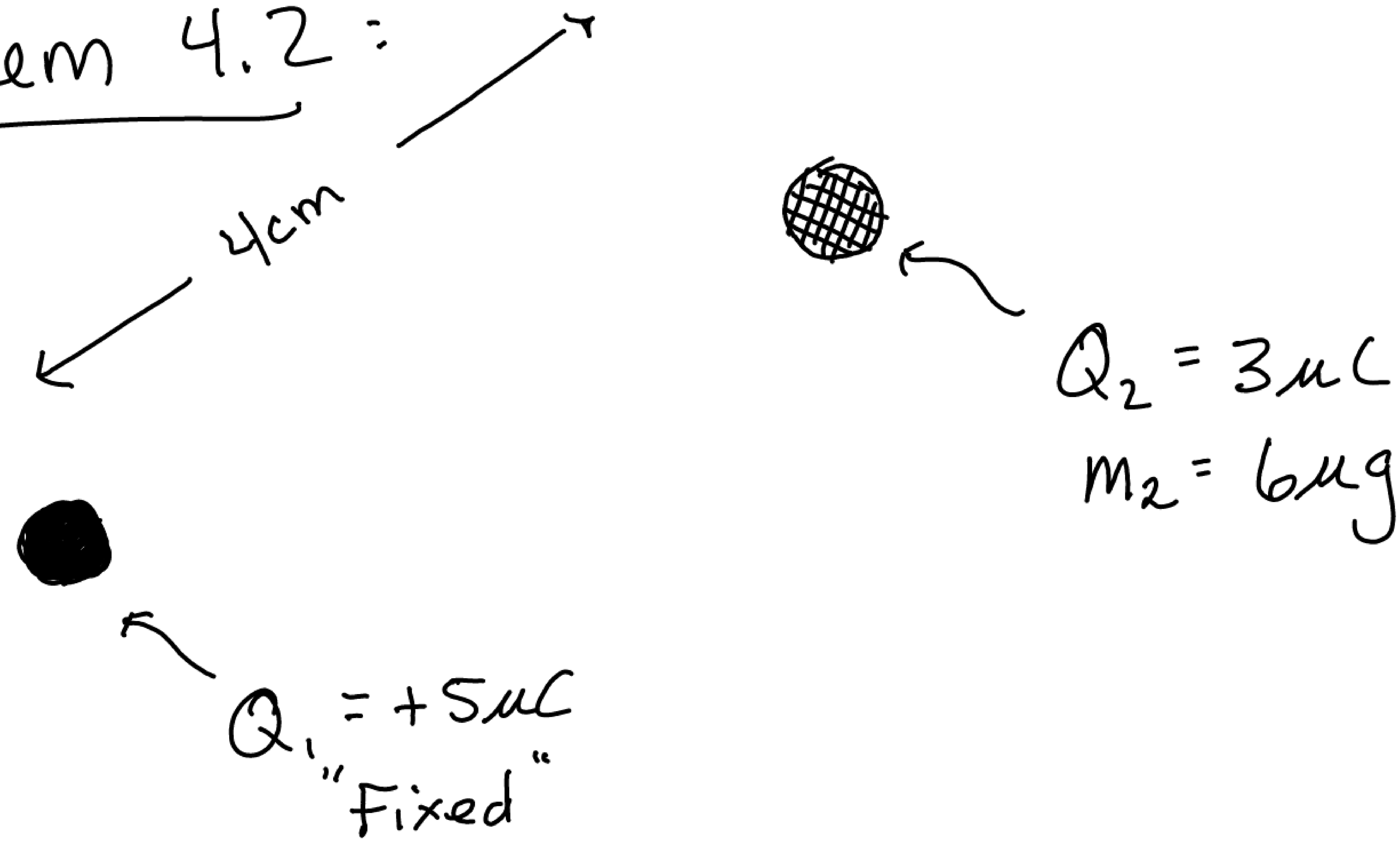
$$V = k_e \frac{q}{r}$$

$$q = \frac{V}{k_e} \cdot r$$

$$q = \frac{500}{9E9} \cdot (15) = \boxed{+833\,nC}$$

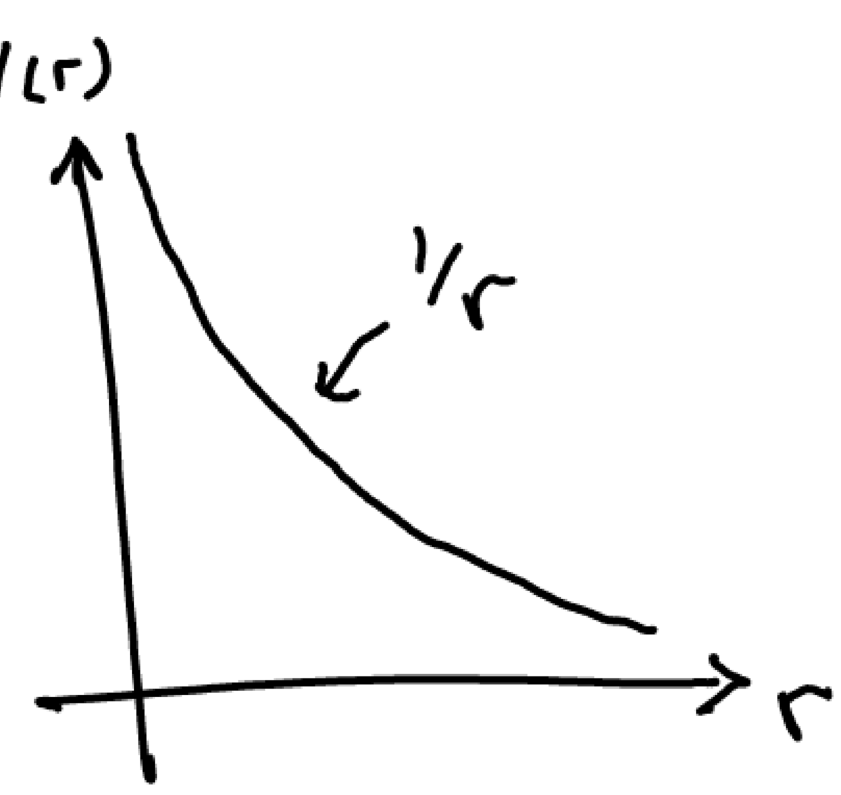
Positive Voltage ⇒ Positive charge.

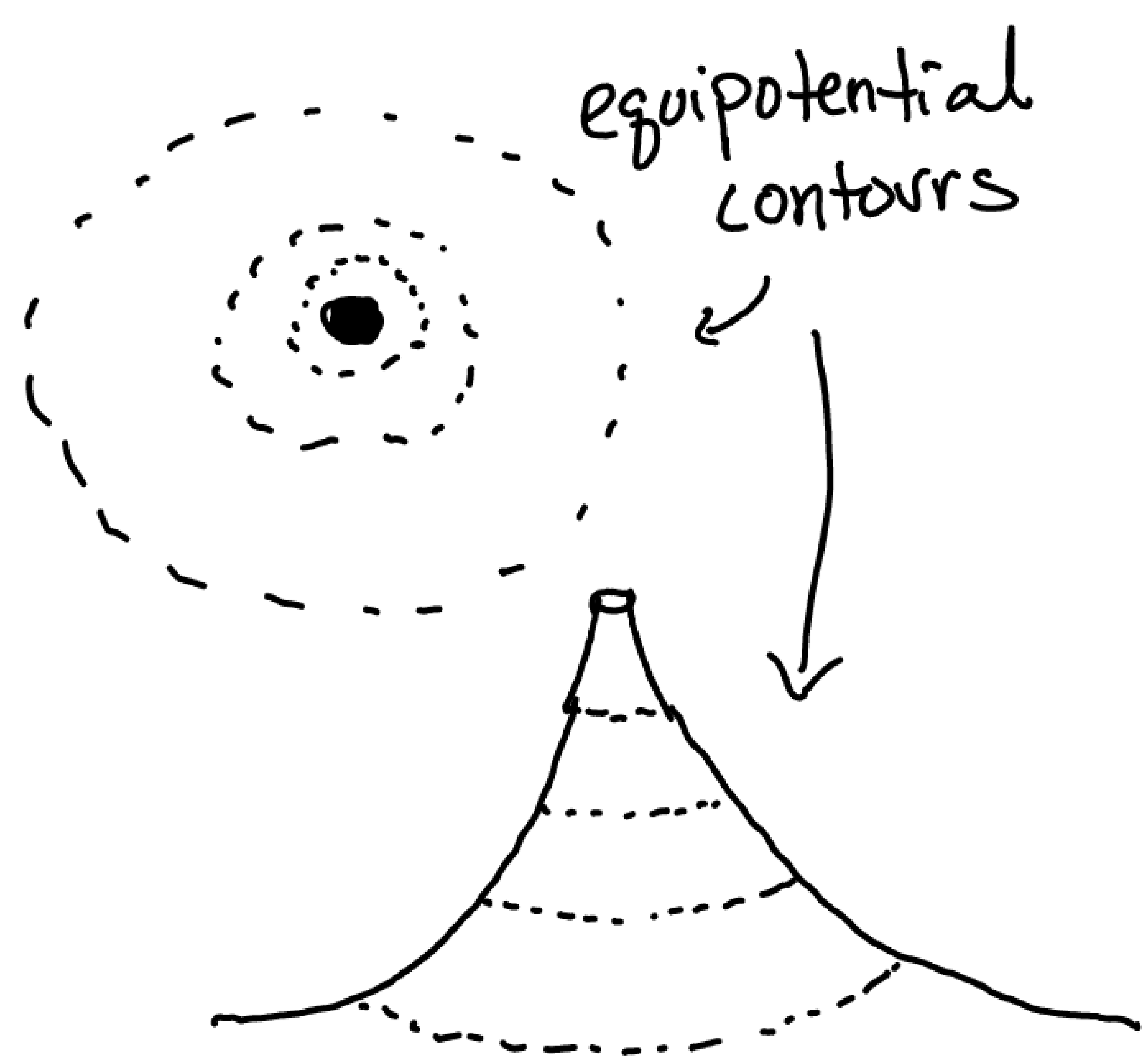
Problem 4.2:



Electric "potential" is set up by Q_1

$$\boxed{V = k\frac{Q_1}{r}}$$





units: [Volts]

slope of the "hill" ⇒ Force

slope of the Voltage ⇒ E-Field

(gradient → $\vec{\nabla}$)

$$-\vec{\nabla}V = -\frac{\partial}{\partial r}\left(k\frac{q}{r}\right)\hat{r} = +k\frac{q}{r^2}\hat{r} = \vec{E}$$

$$\boxed{\vec{E} = -\vec{\nabla}V}$$

Gradient (derivative) is the "easy" direction

It is normally harder to take the integral:

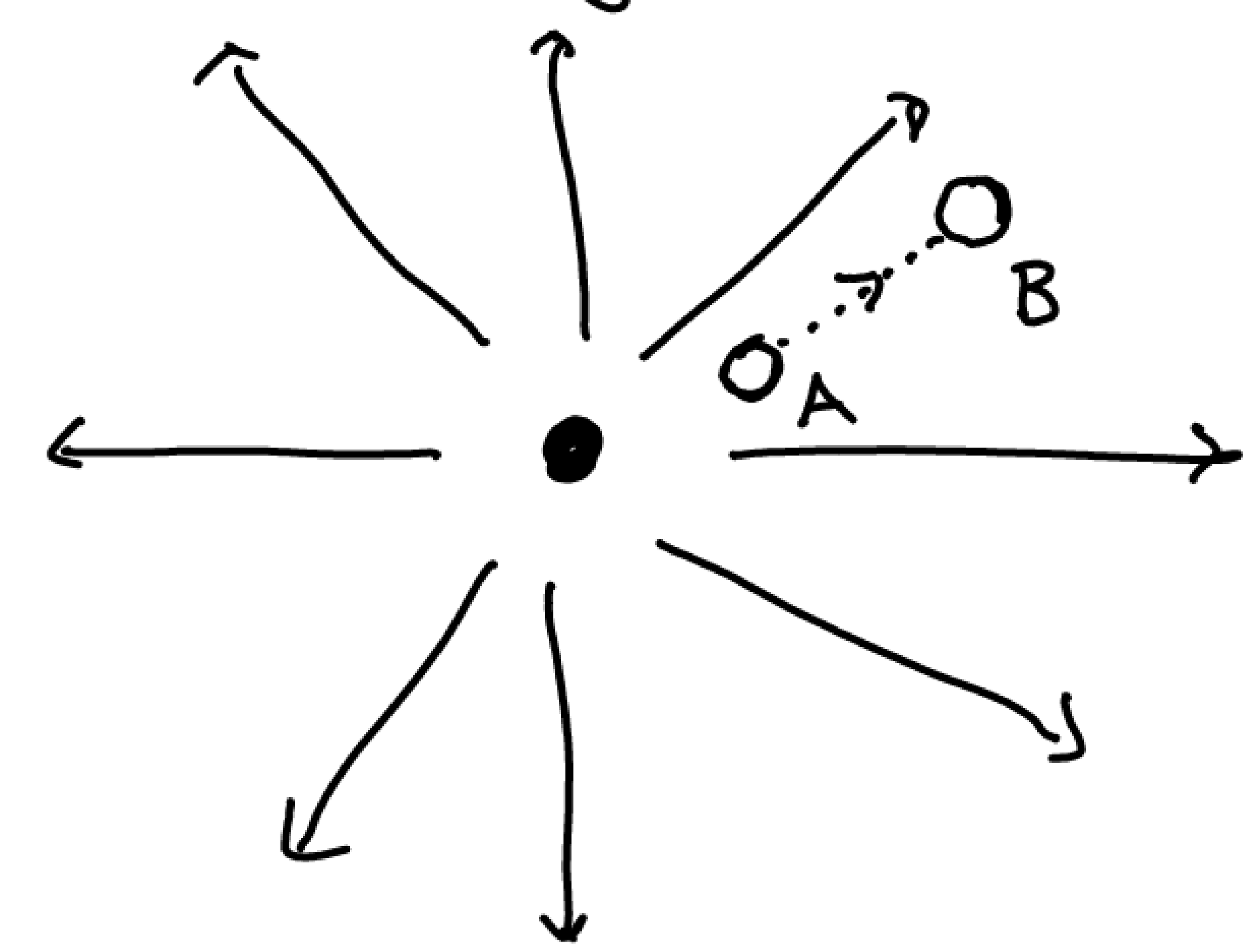
$$\boxed{\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{l}}$$

"line integral"

$$\Delta V_{AB} = -\int_A^B k \frac{q}{|\vec{r}|^2}\, \hat{r} \cdot dr\, \hat{r}$$

$$= -\int_A^B \frac{kq}{r^2}\, dr$$

$$\boxed{\Delta V_{AB} = \frac{kq}{r_B} - \frac{kq}{r_A}}$$



For a point charge:

$d\vec{l}$ is simply radial $\rightarrow dr\,\hat{r}$

Electric Energy (Potential energy) {Name can be confusing}

One charge sets up the contours (equipotentials)

an additional charge can move on the "hill"

ΔV

$$PE = \boxed{U_E = q\,\Delta V}$$

requires two or more charges

this energy adds into the conservation of Energy

$$W_{nc} = \Delta K + \Delta U$$

Woo! now lets solve the problem :)

problem 4.2:

If zero energy (sea level) is very far (∞) off in the distance, the potential Energy is:

$$PE = \frac{KQ_1Q_2}{r} = (9E9)\frac{(5E-6)(3E-6)}{0.04}$$

$$\boxed{= 3.375 \text{ Joules}}$$

remember this is arbitrary could set to zero only ΔU matters

problem 4.2.2:

Now find the change in U

$$\Delta U = K\frac{Q_1Q_2}{r_B} - \frac{KQ_1Q_2}{r_A}$$

$$= 1.6875 - 3.375 = \boxed{-1.6875\text{ J}}$$

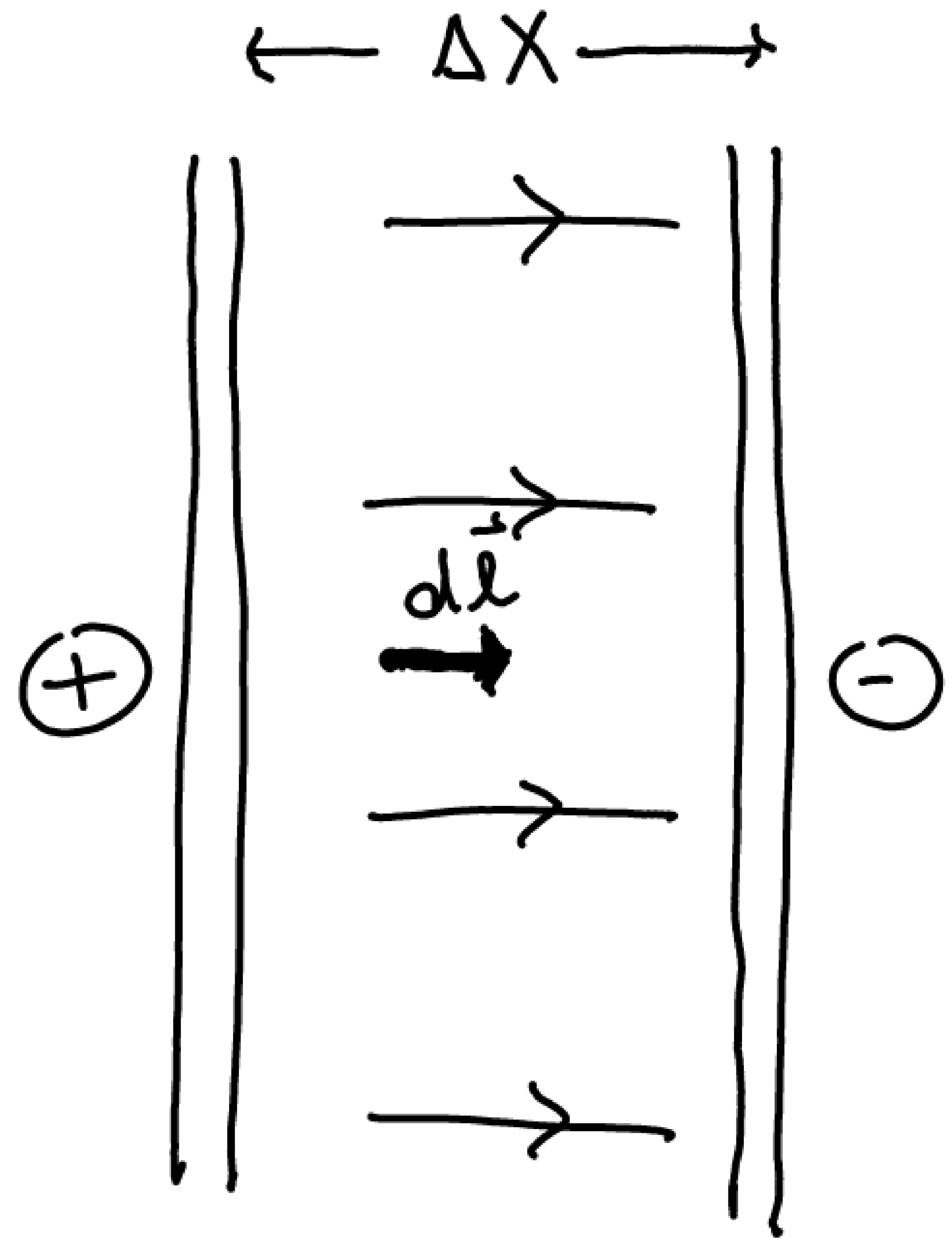
to find speed; use W_{nc} (no friction, no mgh, no $\frac{1}{2}kx^2$)

$$W_{nc} = \Delta K + \Delta U$$

$$0 = \cancel{\tfrac{1}{2}m_1v_{1f}^2}^{\,0} + \tfrac{1}{2}m_2v_{2f}^2 - \cancel{\tfrac{1}{2}m_1v_{1i}^2}^{\,0} - \cancel{\tfrac{1}{2}m_2v_{2i}^2}^{\,0} - 1.6875$$

$$v_{2f} = \sqrt{\frac{2}{m_2}(1.6875)} = \boxed{2.37\times10^4 \text{ m/s}}$$

Problem 4.3:



$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$$

$$\Delta V_{AB} = -\int \vec{E} \cdot d\vec{l}$$

E is constant: pulls out of $\int$

$$\Delta V_{AB} = -\int \frac{\sigma}{\varepsilon_0}\hat{x} \cdot dx\,\hat{x}$$

$$= -\frac{\sigma}{\varepsilon_0}\int dx$$

$$\Delta V = -\frac{\sigma}{\varepsilon_0}\Delta X$$

$$1.5E4 = -\left(\frac{\sigma}{\varepsilon_0}\right)(0.01)$$

$$|\vec{E}| = \frac{\sigma}{\varepsilon_0} = \boxed{1.5 \times 10^{6}\ N/C}$$

problem 4.4

$\Delta V = 4 \times 10^{4}$ Volts

$q = 1.6 \times 10^{-19}$ C

$m = 1.9 \times 10^{-31}$ kg

$$W_{nc} = \Delta K + \Delta U$$

$U = qV$

$$0 = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}^{\,0} + \cancel{qV_f}^{\,0} - qV_i$$

$$\frac{1}{2}mv^2 = qV$$

NEEDS Relativistic correction!

$$v = \sqrt{\frac{2qV}{m}} = \boxed{2.59 \times 10^{8}\ m/s}$$

Speed of light: $c \approx 3E8$ m/s

problem 4.5:

Find potential (Voltage) at a bunch-o-points

$$V = \sum_{i=1}^{2} \frac{K q_i}{r_i}$$

nanocoulombs
$\times 10^{-9}$

P1:
$$V = \frac{(9E9)(5E-9)}{0.02} + \frac{(9E9)(-10E-9)}{0.06}$$

$$\boxed{= 750 \text{ Volts}}$$

P2:
$$V = \frac{(9E9)(5E-9)}{0.06} + \frac{(9E9)(-10E-9)}{0.02}$$

$$\boxed{= -3750 \text{ Volts}}$$

P3:
$$V = \frac{(9E9)(5E-9)}{\sqrt{0.04^2+0.03^2}} + \frac{(9E9)(-10E-9)}{\sqrt{0.04^2+0.03^2}}$$

$$\boxed{= -900 \text{ Volts}}$$

P4: same as P3 (convince yourself: same separation distances)

Problem 4.6:

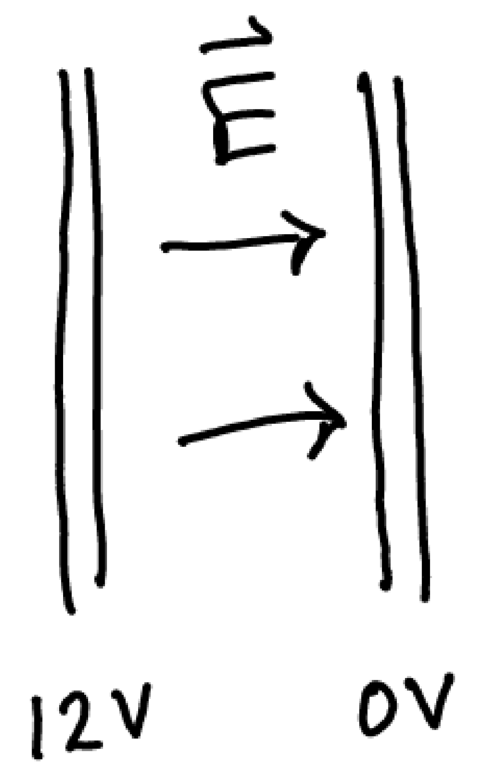
$$V = -xy^2z + 4xy$$

$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) V$$

- $\dfrac{\partial V}{\partial x} = -y^2z + 4y$
- $\dfrac{\partial V}{\partial y} = -2xyz + 4x$
- $\dfrac{\partial V}{\partial z} = -xy^2$

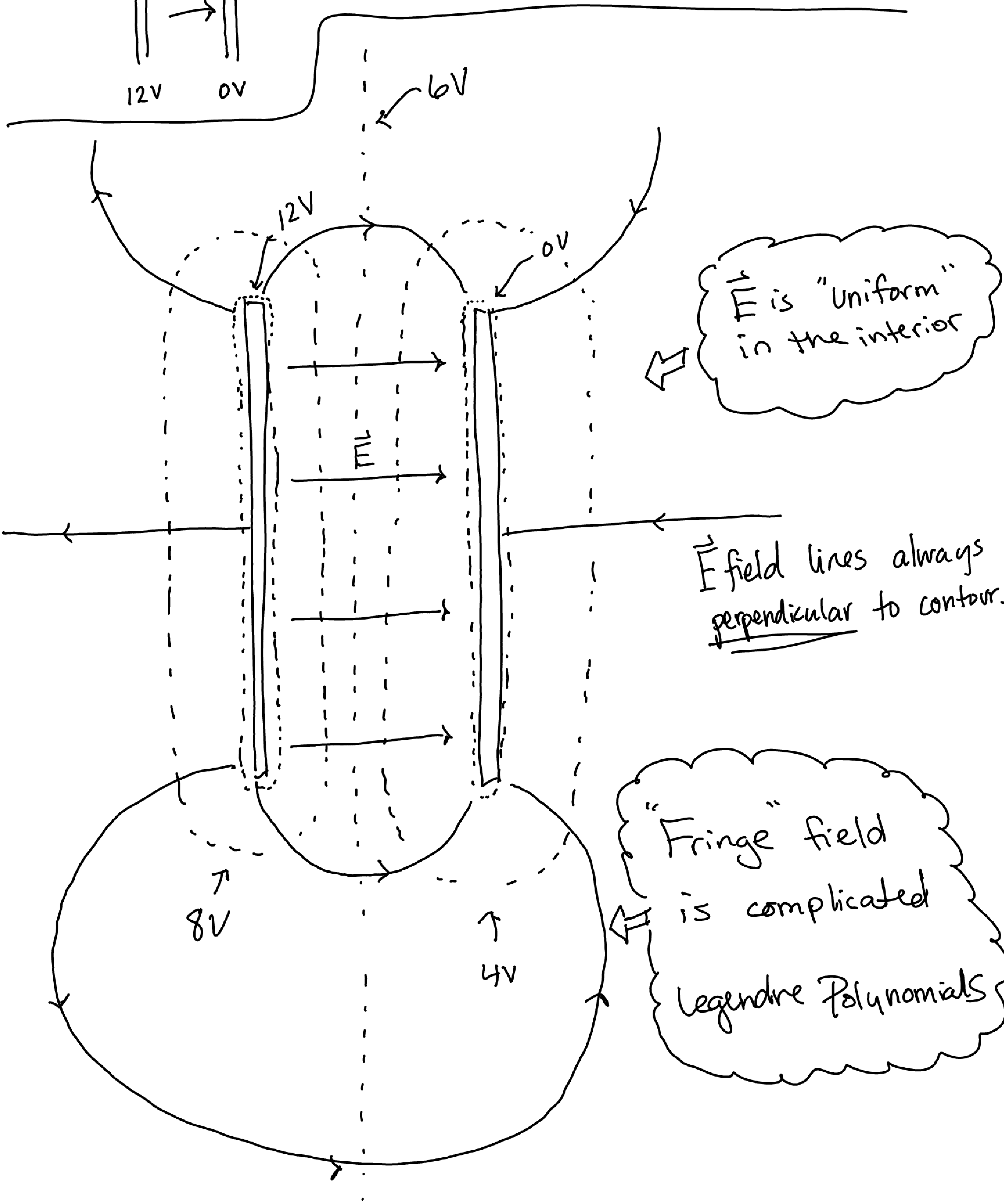
$$\boxed{\vec{E} = \left(y^2z - 4y\right)\hat{x} + \left(2xyz - 4x\right)\hat{y} + \left(xy^2\right)\hat{z}}$$

Problem 4.7:



From Gauss' Law:

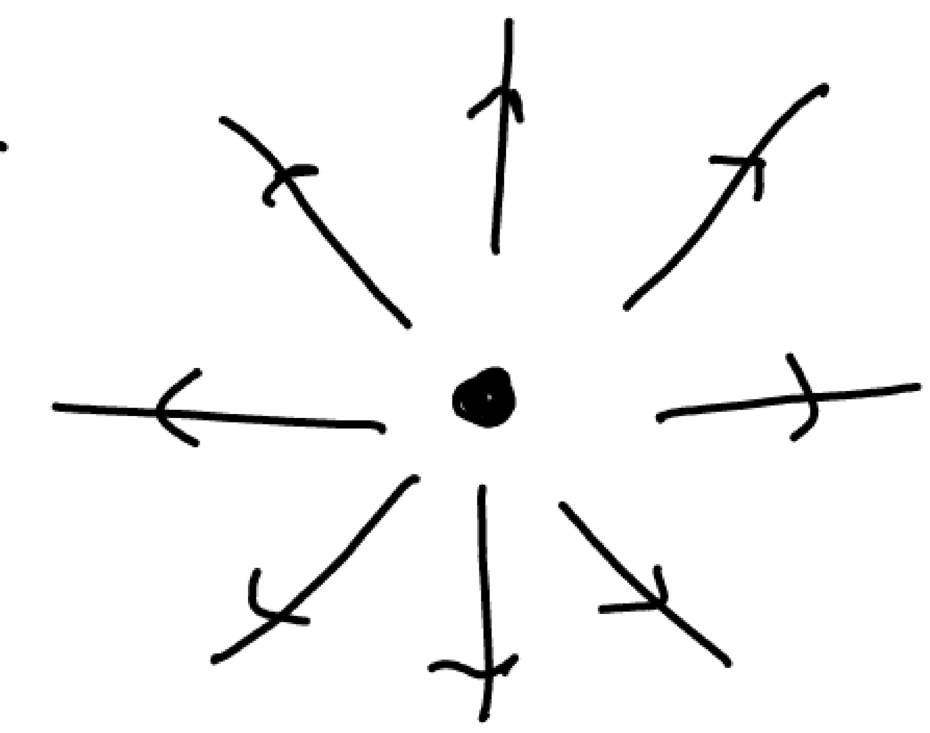
← E field between two plates constant $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$



Problem 4.8:

First find the potential

$\Delta V = -\int E \cdot dl$

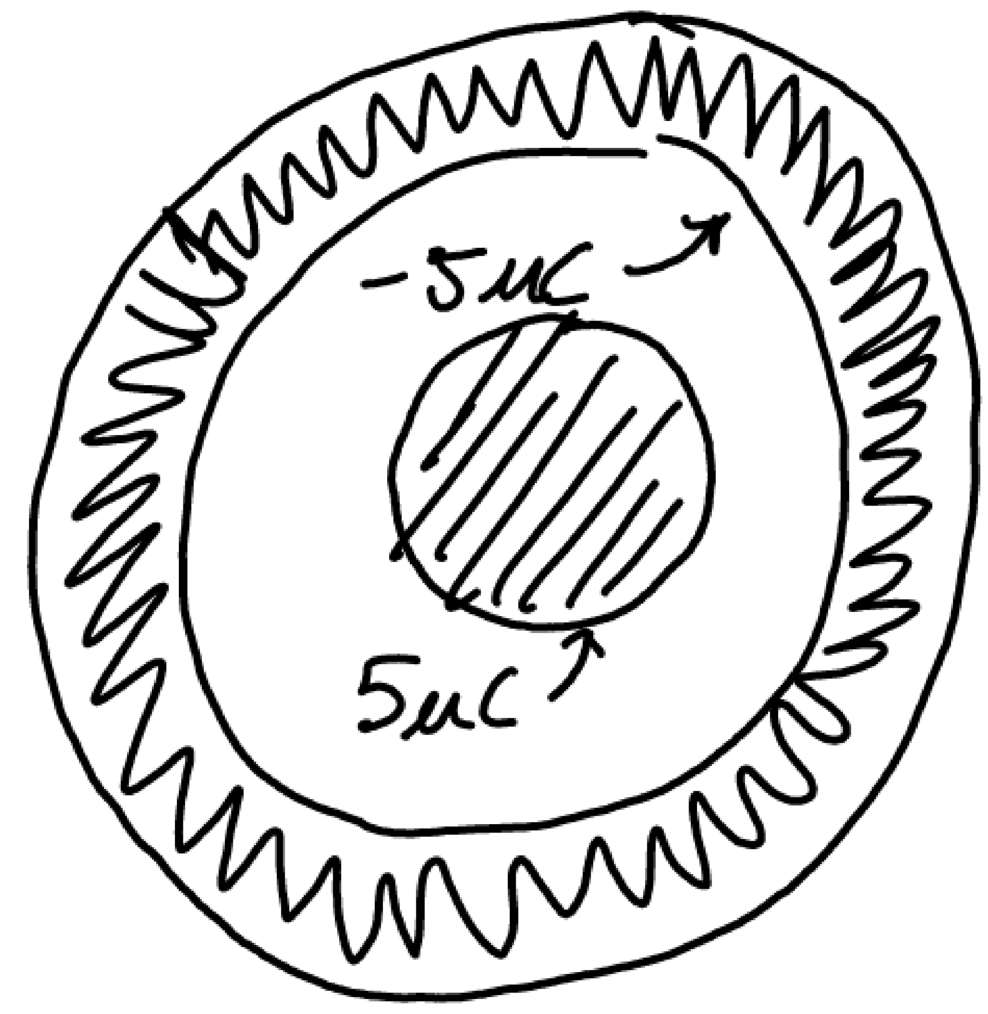


$\Delta V = -\int k \frac{q}{r^2} \hat{r} \cdot dr \hat{r}$ $\qquad E = k \frac{q}{|\vec{r}|^2} \hat{r}$

$\boxed{\Delta V = k\, q/r}$

potential is zero @ infinity

$V(\infty) = 0$



The total charge $q_1 + q_2 = 0$

$\boxed{\vec{E} = 0}$ outside the large shell

$\Delta V = -\int \vec{E}^{\,0} \cdot d\vec{l} = 0$

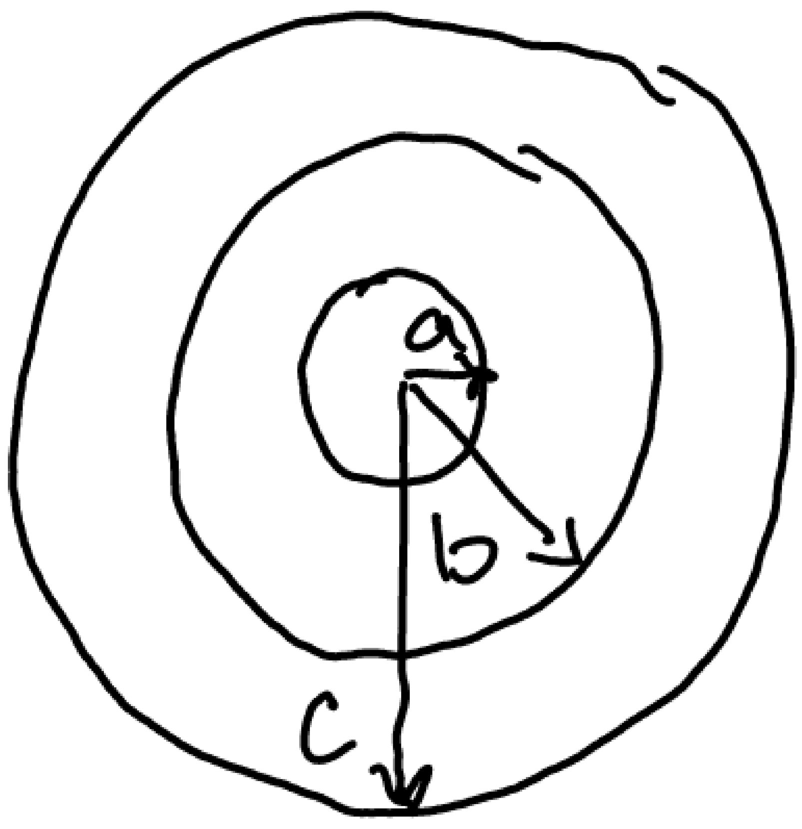
potential cant change when $\vec{E} = 0$ (flat energy "landscape")

so the outer shell $\boxed{V = 0}$ volts

Inside of the conductor $E = 0$

so $\boxed{V = 0}$ all the way to the inside surface of the outer shell.

Now you reach a charged surface



region a: inside of center spherical conductor
$E = 0$ $\quad V =$ unknown constant

region b: void region filled with air (or vacuum)

region c: outside spherical shell $\underline{E = 0}$ $\quad \underline{V = 0}$

discussed on previous page

The potential in region b is equivalent to $+5\,\mu C$ point charge at the center:

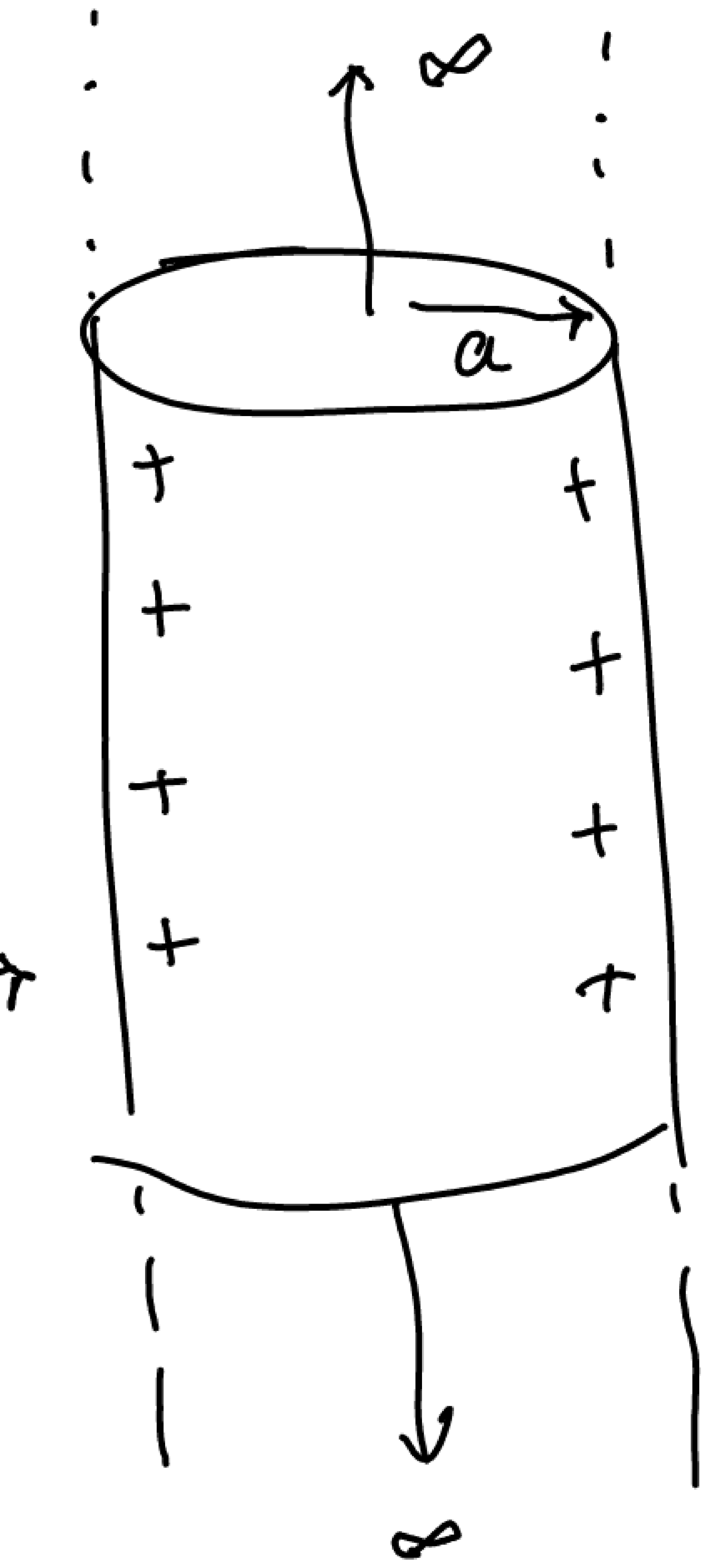
$$V(r) = k\frac{q'}{r} + \text{constant}$$

when $r = b \Rightarrow V = 0$ so subtract this off to keep potential continuous

$$V(r) = \frac{(9E9)(5E\text{-}6)}{r} - \frac{(9E9)(5E\text{-}6)}{0.05}$$

$$V(a) = \frac{(9E9)(5E\text{-}6)}{0.02} - \frac{(9E9)(5E\text{-}6)}{0.05} = \boxed{1.35\times10^{6}\,V}$$

$$V(b) = \frac{(9E9)(5E\text{-}6)}{0.05} - \frac{(9E9)(5E\text{-}6)}{0.05} = \boxed{0\,V}$$



$\sigma = \frac{q}{A}$

Cylindrical Symmetry:

$$\oint_S \vec{E} \cdot \hat{n}\, dA = \frac{q_{in}}{\varepsilon_0}$$

$$E(2\pi r \cancel{l}) = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{r} = \frac{a\sigma}{r\varepsilon_0}\hat{r}$$

$\lambda = q/l$ is the "line" of charge: transform this to a surface charge ⇒

$$\lambda = \frac{q}{l}$$

$$\sigma = \frac{q}{A} = \frac{q}{2\pi a l}$$

$$\sigma = \frac{\lambda}{2\pi a}$$

$$\lambda = 2\pi a \sigma$$

Find potential w/ line integral

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{l}$$

$$= -\int_A^B \frac{a\sigma}{r\varepsilon_0}\hat{r} \cdot dr\,\hat{r}$$

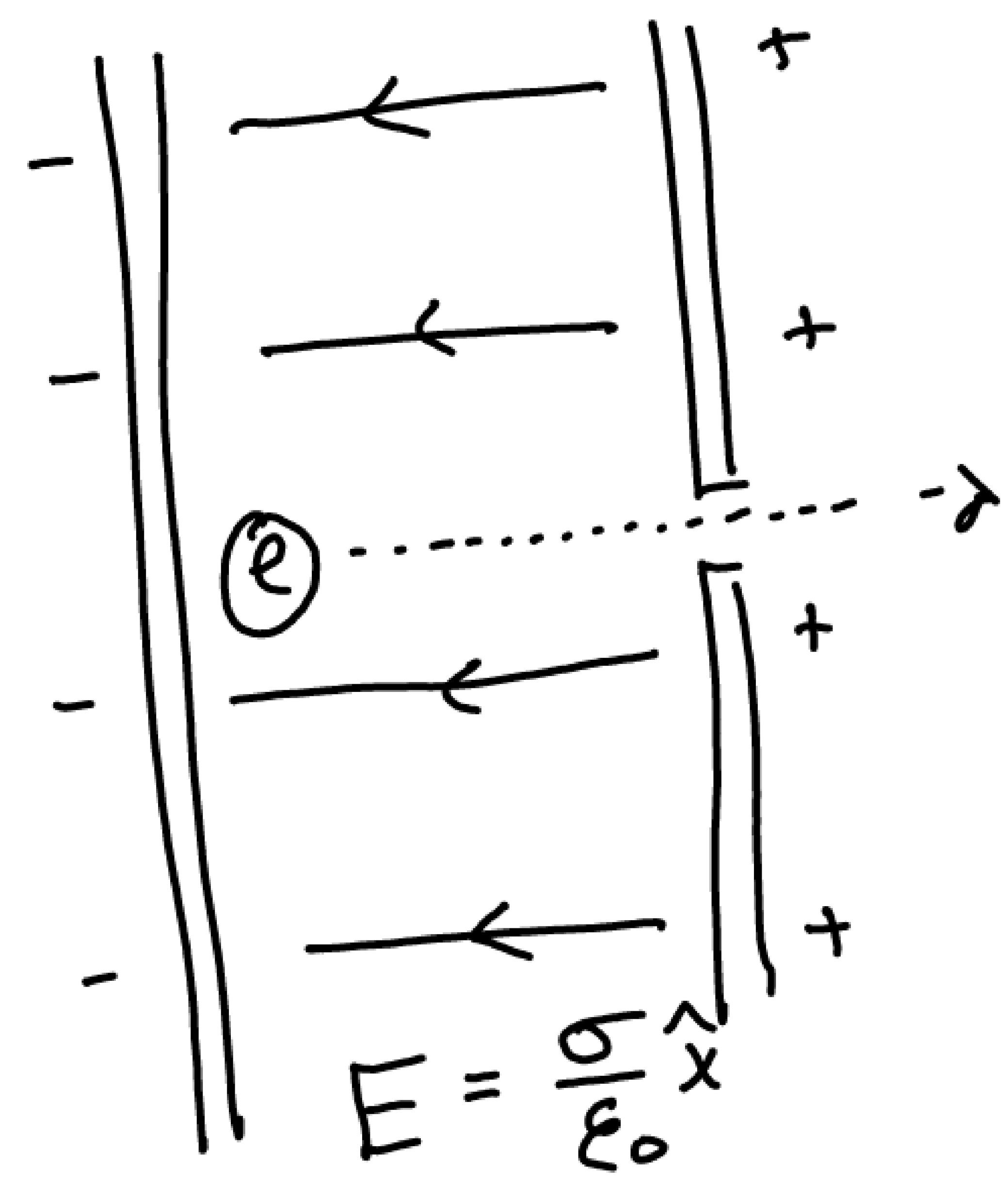
$$= -\frac{a\sigma}{\varepsilon_0}\int_{r_i}^{r_f} \frac{dr}{r} = -\frac{a\sigma}{\varepsilon_0}\left[\ln(r_f) - \ln(a)\right]$$

$$\boxed{V(r) = -\frac{a\sigma}{\varepsilon_0}\ln(r/a)}$$

potential for a cylinder

natural log

problem 4.10:



two options to solve dynamics:

Ⓐ Integrate the force twice (kinematics) $F = qE$, $F = ma$

Ⓑ use conservation of energy $W_{nc} = \Delta K + \Delta U$

$$\vec{F} = m\vec{a} = q\vec{E}$$

$$\vec{a} = \frac{q\vec{E}}{m} = \frac{(-1.6\times10^{-19})(-2.5\times10^{4})}{(1.9\times10^{-31})}\hat{x}$$

$$\boxed{\vec{a} = 2.1\times10^{16}\,\frac{m}{s^2}\hat{x}}$$

part 2: remember the field mosty vanishes outside (minus the "fringe field")

There is no remaining $\vec{E}$ field to pull the electron back.

Each ⊕ charge on right plate is absorbed by a ⊖ charge on the left plate.

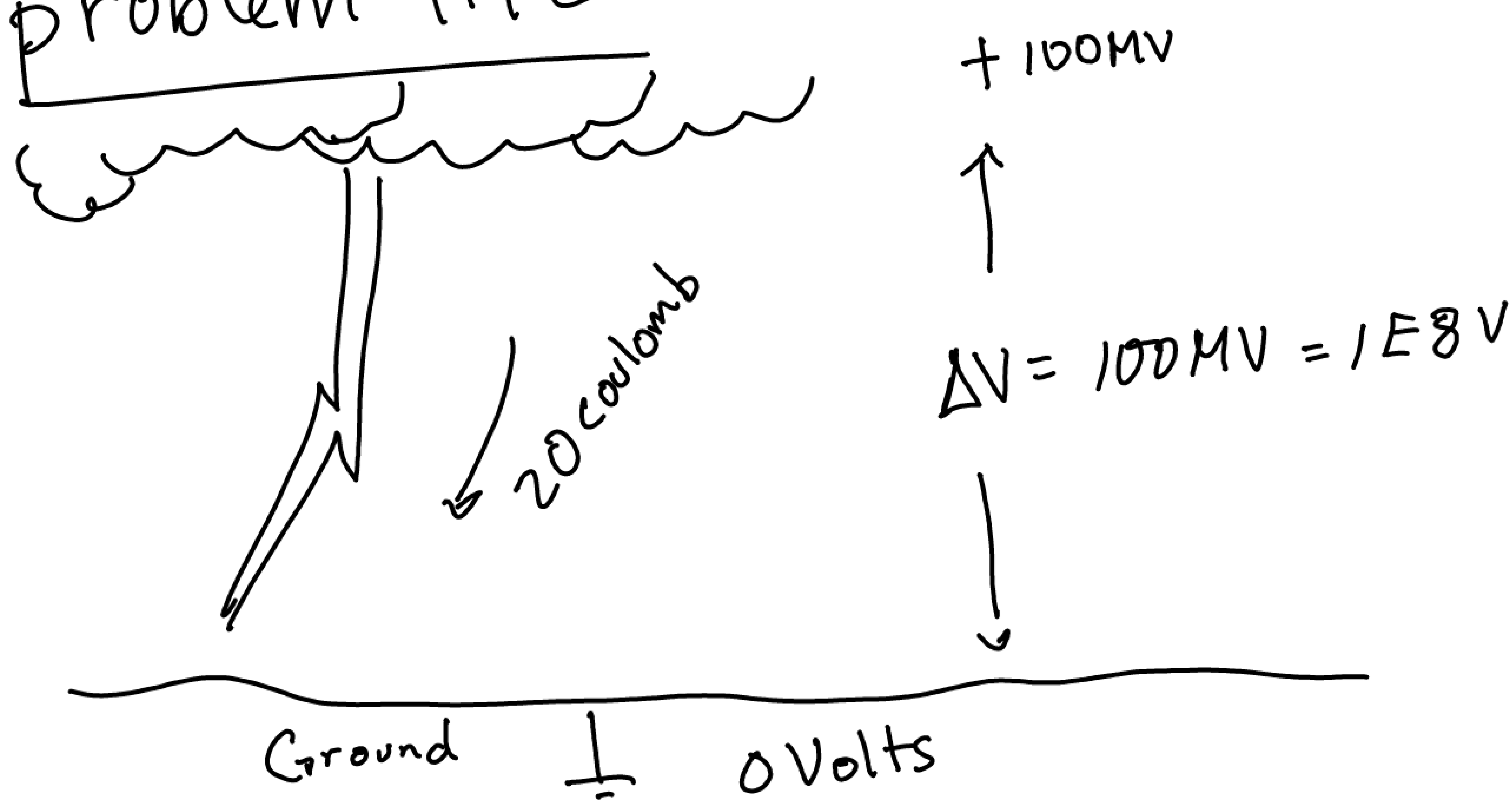
problem 4.11:

$\Delta x = 8\,nm = 8 \times 10^{-9}\,m$

$\vec{E} = 5.5\,\frac{MV}{m}$ $\left(\frac{\text{MegaVolts}}{\text{meter}}\right) = 5.5E6\,\frac{V}{m}$

parallel plates: $V = \frac{\sigma}{\varepsilon_0}\Delta x = E\Delta x = \boxed{0.044 \text{ Volts}}$

problem 4.12:



part 1:

Energy: $\Delta U = q\,\Delta V = \boxed{2.0 \times 10^{9}\,J}$

part 2:

$\Delta Q = 2 \times 10^{9} = mc\Delta T = m(4186)(100-15)$

$\boxed{m = 5600\,Kg}$ raised to $100°C$

Part 3: BOOM!

Problem 4.13:

Electric Potential $\Delta V = 12$ Volts

potential Energy $\Delta U_E = q\,\Delta V$

part 1:

$$W_{nc} = \Delta K + \Delta U$$

voltage

$$0 = \frac{1}{2}mv^2 - qV$$

velocity

$$q = \frac{1}{2}mv^2 \frac{1}{(12V)} = \boxed{19{,}531 \text{ Coulomb}}$$

part 2:

$$W_{nc} = \Delta K + \Delta U$$

$$0 = mgh_f - qV$$

$$q = \frac{mgh}{(12V)} = \boxed{122{,}500 \text{ Coulomb}}$$

part 3:

distance: $\Delta x = V\Delta t = 25\,(3600)$
$= 90\text{km}$

$$W_{nc} = \int \vec{F} \cdot d\vec{r} = (-500)(90\,000)$$

$$= -45000\,000 \text{ Joules}$$

$$W_{nc} = -qV_i \Rightarrow q = \boxed{3{,}750{,}000 \text{ coulomb}}$$