Problem 5.1:

3.3 → 12VDC is common

$$Q = C\,\Delta V$$

Voltage

"Stored" charge

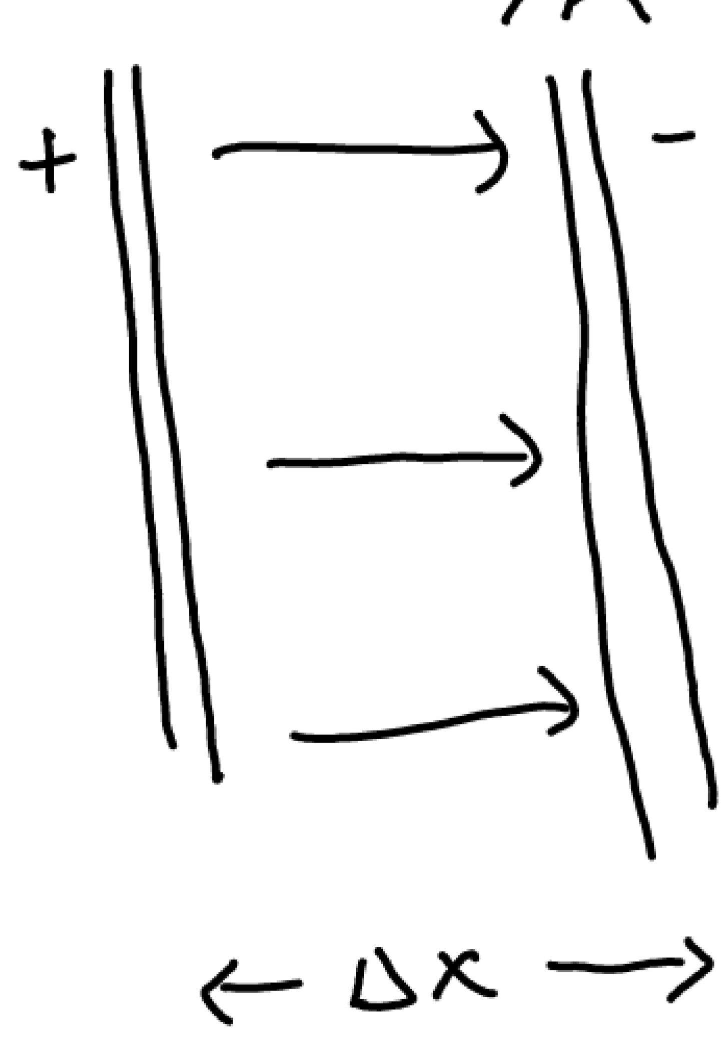
capacitance proportionality w/ Voltage

Larger capacitor will store more charge @ the same voltage.

$$\Delta V = \frac{Q}{C} = \frac{3.1E-6}{2E-6} = \boxed{1.55\ V}$$

Problem 5.2:

$$\sigma = q/A$$



$\Delta V = \int E \cdot dl \Leftrightarrow E\Delta x$ w/ constant $\vec{E}$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$$

$$\Delta V = \frac{\sigma}{\varepsilon_0}\Delta x$$

$$\Delta V = \frac{Q}{\varepsilon_0 A}\Delta x$$

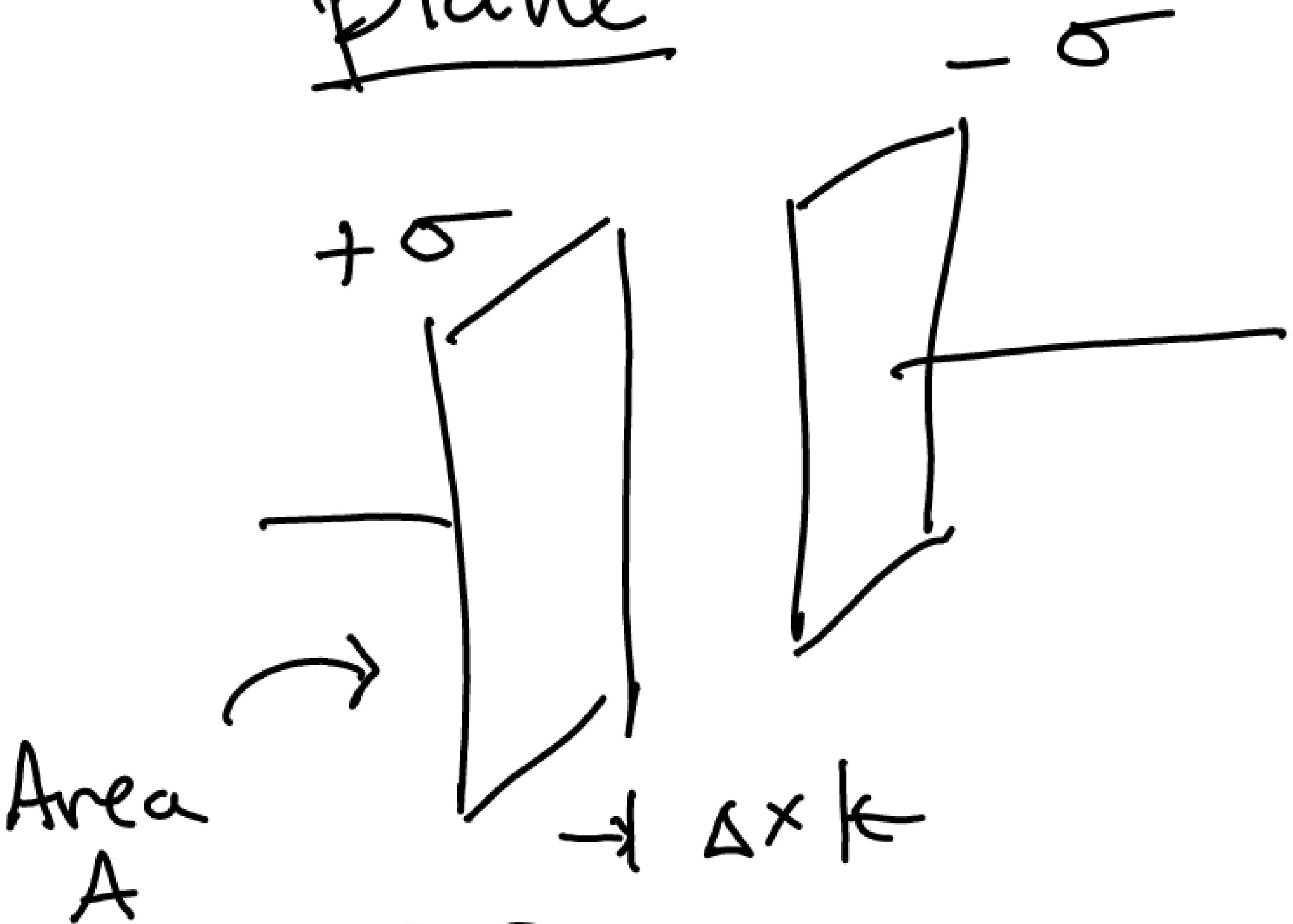
$$Q = \left(\frac{\varepsilon_0 A}{\Delta x}\right)\Delta V$$

capacitance

$$C = \frac{\varepsilon_0 A}{\Delta x}$$

$$A = \frac{C\,\Delta x}{\varepsilon_0} = \frac{(5E-9)(0.002)}{(8.85\,E-12)} = \boxed{1.3\ m^2}$$

plane



set charge as $q = \sigma A$

Evaluate $\vec{E}\cdot d\vec{l} \Rightarrow$ Voltage.

$$-\int E\cdot dl = -E\int dx = -E\,\Delta x = \Delta V$$

Gauss

$$E(2A) = \frac{\sigma A}{\varepsilon_0}$$

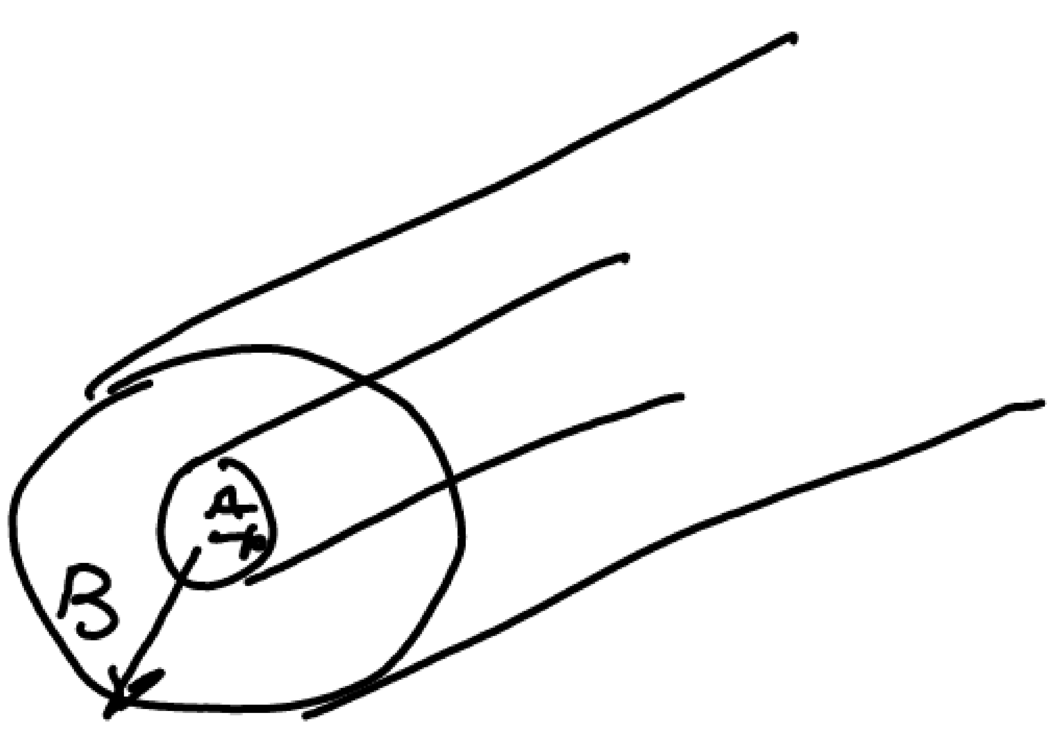
$$E = \sigma/2\varepsilon_0 \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

$$Q = C\Delta V =$$

$$\sigma A = C(E\Delta x) = C\left(\frac{\sigma}{\varepsilon_0}\Delta x\right)$$

$$\boxed{C = \frac{\varepsilon_0 A}{\Delta x}}$$ plane of charge

cylinder



$$q = \lambda l$$

$$\Delta V = -\int \vec{E}\cdot d\vec{l} = \int \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\cdot dr\,\hat{r}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\int \frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln(r)\Big|_A^B$$

Gauss

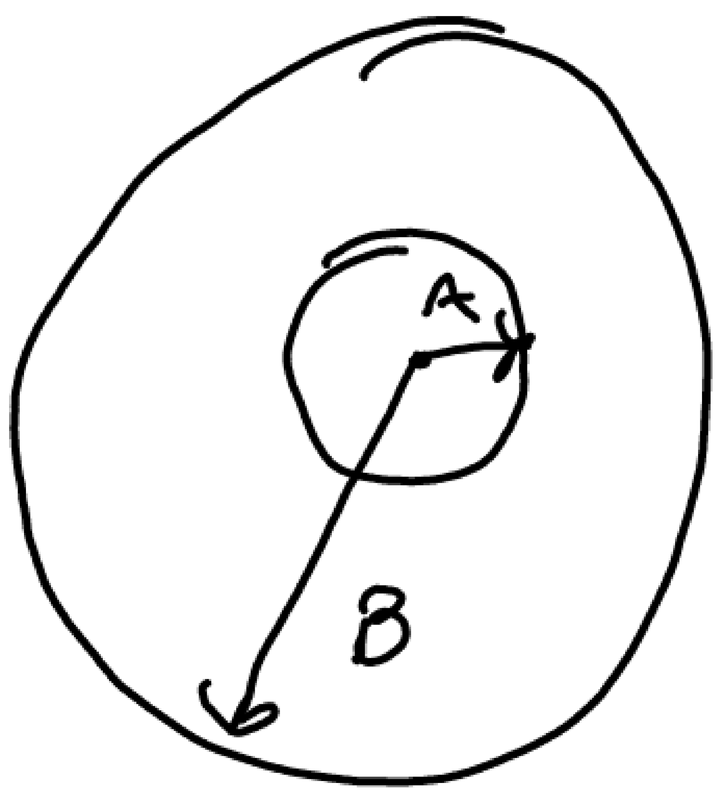
$$E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}}$$

$$Q = C\Delta V = C\left[\frac{Q}{l}\frac{1}{2\pi\varepsilon_0}\ln\left(A/B\right)\right]$$

$$\boxed{C = 2\pi\varepsilon_0 l \ln\left(A/B\right)}$$

sphere:



Gauss:

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$

$$= -\int \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \cdot dr\,\hat{r}$$

$$= -\frac{Q}{4\pi\varepsilon_0 r}\Bigg|_A^B = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{A} - \frac{1}{B}\right]$$

$$Q = C\Delta V$$

$$Q = C\left[\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{A} - \frac{1}{B}\right)\right]$$

$$C = 4\pi\varepsilon_0 \frac{1}{1/A - 1/B}$$

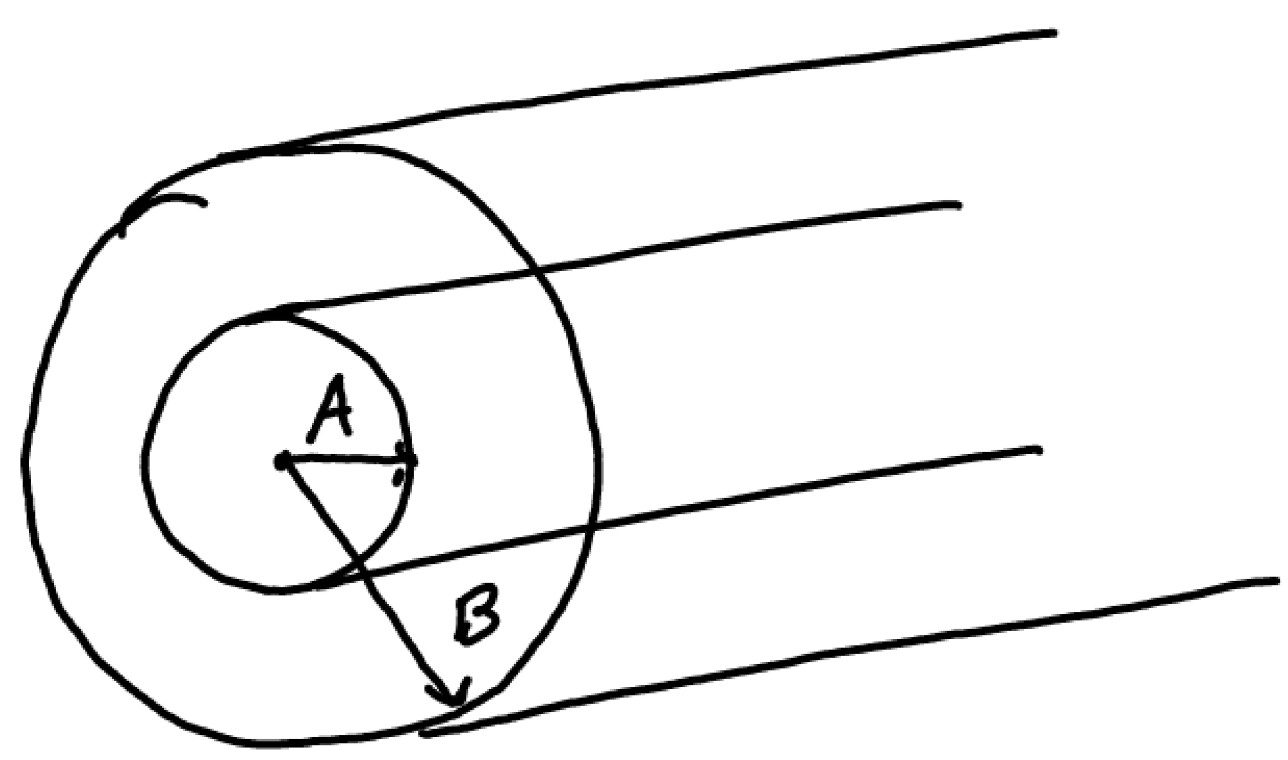
to simplify: $\frac{1}{A} - \frac{1}{B} = \frac{B}{AB} - \frac{A}{AB}$

$$= \frac{B-A}{AB}$$

$$\boxed{C = 4\pi\varepsilon_0\left(\frac{AB}{B-A}\right)}$$

Problem 5.3:

$$C = 2\pi \varepsilon_0 \, l \, \ln\left(\tfrac{A}{B}\right)$$



$$\frac{C}{l} = 20E-12 \ \frac{F}{m}$$

mili -3
micro -6
nano -9
pico -12

$$\frac{C}{l} = 2\pi \varepsilon_0 \ln\left(\tfrac{A}{B}\right)$$

ratio

$$\frac{A}{B} = \exp\left(\frac{C}{2\pi\varepsilon_0 l}\right) = \exp\left(\frac{20E-12}{2\pi\,(8.85E-12)}\right) = \boxed{1.43}$$

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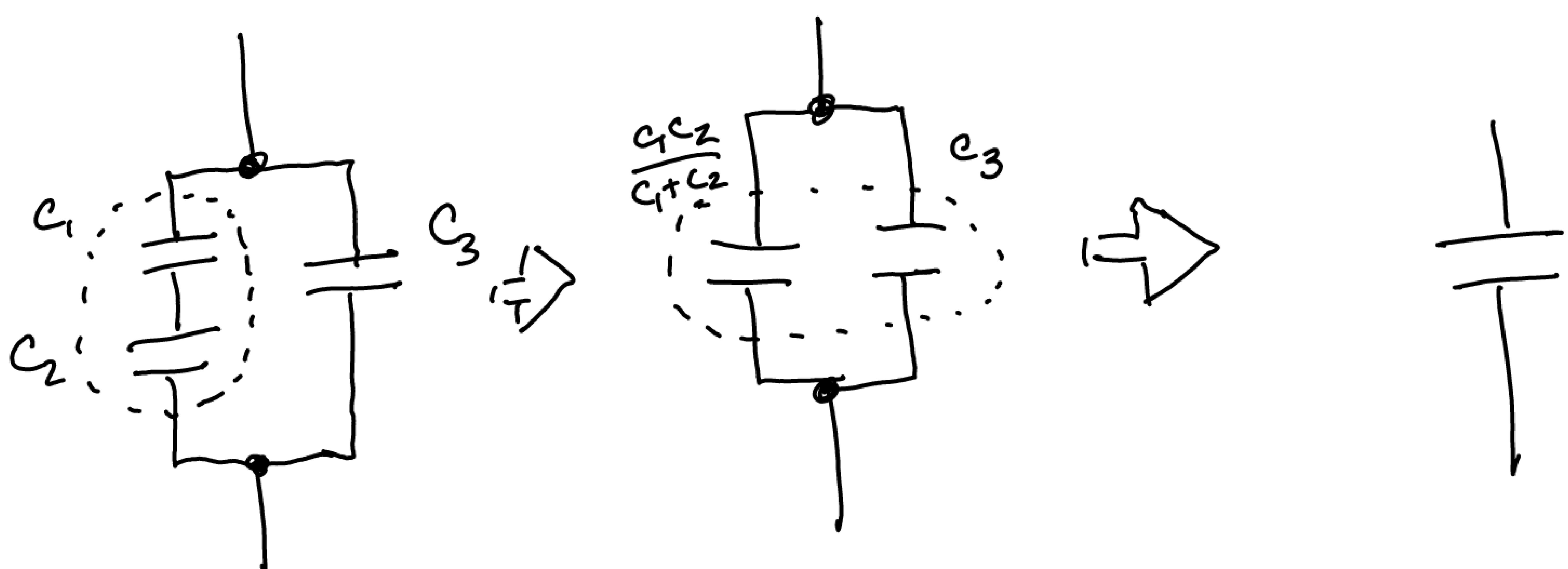
problem 5.4

"series" then "parallel" reduction



series

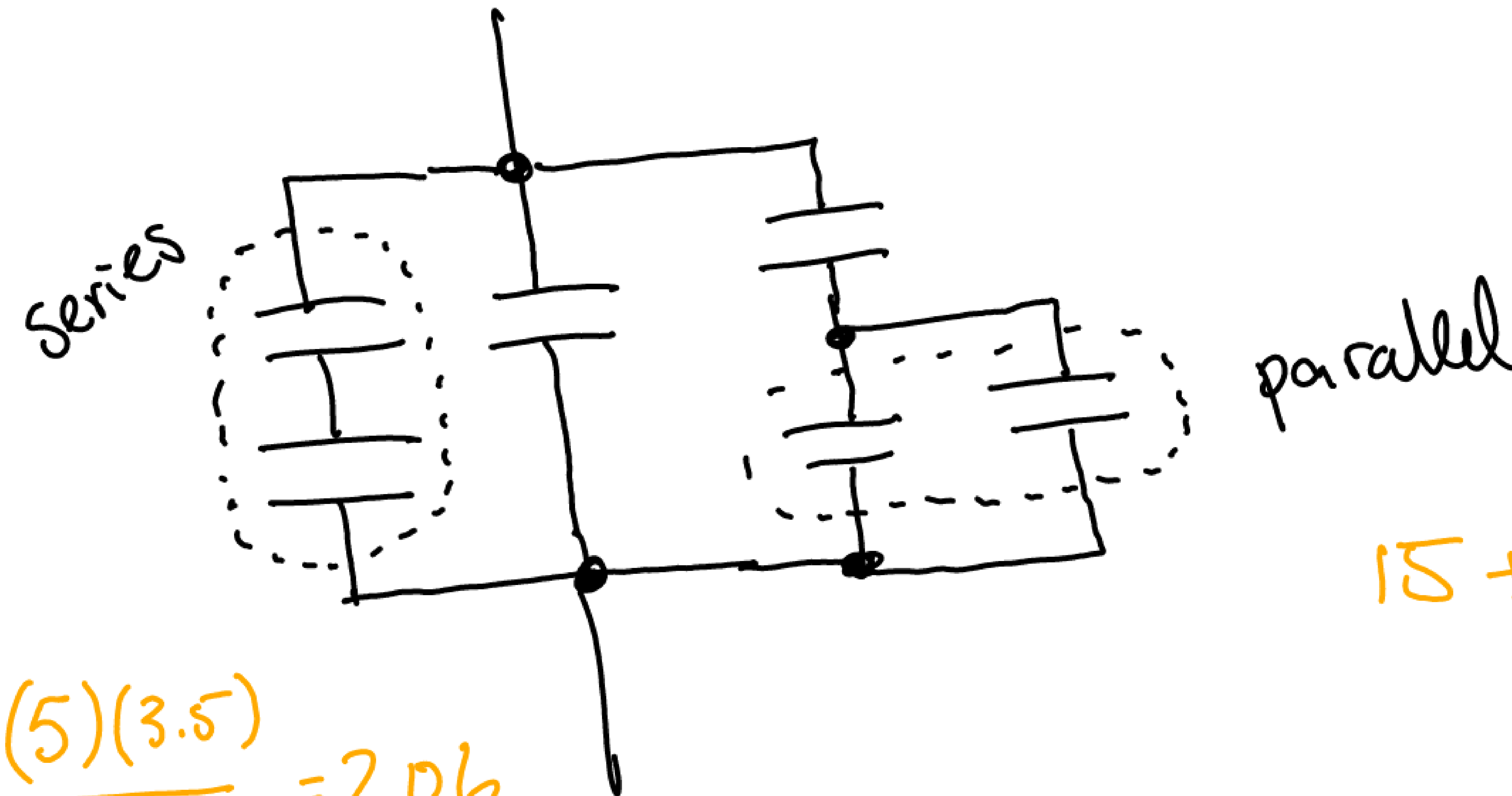
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2}$$

parallel

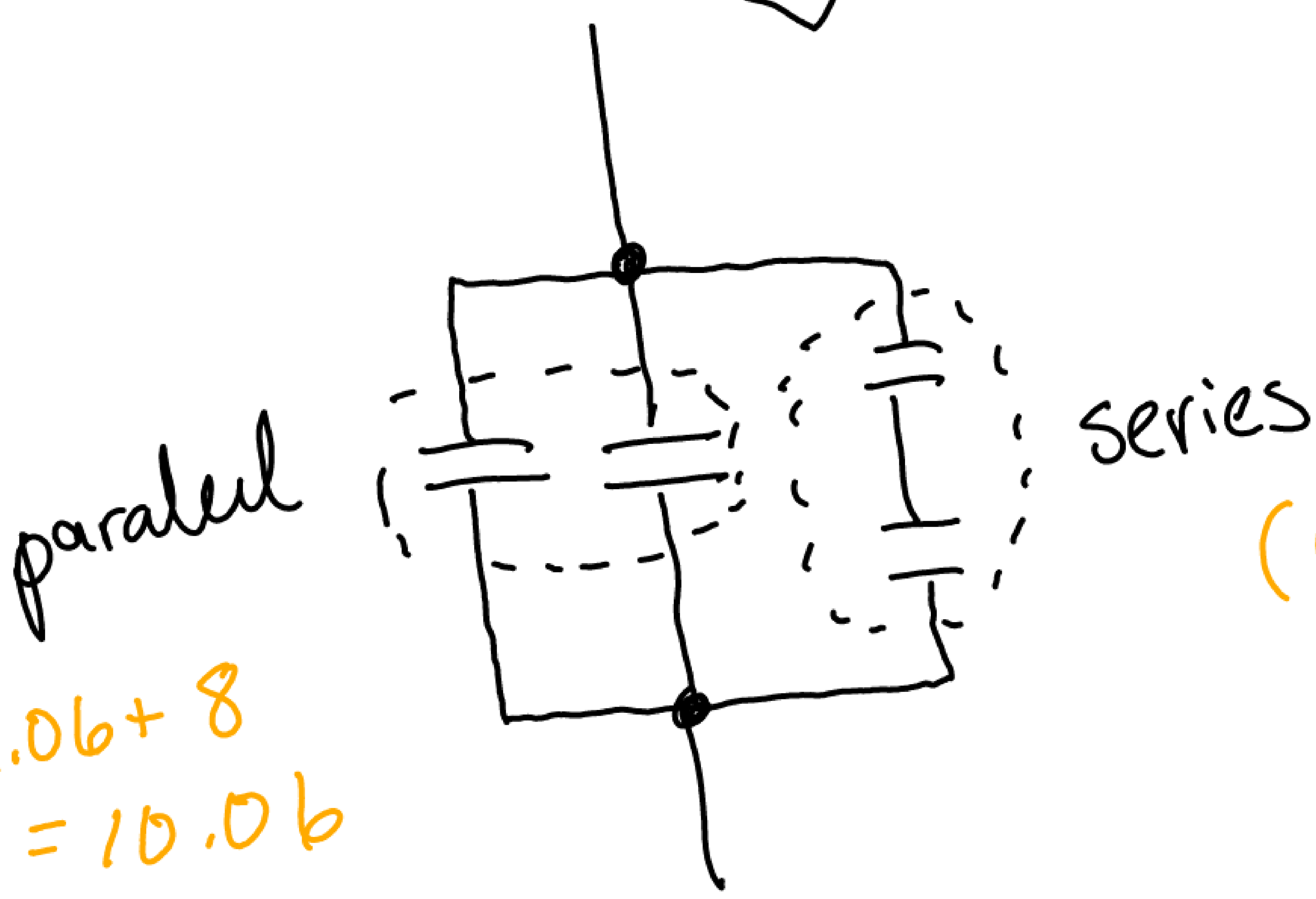
$$C_{eq} = \frac{C_1C_2}{C_1+C_2} + C_3 = \boxed{2.79\ \mu F}$$

problem 5.5



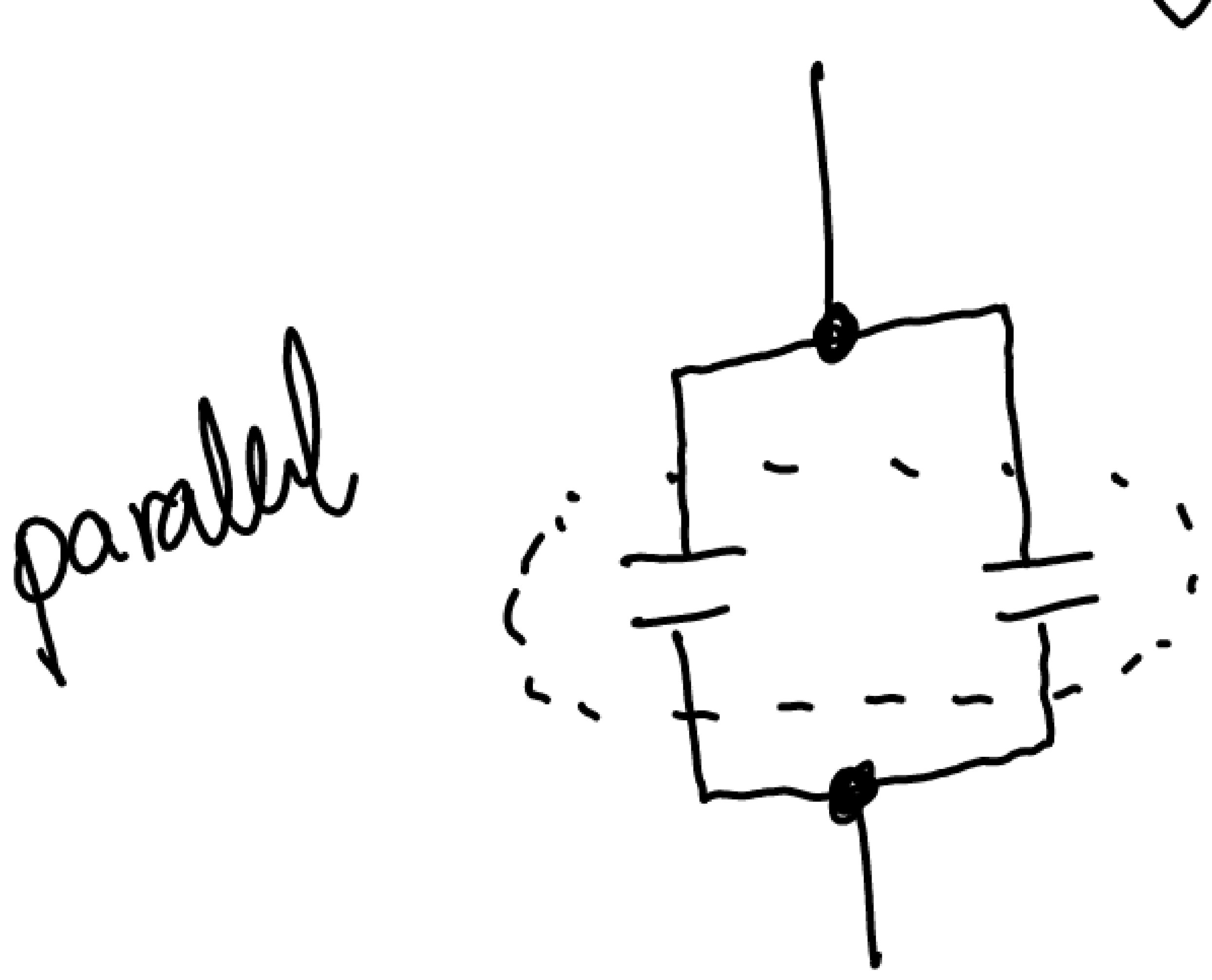
$$\frac{(5)(3.5)}{5+3.5} = 2.06$$

$15 + 0.75 = 15.75$

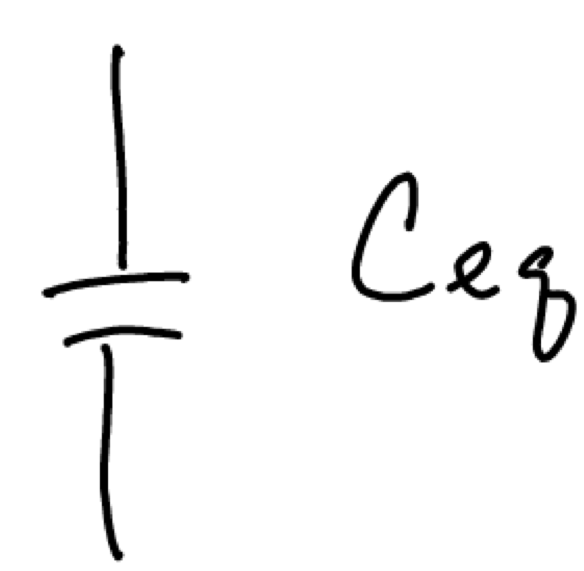


$2.06 + 8$
$= 10.06$

$$\frac{(1.5)(15.75)}{1.5+15.75} = 1.37$$



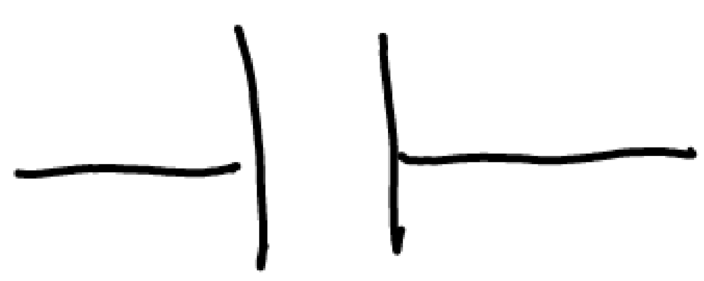
$10.06 + 1.37$



$\boxed{C_{eq} = 11.43\,\mu F}$

problem 5.6:

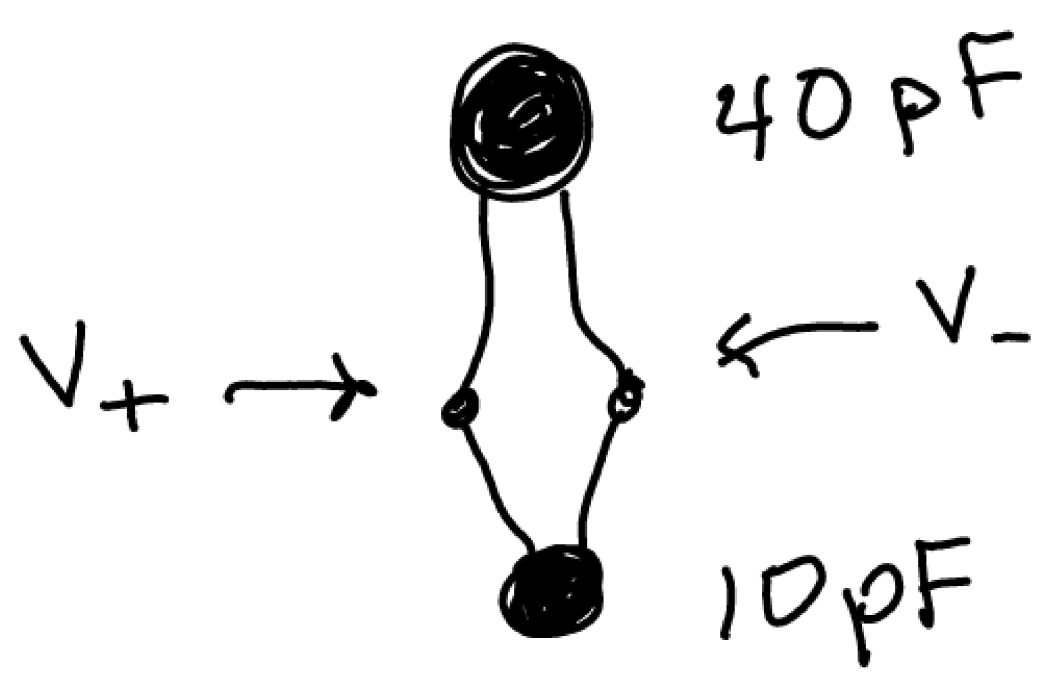
First charge the 40 pF cap at 500 Volts



$$Q = C\Delta V = \boxed{20\,nC}$$

then connect another cap: Two things remain true:

- Charge is shared between both caps
  ↳ the electrons (or "Lack of") can move along the wire, but charge cannot be destroyed.
- Voltage must be equal on each side
  ↳ the voltage drop on a wire is zero if as long as resistance is negligable $R=0$



$$Q = Q_1 + Q_2 = 20\,nC$$

$$\Delta V_1 = \Delta V_2$$

$$Q = Q_1 + Q_2 = C_1\Delta V_1 + C_2\Delta V_2 = (C_1+C_2)\Delta V$$

$$\Delta V = \frac{Q}{C_1+C_2} = \frac{20\,nC}{40pF + 10pF} = \boxed{400V}$$

problem 5.7

$$C = \frac{\varepsilon_0 A}{\Delta x}$$ parallel plate cap

$$C = \frac{(8.85E-12)(0.08)}{0.001} = \boxed{647nF}$$

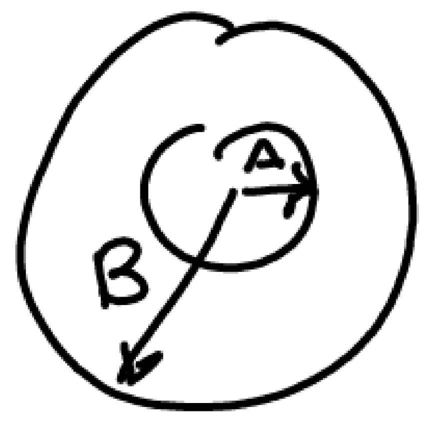
if filled w/ Dielectric:

$$\varepsilon = 6\varepsilon_0$$

$$\boxed{C = 3.88\mu F}$$

problem 5.8:

spherical capacitor



$$C = 4\pi\varepsilon_0 \left[\frac{1}{\frac{1}{A} - \frac{1}{B}}\right]$$

$A = 6.378 \times 10^6\,m$

$B = A + 70\,km$

$B = 6.448 \times 10^6\,m$

$$C = 4\pi\varepsilon_0 \frac{AB}{B-A}$$

$$\boxed{C = 65.3\,mF}$$

$$Q = C\Delta V = (65.3E-3)(3.5E5)$$

$= 22.9$ Kilo Coulomb

$$\boxed{Q = 2.29 \times 10^4 C}$$

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} C \Delta V^2$$

$$\boxed{U_E = 4 \times 10^9 \text{ Joules.}}$$