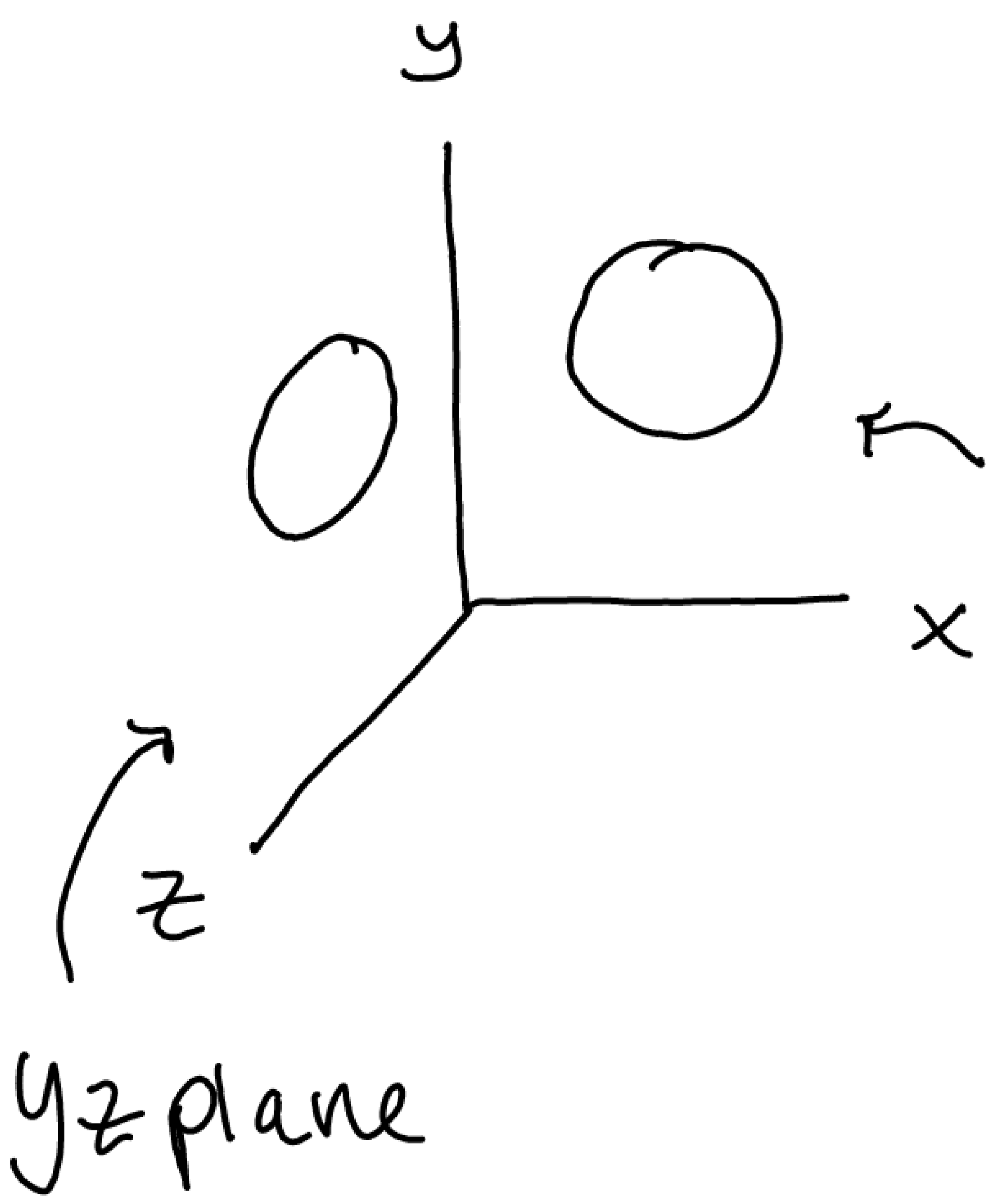
problem 3.1

$\vec{E} = (0, 4000, 3000)\ N/C$

electric field is diagonal "up and out of page"



the normal to surface is

a) $\hat{n} = (0, 0, 1)$

$\Phi_E = \int \vec{E} \cdot \hat{n}\, dA =$

Area $= \pi r^2 = 4\pi$

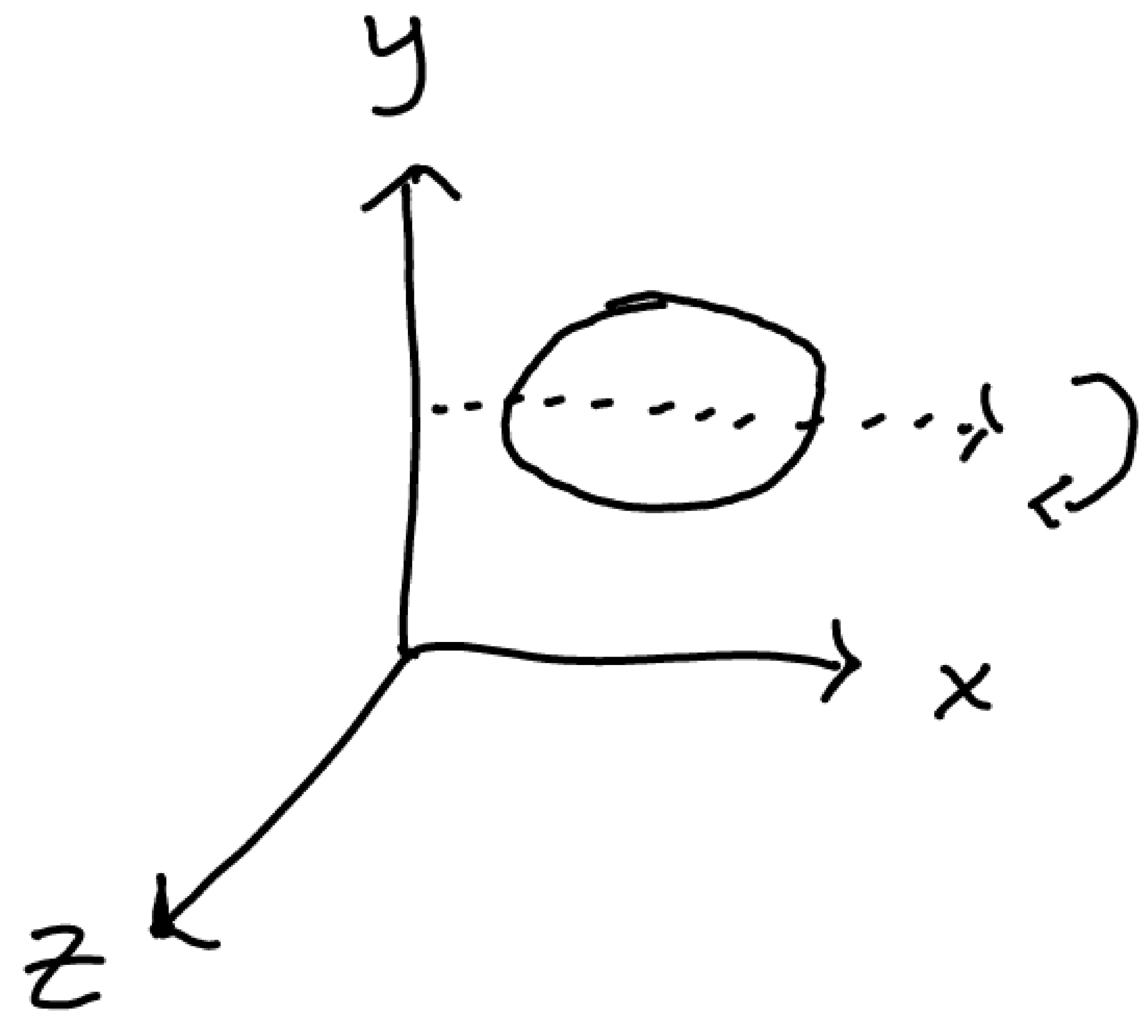
$= \int (0, 4000, 3000) \cdot (0, 0, 1)\, dA$

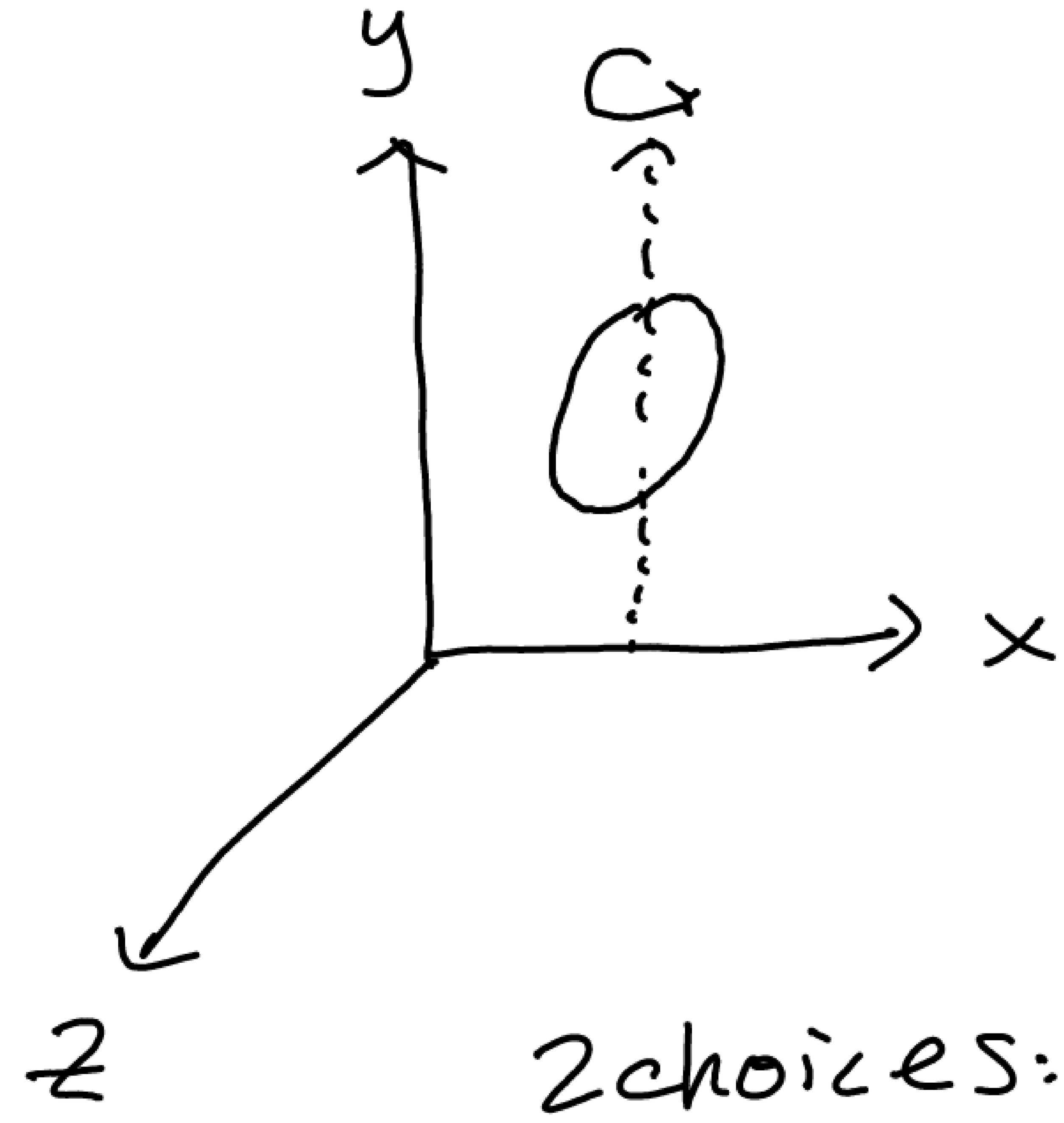
$$\boxed{= 3000\,(4\pi)\ Nm^2/C}$$

b) $\hat{n} = (1, 0, 0)$

$\Phi_E = \int (0, 4000, 3000) \cdot (1, 0, 0)\, dA \boxed{= 0}$

the field lines pass right by the surface part c   this part is ambiguous





2 choices:

rotate x-axis

$$\hat{n} = \frac{(0,1,1)}{\sqrt{0^2+1^2+1^2}}$$

$$= \frac{1}{\sqrt{2}}(0,1,1)$$

$$\underline{\underline{\Phi_E = \frac{4\pi}{\sqrt{2}}(4000+3000)}}$$

$$\hat{n} = \frac{1}{\sqrt{2}}(0,-1,1)$$

$$\underline{\underline{\Phi_E = \frac{4\pi}{\sqrt{2}}(-4000+3000)}}$$

2 choices:

rotate y-axis

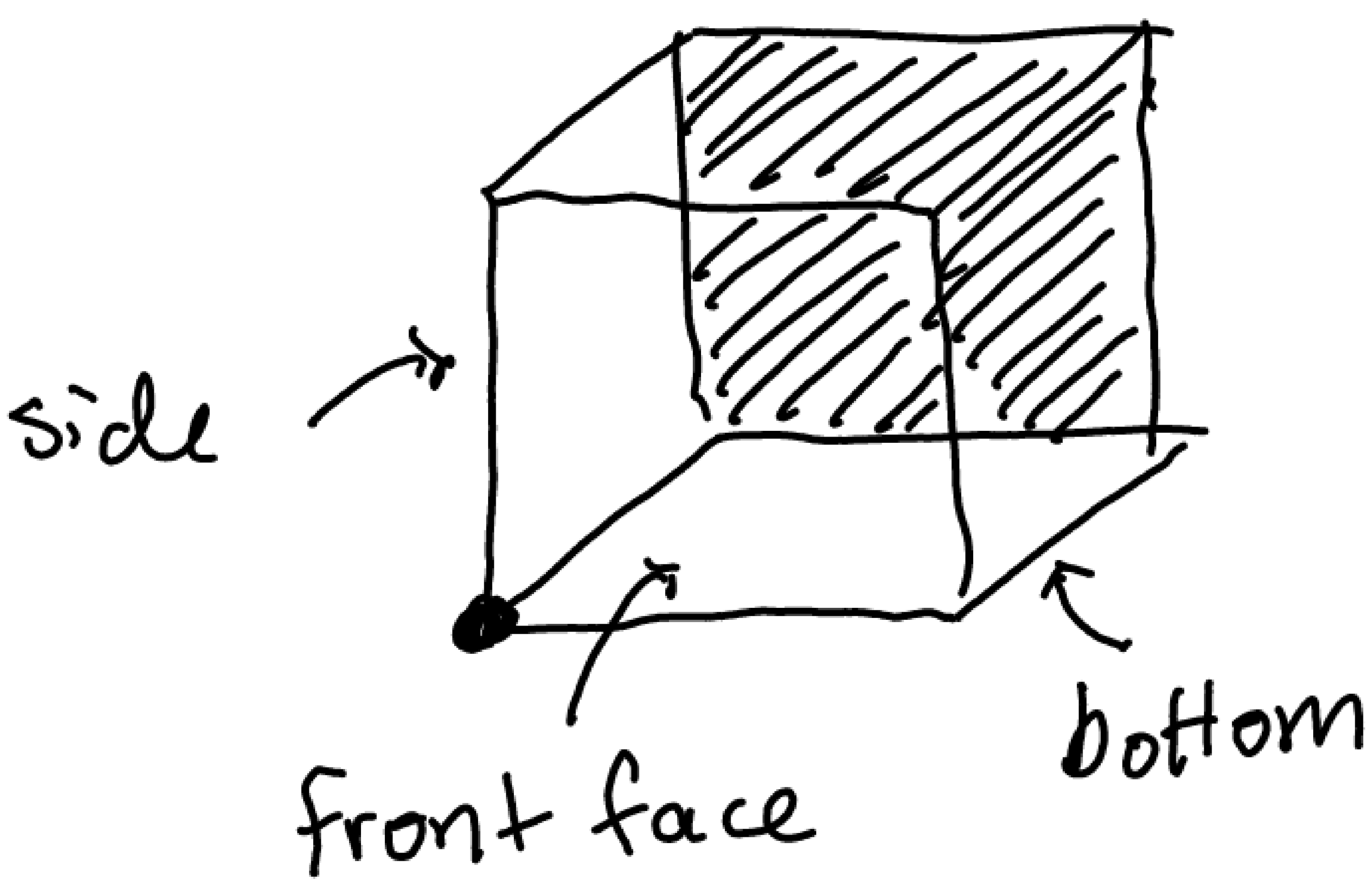
$$\hat{n} = \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}}$$

$$= \frac{1}{\sqrt{2}}(1,0,1)$$

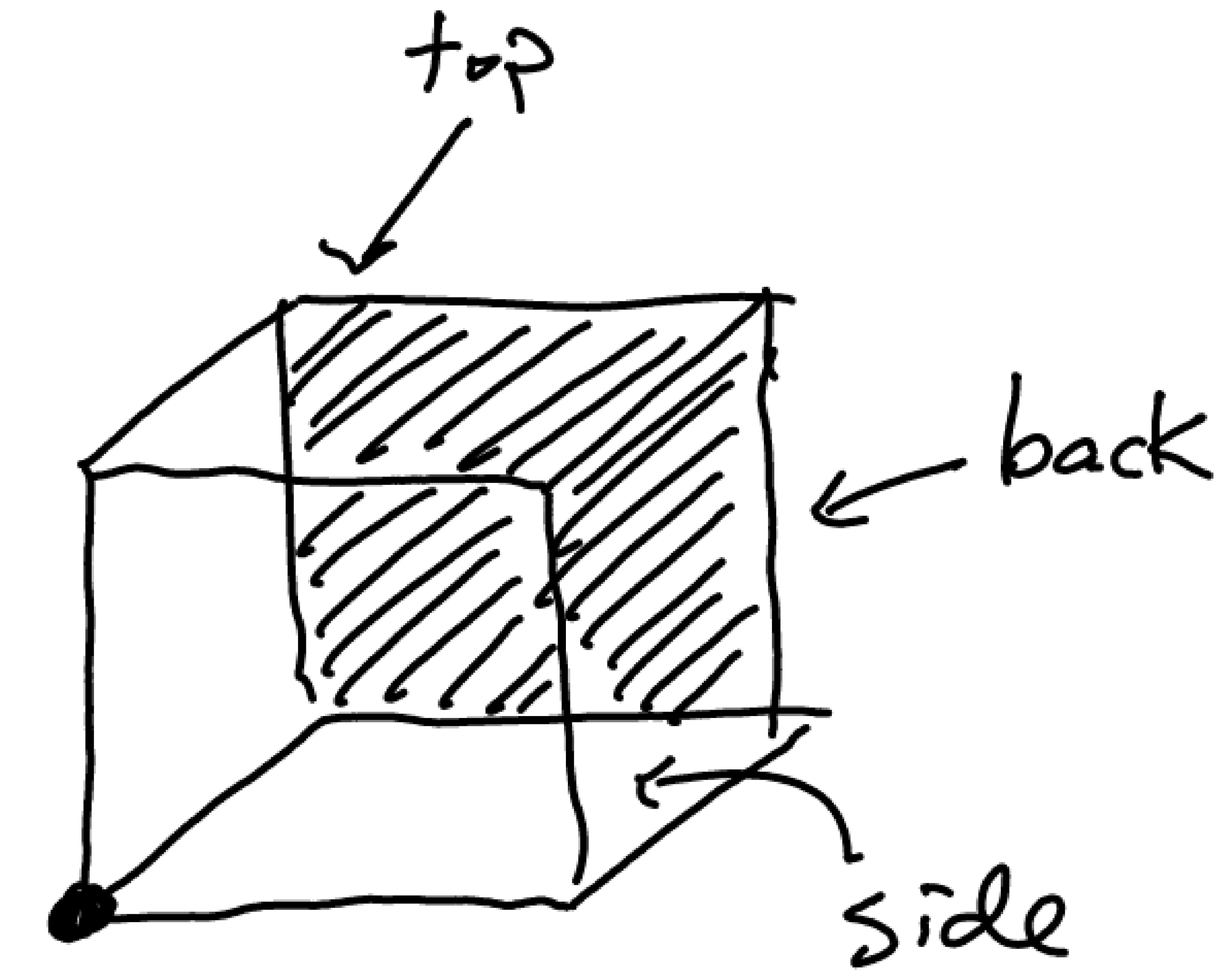
$$\underline{\underline{\Phi_E = \frac{4\pi}{\sqrt{2}}(3000)}}$$

$$\hat{n} = \frac{1}{\sqrt{2}}(-1,0,1)$$

$$\underline{\underline{\Phi_E = \frac{4\pi}{\sqrt{2}}(3000)}}$$



All 3 faces above contribute <u>zero flux</u>



All 3 faces do have flux passing through. (equal amounts, $\frac{1}{3}$ flowing thru each)

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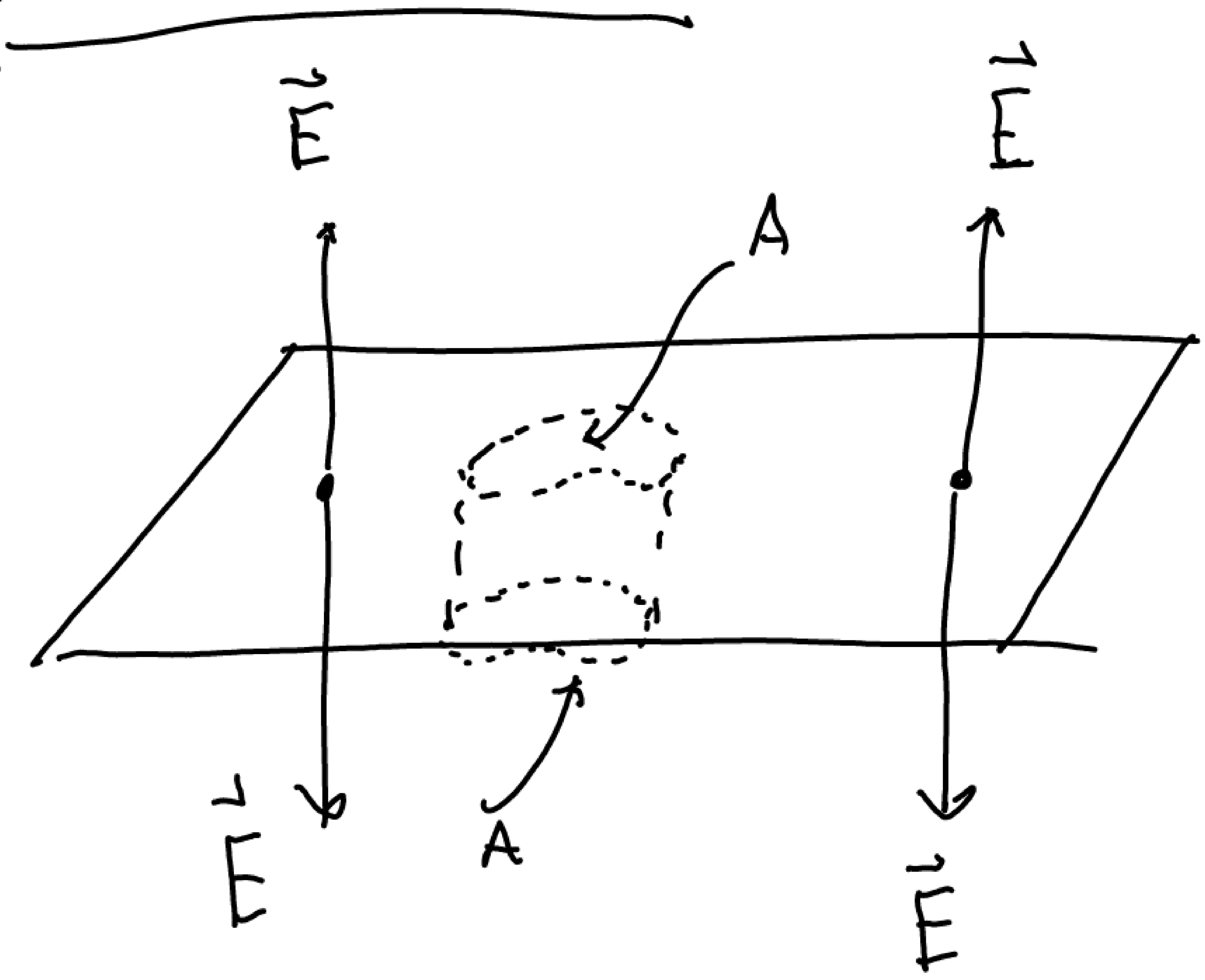
this cube is one of 8 required to enclose the charge

$$\Phi_E = \oint_{\substack{\text{closed}\\\text{surface}}} \vec{E} \cdot \hat{n}\, dA = \frac{q_{in}}{\varepsilon_0}$$

the flux is $\frac{1}{8} \cdot \frac{1}{3}$ of the total

$$\boxed{\Phi_E = \frac{1}{24} \frac{q_{en}}{\varepsilon_0}}$$

problem 3.3:



Gauss' law for a plane

$$\Phi_E = \oiint E \cdot \hat{n}\, dA = E \oiint \hat{n}\, dA$$

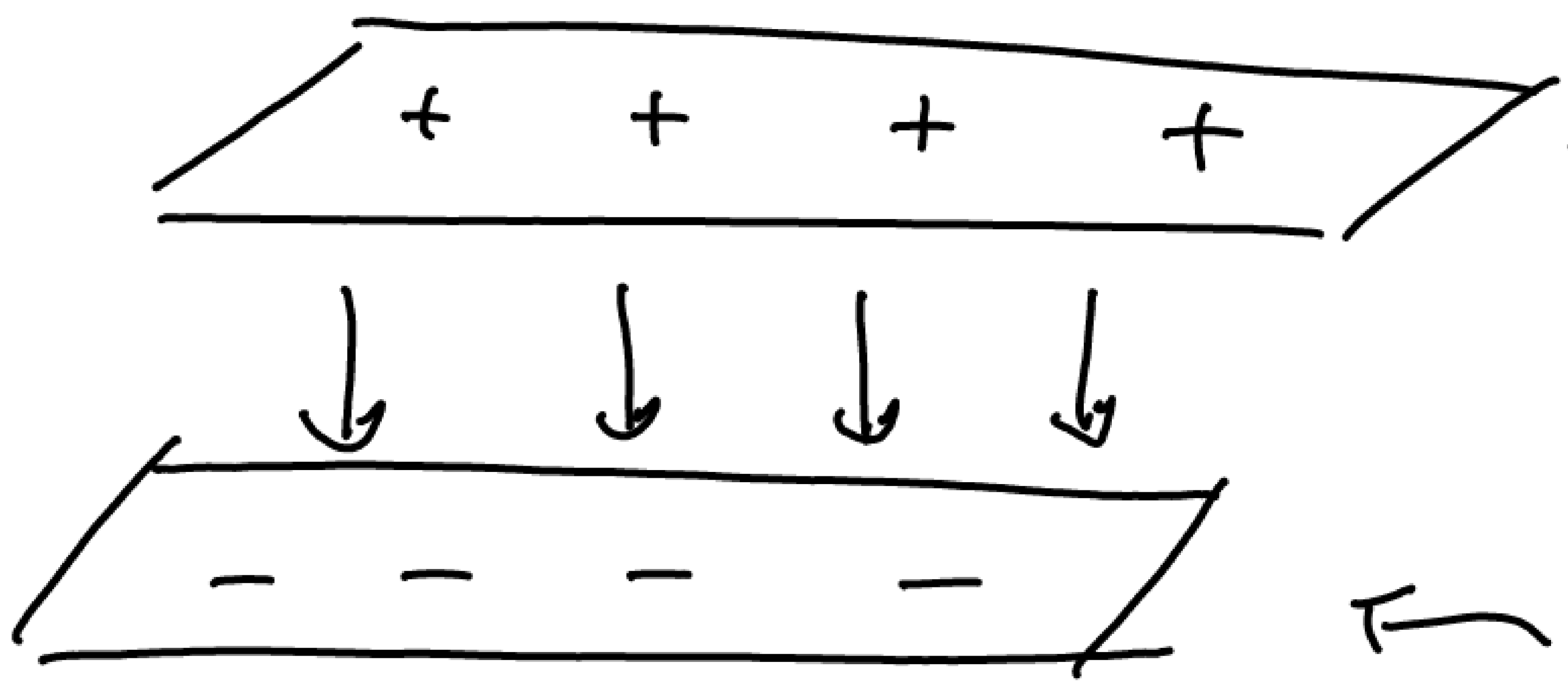
$$= E(2A) = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$ Away from ⊕ charged plane

$$E = \frac{10E-6}{2(8.85E-12)} = \boxed{5.65 \times 10^5\ N/C \text{ away from plane}}$$

problem 3.4

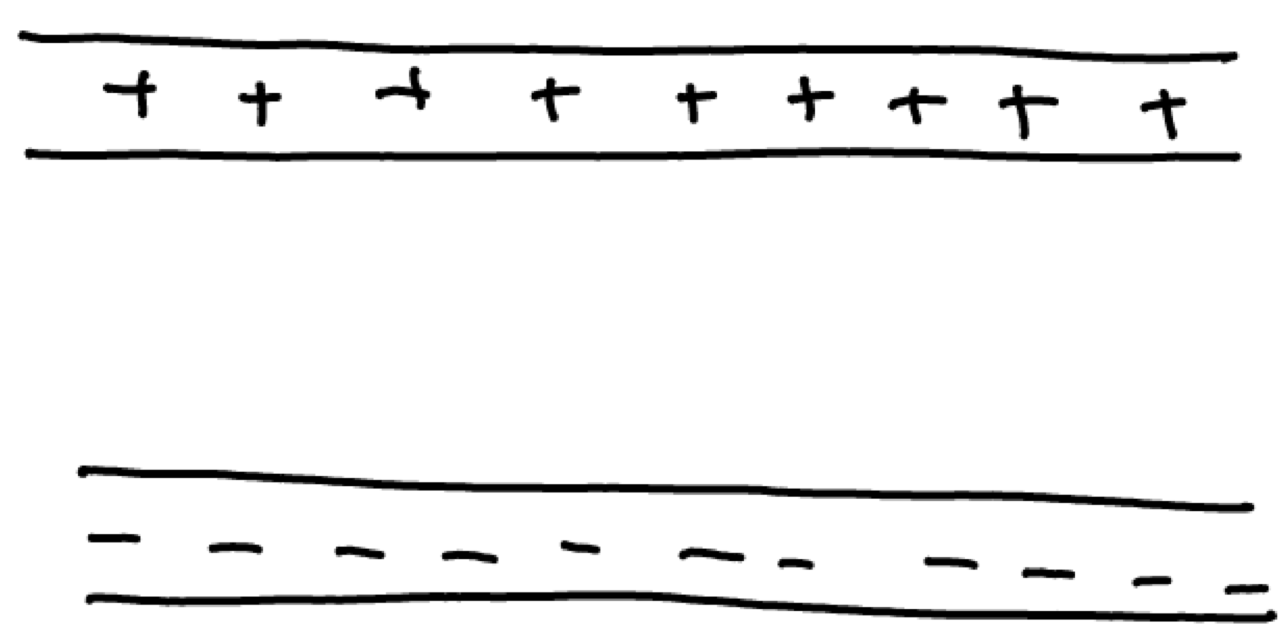
area $A = (0.1)^2 = 0.01\, m^2$



$q = +5nC$

$Q = -5nC$

use superposition to find E-field:

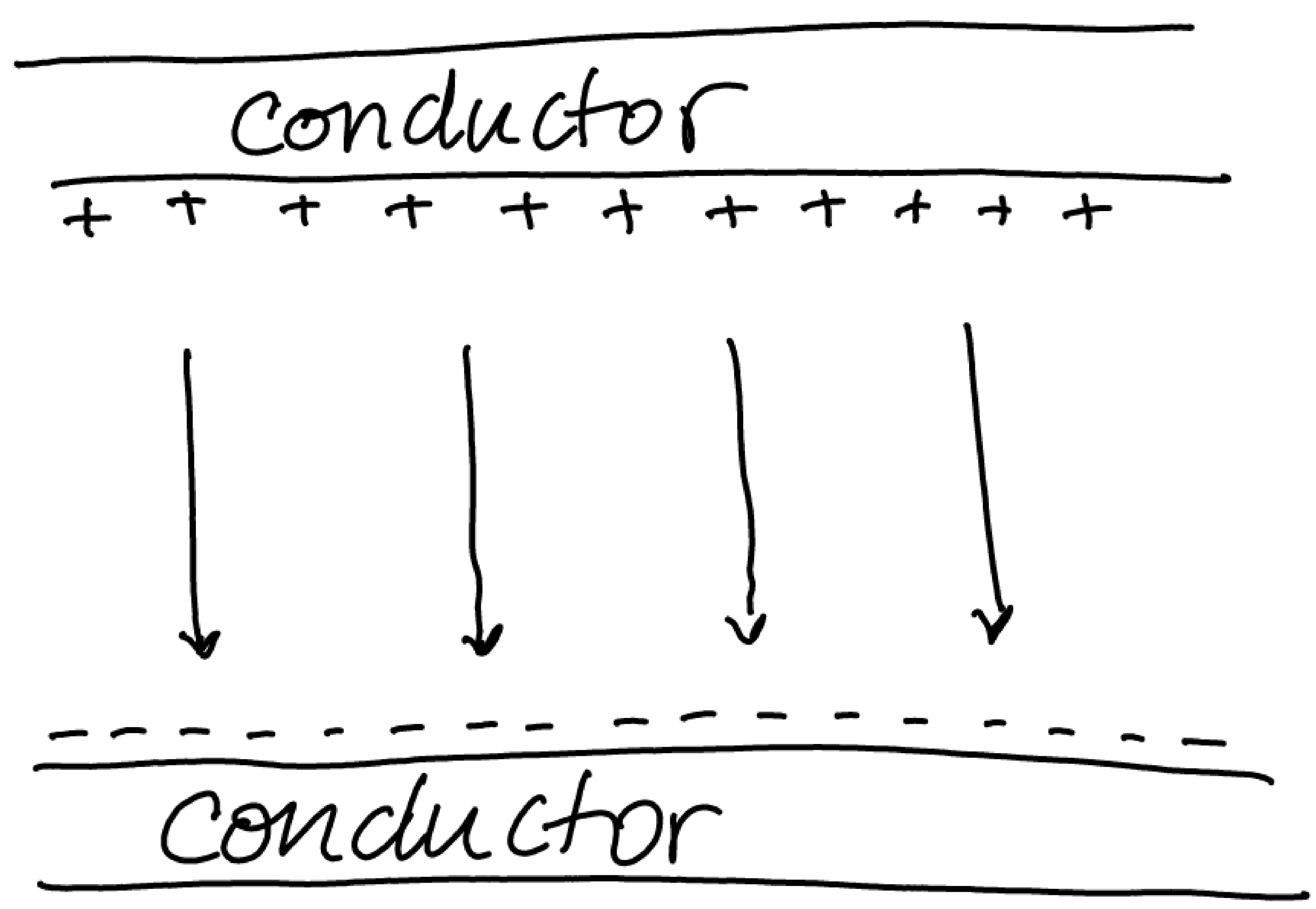


+	−
$\uparrow \frac{\sigma}{2\varepsilon_0}$	$\downarrow \frac{\sigma}{2\varepsilon_0}$
$\downarrow \frac{\sigma}{2\varepsilon_0}$	$\downarrow \frac{\sigma}{2\varepsilon_0}$
$\downarrow \frac{\sigma}{2\varepsilon_0}$	$\uparrow \sigma/2\varepsilon_0$

Add these fields:

- above $\boxed{E = 0}$
- between $E = -\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{-\frac{\sigma}{\varepsilon_0}}$
- below $\boxed{E = 0}$

problem 3.5 :

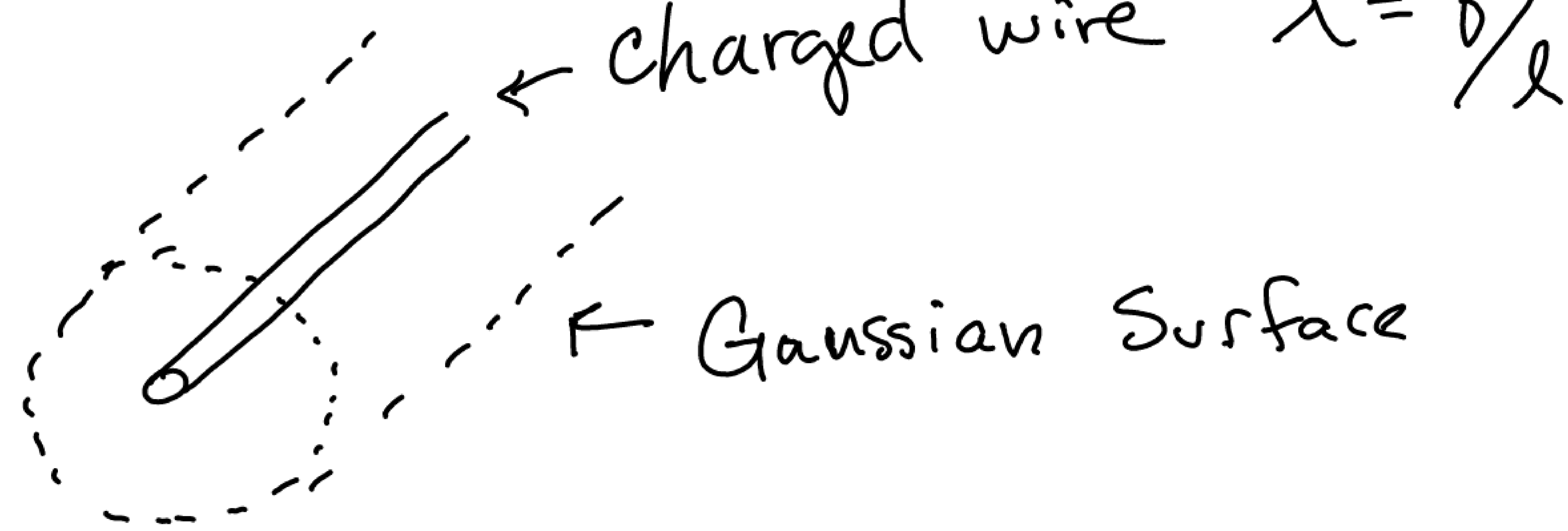


$$E = \frac{\sigma}{\varepsilon_0}$$

Inside of the conductor, the E-field must be zero.

This means all charge must be placed on the inside of the plates.

problem 3.6



$$\underset{\text{closed surface}}{\oiint} \vec{E} \cdot \hat{n}\, dA = \frac{q_{in}}{\varepsilon_0}$$

E is constant over the surface

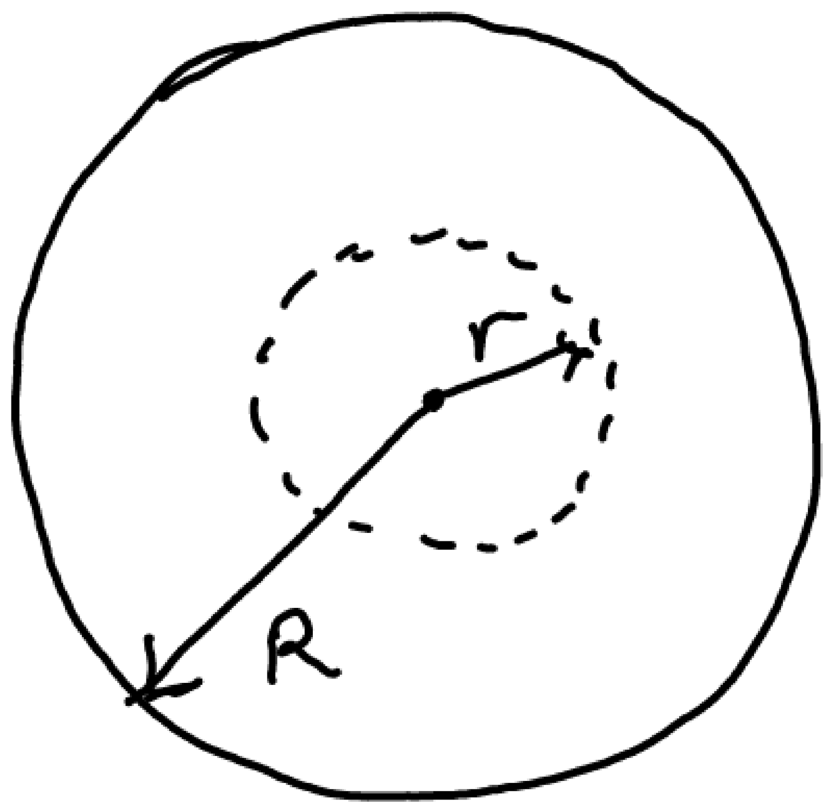
$$E \oiint \hat{n}\, dA = E(2\pi r l) = \frac{q_{en}}{\varepsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

a) $\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}}$ radially outward
$r > 3cm$

b) inside the conductor $\boxed{\vec{E} = 0}$ $r < 3cm$

problem 3.7:



charge distributed uniformly

$$\rho = q/V$$

$$\rho_{in} = \frac{q_{in}}{V_{in}}$$

$$\rho_{in} V_{in} = q_{in}$$

$$\rho(\pi r^2 l) = q_{in}$$

$$\frac{\rho(\cancel{\pi} r^{\cancel{2}} \cancel{l})}{\varepsilon_0} = E(2\cancel{\pi}\cancel{r}\cancel{l})$$

$$\boxed{\frac{\rho r}{2\varepsilon_0} = E} \quad r<R$$

$$\oiint E \cdot \hat{n}\, dA = \frac{q_{in}}{\varepsilon_0}$$

$$\rho = \frac{q_{in}}{V_{in}}$$

$$\rho = \frac{q}{\pi R^2 l}$$

$$q = \pi R^2 l \rho$$

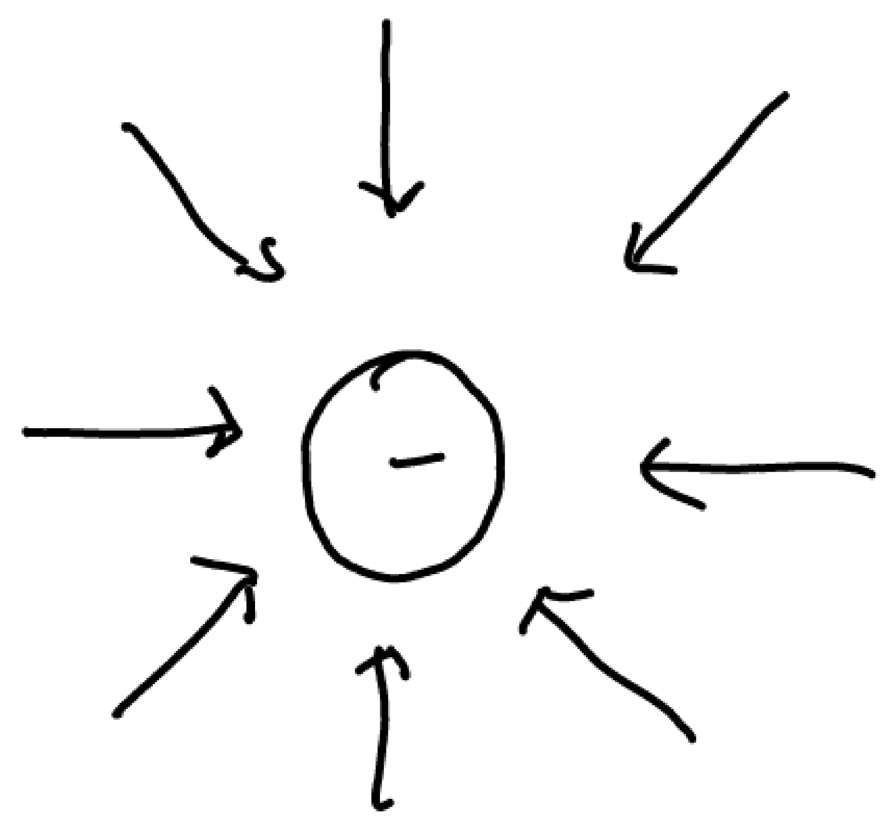
$$E(2\cancel{\pi} r \cancel{l}) = \frac{\cancel{\pi} R^2 \cancel{l} \rho}{\varepsilon_0}$$

$$\boxed{E = \frac{R^2 \rho}{2\varepsilon_0 r}} \quad r>R$$

$E \propto \frac{1}{r}$ just like a line of charge.

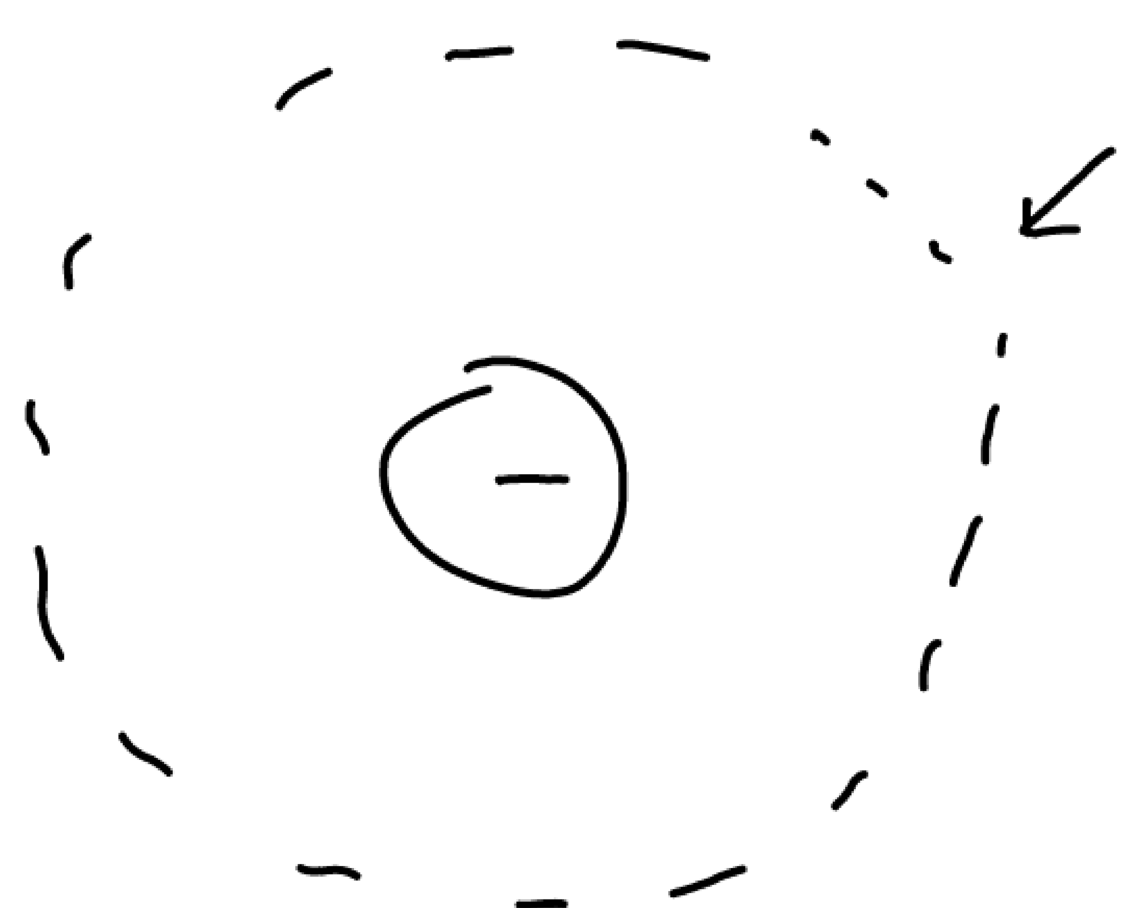
problem 3.8:

E-Field toward center of conductor



Must be a negative charge

spherical Gaussian Surface



$$\oiint_{\text{closed surface}} \vec{E} \cdot \hat{n}\, dA = \frac{q_{enc}}{\varepsilon_0}$$

Symmetry Argument:

$\vec{E}$ is entirely radial and constant over a spherical surface centered on the charge.

$$E\left(\oiint d\vec{A}\right) = \frac{q_{in}}{\varepsilon_0}$$

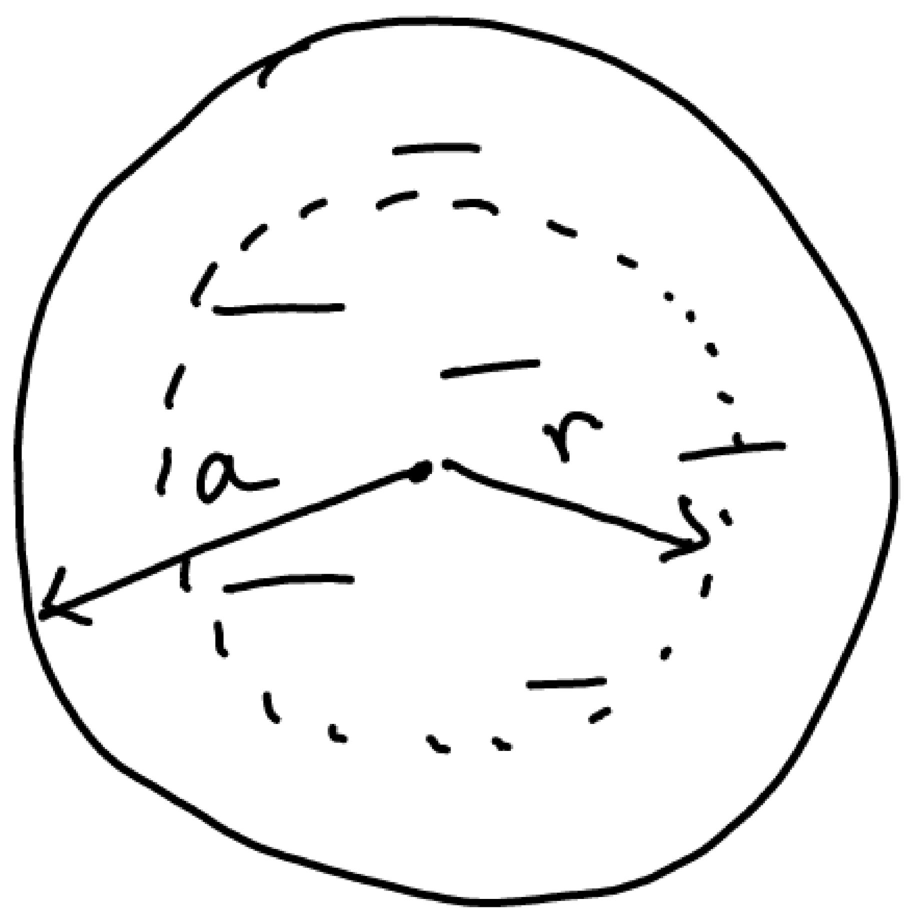
$$E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$

$$q = \varepsilon_0 E(4\pi r^2)$$

$$\boxed{= -0.111\, nC}$$

problem 3.9

$q = -30\,\mu C$ "uniformly" $\Rightarrow$ insulator



E field outside: $r > a$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{r}}$$

E-field Inside:

$$\rho_{in} = \frac{q_{in}}{V_{in}} \Rightarrow \rho = \frac{q_{in}}{\frac{4}{3}\pi r^3}$$

E uniform and radial

$$E\left(\oint\!\!\oint dA\right) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$$

$$E\left(4\pi r^2\right) =$$

inside

$r < a$

$$E = \rho\,\frac{\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0\left(4\pi r^2\right)}$$

$$\boxed{\vec{E} = \frac{\rho r}{3\varepsilon_0}\,\hat{r}}$$

Problem 3.10:

Given the density explicitly

$$\rho = \alpha r^2$$

integrate the density over the volume

$$q_{in} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{r'} dr\, \rho(r)\, r^2 \sin\theta$$

where $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$ in spherical

$$\int_0^{2\pi} d\phi \qquad \int_0^{\pi} \sin\theta\, d\theta$$

$$q_{in} = (2\pi - 0)(-(-1)+(1)) \int_0^{r'} \alpha r^4\, dr$$

$$= 4\pi \left(\frac{1}{5} \alpha r^5 \right)$$

Gauss Law:

$$E \oint dA = \frac{4\pi\alpha}{5\varepsilon_0} r^5$$

r<R:

$$E(\cancel{4\pi}\cancel{r^2}) = \frac{\cancel{4\pi}\alpha}{5\varepsilon_0} r^{\cancel{5}3}$$

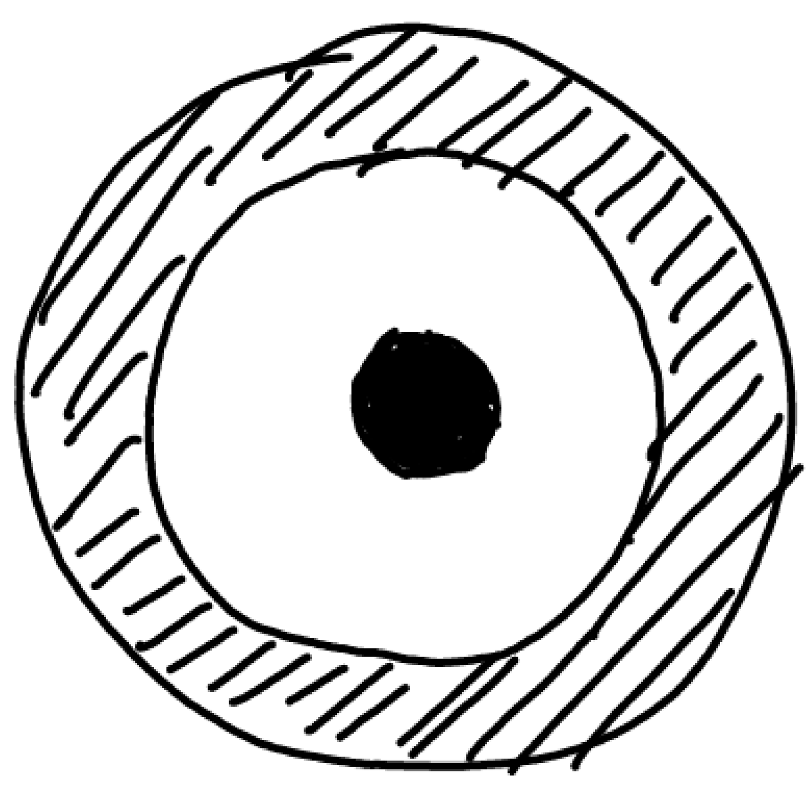
$$\boxed{E = \alpha r^3 / 5\varepsilon_0 \; \hat{r}}$$

r>R:

$$E(\cancel{4\pi} r^2) = \frac{\cancel{4\pi}\alpha R^5}{5\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\alpha R^5}{5\varepsilon_0} \frac{1}{r^2} \hat{r}}$$

problem 3.11:



Inner sphere: $r = 0.04m$
charged $q = 5\mu C$

Outer sherical shell $r_{in} = 0.06m$
$r_{out} = 0.08m$
$q = -8\mu C$

4 total regions:

$r < 0.04m$ $\boxed{E = 0}$ (inside conductor)

$0.04 < r < 0.06m$ $\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{(5E-6)}{r^2} \hat{r}}$

$0.06 < r < 0.08m$ $\boxed{E = 0}$ (inside conductor)

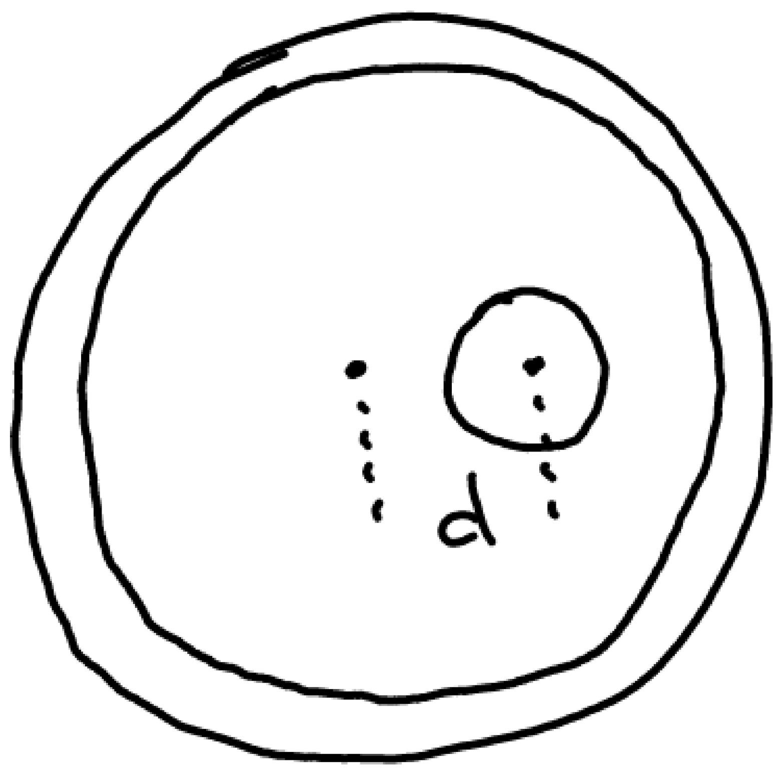
$r > 0.08m$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{(5E-6) - (8E-6)}{r^2} \hat{r}$$

$$\boxed{\vec{E} = \frac{-3E-6}{4\pi\varepsilon_0 r^2} \hat{r}}$$

problem 3.11.2:

Find E-field everywhere when " " the centers are separated by distance d



Inside the conductors:

$$\boxed{E = 0}$$

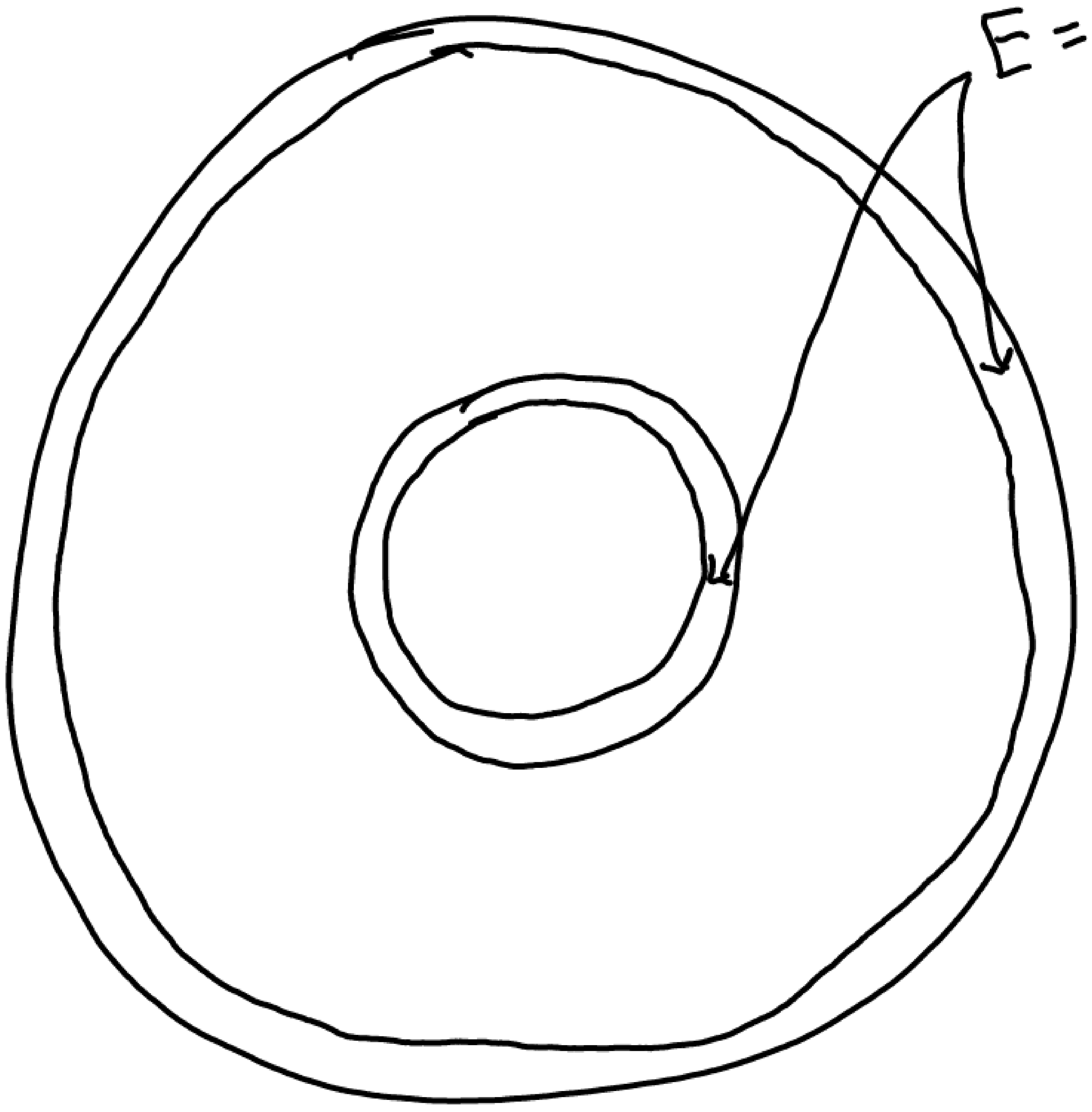
outside of the outer shell

$$\boxed{E = k \frac{q_1 + q_2}{|\vec{r}|^2} \hat{r}}$$

This is the same as part 1. Now comes the hard part: $(+q_1)$ charge on the inner Shell will draw $(-q_1)$ charge to the inside of the outer shell, but the charges will not be uniform. More charge will gather close to the inner sphere.

- Full solution requires expansion of Spherical harmonics $\Psi_{ylm}(r, \theta, \phi)$ as seen in electron orbitals (HARD MATH!)

problem 3.12:



$r < R_1$ $\boxed{E = 0}$

$R_1 < r < R_2$ $\boxed{\vec{E} = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{r}}$

$r > R_2$ $\boxed{\vec{E} = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2}\hat{r}}$

Charge on:

Inner surface of innershell:

$q = 0$ $\boxed{\sigma = 0}$

outer surface of inner shell:

$q_{in} = q_1$ $\boxed{\sigma = \frac{q_1}{4\pi R_1^2}}$

Inner surface outer shell:

$q_{in} = 0$ so $q = -q_1$ $\boxed{\sigma = \frac{-q_1}{4\pi R_2^2}}$

outer surface outer shell

$q_{in} = q_1 + q_2$ $q = q_2 + q_1$ $\boxed{\sigma = \frac{q_2 + q_1}{4\pi R_2^2}}$