problem 1.1:

1 electron $= -1.6 \times 10^{-19}$ Coulomb

nano $\rightarrow 10^{-9}$

micro $\rightarrow 10^{-6}$

a) $-2nC = -2 \times 10^{-9} C$ then do unit conversion

$$-2E-9\cancel{C}\left(\frac{1 \text{ electron}}{-1.6E-19\cancel{C}}\right) = 1.25 E10 \text{ electrons}$$

$\boxed{= 12.5 \text{ billion electrons}}$

b) $0.5 \mu C = 0.5 E-6 C$

$$0.5E-6C\left(\frac{1 \text{ electron}}{-1.6E-19\cancel{C}}\right) = 3.13 E12$$

$\boxed{= 3.13 \text{ trillion electrons}}$

problem 1.2:

$$3.75 E21 \text{ electrons}\left(\frac{1.6E-19C}{1 \text{ electron}}\right) = \boxed{600 C}$$

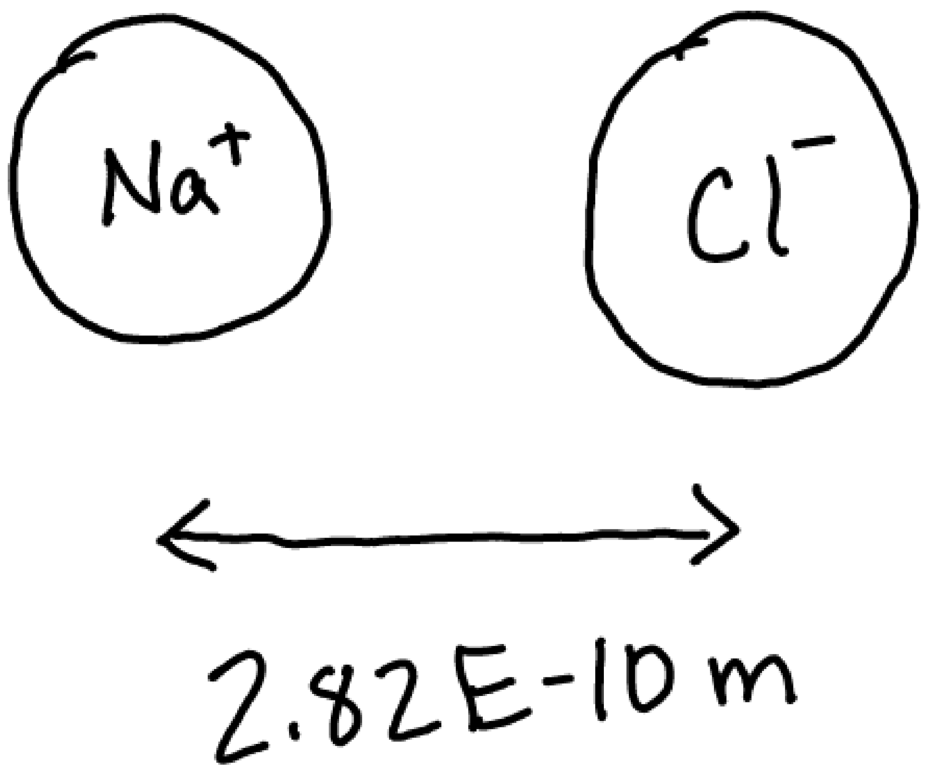
problem 1.3: pico $\rightarrow 10^{-12}$

$$0.300 pC = \frac{0.3E-12C}{1.6E-19} = \boxed{1.875 E6 \text{ electrons}}$$

$$\frac{\text{ionized atoms}}{\text{total atoms}} = \frac{1.875 E6}{1.0 E16} \approx \boxed{\text{1 in 10 billion are ionized}}$$

# Problem 1.4

Salt $\rightarrow Na^+Cl^-$




$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = 9E9\,\frac{(1.6E-19)^2}{(2.82E-10)^2}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9E9$$

$$= 2.89E-9\,N$$

$$\boxed{= 2.9 \text{ nano Newtons}}$$

# Problem 1.5

$$m_{electron} = 9.1E-31\,kg$$

$\vec{E}$ - Field $\quad \vec{F} = q\vec{E}$

$[N] = [C][N/C]$ ← units

$$\Sigma F = ma \Rightarrow ma = qE$$

$$a = \frac{qE}{m} = \frac{(1.6E-19)(2E5)}{(9.1E-31)}$$

$$\boxed{a = 3.5\times10^{16}\ m/s^2}$$

## Problem 1.6

a) Looks correct

e) Looks correct

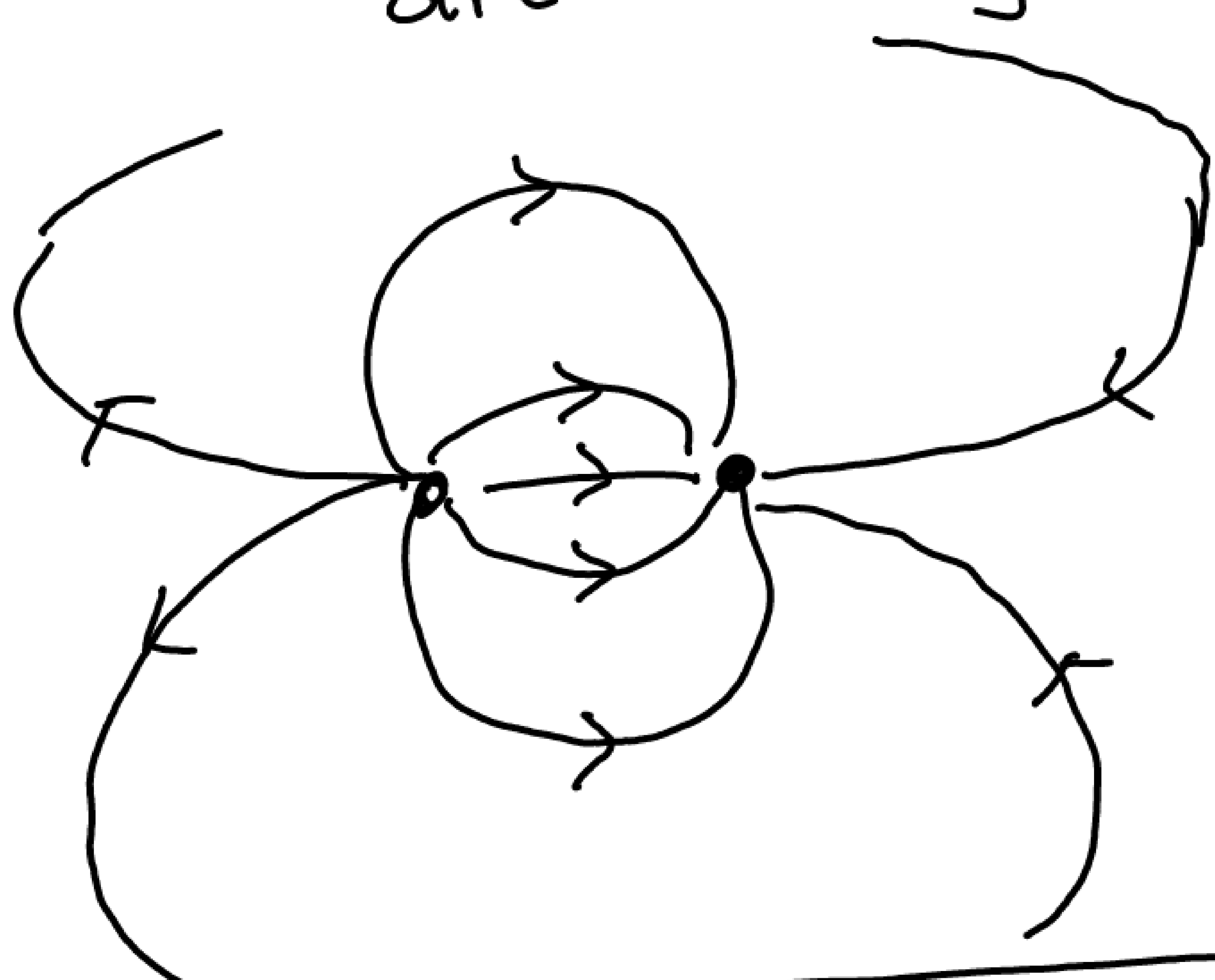
b) − is a "sink"
lines must go inwards

c) lines must emerge
symmetrically

g) Dipole!
the density of the lines
are wonky

d) + is a "source"
lines must go outwards

f) lines are crossing!
Never cross the
<u>streams</u>



## Problem 1.7

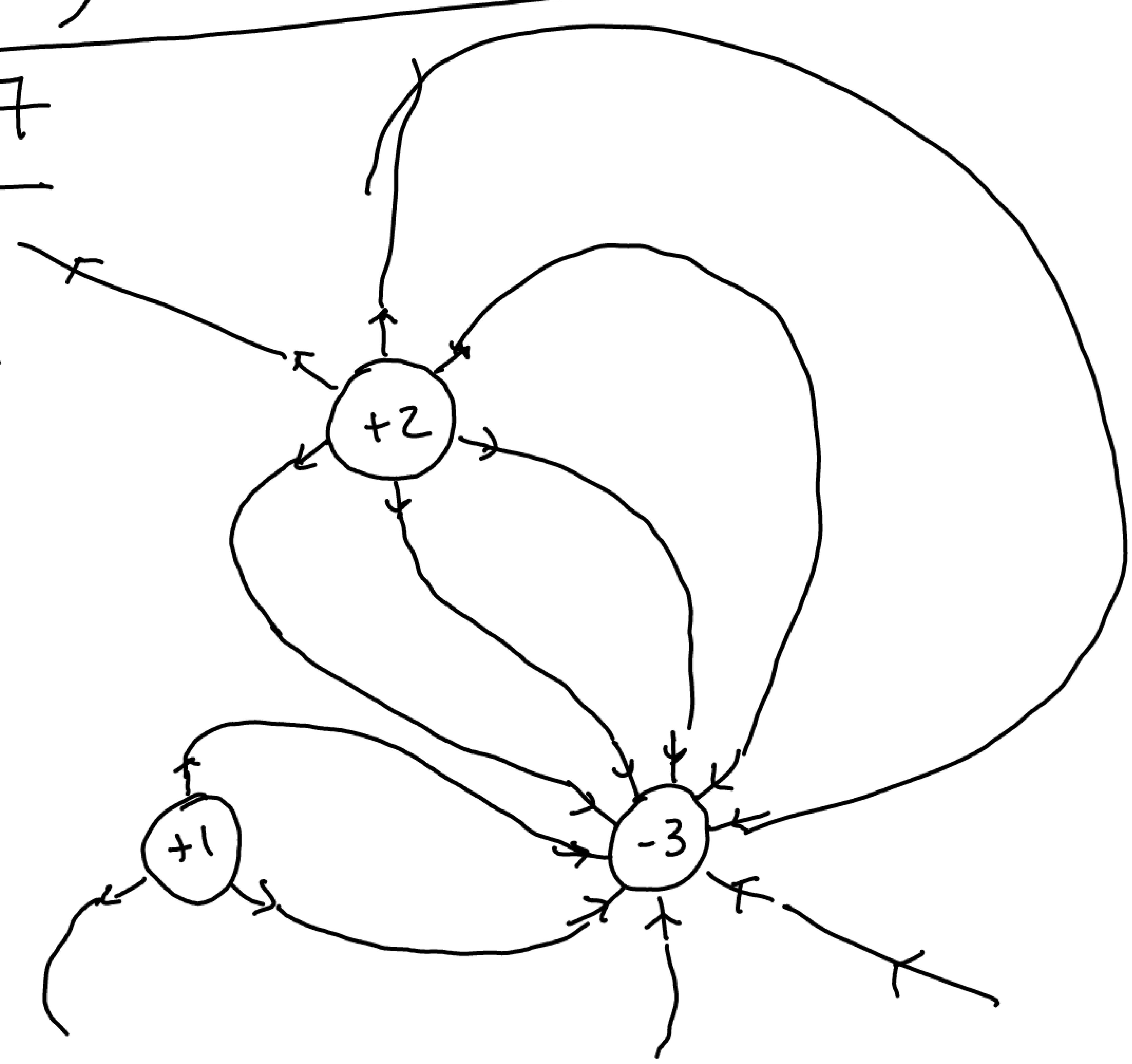
"Artistic" E-Field

Key Point:

$\boxed{+1}$ → 3 <u>out</u> lines

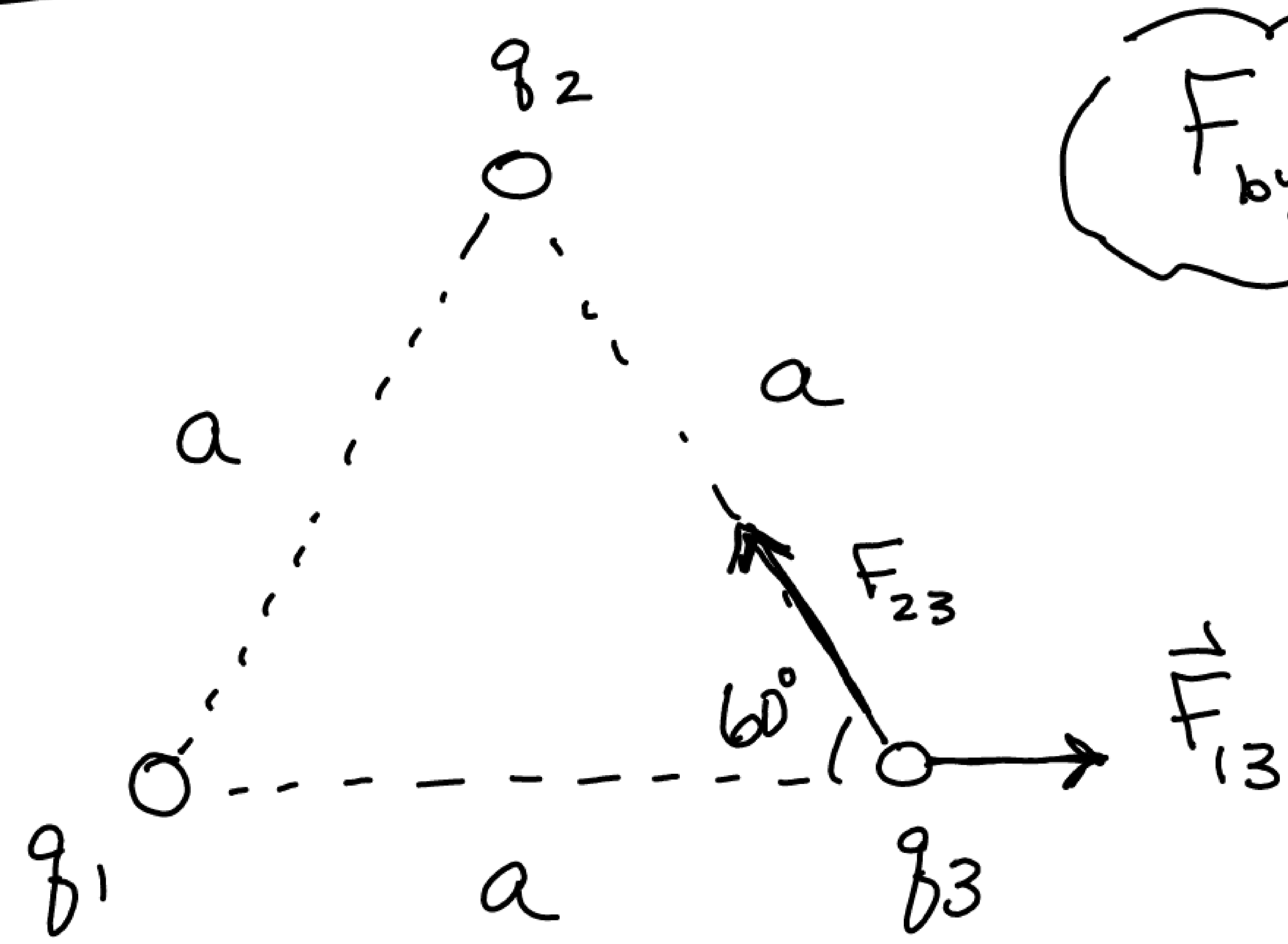
$\boxed{+2}$ → 6 <u>out</u> lines

$\boxed{-3}$ → 9 <u>in</u> lines



## problem 1.8

$F_{by,on}$




$|\vec{F}_{13}| = k_e \dfrac{q_1 q_3}{a^2}$ only in $\hat{x}$ direction

$|\vec{F}_{23}| = k_e \dfrac{q_2 q_3}{a^2}$ directed up & left.

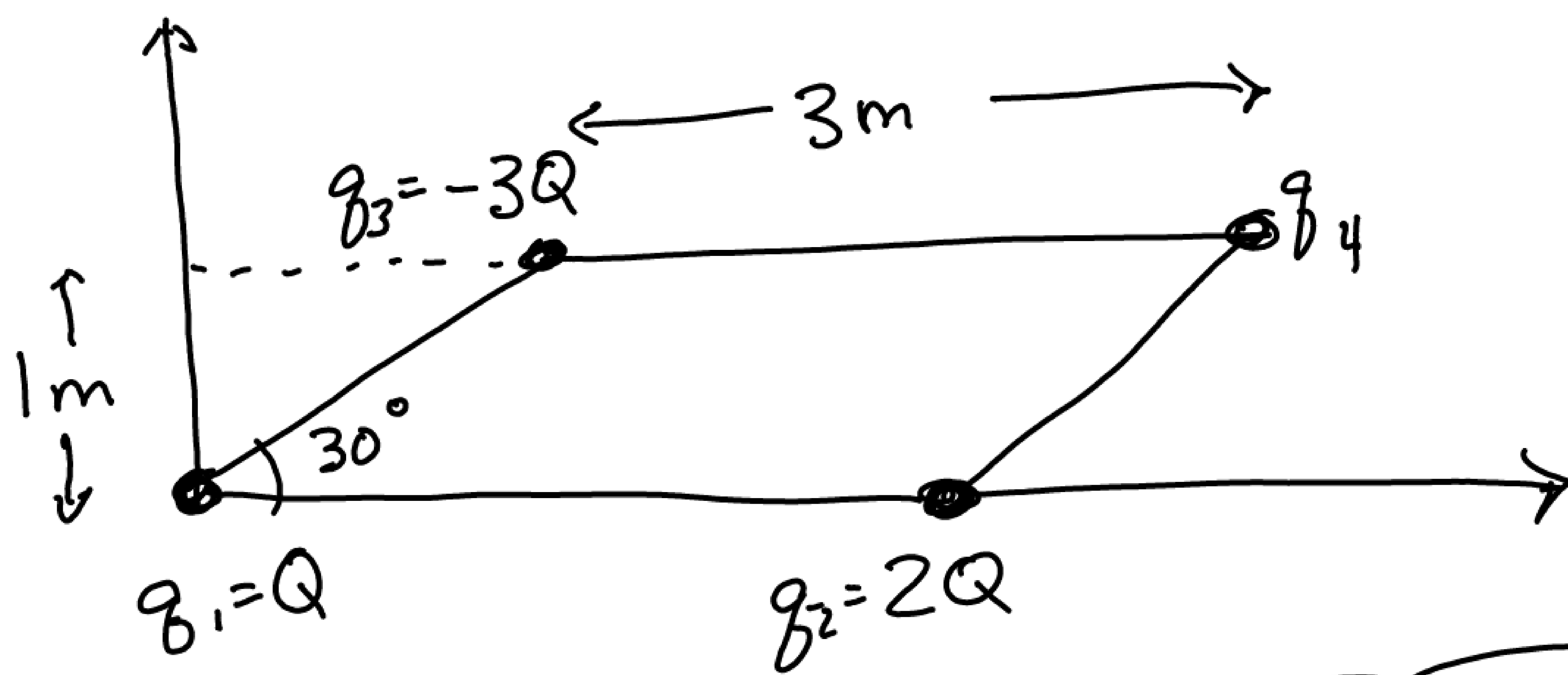
$$\vec{F}_{13} = \left( k_e \frac{q^2}{a^2} , 0 \right)$$

$$\vec{F}_{23} = \left( -k_e \frac{2q^2}{a^2} \cos 60° , k_e \frac{2q^2}{a^2} \sin 60° \right)$$

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$\boxed{\vec{F}_3 = k_e \frac{q^2}{a^2} \left( 1 - 2\cos 60 \hat{x} + \sin 60 \hat{y} \right)}$$

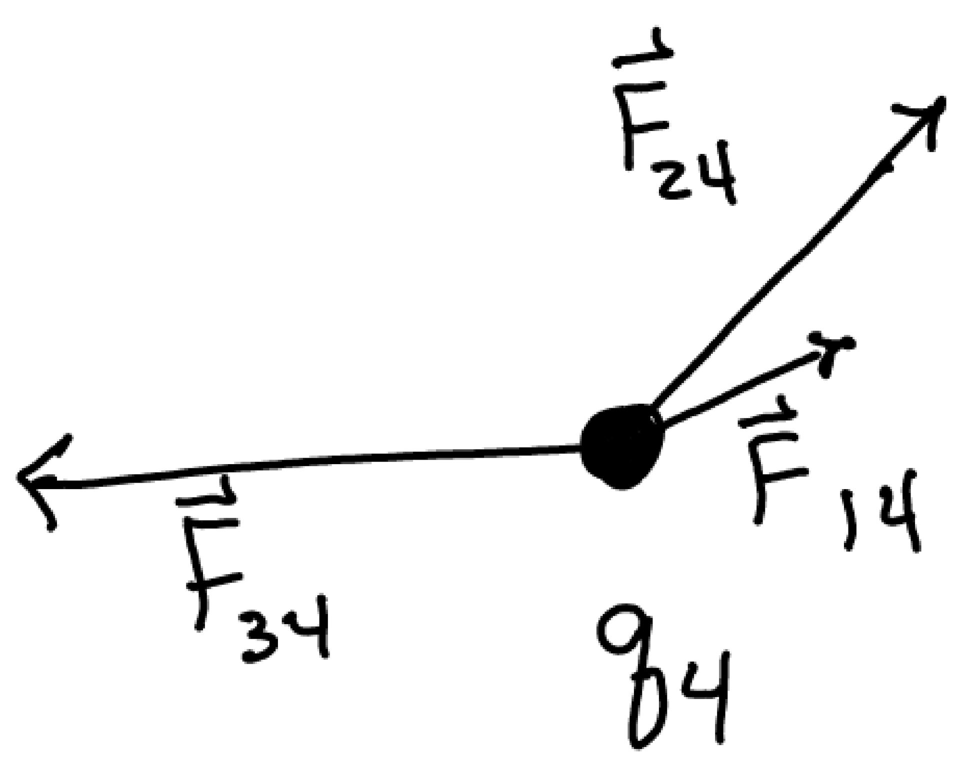
# Problem 1.9 (pg 1 of 3)




$q = 5\mu C$

$Q = 8\mu C$

## FBD-4




the task is now to find these vectors and add them all up. Lets do this with

$$\boxed{\vec{F}_4 = \frac{q_4}{4\pi\varepsilon_0}\sum_{i=1}^{3}\frac{q_i}{|\vec{r}_{i4}|^2}\hat{r}_{i4}}$$

This vector eqn requires

$$|\vec{r}_i| \ \& \ \hat{r}_i$$

where $\boxed{\hat{r}_i = \dfrac{\vec{r}_i}{|\vec{r}_i|}}$

Start w/ the easy one

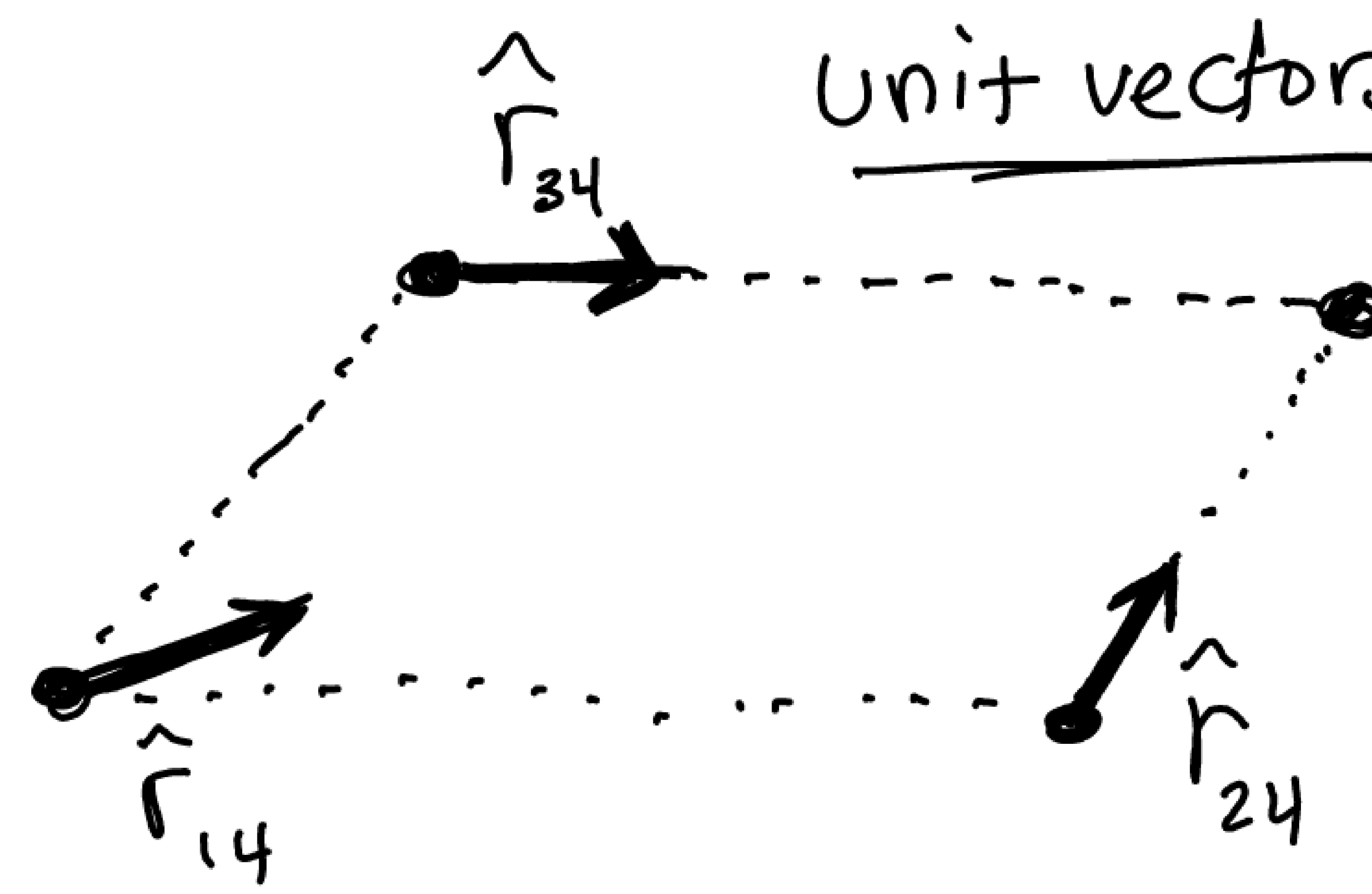
$$\vec{r}_{34} = (3,0)$$

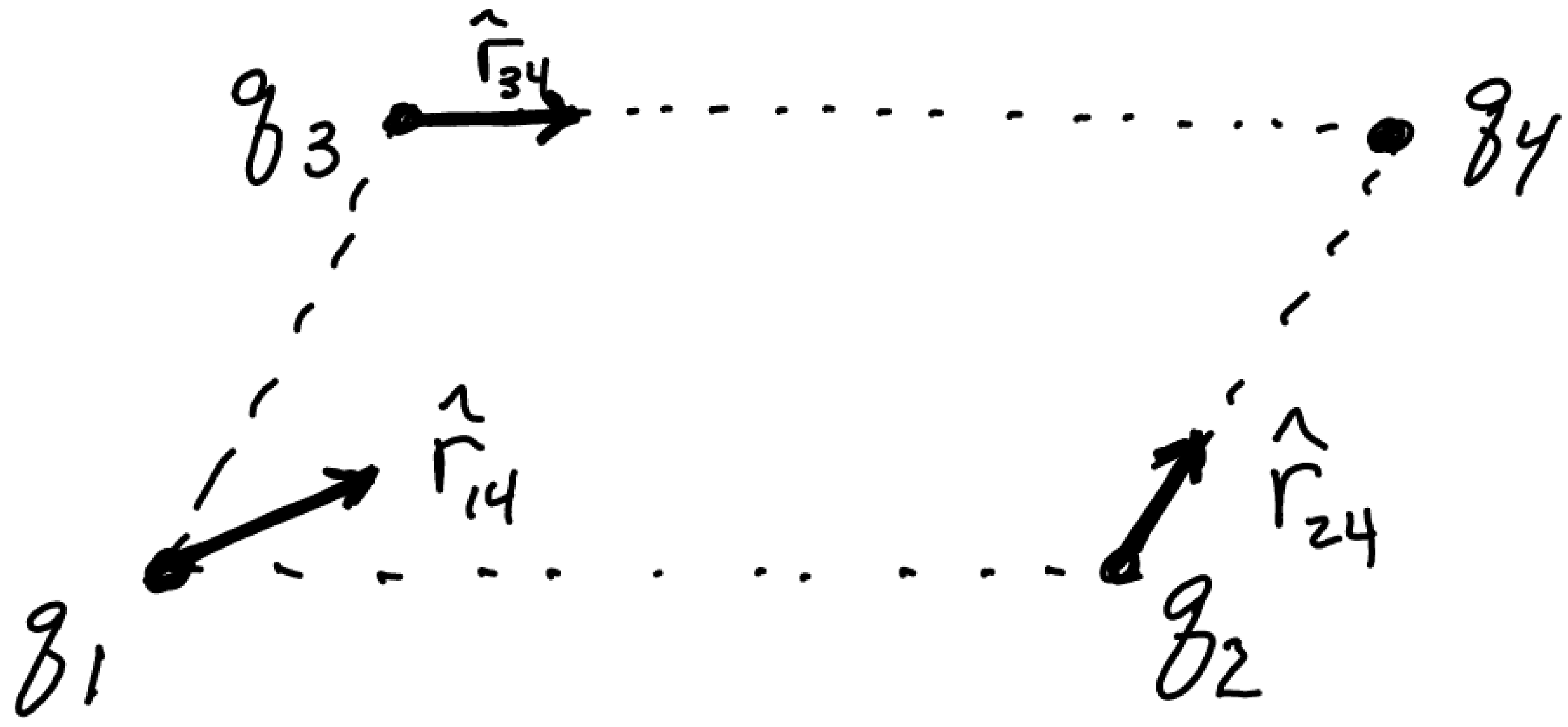
$$|\vec{r}_{34}| = \sqrt{3^2+0^2} = 3$$

$$\hat{r}_{34} = \frac{(3,0)}{3} = (1,0)$$

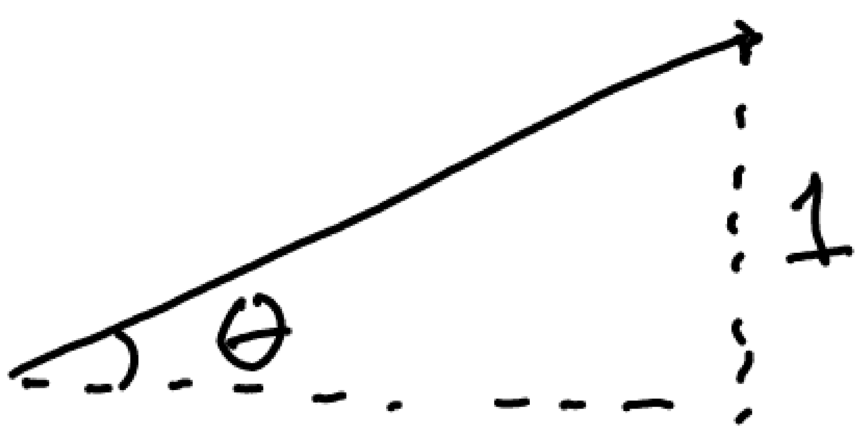
see next page.

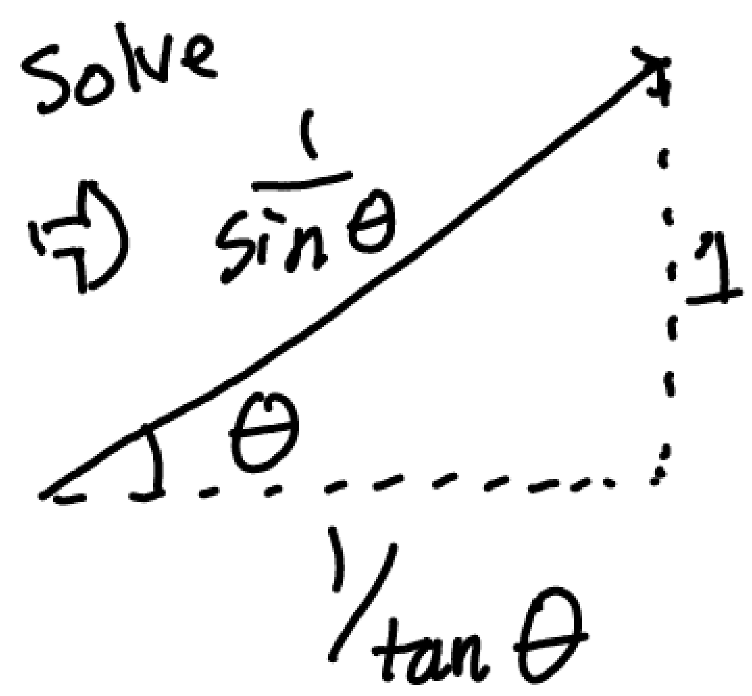
unit vectors:






## 1 → 4 vector



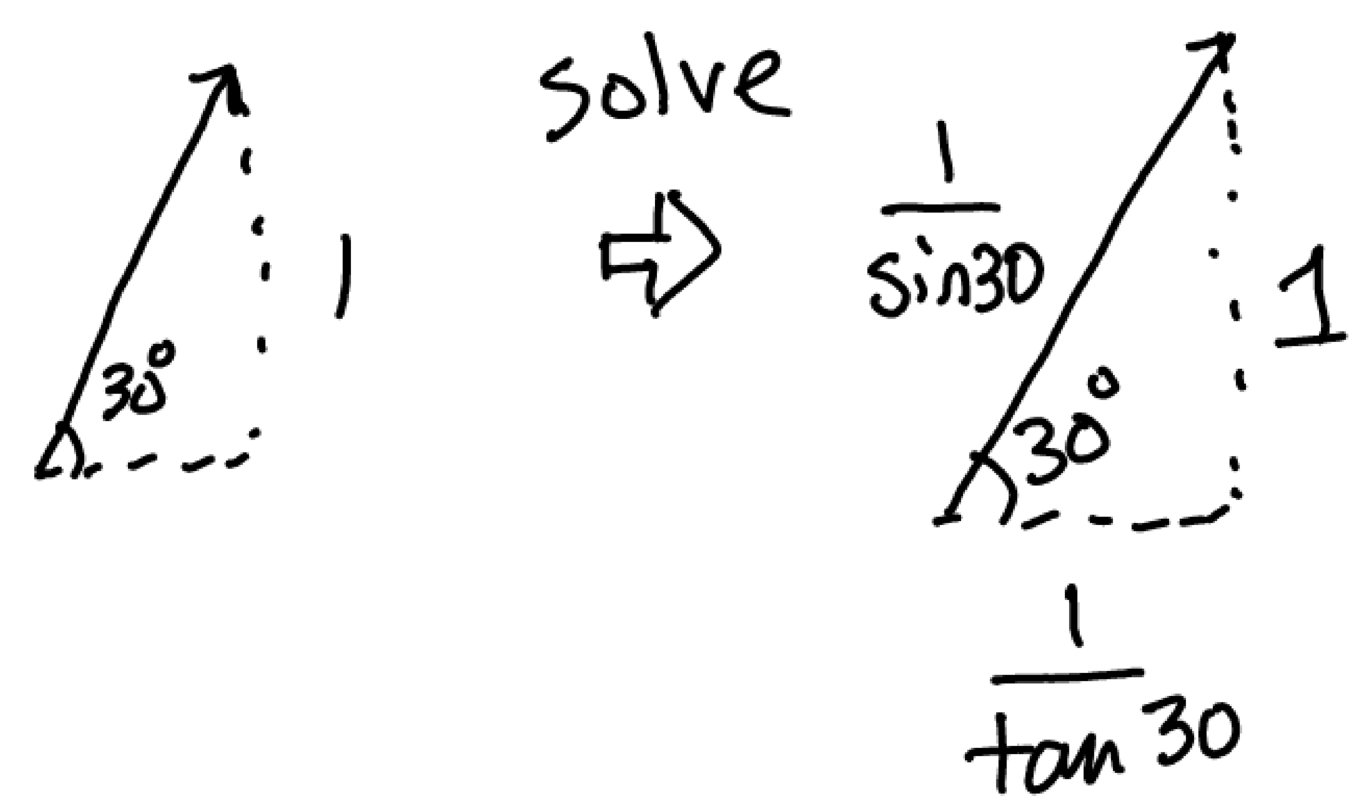

$\theta = 11.932°$

$$\vec{r}_{14} = \left(\frac{1}{\tan\theta}, 1\right)$$

$$|\vec{r}_{14}| = \frac{1}{\sin\theta}$$

$$\hat{r}_{14} = \frac{\vec{r}_{14}}{|\vec{r}_{14}|} = (\cos\theta, \sin\theta)$$

## 2 → 4 Vector




$$\vec{r}_{24} = \left(\frac{1}{\tan 30}, 1\right)$$

$$|\vec{r}_{24}| = \frac{1}{\sin 30}$$

$$\hat{r}_{24} = \frac{\vec{r}_{24}}{|\vec{r}_{24}|} = (\cos 30, \sin 30)$$

then construct the force from all 3

$$\vec{F}_{q_4} = \frac{q_4}{4\pi\varepsilon_0} \sum_{i=1}^{3} \frac{q_i}{|r_i|^2} \hat{r}_i$$

1.9 (pg 3 of 3)

$$F_4 = \frac{q_4}{4\pi\varepsilon_0}\left[\frac{q_1}{(1/\sin 15)^2}(\cos\theta, \sin\theta)\right] \Leftarrow F_{14}$$

$$+ \frac{q_4}{4\pi\varepsilon_0}\left[\frac{q_2}{(1/\sin 30)^2}\left(\cos 30, \frac{1}{\sin 30}\right)\right] \Leftarrow F_{24}$$

$$+ \frac{q_4}{4\pi\varepsilon_0}\left[\frac{q_3}{3^2}(1, 0)\right] \Leftarrow F_{34}$$

Finally evaluate

$q_1 = 5\mu C$ $q_2 = 10\mu C$ $q_3 = -15\mu C$ $q_4 = 8\mu C$

$\frac{1}{4\pi\varepsilon_0} = 9E9$ $\mu \to 10^{-6}$

micro

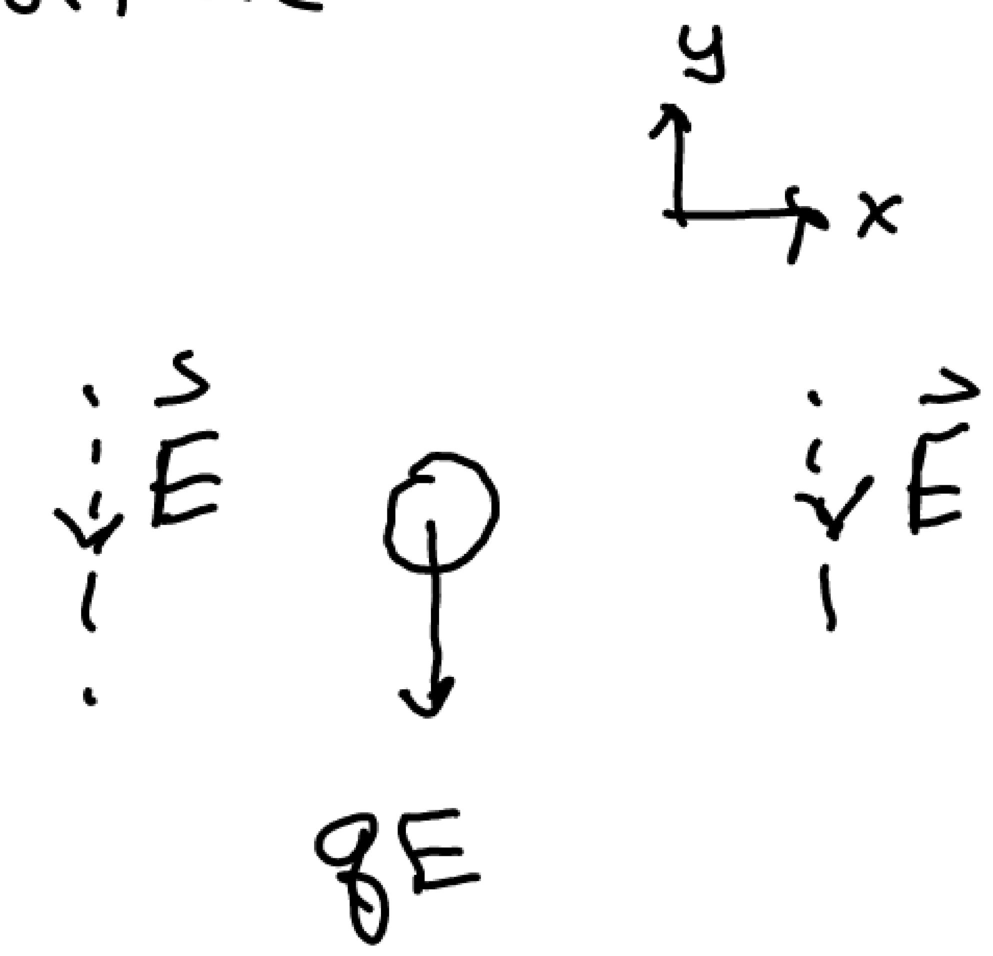
$$\left.\begin{aligned} F_4 &= 0.0241(\cos\theta, \sin\theta) \\ &+ 0.18(\cos 30, \sin 30) \\ &- 0.12(1, 0) \end{aligned}\right\} \begin{matrix} F_{14} \\ F_{24} \\ F_{34} \end{matrix}$$

$$\boxed{\begin{aligned} F_4 &= (0.059, 0.094) \text{ Newtons} \\ &= 0.11\,[N] @ 57.8^\circ \end{aligned}}$$

## problem 1.10

this is a throwback to physics 1 : Kinematics!

A unifom $\vec{E}$-field will behave the same as gravity on the surface of Earth.




$$\vec{F} = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q}{m}\vec{E}$$

$$\vec{a} = \frac{1.6E-19}{1.67E-27}(0, -4E5) = (0, -3.83E10)\frac{m}{s^2}$$

this is the new "9.8"

Find the time using x-direction

$$x_f = x_i + v_{ix} t \qquad (0.12) = 0 + (1.5E7)t$$

$$t = 8E-9s$$

then solve y-direction

$y_i = 0$

$y_f = ?$ $\qquad a_y = -3.83E10 \frac{m}{s^2}$ $\quad \Rightarrow$

$v_{iy} = 0$ $\qquad t = 8E-9s$

$v_{fy} = ?$

$$y_f = \cancel{y_i}^{0} + \cancel{v_{iy}t}^{0} + \frac{1}{2}a_y t^2$$

$$= \frac{1}{2}a_y t^2$$

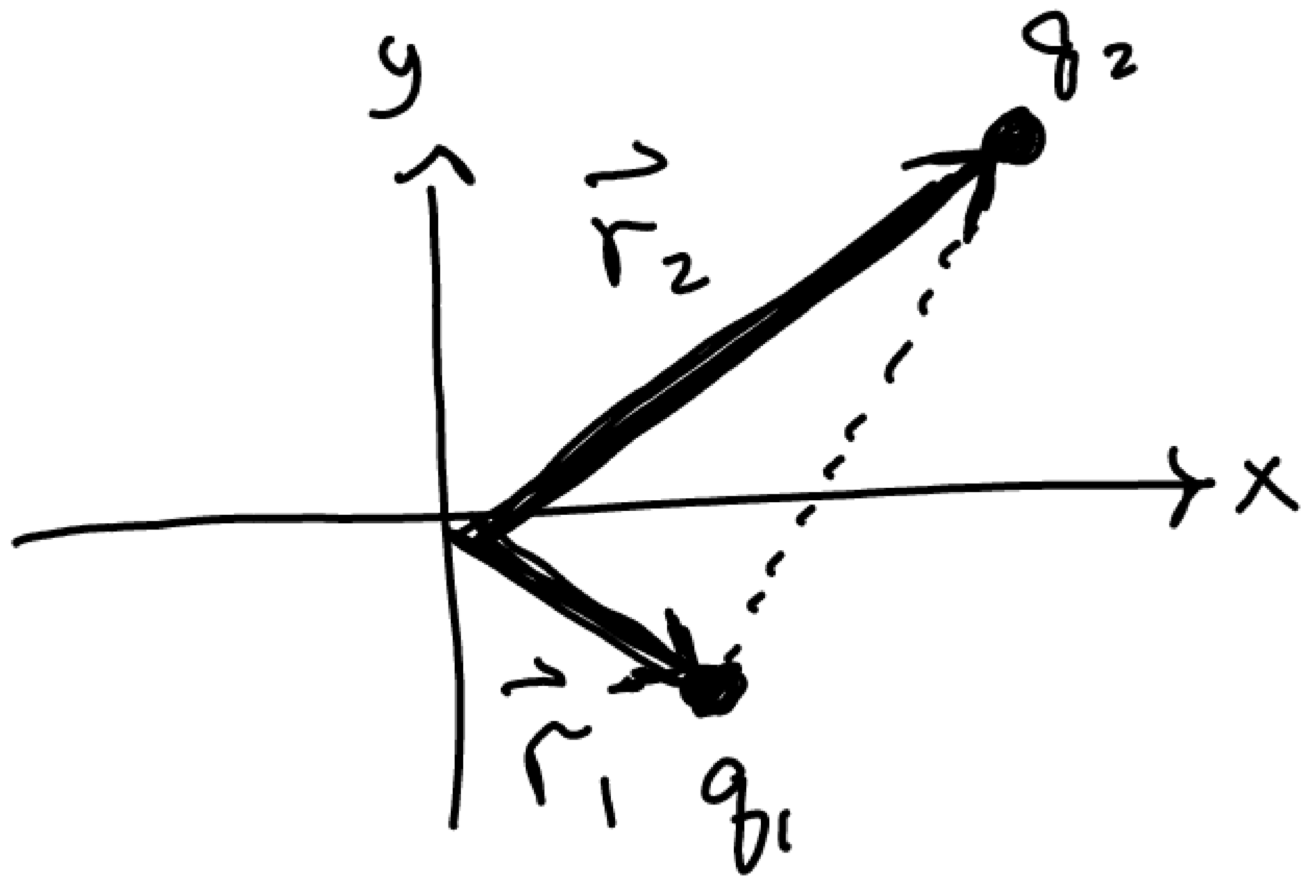
$$\boxed{y_f = -1.23 \text{ microns}}$$

micron $\rightarrow 10^{-6}$m

## problem 1.11

$\vec{r}_1 = (4, -2, 2)\,m$ $\qquad q_1 = 2\mu C$

$r_2 = (8, 5, -9)\,m$ $\qquad q_2 = 4\mu C$




Coulombs law

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|r|^3}\vec{r}$$

Method 1: move charge 1 to the origin
(this means you must also transform $q_2 \rightarrow$ origin)

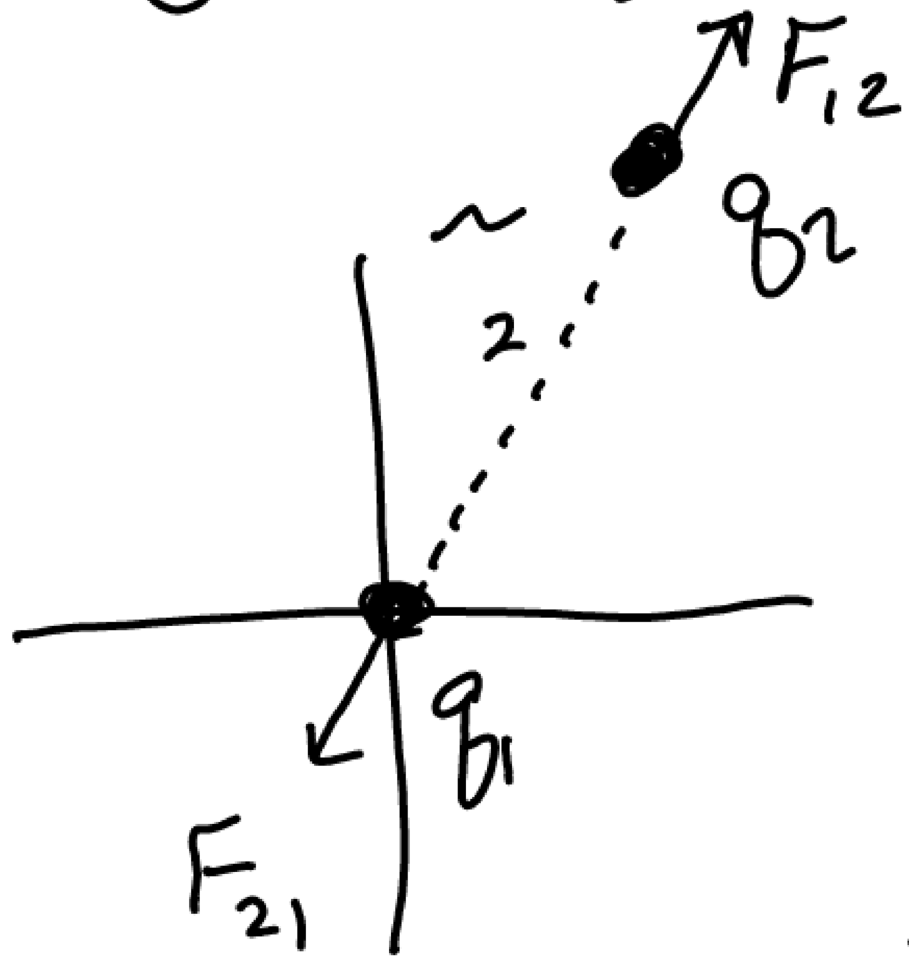
I call this → "r twiddle"

$$\tilde{r}_1 = \vec{r}_1 - \vec{r}_1 = (0, 0)$$ ← $q_1$ @ origin

$$\tilde{r}_2 = \vec{r}_2 - \vec{r}_1 = (8, 5, -9) + (-4, 2, -2)$$

$$\tilde{r}_2 = (4, 7, -11)$$ ← transformed position of $q_2$

this transforms $\tilde{r}_1$ to the origin
and $\tilde{r}_2$ to its <u>new</u> "transformed" coordinate.




$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{(2E\text{-}6)(4E\text{-}6)}{(4^2+7^2+11^2)^{3/2}}(4, 7, -11)$$

$$\boxed{\vec{F}_{12} = (1.13,\ 1.98,\ -3.12)\times 10^{-4}\,N}$$

Method 2: Dont more the origin, instead include shift inside coulombs law itself (this is better for computer evaluation)

$$F = \frac{1}{4\pi\varepsilon_0} \frac{(q_1)(q_2)\,(\vec{x}-\vec{x}_0,\ \vec{y}-\vec{y}_0,\ \vec{z}-\vec{z}_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}}$$

where $x, y, z$ are the coords of $q_2$

and $x_0, y_0, z_0$ are the coords of $q_1$

* this does not force a coord shift, but increases the amount of calculator work. No problem for a computer

You should get identical answers using this alternative formula.