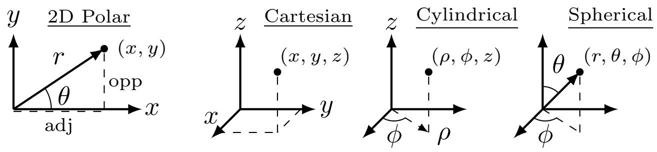
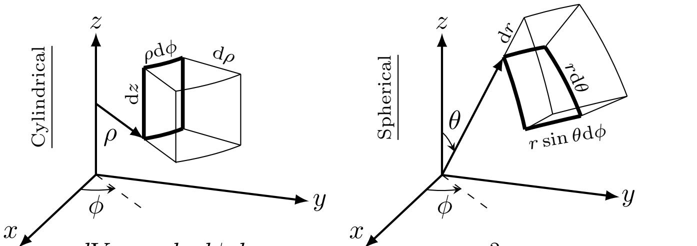
Coordinate Systems





$dV = \rho\, d\rho\, d\phi\, dz$

$d\vec{\boldsymbol{\ell}} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$

$\vec{\nabla} = \hat{\boldsymbol{\rho}}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{z}}\frac{\partial}{\partial z}$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$d\vec{\boldsymbol{\ell}} = dr\hat{\boldsymbol{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$

$\vec{\nabla} = \hat{\boldsymbol{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$

Trigonometry

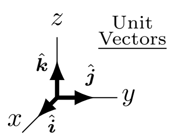
$\sin(-x) = -\sin x$ (odd)

$\cos(-x) = \cos x$ (even)

$\sin^2 x + \cos^2 x = 1$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$



$\vec{\boldsymbol{r}} = (x,y,z)$

$\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$

$\hat{\boldsymbol{r}} = \frac{x}{|\vec{r}|}\hat{\boldsymbol{\imath}} + \frac{y}{|\vec{r}|}\hat{\boldsymbol{\jmath}} + \frac{z}{|\vec{r}|}\hat{\boldsymbol{k}}$

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = 1 - 2\sin^2 x$

$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$\sin(x/2) = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos(x/2) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan(x/2) = \frac{1-\cos x}{\sin x}$

⇐ Derivatives and Integrals ⇒

df/dx	$\boldsymbol{f(x)}$	$\int f dx$	integration strat
nx^{n-1}	x^n	$\left(\frac{1}{n+1}\right)x^{n+1}$	$n \neq -1$
ae^{ax}	e^{ax}	$(1/a)\, e^{ax}$	$u = ax$
$e^{ax}(ax+1)$	xe^{ax}	$e^{ax}\frac{(ax-1)}{a^2}$	$u = x \quad dv = e^{ax}dx$
$1/x$	$\ln(ax)$	$x\ln(ax) - x$	$u = \ln(ax) \quad dv = dx$
$\ln(a)a^x$	a^x	$\frac{a^x}{\ln(a)}$	$e^{\ln a^x}; u = x\ln(a)$
$\cos(x)$	$\sin(x)$	$-\cos(x)$	
$-\sin(x)$	$\cos(x)$	$\sin(x)$	
$\sec^2(x)$	$\tan(x)$	$-\ln\lvert\cos x\rvert$	$u = \cos(x)$
$2a\sin(ax)\cos(ax)$	$\sin^2(ax)$	$\frac{x}{2} - \frac{\sin(2ax)}{4a}$	trig identity
$-2a\sin(ax)\cos(ax)$	$\cos^2(ax)$	$\frac{x}{2} + \frac{\sin(2ax)}{4a}$	trig identity

$\frac{d}{dx}\sin^{-1}(x) = 1/\sqrt{1-x^2}$

$\frac{d}{dx}\cos^{-1}(x) = -1/\sqrt{1-x^2}$

$\frac{d}{dx}\tan^{-1}(x) = 1/(1+x^2)$

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right|$

$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$

$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$

Vector Calculus

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$ Cartesian

$\vec{\nabla} f$	Gradient	$\int_{\vec{a}}^{\vec{b}}(\vec{\nabla} f)\cdot d\vec{\ell} = f(\vec{b}) - f(\vec{a})$
$\vec{\nabla}\cdot\vec{F}$	Divergence	$\iiint(\vec{\nabla}\cdot\vec{F})dV = \oint_S \vec{F}\cdot d\vec{A}$
$\vec{\nabla}\times\vec{F}$	Curl	$\iint(\vec{\nabla}\times\vec{F})\cdot d\vec{A} = \oint_C \vec{F}\cdot d\vec{\ell}$

Vector Multiplication

Dot Product → $\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

$$\vec{A}\times\vec{B} = \left|\begin{array}{ccc|cc} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ A_x & A_y & A_z & A_x & A_y \\ B_x & B_y & B_z & B_x & B_y \end{array}\right.$$

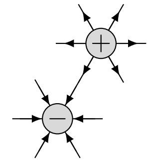
$= AB\sin\theta$

← Cross Product

down right$(+)$

down left$(-)$

Electric Field $[N/C]$ $[V/m]$



$\vec{F} = q\vec{E}$

$\left|\vec{F}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_1\, q_2}{r^2}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$

$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i$

Gauss' Law $\varepsilon_0 = 8.85\times 10^{-12}[F/m]$ permittivity

$\Phi_E = \iint_S \vec{E}\cdot\hat{n}\,dA$

Flux $\Phi_E = EA\cos\theta$

$\oiint_S \vec{E}\cdot\hat{n}\,dA = \frac{q_{\text{in}}}{\varepsilon_0}$

$E\cdot dA = \frac{q_{\text{in}}}{\varepsilon_0}$

Electric Potential [Volts] $[V]$ $[J/C]$

$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{\ell}$

$\vec{E} = -\vec{\nabla} V$

$\vec{E}_x = -\frac{dV}{dx}\hat{x}$

$\Delta V_{\text{point}} = \frac{kq}{r}\Big|_A^B$

$W_{nc} = \Delta K + \Delta U$

$\Delta V_{\text{line}} = \frac{-\lambda}{2\pi\varepsilon_0}\ln(r)\Big|_A^B$

$K = \frac{1}{2}mv^2$

$U_g = mgh$

$\Delta V_{\text{plane}} = -E\Delta x\Big|_{x_i}^{x_f}$

$U_E = qV$

$U_s = \frac{1}{2}kx^2$

Magnetic Field $\vec{\boldsymbol{B}}$ [Tesla] $[T]$ $[10^4$ Gauss$]$

$\vec{F} = q\,\vec{v}\times\vec{B} = I\vec{\ell}\times\vec{B}$

$\vec{\mu} = I\,\vec{A} = \frac{q}{2m}\vec{L}$

cyclotron

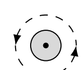
$\vec{\tau} = \vec{\mu}\times\vec{B}$ ← torque

$r = \frac{mv}{qB}$

$T = \frac{2\pi m}{qB}$

$U_B = -\vec{\mu}\cdot\vec{B}$

Ampere's Law $\mu_0 = 4\pi\times 10^{-7}[N/A^2]$ permeability

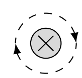


$\Phi_B = \iint_S \vec{B}\cdot\hat{n}\,dA$

Flux $\Phi_B = BA\cos\theta$

$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I$

$\oiint_S \vec{B}\cdot d\vec{A} = 0$



$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$

$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$

current line $B = \frac{\mu_0 I}{2\pi r}$

$B = \mu_0\frac{N}{\ell}I$ solenoid

Faraday's Law of Induction [Weber] $[Wb]$ $[V\cdot s]$

$\mathcal{E} = \oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

nature abhors change in magnetic flux

$\mathcal{E} = NBA\omega\sin(\omega t)$

$\mathcal{E} = B\ell v$

Maxwells Equations

$\vec{\boldsymbol{F}} = q\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}\right)$ Lorrentz

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$	Gauss	$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$
$\vec{\nabla}\cdot\vec{B} = 0$	NoName	$\oint_S \vec{B}\cdot d\vec{A} = 0$
$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	Faraday	$\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{\partial\Phi_B}{dt}$
$\vec{\nabla}\times\vec{B} = \mu_0\vec{I} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$	Ampere	$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I + \varepsilon_0\mu_0\frac{\partial\Phi_E}{dt}$
$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$	EM-Wave	$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0\mu_0\frac{\partial^2 E_y}{\partial t^2}$

Physics II - Electromagnetism

Conversions

$1\,in = 2.54\,cm$	$1\,ft\text{-}lb = 1.356\,N{\cdot}m$	tera	T	10^{12}
$1\,ft = 0.3048\,m$	$1\,cal = 4.186\,J$	giga	G	10^{9}
$1\,yard = 3\,ft$	$1\,kWh = 3.60\,MJ$	mega	M	10^{6}
$1\,mi = 5280\,ft$	$1\,hp = 746\,J/s$	kilo	k	10^{3}
$1\,\frac{mi}{hr} = 0.447\,\frac{m}{s}$	$1\,C = 6.25\cdot 10^{18}\,e$	centi	c	10^{-2}
$1\,L = 1000\,cm^3$	$1eV = 1.60\cdot 10^{-19}\,J$	milli	m	10^{-3}
$1\,gal = 3.786\,L$	$T_C = 5/9(T_F - 32)$	micro	μ	10^{-6}
$1\,ft^3 = 28.32\,L$	$T = T_C + 273$	nano	n	10^{-9}
$1\,lb = 4.448\,N$	$1\,atm = 101325\,Pa$	pico	p	10^{-12}

Greek Symbols

alpha	α		iota	ι		rho	ρ	
beta	β		kappa	κ		sigma	σ	Σ
gamma	γ	Γ	lambda	λ	Λ	tau	τ	
delta	δ	Δ	mu	μ		upsilon	υ	Υ
epsilon	ϵ		nu	ν		phi	ϕ	Φ
zeta	ζ		xi	ξ	Ξ	chi	χ	
eta	η		omicron	o		psi	ψ	Ψ
theta	θ	Θ	pi	π	Π	omega	ω	Ω

Physical Constants

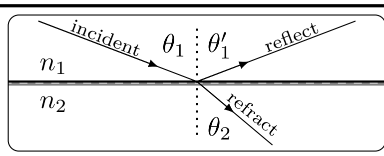
Name	Symbol	Value
Speed of Light	c	$2.99792458\cdot 10^{8}\ m/s$
Electrostatic Constant	k_e	$8.99\cdot 10^{9}\ N{\cdot}m^2/C^2$
Electric Permittivity	ε_0	$8.854\cdot 10^{-12}\ C^2/(N{\cdot}m^2)$
Magnetic Permeability	μ_0	$4\pi\cdot 10^{-7}\ T{\cdot}m/A$
Elementary Charge	e	$1.60\cdot 10^{-19}\ C$
Planck Constant	h	$6.63\cdot 10^{-34}\ J\cdot s$
Bohr Magneton	μ_B	$9.274\cdot 10^{-24}\ J/T$
Rest mass of Electron	m_e	$9.1093\cdot 10^{-31}\ kg$
Rest mass of Proton	m_p	$1.6726\cdot 10^{-27}\ kg$
Atomic Mass Unit	u	$931.49\ MeV/c^2$

Ray Optics $n = c/v$

$\theta_1 = \theta_1'$

$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

$\sin(\theta_{\text{critical}}) = n_2/n_1$



Thin Lens:

$f \to$ focus

$p \to$ object

$q \to$ image

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad M = \frac{h'}{h} = -\frac{q}{p}$$

$$1/f = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Polarization:

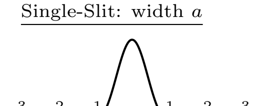
$$\tan(\theta_{\text{Brewster}}) = n_2/n_1 \qquad I = I_0 \cos^2(\theta)$$

Refractive Index $n = c/v$ $\lambda = 590nm$

Material	n	Material	n	Material	n
Vacuum	1 exact	Ice	1.31	Window	1.52
Air	1.000293	Water	1.333	Polycarb	1.58
He	1.000036	Ethanol	1.36	Sapphire	1.77
H2	1.000132	OliveOil	1.47	Zirconia	2.15
CO2	1.00045	Silica	1.46	Diamond	2.417

Wave Optics

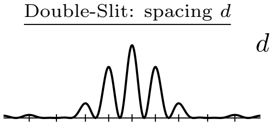
Single-Slit: width a



$$a\,\sin(\theta_{\text{dark}}) = m\lambda \qquad m = \pm 1, \pm 2, \ldots$$

$$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2 \qquad \beta = \frac{\phi}{2} = \frac{\pi a \sin\theta}{\lambda}$$

Double-Slit: spacing d



$$d\,\sin(\theta_{\text{bright}}) = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

$$I = I_0 \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right)$$

Capacitance C [Farad] [F]

$$\Delta Q = C\Delta V \qquad C = \epsilon_0 \frac{A}{d} \qquad C = \kappa C_0 \qquad \kappa = \frac{\varepsilon_0}{\varepsilon}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \leftarrow\text{series} \qquad \text{parallel}\rightarrow \quad C_{eq} = C_1 + C_2$$

$$U_C = \tfrac{1}{2}V^2 C = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}QV$$

$$q_\uparrow(t) = Q\left(1 - e^{-t/\tau}\right) \qquad q_\downarrow(t) = Qe^{-t/\tau} \qquad \tau = RC$$

Resistance R [Ohm] [Ω]

$$R_{eq} = R_1 + R_2 \quad \leftarrow\text{series} \qquad \text{parallel}\rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \rho\frac{\ell}{A} \qquad \rho = \rho_0\left[1 + \alpha(T - T_0)\right]$$

Current I [Amp] [A] [C/s]

$$I = \frac{dQ}{dt} \qquad I = \iint \vec{J}\cdot d\vec{A} \qquad I = nqAv_d$$

Direct-Current Circuits

$$\Delta V = IR \qquad \mathbb{P} = IV = I^2R = \frac{V^2}{R}$$

junction $\rightarrow I_{in} = I_{out}$

loop $\rightarrow \Sigma\Delta V = 0$

Inductance L [Henry] [H]

$$\mathcal{E} = -L\frac{dI}{dt} \qquad L = \frac{N\Phi_B}{I} = \mu_0\frac{N^2}{\ell}A$$

$$I_\uparrow(t) = \frac{\mathcal{E}}{R}\left(1 - e^{\frac{-t}{\tau}}\right) \qquad I_\downarrow(t) = \frac{\mathcal{E}}{R}e^{\frac{-t}{\tau}} \qquad \tau = \frac{L}{R}$$

Alternating-Current Circuits $\omega = 2\pi f$

$$X_L = \omega L \qquad X_C = \frac{1}{\omega C} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} \qquad \Delta V_{\text{rms}} = I_{\text{rms}}Z \qquad \mathbb{P}_{avg} = I_{\text{rms}}\Delta V_{\text{rms}}\cos(\phi)$$

$$e^{i\phi} = \cos(\phi) + i\sin(\phi) \qquad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

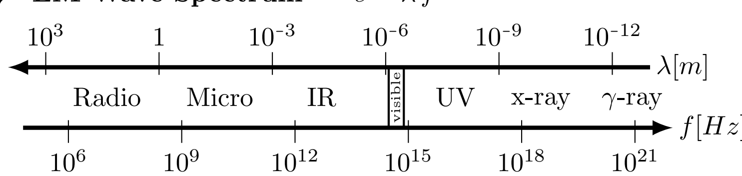
$$q(t) = Q_{\max}\exp\left(\tfrac{-RC}{2L}\right)\cos\omega_d t \qquad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \Delta V_{\text{sec}} = \frac{N_{\text{sec}}}{N_{\text{pri}}}\Delta V_{\text{pri}}$$

Electromagnetic Material Properties

Material	Resistivity ρ ($\Omega{\cdot}m$)	Dielectric $\varepsilon/\varepsilon_0$	Permeability μ/μ_0
Vacuum	-	1 exact	1 exact
Aluminium	2.82×10^{-8}	-	1.00002
Copper	1.7×10^{-8}	-	0.999991
Iron (99.8%)	10×10^{-8}	-	5×10^{3}
Pure Iron	10×10^{-8}	-	2×10^{5}
Pure Silicon	2.3×10^{3}	3.78	0.99837
Paper	10^{12}	3.7	-
Rubber	10^{13}	6.7	-
Porcelain	10^{14}	6	-
Quartz	7.5×10^{17}	3.78	1.000014

EM Wave Spectrum $c = \lambda f$



Physics II - Circuits & Light