

TUTORIAL SESSION 1:

1. Addition Rule of Probability

Concept:

The Addition Rule allows us to calculate the probability that either event A or event B (or both) occurs.

Formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where:

- $P(A \cup B)$: Probability that A or B or both occur
- $P(A \cap B)$: Probability that both A and B occur

Real-World Example:

Consider a standard deck of 52 cards:

- Event A: Drawing a red card ($P(A) = 26/52$)
- Event B: Drawing a face card ($P(B) = 12/52$)
- Both red and face card ($P(A \cap B) = 6/52$)

Calculation:

$$P(A \cup B) = 26/52 + 12/52 - 6/52 = 32/52 \approx 0.615$$

Python Code:

```
def addition_rule(p_a, p_b, p_a_and_b):  
    return p_a + p_b - p_a_and_b  
  
p_red = 26 / 52  
p_face = 12 / 52  
p_red_and_face = 6 / 52  
  
result = addition_rule(p_red, p_face, p_red_and_face)  
print(f"P(Red or Face Card): {round(result, 3)}")
```

2. Multiplication Rule of Probability

Concept:

The Multiplication Rule helps calculate the probability of the intersection of two events. It differs based on whether the events are independent or dependent.

Independent Events Formula:

$$P(A \cap B) = P(A) \times P(B)$$

Dependent Events Formula:

$$P(A \cap B) = P(A) \times P(B | A)$$

Example 1: Independent Events

- Event A: Tossing a coin and getting heads ($P(A) = 0.5$)
- Event B: Rolling a 4 on a die ($P(B) = 1/6$)

Python Code:

```
def multiplication_independent(p_a, p_b):  
    return p_a * p_b  
  
p_heads = 0.5  
p_4 = 1 / 6  
print("P(Heads and 4):", round(multiplication_independent(p_heads, p_4), 4))
```

Example 2: Dependent Events

- Event A: Drawing an ace on the first draw ($P(A) = 4/52$)
- Event B: Drawing a second ace without replacement ($P(B|A) = 3/51$)

Python Code:

```
def multiplication_dependent(p_a, p_b_given_a):  
    return p_a * p_b_given_a  
  
p_ace1 = 4 / 52  
p_ace2_given_ace1 = 3 / 51  
print("P(Ace1 and Ace2):", round(multiplication_dependent(p_ace1, p_ace2_given_ace1), 4))
```

3. Bayes' Theorem

Concept:

Bayes' Theorem is used to reverse conditional probabilities. It calculates the probability of event A given that B has occurred.

Formula:

$$P(A | B) = (P(B | A) \times P(A)) / P(B)$$

Real-World Example:

- $P(\text{Disease}) = 0.01$
- $P(\text{Positive} | \text{Disease}) = 0.99$
- $P(\text{Positive}) = 0.05$
- Result: $P(\text{Disease} | \text{Positive}) = (0.99 \times 0.01) / 0.05 = 0.198$

Python Code:

```
def bayes_theorem(p_b_given_a, p_a, p_b):  
    return (p_b_given_a * p_a) / p_b  
  
p_disease = 0.01  
p_positive_given_disease = 0.99  
p_positive = 0.05  
  
p_disease_given_positive = bayes_theorem(p_positive_given_disease, p_disease, p_positive)  
print(f"P(Disease | Positive Test): {round(p_disease_given_positive, 3)}")
```