Natural Language Processing

Lecture VII. Word Embedding – a 30-year journey

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Something about graduate school

"The path to real success is not to compete, but to invent a new game, and then master it." Reid Hoffman,

https://www.linkedin.com/pulse/dont-just-compete-invent-new-game-master-reid-hoffman

Salute to the NLP pioneers, including but not limited to: Elman (1990), Bengio (2003), Collobert & Weston (2008, look-up table), Mnih & Hinton (2007 & 2009, tree).

Language model review

- BOW or unigram: no order of words
- N-gram where $N > 1$: a sequence of words, some structural information.
- A classical language model estimates a cost function (e.g., likelihood) of word sequences.
- Problem?
  - "curse of dimensionality" $P(w_1, \ldots, w_n) = P(w_2|w_1) \times P(w_3|w_1, w_2) \times \cdots \times P(w_n|w_1, \ldots, w_{n-1}) = \prod_{i=2}^{n} P(w_i|w_1, \ldots, w_{i-1}) \approx \prod_{i=2}^{n} P(w_i|w_{i-\tau-1}, \ldots, w_{i-1})$ Too many probablities!
  - What about unseen combinations (not just words)? Smoothing is not enough.
  - How to plug into neural networks?

How to plug words into an ANN

- A very good explanation from TF v1 tutorial about why word needs to be vectorized.
  `https://github.com/tensorflow/docs/blob/r1.15/site/en/tutorials/representation/word2vec.md`
- How do we send sequences of words into an NN?
- Using ASCII code? Using UTF code?
- Turning words into vectors (the simple ways):
  - one-hot encoder (Finding Structure in Time, Elman, 1990, Table 5, each word is a 31-bit vector)
  - co-occurrence matrix (e.g., P("fox", "jump"), P("lazy", "dog"))
  - factorization on the co-occurrence matrix (to reduce dimensionality) such as SVD

Word representation

- “A word representation is a mathematical object associated with each word, often a vector.” [1]
- “Each dimension’s value corresponds to a feature and might even have a semantic or grammatical interpretation, so we call it a word feature.”
- One-hot (aka 1-of-N) encoding is one, but obviously not good.
- Word embedding: a distributed representation

  Ref [1]: Turian, Ratinov, and Bengio, Word representations: A simple and general method for semi-supervised learning, ACL 2010

Embedding via Factorization

- Intuition: Semantically (dis)similar/(un)related words should co-occur in documents (in)frequently.
- A word-document co-occurence matrix can be considered as a composition of a series of transforms.
  1. Each word is a distribution over given semantic dimensions, e.g., “water” covers “liquid”, “clear”, and “oderless”.
  2. A document is generated by sampling words in different semantic dimensions, thus transform the word probabilities to their distributions in documents.
  3. Some semantic dimensions are more important.
- This is the idea behind Singular Vector Decomposition (SVD): $M = U\Sigma V$, where all rows of U or all columns of V are orthorgonal, and Σ a diagnoal matrix of signular values (importances of semantic dimensions).
- The dimension of U is high. Usually we zero out some dimensions in Σ to focus on only the important ones. And thus, the resulting U is no longer orthornormal.
- SVD on word-document co-occurence [2]. Note that the SVD in this tutorial is called compact SVD.

Problems of factorization-based embedding

- You have to re-create the co-occurence matrix when updating or fine-tuing.
- A large sparse matrix.
- Starting from scratch in computing SVD each time. Cannot reuse previous results, i.e., fine-tuning.

History

- "A Neural Probabilistic Language Model", Bengio et al, JMLR, 2003
- "Three New Graphical Models for Statistical Language Modelling", Mnih & Hinton, ICML 2007
- "A Unified Architecture for Natural Language Processing: Deep Neural Networks with Multitask Learning", Collobert and Weston, ICML 2008
- "Neural network based language models for higly inflective languages", Mikolov et al., ICASSP 2009, separating the training of embeddings and that of the LM/task NN.
- Finally, Word2vec, "Distributed Representations of Words and Phrases and their Compositionality", Mikolov et al., NIPS 2013
- And, "GloVe: Global Vectors for Word Representation", Pennnington et al., EMNLP 2014

First neural language model

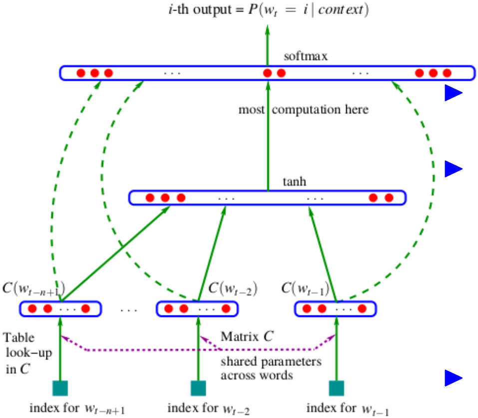
- Bengio et al., NIPS 2003, A neural probablistic language model
- Joint probability as a function of words:
  $P(\underbrace{w_i}_{target} | \underbrace{w_{i-\tau-1}, \ldots, w_{i-1}}_{context}) = f(\underbrace{w_i}_{target}, \underbrace{w_{i-\tau-1}, \ldots, w_{i-1}}_{context})$ in 2 steps:
  1. each word $w_{j\in[i-\tau-1..i]}$ into a vector $C(w_j) \in \mathbb{R}^D$ thru a look-up table C, which is also a function, and
  2. a function $g(\underbrace{C(w_x)}_{\text{any word } w_x,}, \underbrace{C(w_{i-\tau-1}), \ldots, C(w_{i-1})}_{\text{given the same context}})$ such that

  $w_i = \operatorname{argmax}_x g$, e.g.,

$$
\begin{array}{lcr}
g(\text{“}dog\text{''}|\text{“a brown fox jumps over a lazy”}) & > & g(\text{“}penguin\text{''}|\text{“a brown fox jumps over a lazy”}) \\
g(\text{“}dog\text{''}|\text{“a brown fox jumps over a lazy”}) & > & g(\text{“}wheel\text{''}|\text{“a brown fox jumps over a lazy”}) \\
g(\text{“}dog\text{''}|\text{“a brown fox jumps over a lazy”}) & > & g(\text{“}homework\text{''}|\text{“a brown fox jumps over a lazy”}) \\
g(\text{“}dog\text{''}|\text{“a brown fox jumps over a lazy”}) & > & g(\text{“}salary\text{''}|\text{“a brown fox jumps over a lazy”}) \\
g(\text{“}dog\text{''}|\text{“a brown fox jumps over a lazy”}) & > & \ldots g \text{ on any constructed fake/negative examples} \ldots
\end{array}
$$

- $\mathcal{V}$ is the vocabulary. For computing sake, g is simplifed into $g(x, C(w_{i-\tau-1}), \ldots, C(w_{i-1}))$ where x is the index of the target word (correct or fake) in the vocabulary.

First neural language model (cont.)



- Bengio et al. (2003) used a feedforward network to do it.
- Just 3 layers:
  1. input (vectors of context words),
  2. middle,
  3. output (each neuron in the output corresponds to (the vocabulary index of) a word)
- g at "most computation here".

Note the subscripts are different.

Cost function of a neural language model

- Maximize the probablities of correct examples, e.g., $g(\text{“}dog''|\text{“a brown fox jumps over a lazy”})$
- Minimize those of all fake exampels, e.g., $g(\text{“}penguin''|\text{“a brown fox jumps over a lazy”})$, $g(\text{“}homework''|\text{“a brown fox jumps over a lazy”})$
- Put them all together: $J = g(\text{correct target}, context) - g(\text{fake target}, context)$ Max it!
- During the training, the g for correct targets grow, while the g for fake targets drop, because the C for them is being updated.
- Actually Bengio et al. 2003 has only the first term, lack of the part for fake targets.
- For computational stablity, usually $\log g$.
- How to generate fake samples? Let all words but “dog” to pair with context “a brown fox jumps over a lazy”?

Negative sampling

- Too many fake samples. For each correct target (e.g., "a quick brown fox jumps over a lazy dog"), we can have $|\mathcal{V}| - 1$ fake/negative samples (e.g., "a quick brown fox jumps over a lazy cat/penguin/car/code...").
- A better strategy is to just sample some of them.

Recap: Bengio 2003, first NNLM

- Words are represented into vectors using a look-up table (embedding matrix)
- The look-up table is updated using backpropgragation (thus word embeddings are updated)
- Context words are mapped together into the output layer
- Forward (not to be confused with feedforward): using history words to predict next word

Mnih & Hinton, ICML 2007, bi-linear word-interaction model

- The goal is still to use left history words w_1 to w_{n-1} to predict the n-th word w_n
- $$E(w_n; w_{1:n-1}) = -\left(\sum_{i=1}^{n-1} v_i^T R C_i\right) R^T v_n - bias$$

  where E is not error by energy, v_i is the embedding of the i-th word.
- R is the embedding matrix while C_i's are the weights of the language model.
- Bi-linear: embeddings of context words $v_i^T R$ are linear projected by C_i's, and then the summation of projections dot product with the embedding of the target word $R^T v_n$.
- Purely linear: faster than tanh used in Bengio 2003.

Collobert & Weston, ICML 2008, A Unified Architecture for Natural Language Processing: Deep Neural Network with Multitask Learning

- Lookup-table layer: embedding layer
- Convolutional layers to extract features
- TDNNs to deal with variable lengths of sentences.
- Position encoding: encode the distance between every word in the sentence and the word to be predicted

Separating word embeddings and language models

- All work up to this point tries to learn word embeddings and language models together: one network with both the projection/LUT layer and the language model layer (for the task).
- In Mokolov et al., ICASSP 2009, “Neural network based language models for higly inflective languages”, the authors noticed that separating the two can be better.
- This becomes the foundation of the word2vec.

Mokolov et al., ICASSP 2009, Neural network based language models for higly inflective languages

- Did not refer to Bengio's or Hinton's models at all.
- Words like "embedding" or "look-up table" do not appear in this paper.
- Just brutal force: one-hot encoding as the input, which does the propose of embedding layer (maybe) unintentionally.
- Per the authors, they did so hoping to form a clustering of word representations at the hidden layer, e.g., "see", "saw", "seen" would be mapped to similar vectors.
- Separating the training of word embeddings and language models.
- First train word embeddings by predicting the next word based on the previous word, called bigram network in the paper.
- Then train n-gram LM using the word embeddings just trained.

Some thoughts about research

- Mikolov et al. used extremely simple networks in their ICASSP 2009 paper.
- Nothing fancy like bi-linear interactions.
- Their terminology differs from those appearing in Bengio's or Hinton's.
- Their InterSpeech 2010 paper "Recurrent neural network based language model" is just Elman's network. Again, one-hot encoding as input.
- Hypothetically, if they submitted the papers to ACL/NAACL/EMNLP/COLING, what feedback would they receive? "Trivial model," "nothing new," "lack of comparison with X,Y,Z".
- The beauty of science is to make things simple.

Word2vec (2013)

- Two models: CBOW (similar to Bengio et al. 2003) and Skip-gram.
- CBOW: use context to predict target word.
- Skip-gram: use target word to predict context words:
- Very simple network architecture: For CBOW, see `https://www.tensorflow.org/tutorials/representation/word2vec` For Skip-gram, see `http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model/`
- For math, see `https://papers.nips.cc/paper/5021-distributed-representations-of-words-and-phras` `pdf`

Why word2vec succeeded

- Separating word embedding learning and language model learning.
- A super simple linear layer – faster training.
- Skip-gram – updating the embedding of only one word each time.
- Bidirectional context: forward and backword

Limitations of word2vec

- It only uses local context information.
- However, some local context words do not contain much semantics of the center word, e.g., “the” in “The cat sat on the mat,” because they have lots co-occurence with other words.
- Solution: remove stop words from the corpus.
- But that’s arbitrary and relies on manual rules.
- Better solution: make use of word-word co-occurence in a global scope.

GloVe

- We have seen two approaches to word embedding: factorization on co-cooccurence matrixes and neural network-based embedding using local context.
- GloVe combines the benefit of the two.
- Key observation: ratios of co-occurrence probabilities reveal semantics better than co-occurence probabilities.

Probability and Ratio	$k = solid$	$k = gas$	$k = water$	$k = fashion$
$P(k\|ice)$	1.9×10^{-4}	6.6×10^{-5}	3.0×10^{-3}	1.7×10^{-5}
$P(k\|steam)$	2.2×10^{-5}	7.8×10^{-4}	2.2×10^{-3}	1.8×10^{-5}
$P(k\|ice)/P(k\|steam)$	8.9	8.5×10^{-2}	1.36	0.96

- "water" and "fashion" both have little power to tell the difference between "ice" and "steam": ratios around 1. They are both about water and have little connection with fashion. But "solid" and "gas" can: ratios much larger or smaller than 1.
- $P(k|\text{ice})P(k|\text{steam}) >> 1$ if k is closer to "ice", and $<< 1$ if k is closer to "steam".
- Challenge: how to define a loss function to achieve the t

The loss of function of GloVe

- GloVe wants to archive a relation F of two words w_i and w_j and a context word w'_k ($'$ indicating the context word, not an operation) such that $F(w_i, w_j, w'_k) = P_{ik}/P_{jk}$ where P_{ik} and P_{jk} are the co-occuring probability of w_i and w'_k and that of w_j and w'_k.
- First, the difference between w_i and w_j is expected to be characterized linearly. The simplest linear difference is vector subtraction. Hence, $F(w_i - w_j, w'_k) = P_{ik}/P_{jk}$. F is overloaded.
- Second, the difference $w_i - w_j$ with respect to the context word w'_k to be linearly characterized as well. The simplest form is dot product. Hence, $F((w_i - w_j)^T w'_k) = P_{ik}/P_{jk}$. F is overloaded again.
- Third, we want to characterize the difference between any two words using their co-occourence, regardless of whether a word is a context word or not. Hence, we want $F((w_i - w_j)^T w'_k) = F(w_i^T w'_k + (-w_j^T w'_k)) = F(w_i^T w'_k) \circ F(-w_j^T w'_k)$ where $\circ$ is an operation to be found. Such an F is known as a group homomorphism in discrete math.

The loss of function of GloVe II

- We can make $\circ$ to be super simple, just multiplication. Thus, $F((w_i - w_j)^T w'_k) = F(w_i^T w'_k) \cdot F(-w_j^T w'_k)$ Then F is a homomorphism between groups $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \times)$.
- Exponential functions are such homomorphism, i.e., $e^{a+b} = e^a \cdot e^b$, thus $F = \exp$.
- Based on the definition, $F((w_i - w_j)^T w'_k) = P_{ik}/P_{jk}$ and $F((w_j - w_i)^T w'_k) = P_{jk}/P_{ik}$ (i and j flipped in the second equation). Their product $F(x)F(-x) = \frac{P_{ik}}{P_{jk}}\frac{P_{jk}}{P_{ik}} = 1$ or $F(-x) = \frac{1}{F(x)}$.
- Using this property, we have $F(w_i - w_j)^T w'_k) = F(w_i^T w'_k) \cdot F(-w_j^T w'_k) = \frac{F(w_i^T w'_k)}{F(w_j^T w'_k)}$.

The loss of function of GloVe III

- Recalling that $F((w_i - w_j)^T w'_k) = P_{ik}/P_{jk}$, we have $F(w_i^T w'_k) = \exp(w_i^T w'_k) = P_{ik} = X_{ik}/Xi$ where X_{ik} is the global cooccurence of w_i and w_k and X_i is the global occurence of w_i.
- Log on both sides, we have $w_i^T w'_k = \log X_{ik} - \log X_i$ where $\log X_i$ has nothing to do with w'_k and hence is absorbed into a bias: $w_i^T w'_k = \log X_i k + b_i$.
- Last tuning: the authors want the formula above to be symmetric to both w'_k and w_i, thus the bias term is not only there for non-context word. Hence they add a bias for the context word: $w_i^T w'_k = \log X_{ik} + b_i + b'_k$.
- Then the loss function is $(\log X_{ik} - w_i^T w'_k - b_i - b'_k)^2$ and the goal is to minimize it.
- Not really. One more thing.

The loss of function of GloVe IV

- Word pairs have different frequencies in a corpus. So they should have different contributions to the loss function.
- $J = \sum_{i,j=1}^{|\mathcal{V}|} W(X_{i,j})(\log X_{ij} - w_i^T w_j - b_i - b_j)^2$
- Two goals of the weight function W: $W(X_{i,j})$ cannot be too large if $X_{i,j}$ is small whereas it cannot be too large also for frequenty w_i and w_j pairs.
- An implementation:

$$W(X_{i,j}) = \begin{cases} (X_{i,j}/X_{max})^{\alpha} & if \;\; X_{i,j} < X_{max} \\ 1 & o/w \end{cases}$$

  where X_{max} is the maximal cooccurence of two words in the corpus.
- Empirical study finds that $\alpha = 3/4$ is a good number.
- See also: `http://mlexplained.com/2018/04/29/paper-dissected-glove-global-vectors-for-word-repre` and `http://text2vec.org/glove.html`

Sentence embedding

- DAN
- Skip-thought
- Transformer