Math Notes

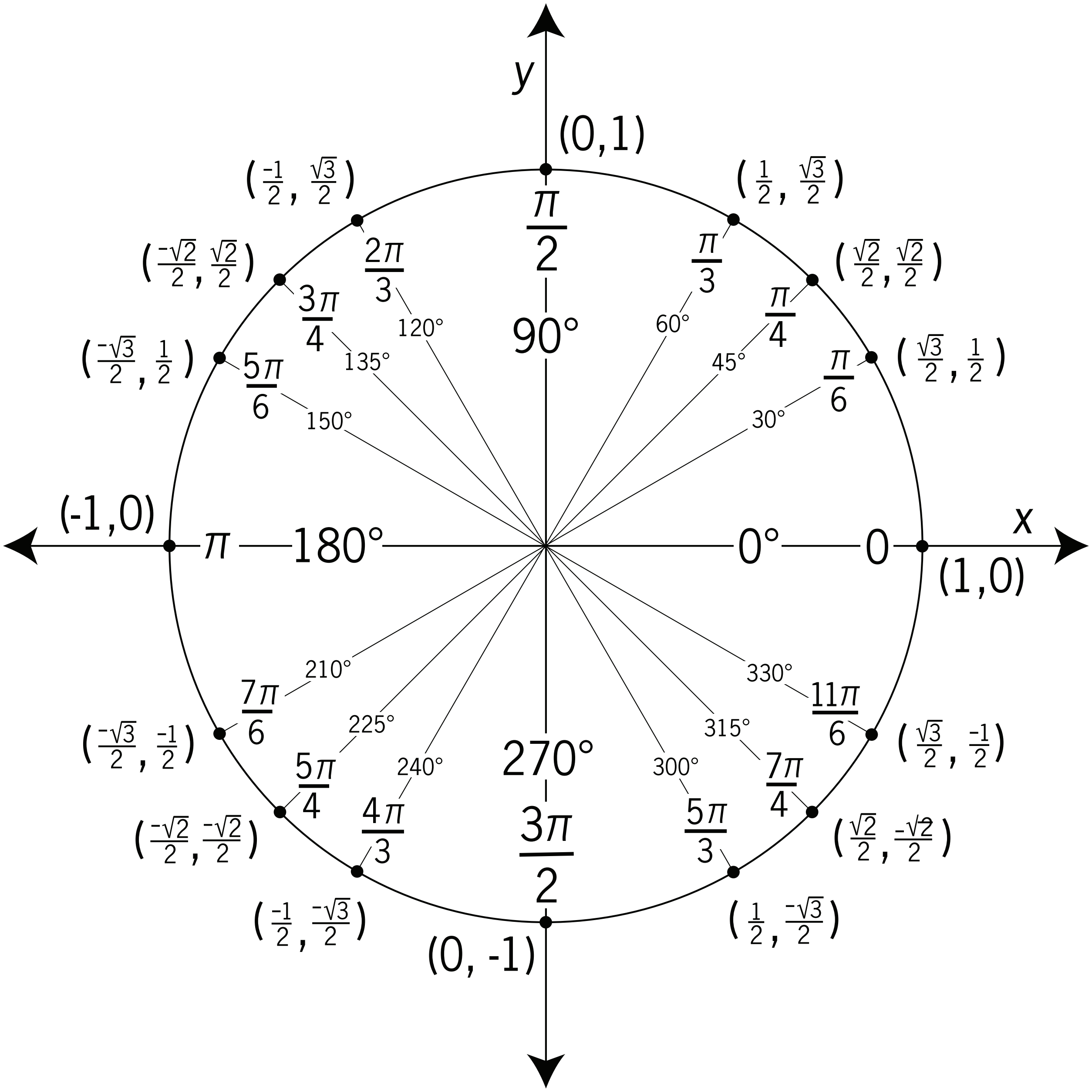
K

March 6, 2020

Contents

1 Trigonometric Concepts

1.1 Unit Circle



Source: https://etc.usf.edu/clipart/43200/43215/unit-circle7_43215.htm

1.2 Basic Trigonometry

$$sin(\theta) = \frac{Opposite}{Hypotenuse} = \frac{y}{1} = y \qquad cos(\theta) = \frac{Adjacent}{Hypotenuse} = \frac{x}{1} = x \qquad tan(\theta) = \frac{Opposite}{Adjacent} = \frac{y}{x} = \frac{sin(\theta)}{cos(\theta)}$$

$$csc(\theta) = \frac{1}{sin(\theta)} \qquad sec(\theta) = \frac{1}{cos(\theta)} \qquad cot(\theta) = \frac{cos(\theta)}{sin(\theta)}$$

1.3 Pythagorean Theorem Identities

$$sin^2\theta + cos^2\theta = 1$$
$$tan^2\theta + 1 = sec^2\theta$$
$$1 + cot^2\theta = csc^2\theta$$

1.4 Half Angle Identities

$$cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+cos(\theta)}{2}} \qquad tan(\frac{\theta}{2}) = \pm\sqrt{\frac{1-cos(\theta)}{2}}$$
$$sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-cos(\theta)}{2}} \qquad \quad = \frac{sin(\theta)}{1+cos(\theta)} = \frac{1-cos(\theta)}{sin(\theta)}$$

1.5 Double Angle Identities

$$\begin{aligned} sin(2\theta) &= 2sin\theta cos\theta \\ cos(2\theta) &= cos^2\theta - sin^2\theta \\ &= 2cos^2\theta - 1 \\ &= 1 - 2sin^2\theta \\ tan(2\theta) &= \frac{2tan\theta}{1-tan^2\theta} \end{aligned}$$

1.6 Power Reduction Identities

$$sin^2\theta = \frac{1 - cos2\theta}{2} \qquad csc^2\theta = \frac{2}{1 - cos2\theta}$$

$$cos^2\theta = \frac{1 + cos2\theta}{2} \qquad sec^2\theta = \frac{2}{1 + cos2\theta}$$

$$tan^2\theta = \frac{1 - cos2\theta}{1 + cos2\theta} \qquad cot^2\theta = \frac{1 + cos2\theta}{1 - cos2\theta}$$

2 Trigonometric Substitution

Expression	Substitution	Identity
$\sqrt{a^2 - b^2x^2}$	$x = \frac{a}{b}sin\theta$	$1 - sin^2\theta = cos^2\theta$
$\sqrt{a^2 + b^2x^2}$	$x = \frac{a}{b}tan\theta$	$1 - tan^2\theta = sec^2\theta$
$\sqrt{b^2x^2 - a^2}$	$x = \frac{a}{b}sec\theta$	$sec^2\theta - 1 = tan^2\theta$

3 Table of Basic Derivatives

y	$\frac{dy}{dx}$
C	0
x	1
$ax^2 + bx + c$	$2ax + b$
x^n	nx^{n-1}
$x^{-1}, \frac{1}{x}$	$-\frac{1}{x^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
$ln(x)$	$\frac{1}{x}$
$log_a(x)$	$\frac{1}{xln(a)}$
a^x	$a^x ln(a)$
e^x	e^x
$sin(x)$	$cos(x)$
$cos(x)$	$-sin(x)$
$tan(x)$	$\frac{1}{cos^2x}, sec^2x$
$cot(x)$	$-\frac{1}{sin^2x}, -csc^2x$
$sec(x)$	$tan(x)sec(x)$
$csc(x)$	$-cot(x)csc(x)$

4 Table of Basic Integrals

$f(x)$	$\int f(x)dx = F(x) + C$
x^α $\alpha \neq 0$	$\frac{x^{\alpha+1}}{\alpha+1} + C$
$sin(kx)$	$-\frac{cos(kx)}{k} + C$
$cos(kx)$	$\frac{sin(kx)}{k} + C$
$sec^2(kx)$	$\frac{tan(kx)}{k} + C$
$csc^2(kx)$	$-\frac{cot(kx)}{k+C}$
e^{kx}	$\frac{e^{kx}}{k} + C$
$x^-1, \frac{1}{x}$	$ln\|x\| + C$
$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1} + C$
$\frac{1}{1+x^2}$	$tan^{-1}x + C$
a^{kx}	$\frac{1}{klna}a^{kx} + C$
$\frac{1}{\sqrt{a^2-x^2}}$	$sin^{-1}\frac{x}{a} + C$
$\frac{1}{\sqrt{a^2+x^2}}$	$\frac{1}{a}tan^{-1}(x/a) + C$

5 Substitution Techniques

5.1 U Substitution

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$
$$\text{where: } u = g(x), du = g'(x)$$

5.2 Integration by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$
$$\int udv = uv - \int vdu$$

$$\int_a^b f(x)g'(x) = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)dx$$

6 Engineering Formulas

6.1 Spring Formulas

$$F = kx$$
$$W = \int_a^b kxdx$$

$$F = \text{Force (Newtons [N])}$$
$$k = \text{spring constant (Newton meters } {}^{N}/_{m}\text{)}$$
$$x = \text{change in distance (meters [m])}$$
$$W = \text{Work (Joules [J])}$$
$$a = \text{initial length (meters [m])}$$
$$b = \text{final length (meters [m])}$$

6.2 Fluid Formulas

$$W = F \cdot d = \int F dx$$
$$V = \pi r^2 h (\text{apply to cylinders})$$
$$F = m \cdot a = V \cdot \rho$$

$$W = \text{Weight ()}$$
$$F = \text{Force (Newtons [N])}$$
$$d = \text{distance (meters [d])}$$
$$m = \text{mass (meters}^3 [m^3])$$
$$a = \text{acceleration (meters per second}^2\ [{}^m/{}_{s^2}])$$
$$\rho = \text{Something}$$

7 Method of Partial Fractions

$$\int \frac{P_n(x)}{Q_m(x)} dx \text{ when } m > n$$

n and m are defined as the degree of the numerator and the denominator.

7.1 Decomposition Types

Type	Factor Example	Decomposition
Linear Factor	$(x-4)$	$\frac{A}{x-4}$
Repeated Linear Factor	$(x-4)^2$	$\frac{A}{(x+4)} + \frac{B}{(x+4)^2}$
Quadratic Irreducible Factor	(x^2+4)	$\frac{Ax+B}{x^2+4}$
Repeated Quadratic Irreducible Factor	$(x^2+4)^2$	$\frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$

7.2 Numerical Integration

7.2.1 Midpoint Rule

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(\overline{x_i})\Delta x$$
$$x_i* = \frac{x_{i-1} + x_i}{2} = \overline{x_i}$$
$$x_i* \in [x_{i-1}, x]$$

7.2.2 Trapezoidal Rule

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x$$
$$\approx \sum_{i=1}^{n} \frac{\Delta x}{2} [f(x_{i-1}) + f(x_i)]$$

7.2.3 Error Approximation (Actual Error)

$$E_T = \int_a^b f(x)dx - T_n$$

$$E_H = \int_a^b f(x)dx - M_n$$

7.2.4 Error Bounds

$$|f''(x)| \leq K \text{ on } [b, a] f(x) = \frac{1}{x}, f''(x) = -\frac{2}{x^3}$$
$$\left| -\frac{2}{x^3} \right| \leq 2 = K$$
$$|E_T| \leq \frac{K(b-a)^3}{12n^2} = \frac{2(2-1)^3}{12 \cdot 5^2} = 0.0012$$
$$|E_M| \leq \frac{K(b-a)^3}{24n^2} = \frac{2(2-1)^3}{24 \cdot 5^2} = 0.0012$$
$$E_T \leq 0.6931 - 0.6956$$

7.3 Improper Integrals

7.3.1 Type 1

$x \in [a, b]$, $f(x)$ is continuous on $[a, b)$ at $x = b$ $f(x)$ discontinues

$$\int_a^\infty f(x) = \lim_{x \to \infty} \left(\int_a^b f(x)dx \right)$$

7.3.2 Type 2

$f(x)$ dicontinues at $x = a$, continuous on $(a, b]$

$$\int_a^b f(x)dx = \lim_{t \to\ a} \int_t^b f(x)dx$$

7.3.3 Type 3

$f(x)$ discontinues at $x = c$, $a < c < b$ everywhere else on $[a, b]$
$f(x)$ is continuous

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

7.4 The Length of the Curve

$x = x(t) \qquad y = y(t) \qquad a \leq t \leq b$

$$S' = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}$$

8 Area of Revolution

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$dA = 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$1) \int_a^b dA = A = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$2) x = xb$$

9 Separable Equations

$$y'(x) = g(x)h(y), \frac{dy}{dx} = g(x)h(y)\Big| dx$$

$$dy = h(y)g(x)dx \Big| \frac{1}{h(y)}$$

$$\frac{1}{h(y)} dy = g(x)dx, H(y) = \frac{1}{h(y)} \longrightarrow H(y)dy = g(x)dx$$

$$\int H(y) = \int g(x)dx + C$$

Implicit Form of Solution — $F(y) = G(x) + C$

Explicit Form of Solution — $y = F^{-1}(G(x) + C)$

10 Additional Resources

Print
Calculus Study Guide: https://mt-jfk.com/ap-calculus-study-guide.pdf

Video
The Organic Chemistry Tutor: https://www.youtube.com/channel/UCEWpbFLzoYGPfuWUMFPSaoA
Black Pen Red Pen: https://www.youtube.com/user/blackpenredpen