Circuit Analysis

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1. ***DC CIRCUITS***

1.1. **RESISTOR ONLY CIRCUITS**

1.1.1. -What does it mean for a resistor to be in series or in parallel?

SERIES

Components that share a single node *only with* each other.

PARALLEL

Components that share two nodes between each other.

1.1.2. -How do you combine resistors that are in parallel or in series?

FOR SERIES	**FOR PARALLEL**
Addition	Inverse Addition/Product-Over-Sum
$R_{Total} = R_1 + R_2 + R_3 ... R_Z$	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} ... \frac{1}{R_Z}$
	OR

$$\overbrace{R_T = \frac{R_1 R_2}{R_1 + R_2}}^{\text{For Combining Two Resistors At A Time Only}}$$

1.1.3. -How do you perform calculations for basic circuits?

For basic circuits, we use the following laws:

KVL

$$\text{In a closed loop:}$$
$$\sum_0^N V_x = 0$$
$$\text{N is the number of voltage drops}$$

KCL

$$\text{For a node:}$$
$$\sum_0^N I_x = \sum_0^K I_y$$
$$\text{N and K are the number of currents}$$
$$\text{going into the node and out of the node, respectively.}$$

These formulas are exclusive to TWO resistors circuit:

If you know the total voltage drop between two components in series and their resistances
You can find the individual voltage drops of each component:

$$V_1 = \frac{V_T R_1}{R_1 + R_2}$$

If you know the total current between two components in parallel and their resistances
You can find the individual current going throuigh each component:

$$C_2 = \frac{C_T R_1}{R_1 + R_2}$$

We can also change the circuits a bit to make calculations a bit easier

A voltage source in series with a resistor is equal to a current source in parallel with the same value resistor.
Thus we just do a simple ohm's law and change values and the placements accordingly.

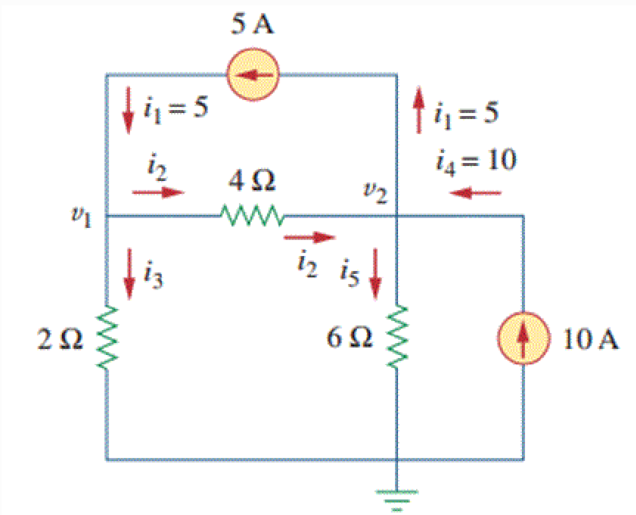
$$I_S = \frac{V_S}{R_{series/parallel}}$$

1.1.4. -Are there any tricks to do calculations faster on more complicated circuits?

There are two main methods for solving complicated circuits: **Nodal Analysis** or **Mesh Analysis**.

Nodal Analysis

finds the node voltages of the circuit through equation manipulation of Kirchoff's Current Law.



Instructions:

1. Decide on an arbitrary current direction for each component.
2. Create KCL equations at each node. For X complex nodes, you need atleast X-1 equations.
3. For currents not already given, transform each current into the following equation:

$$I = \frac{V_{high} - V_{low}}{R}$$

Note: Current goes from High Voltage to Low Voltage.

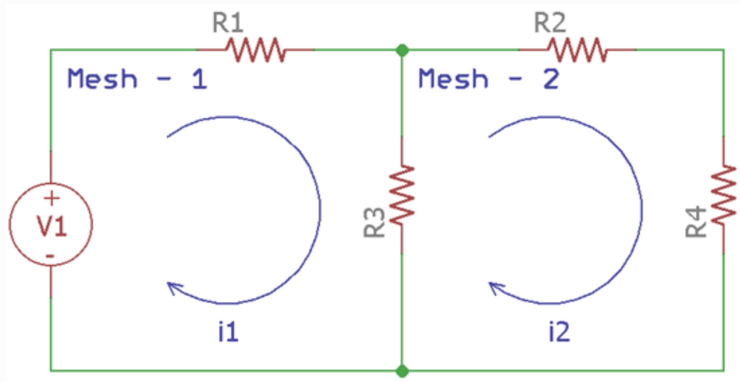
SPECIAL CASES

Voltage source between two unknown nodes:

Treat the voltage source as one giant node

Mesh Analysis

finds the currents of the circuit through equation manipulation of Kirchoff's Voltage Law.



Instructions:

1. For every mesh, create an arbitrary current loop. All loops must go in the same direction.
2. Do a KVL for each loop replacing every voltage drop with the following equation:

$$V = R_1 \times \underbrace{(I_{IN} - I_{OUT})}_{I_{IN} \text{ is the current loop you're working on}}$$

NOTE: If there is a current source and it is *ONLY* part of one loop, the current source is the value of the loop when you align the directions of the loop.

SPECIAL CASES

Current source in between two meshs:

Combine the two meshs together and do the same steps but add this extra equation:

$$\underbrace{I_{source} = I_1 - I_2}_{I_1 \text{ should be the loop going in the same direction as the source.}}$$

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and do the KCL for it. In addition to making one more equation where

$$V_{source} = V_{high} - V_{low}$$

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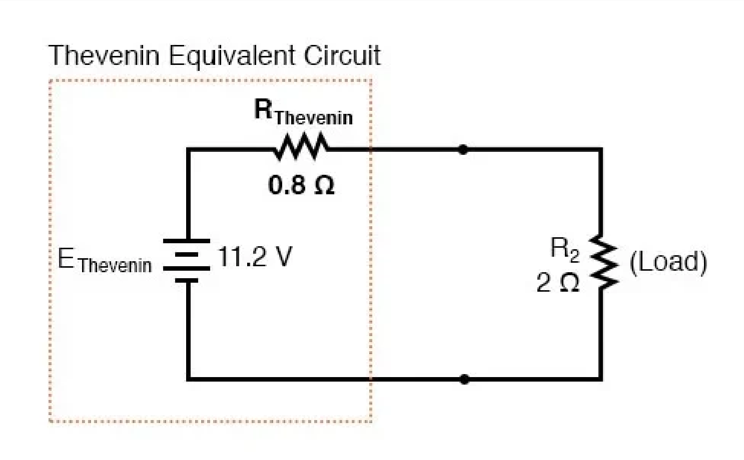
4. Solve the system of equations in order to find each nodal voltage.

3. Solve the system of equations to find the current of each loop.
For N meshs you need N equations.

1.1.5. -Are there any circuit simplification tricks to make future calculations easier?

Convert the circuit into it's **Thevenin** or **Norton Equivalent**

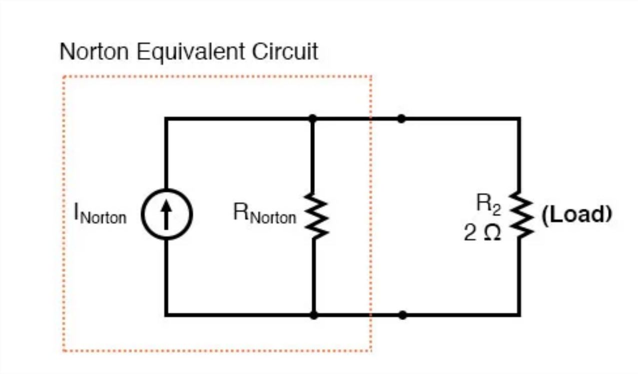
Thevenin Circuit:



Finding $V_{Thevnin}$ or V_{th}.

At the point where there is going to be changes, find the open circuit voltage drop at that point.

Norton Circuit:



Finding I_N:

At the points of change, find the short circuit current.

Finding R_{th}:

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1. Turn off all sources *except* dependent sources.
2. At the same open circuit as before, now find the R_{eq}.

Special Cases:

The circuit has dependent sources.

1. Put a voltage/current source at the open circuit.
    - Voltage Source: If it will be in series with the other components.
    - Current Source: If it will be in parallel with the other components.
2. Find the voltage of the source if you put a current source or the current if you put a voltage source.
3. $$R_{th} = \frac{V_T}{I_T}$$

1.2. **CAPACITOR/INDUCTOR CIRCUITS**

1.2.1. How do I simplify Capacitors/Inductors?

SERIES:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ... \frac{1}{C_Z}$$

OR

$$\overbrace{C_T = \frac{C_1 C_2}{C_1 + C_2}}^{\text{For Combining Two Capacitors At A Time Only}}$$

PARALLEL:

$$C_{Total} = C_1 + C_2 + C_3 ... C_Z$$

Inductors simplify the exact same way as resistors.

1.2.2. How to perform calculations with capacitors?

To find the Energy stored in a capacitor/Inductor:

$$U = \frac{1}{2} C V^2 = \frac{1}{2} L I^2$$

To find the current of a capacitor/Inductor:

$$I = C \frac{dv}{dt} = \frac{1}{L} \int V dt$$

To find the voltage of a capacitor/Inductor:

$$V = \frac{1}{C} \int I dt = L \frac{dI}{dt}$$

1.2.3. How to do calculations when there are switches?

Main Formula:

$$V = V(\infty) + [V(0) - V(\infty)]e^{\frac{t}{\tau}}$$

NOTE: You can use interchange V with I

$$\tau = CR = \frac{L}{R}$$

2. ***AC CIRCUITS***

2.1 RESISTOR, CAPACITOR, AND INDUCTOR CIRCUITS

2.1.1 What is the difference between AC and DC, mathematically?

Everything basically acts like a resistor and has an impedance(Z) instead of a resistance.
To make the math simplier, we transform the equations from trigonometry to the phasor/rectangular.

$$\omega = 2\pi f$$

$$\underbrace{Acos(\omega x + \phi)}_{\text{Trigonometric}} = \underbrace{A\angle\phi}_{\text{Phasor}} = \underbrace{Acos(\phi) + jsin(\phi)}_{\text{Rectangular}}$$

Resistance Conversion to Impedance

Resistors	Capacitors	Inductors
$Z = R$	$Z = \frac{-j}{\omega C}$	$Z = jwL$

You combine Impedance together as if they are resistors. The mathematics is almost the same as DC circuits, the only difference is that you have imaginary numbers.