Math Notes

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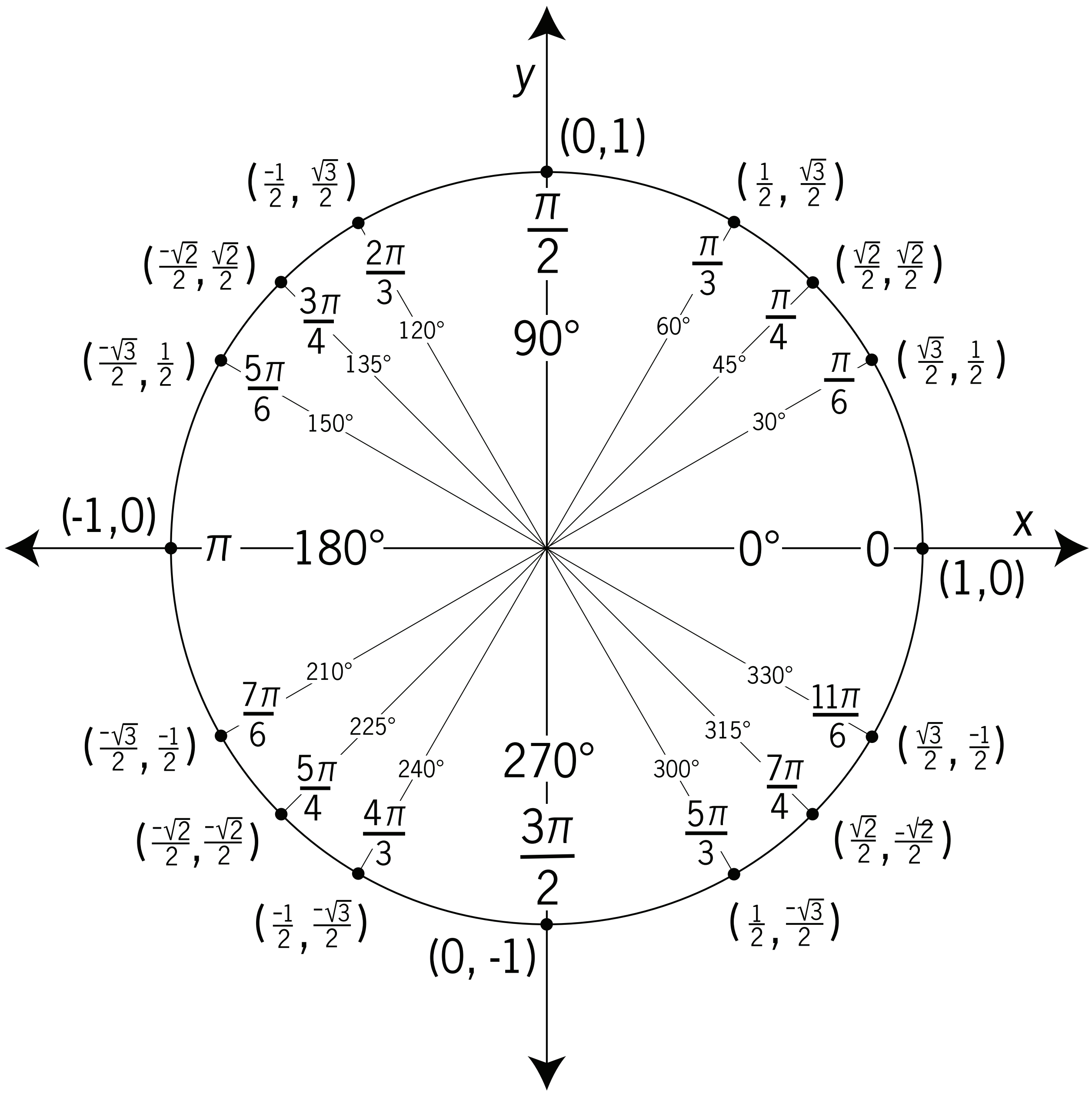
March 6, 2020

Contents

1 Trigonometric Concepts

1.1 Unit Circle



Source: https://etc.usf.edu/clipart/43200/43215/unit-circle7_43215.htm

1.2 Basic Trigonometry

$$sin(\theta) = \frac{Opposite}{Hypotenuse} = \frac{y}{1} = y \qquad cos(\theta) = \frac{Adjacent}{Hypotenuse} = \frac{x}{1} = x \qquad tan(\theta) = \frac{Opposite}{Adjacent} = \frac{y}{x} = \frac{sin(\theta)}{cos(\theta)}$$

$$csc(\theta) = \frac{1}{sin(\theta)} \qquad sec(\theta) = \frac{1}{cos(\theta)} \qquad cot(\theta) = \frac{cos(\theta)}{sin(\theta)}$$

1.3 Pythagorean Theorem Identities

$$\begin{aligned} sin^2\theta + cos^2\theta &= 1 \\ tan^2\theta + 1 &= sec^2\theta \\ 1 + cot^2\theta &= csc^2\theta \end{aligned}$$

1.4 Half Angle Identities

$$\begin{aligned} cos(\frac{\theta}{2}) &= \pm\sqrt{\frac{1+cos(\theta)}{2}} \\ sin(\frac{\theta}{2}) &= \pm\sqrt{\frac{1-cos(\theta)}{2}} \end{aligned} \qquad \begin{aligned} tan(\frac{\theta}{2}) &= \pm\sqrt{\frac{1-cos(\theta)}{2}} \\ &= \frac{sin(\theta)}{1+cos(\theta)} \\ &= \frac{1-cos(\theta)}{sin(\theta)} \end{aligned}$$

1.5 Double Angle Identities

$$\begin{aligned} sin(2\theta) &= 2sin\theta cos\theta \\ cos(2\theta) &= cos^2\theta - sin^2\theta \\ &= 2cos^2\theta - 1 \\ &= 1 - 2sin^2\theta \\ tan(2\theta) &= \frac{2tan\theta}{1-tan^2\theta} \end{aligned}$$

1.6 Power Reduction Identities

$$sin^2\theta = \frac{1-cos2\theta}{2} \qquad csc^2\theta = \frac{2}{1-cos2\theta}$$
$$cos^2\theta = \frac{1+cos2\theta}{2} \qquad sec^2\theta = \frac{2}{1+cos2\theta}$$
$$tan^2\theta = \frac{1-cos2\theta}{1+cos2\theta} \qquad cot^2\theta = \frac{1+cos2\theta}{1-cos2\theta}$$

2 Trigonometric Substitution

Expression	Substitution	Identity
$\sqrt{a^2-b^2x^2}$	$x = \frac{a}{b}sin\theta$	$1 - sin^2\theta = cos^2\theta$
$\sqrt{a^2+b^2x^2}$	$x = \frac{a}{b}tan\theta$	$1 - tan^2\theta = sec^2\theta$
$\sqrt{b^2x^2-a^2}$	$x = \frac{a}{b}sec\theta$	$sec^2\theta - 1 = tan^2\theta$

3 Table of Basic Derivatives

y	$\frac{dy}{dx}$
C	0
x	1
ax^2+bx+c	$2ax+b$
x^n	nx^{n-1}
$x^{-1}, \frac{1}{x}$	$-\frac{1}{x^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
$ln(x)$	$\frac{1}{x}$
$log_a(x)$	$\frac{1}{xln(a)}$
a^x	$a^x ln(a)$
e^x	e^x
$sin(x)$	$cos(x)$
$cos(x)$	$-sin(x)$
$tan(x)$	$\frac{1}{cos^2x}, sec^2x$
$cot(x)$	$-\frac{1}{sin^2x}, -csc^2x$
$sec(x)$	$tan(x)sec(x)$
$csc(x)$	$-cot(x)csc(x)$

4 Table of Basic Integrals

$f(x)$	$\int f(x)dx = F(x)+C$
x^α $\alpha \neq 0$	$\frac{x^{\alpha+1}}{\alpha+1}+C$
$sin(kx)$	$-\frac{cos(kx)}{k}+C$
$cos(kx)$	$\frac{sin(kx)}{k}+C$
$sec^2(kx)$	$\frac{tan(kx)}{k}+C$
$csc^2(kx)$	$-\frac{cot(kx)}{k+C}$
e^{kx}	$\frac{e^{kx}}{k}+C$
$x^-1, \frac{1}{x}$	$ln\|x\|+C$
$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1}+C$
$\frac{1}{1+x^2}$	$tan^{-1}x+C$
a^{kx}	$\frac{1}{klna}a^{kx}+C$
$\frac{1}{\sqrt{a^2-x^2}}$	$sin^{-1}\frac{x}{a}+C$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}tan^{-1}(x/a)+C$

5 Substitution Techniques

5.1 U Substitution

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$
$$\text{where: } u = g(x), du = g'(x)$$

5.2 Integration by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$
$$\int udv = uv - \int vdu$$

$$\int_a^b f(x)g'(x) = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)dx$$

6 Engineering Formulas

6.1 Spring Formulas

$$F = kx$$
$$W = \int_a^b kxdx$$

F = Force (Newtons [N])
k = spring constant (Newton meters $^N/_m$)
x = change in distance (meters [m])
W = Work (Joules [J])
a = initial length (meters [m])
b = final length (meters [m])

6.2 Fluid Formulas

$$W = F \cdot d = \int F dx$$
$$V = \pi r^2 h \text{(apply to cylinders)}$$
$$F = m \cdot a = V \cdot \rho$$

$$W = \text{Weight ()}$$
$$F = \text{Force (Newtons [N])}$$
$$d = \text{distance (meters [d])}$$
$$m = \text{mass (meters}^3 [m^3])$$
$$a = \text{acceleration (meters per second}^2\ [{}^m/{}_{s^2}])$$
$$\rho = \text{Something}$$

7 Method of Partial Fractions

$$\int \frac{P_n(x)}{Q_m(x)} dx \text{ when } m > n$$

n and m are defined as the degree of the numerator and the denominator.

7.1 Decomposition Types

Type	Factor Example	Decomposition
Linear Factor	$(x-4)$	$\frac{A}{x-4}$
Repeated Linear Factor	$(x-4)^2$	$\frac{A}{(x+4)} + \frac{B}{(x+4)^2}$
Quadratic Irreducible Factor	(x^2+4)	$\frac{Ax+B}{x^2+4}$
Repeated Quadratic Irreducible Factor	$(x^2+4)^2$	$\frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$

7.2 Numerical Integration

7.2.1 Midpoint Rule

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(\overline{x_i})\Delta x$$
$$x_i* = \frac{x_{i-1} + x_i}{2} = \overline{x_i}$$
$$x_i* \in [x_{i-1}, x]$$

7.2.2 Trapezoidal Rule

$$\begin{aligned}\int\limits_a^b f(x)dx &\approx \sum_{i=1}^{n} \frac{f(x_{i-1})+f(x_i)}{2}\Delta x\\ &\approx \sum_{i=1}^{n} \frac{\Delta x}{2}[f(x_{i-1})+f(x_i)]\end{aligned}$$

7.2.3 Error Approximation (Actual Error)

$$E_T = \int\limits_a^b f(x)dx - T_n$$

$$E_H = \int\limits_a^b f(x)dx - M_n$$

7.2.4 Error Bounds

$$|f''(x)| \leq K \text{ on } [b,a] f(x) = \frac{1}{x}, f''(x) = -\frac{2}{x^3}$$

$$\left|-\frac{2}{x^3}\right| \leq 2 = K$$

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} = \frac{2(2-1)^3}{12 \cdot 5^2} = 0.0012$$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2} = \frac{2(2-1)^3}{24 \cdot 5^2} = 0.0012$$

$$E_T \leq 0.6931 - 0.6956$$

7.3 Improper Integrals

7.3.1 Type 1

$x \in [a,b]$, $f(x)$ is continuous on $[a,b)$ at $x = b$ $f(x)$ discontinues

$$\int\limits_a^\infty f(x) = \lim_{x\to\infty}\left(\int\limits_a^b f(x)dx\right)$$

7.3.2 Type 2

$f(x)$ sicontinues at $x = a$, continuous on $(a,b]$

$$\int\limits_a^b f(x)dx = \lim_{t\to\ a}\int\limits_t^b f(x)dx$$

7.3.3 Type 3

$f(x)$ discontinues at $x = c$, $a < c < b$ everywhere else on $[a, b]$
$f(x)$ is continuous

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

7.4 The Length of the Curve

$x = x(t)$ $\qquad$ $y = y(t)$ $\qquad$ $a \leq t \leq b$

$$S' = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}$$

8 Additional Resources

Print

Calculus Study Guide: https://mt-jfk.com/ap-calculus-study-guide.pdf

Video

The Organic Chemistry Tutor: https://www.youtube.com/channel/UCEWpbFLzoYGPfuWUMFPSaoA
Black Pen Red Pen: https://www.youtube.com/user/blackpenredpen