

Math Notes

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1 Time Analysis

1.1 Logarithms

<http://dl.uncw.edu/digilib/Mathematics/Algebra/mat111hb/EandL/logprop/logprop.html>

$$\begin{aligned}\log_a(uv) &= \log_a u + \log_a v \\ \ln(uv) &= \ln u + \ln v\end{aligned}$$

$$\begin{aligned}\log_a\left(\frac{u}{v}\right) &= \log_a u - \log_a v \\ \ln\left(\frac{u}{v}\right) &= \ln u - \ln v\end{aligned}$$

$$\begin{aligned}\log_a u^n &= n \log_a u \\ \ln u^n &= n \ln u\end{aligned}$$

2 Classification of Edges

- **Tree Edge** (u, v) if v was WHITE while exploring (u, v)

$$v = \text{Adj}[u][v]$$

(u, v) is a Tree Edge if v is white

- discover a tree for the first time
- A DFS tree consists of Tree edges and nodes involved

- **Back Edge** (u, v) connects vertex u to its ancestor v in a depth-first tree (v is GRAY)
 - If v is Grey, then (u, v) is Back Edge
- **Forward Edge** (u, v) is a non-tree edge connecting u to a descendant v in a depth-first tree (v is BLACK)

- **Cross Edge** (u, v) is all other edges
- **Summary**
 - **Tree Edge** (u, v) (v is WHITE)
 - **Back Edge** (u, v) (v is GRAY)
 - **Forward Edge** (u, v) (v is Black, $stamps[u].d < stamps[v].d$)
 - **Cross Edge** (u, v) (v is Black, $stamps[u].d > stamps[v].d$)
- All nodes are white initially
- Given a directed graph $G = (V, E)$, write a function **isCycle()** that returns true if the given graph has a cycle
 - If we discover a Grey node of by DFS, then there is a cycle in the graph