

Math Notes

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1 Table of Integrals

1.1 Basic Integrals

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
2. $\int \frac{du}{u} = \ln |u| + C$
3. $\int e^u du = e^u + C$
4. $\int a^u du = \frac{a^u}{\ln a} + C$
5. $\int \sin u du = -\cos u + C$
6. $\int \cos u du = \sin u + C$
7. $\int \sec^2 u du = \tan u + C$
8. $\int \csc^2 u du = -\cot u + C$
9. $\int \sec u \tan u du = \sec u + C$
10. $\int \csc u \cot u du = -\csc u + C$
11. $\int \tan u du = \ln |\sec u| + C$
12. $\int \cot u du = \ln |\sin u| + C$
13. $\int \sec u du = \ln |\sec u + \tan u| + C$
14. $\int \csc u du = \ln |\csc u + \cot u| + C$
15. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$
16. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
17. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$

1.2 Trigonometric Integrals

1. $\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
2. $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
3. $\int \tan^2 u du = \tan u - u + C$
4. $\int \cot^2 u du = -\cot u - u + C$
5. $\int \sin^3 u du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$
6. $\int \cos^3 u du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$

7. $\int \tan^3 u du = \frac{1}{2} \tan^2 u \ln |\cos u| + C$
8. $\int \cot^3 u du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$
9. $\int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$
10. $\int \csc^3 u du = -\frac{1}{2} \sec u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$
11. $\int \sin^n u du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u du$
12. $\int \cos^n u du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u du$
13. $\int \tan^n u du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u du$
14. $\int \cot^n u du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u du$
15. $\int \sec^n u du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u du$
16. $\int \csc^n u du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u du$
17. $\int \sin au \cos b u du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
18. $\int \cos au \cos b u du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
19. $\int \sin au \cos b u du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\cos(a+b)u}{2(a+b)} + C$
20. $\int u \sin u du = \sin u - u \cos u + C$
21. $\int u \cos u du = \cos u + u \sin u + C$
22. $\int u^n \sin u du = -u^n \cos u + n \int u^{n-1} \cos u du$
23. $\int u^n \cos u du = u^n \sin u - n \int u^{n-1} \sin u du$
24. $\int \sin^n u \cos^m u du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u du$
 $= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u du$

2 Exponential and Logarithms Integrals

1. $\int u e^{au} du = \frac{1}{a^2} (au - 1) e^{au} + C$
2. $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$
3. $\int e^{au} \sin b u du = \frac{e^{au}}{a^2 + b^2} (a \sin b u - b \cos b u) + C$
4. $\int e^{au} \cos b u du = \frac{e^{au}}{a^2 + b^2} (a \cos b u + b \sin b u) + C$
5. $\int \ln u du = u \ln u - u + C$
6. $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$
7. $\int \frac{1}{u \ln u} du = \ln |\ln u| + C$

3 Hyperbolic Integrals

1. $\int \sinh u du = \cosh u + C$
2. $\int \cosh u du = \sinh u + C$
3. $\int \tanh u du = \ln \cosh u + C$
4. $\int \coth u du = \ln |\sinh u| + C$
5. $\int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$
6. $\int \operatorname{csch} u du = \ln |\tanh \frac{1}{2}u| + C$
7. $\int \operatorname{sech}^2 u du = \tanh u + C$
8. $\int \operatorname{csch}^2 u du = -\coth u + C$
9. $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$
10. $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

4 Inverse Trigonometric Integrals

1. $\int \sin^{-1} u du = u \sin^{-1} u + \sqrt{1-u^2} + C$
2. $\int \cos^{-1} u du = u \cos^{-1} u - \sqrt{1-u^2} + C$
3. $\int \tan^{-1} u du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$
4. $\int u \sin^{-1} u du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$
5. $\int u \cos^{-1} u du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$
6. $\int u \tan^{-1} u du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$
7. $\int u^n \sin^{-1} u du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], n \neq -1$
8. $\int u^n \cos^{-1} u du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], n \neq -1$
9. $\int u^n \tan^{-1} u du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1}}{1+u^2} du \right], n \neq -1$

5 Integrals Involving $a^2 + u^2, a > 0$

1. $\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
2. $u^2 \int \sqrt{a^2 + u^2} du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
3. $\int \frac{\sqrt{a^2 + u^2}}{u} du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$

6 Additional Resources

Textbook

Text: <https://openstax.org/details/books/calculus-volume-2>

Print

Calculus Study Guide: <https://mt-jfk.com/ap-calculus-study-guide.pdf>

Video

The Organic Chemistry Tutor: <https://www.youtube.com/channel/UCeWpbFLzoYGPfuWUMFPSaoA>

Black Pen Red Pen: <https://www.youtube.com/user/blackpenredpen>

Khan Academy: <https://www.youtube.com/c/khanacademy>

Websites

Khan Academy - AP/College Calculus BC: <https://www.khanacademy.org/math/ap-calculus-bc>