

Math Notes

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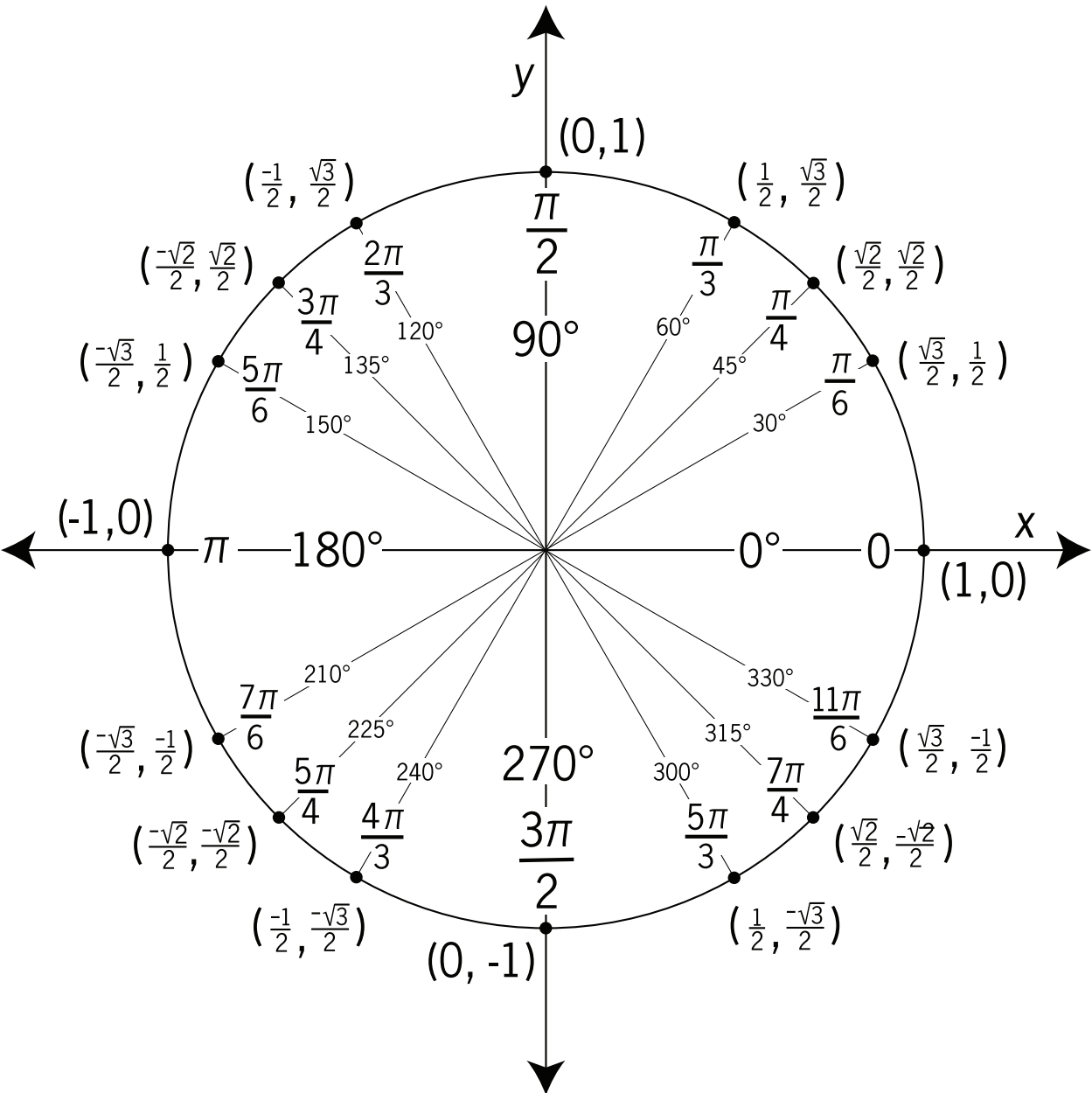
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Contents

1	Trigonometric Concepts	2
1.1	Unit Circle	2
1.2	Basic Trigonometry	2
1.3	Pythagorean Theorem Identities	2
1.4	Half Angle Identities	3
1.5	Double Angle Identities	3
1.6	Power Reduction Identities	3
2	Trigonometric Substitution	3
3	Table of Basic Derivatives	3
4	Table of Basic Integrals	4
5	Substitution Techniques	4
5.1	U Substitution	4
5.2	Integration by Parts	4
6	Engineering Formulas	4
6.1	Spring Formulas	4
6.2	Fluid Formulas	5
7	Method of Partial Fractions	5
7.1	Decomposition Types	5
8	Additional Resources	6

1 Trigonometric Concepts

1.1 Unit Circle



Source: https://etc.usf.edu/clipart/43200/43215/unit-circle7_43215.htm

1.2 Basic Trigonometry

$$\begin{aligned} \sin(\theta) &= \frac{\textit{Opposite}}{\textit{Hypotenuse}} \\ &= \frac{y}{1} = y \end{aligned}$$

$$\begin{aligned} \cos(\theta) &= \frac{\textit{Adjacent}}{\textit{Hypotenuse}} \\ &= \frac{x}{1} = x \end{aligned}$$

$$\begin{aligned} \tan(\theta) &= \frac{\textit{Opposite}}{\textit{Adjacent}} \\ &= \frac{y}{x} \\ &= \frac{\sin(\theta)}{\cos(\theta)} \end{aligned}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

1.3 Pythagorean Theorem Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

1.4 Half Angle Identities

$$\cos(\frac{\theta}{2}) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$$
$$\sin(\frac{\theta}{2}) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\tan(\frac{\theta}{2}) = \pm \sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$
$$= \frac{\sin(\theta)}{1 + \cos(\theta)}$$
$$= \frac{1 - \cos(\theta)}{\sin(\theta)}$$

1.5 Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

1.6 Power Reduction Identities

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\csc^2\theta = \frac{2}{1 - \cos 2\theta}$$
$$\sec^2\theta = \frac{2}{1 + \cos 2\theta}$$
$$\cot^2\theta = \frac{1 + \cos 2\theta}{1 - \cos 2\theta}$$

2 Trigonometric Substitution

Expression	Substitution	Identity
$\sqrt{a^2 - b^2x^2}$	$x = \frac{a}{b}\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + b^2x^2}$	$x = \frac{a}{b}\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{b^2x^2 - a^2}$	$x = \frac{a}{b}\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

3 Table of Basic Derivatives

y	$\frac{dy}{dx}$
C	0
x	1
$ax^2 + bx + c$	$2ax + b$
x^n	nx^{n-1}
$x^{-1}, \frac{1}{x}$	$-\frac{1}{x^2}$
\sqrt{x}	$\frac{1}{2\sqrt{x}}$
$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
$\ln(x)$	$\frac{1}{x}$
$\log_a(x)$	$\frac{1}{x\ln(a)}$
a^x	$a^x\ln(a)$
e^x	e^x
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2x}, \sec^2x$
$\cot(x)$	$-\frac{1}{\sin^2x}, -\csc^2x$
$\sec(x)$	$\tan(x)\sec(x)$
$\csc(x)$	$-\cot(x)\csc(x)$

4 Table of Basic Integrals

	$f(x)$	$\int f(x)dx = F(x) + C$
1	x^α $\alpha \neq 0$	$\frac{x^{\alpha+1}}{\alpha+1} + C$
2	$\sin(kx)$	$-\frac{\cos(kx)}{k} + C$
3	$\cos(kx)$	$\frac{\sin(kx)}{k} + C$
4	$\sec^2(kx)$	$\frac{\tan(kx)}{k} + C$
5	$\csc^2(kx)$	$-\frac{\cot(kx)}{k} + C$
6	e^{kx}	$\frac{e^{kx}}{k} + C$
7	$x^{-1}, \frac{1}{x}$	$\ln x + C$
8	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}x + C$
9	$\frac{1}{1+x^2}$	$\tan^{-1}x + C$
10	a^{kx}	$\frac{1}{k \ln a} a^{kx} + C$
11	$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \frac{x}{a} + C$
12	$\frac{1}{a^2+x^2}$	$\frac{1}{a} \tan^{-1}(x/a) + C$

5 Substitution Techniques

5.1 U Substitution

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$

where: $u = g(x), du = g'(x)$

5.2 Integration by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

$$\int u dv = uv - \int v du$$

$$\int_a^b f(x)g'(x)dx = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)dx$$

6 Engineering Formulas

6.1 Spring Formulas

$$F = kx$$

$$W = \int_a^b kx dx$$

$$F = \text{Force (Newtons [N])}$$

$$k = \text{spring constant (Newton meters } ^N/_m)$$

$$x = \text{change in distance (meters [m])}$$

$$W = \text{Work (Joules [J])}$$

$$a = \text{initial length (meters [m])}$$

$$b = \text{final length (meters [m])}$$

6.2 Fluid Formulas

$$W = F \cdot d = \int F dx$$
$$V = \pi r^2 h (\text{apply to cylinders})$$
$$F = m \cdot a = V \cdot \rho$$

W = Weight ()
 F = Force (Newtons [N])
 d = distance (meters [d])
 m = mass (meters³[m³])
 a = acceleration (meters per second² [^m/_s²])
 ρ = Something

7 Method of Partial Fractions

$$\int \frac{P_n(x)}{Q_m(x)} dx \text{ when } m > n$$

n and m are defined as the degree of the numerator and the denominator.

7.1 Decomposition Types

Type	Factor Example	Decomposition
Linear Factor	$(x - 4)$	$\frac{A}{x-4}$
Repeated Linear Factor	$(x - 4)^2$	$\frac{A}{(x+4)} + \frac{B}{(x+4)^2}$
Quadratic Irreducible Factor	$(x^2 + 4)$	$\frac{Ax+B}{x^2+4}$
Repeated Quadratic Irreducible Factor	$(x^2 + 4)^2$	$\frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$

8 Additional Resources

Print

Calculus Study Guide: <https://mt-jfk.com/ap-calculus-study-guide.pdf>

Video

The Organic Chemistry Tutor: <https://www.youtube.com/channel/UCEWpbFLzoYGPfuWUMFPSaoA>

Black Pen Red Pen: <https://www.youtube.com/user/blackpenredpen>