

We know that the standard model of a logistic sigmoid growth curve is:

$$f(x) = \frac{M}{1 + e^{-k(x-x_0)}}$$

Using some basic algebra, we can see that:

$$\begin{aligned} f(x) &= \frac{M}{1 + e^{-kx + kx_0}} \\ &= \frac{M}{1 + e^{kx_0} \times e^{-kx}} \end{aligned}$$

The equation above is now in the desired form:

$$y = \frac{a}{1 + be^{-cx}}$$

Where  $a = M$ ,  $b = e^{kx_0}$  and  $c = k$ .

In the previous easy test, we had calculated values for  $M$ ,  $k$ , and  $x_0$  as 11.0421165, 3.349129 and 5.02105825. Therefore, if we were to directly substitute these values into the expressions for  $a$ ,  $b$  and  $c$ , then we would get:

$$a = 11.0421165, b = 2.009882 \times 10^7 \text{ and } c = 5.02105825$$

However, these values for  $a$ ,  $b$  and  $c$  would not be completely accurate. This is because when performing the previous easy test, we had transformed the standard logistic function slightly by adding -1 to the end of the function. The reason we had done this is because the standard logistic function cannot be negative for real values of  $x$ , whereas in the sample data given, we had negative values for  $y$  at certain times. We had calculated  $M$ ,  $k$  and  $x_0$  for this adjusted function, and therefore not in the form desired. This means that the values for  $a$ ,  $b$  and  $c$  are only rough estimates, and are not completely accurate. This is why the parameters are difficult to determine, which effectively comes down to the fact that the function given for the medium test can only be positive for real values of time however we have  $y$  values that are negative.