Logarithmic Function Definition

In mathematics, the logarithmic function is an inverse function to exponentiation. The logarithmic function is defined as

For x > 0 , a > 0, and a ≠1,

y = $\log_a x$ if and only if $x = a^y$

Then the function is given by

$f(x) = \log_a x$

The base of the logarithm is a. This can be read it as log base a of x. The most 2 common bases used in logarithmic functions are base 10 and base e.

Common Logarithmic Function

The logarithmic function with base 10 is called the common logarithmic function and it is denoted by $\log_{10}$ or simply log.

$f(x) = \log_{10} x$

Natural Logarithmic Function

The logarithmic function to the base e is called the natural logarithmic function and it is denoted by $\log_e$.

$f(x) = \log_e x$

Logarithmic Functions Properties

Logarithmic Functions have some of the properties that allow you to simplify the logarithms when the input is in the form of product, quotient or the value taken to the power. Some of the properties are listed below.

Product Rule

$$\log_b MN = \log_b M + \log_b N$$

The logarithm of the product is the sum of the logarithms of the factors

Example:

$\log_2 8 + \log_2 4$

$\log_2 8 + \log_2 4 = \log_2 2^3 + \log_2 2^2$

$\log_2 8 + \log_2 4 = 3\log_2 2 + 2\log_2 2$

$\log_2 8 + \log_2 4 = 3(1) + 2(1)$

$\log_2 8 + \log_2 4 = 5$

Quotient Rule

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

The logarithm of the ratio of the quantities/values is the logarithm of the numerator minus the logarithm of the denominator

Divide two numbers with the same base, subtract the exponents.

Example:

$\log_3 162 - \log_3 2$

$\log_3 162 - \log_3 2 = \log_3 \frac{162}{2}$

$\log_3 162 - \log_3 2 = \log_3 81$

$\log_3 162 - \log_3 2 = \log_3 3^4$

$\log_3 162 - \log_3 2 = 4\log_3 3$

$\log_3 162 - \log_3 2 = 4$

Power Rule

Raise an exponential expression to power and multiply the exponents.

$\log_b M^k = k.\log_b M$

Example : log 100^3 = 3. Log 100 = 3 x 2 = 6

Zero Exponent Rule

$\log_a 1 = 0$

Change of Base Rule

$\log_b(x) = \frac{\ln x}{\ln b}$

OR

$\log_b(x) = \frac{\log_{10}(x)}{\log_{10} b}$

Other Important Rules of Logarithmic Function

$\log_b(b) = 1$ eg $\log_{10}(10) = 1$

$\log_b(b^x) = x$ eg $\log_{10}(10^2) = 2$

There are also some of the logarithmic function with fractions. It has a useful property to find the log of a fraction by applying the identities

$\ln(ab) = \ln(a) + \ln(b)$

$\ln(a^x) = x\ln(a)$

Logarithmic Equations

Logarithmic Function Examples

Here you are provided with some logarithmic functions example.

Example 1:

Use the properties of logarithms to write as a single logarithm for the given equation: $5 \log_9 x + 7 \log_9 y - 3 \log_9 z$

Solution:

By using the power rule , $\text{Log}_b M^p = P \log_b M$, we can write the given equation as

$5 \log_9 x + 7 \log_9 y - 3 \log_9 z = \log_9 x^5 + \log_9 y^7 - \log_9 z^3$

From product rule, $\log_b MN = \log_b M + \log_b N$

$5 \log_9 x + 7 \log_9 y - 3 \log_9 z = \log_9 x^5y^7 - \log_9 z^3$

From Quotient rule, $\log_b M/N = \log_b M - \log_b N$

$5 \log_9 x + 7 \log_9 y - 3 \log_9 z = \log_9 (x^5y^7 / z^3)$

Therefore, the single logarithm is $5 \log_9 x + 7 \log_9 y - 3 \log_9 z = \log_9 (x^5y^7 / z^3)$

Question 2:

Use the properties of logarithms to write as a single logarithm for the given equation: $1/2 \log_2 x - 8 \log_2 y - 5 \log_2 z$

Solution:

By using the power rule , $\text{Log}_b M^p = P \log_b M$, we can write the given equation as

$1/2 \log_2 x - 8 \log_2 y - 5 \log_2 z = \log_2 x^{1/2} - \log_2 y^8 - \log_2 z^5$

From product rule, $\log_b MN = \log_b M + \log_b N$

Take minus '- ' as common

$1/2 \log_2 x - 8 \log_2 y - 5 \log_2 z = \log_2 x^{1/2} - \log_2 y^8z^5$

From Quotient rule, $\log_b M/N = \log_b M - \log_b N$

$1/2 \log_2 x - 8 \log_2 y - 5 \log_2 z = \log_2 (x^{1/2} / y^8z^5)$

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