GET 209 [Part II]

Course Outline

- DC/AC Bridges : Resistance, Capacitance and Inductance Measurements
- Transducers
- Single phase Circuits
- Complex j - Notation
- AC Circuits
- Impedance
- Admittance
- Susceptance

Texts

- J. A. Svoboda and R. C. Dorf, Introduction to Electric Circuits, 9th Edition, Wiley

— William H. Hayt, Jack E. Kemmerly and Steven M. Durbin, Engineering Circuit Analysis

~ C.K. Alexander and Matthew N. O. Sadiku, Fundamentals of Electric Circuits

~ B.L. Theraja and A. K. Theraja, Electrical Technology.

DC/AC Bridges

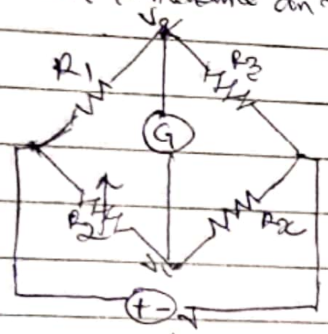
Bridge circuits are used very commonly as a variable conversion element in measurement systems and produce an output in the form of a voltage level that changes as the measured physical quantity changes.
- They provide an accurate method of measuring resistance, inductance and capacitance values, and enable the detection of very small changes in these quantities about a nominal value.
- They are of immense importance in measurement system technology because so many transducers measuring physical quantities have an output that is expressed as a change in resistance, inductance or capacitance.

Resistance Measurement with Wheatstone Bridge.

- While ohmmeters are designed to measure resistance in low, mid or high range, a Wheatstone bridge is used to measure resistance in the mid range, say, between 1Ω and 1MΩ.
- Very low values of resistances are measured with a milliohmmeter while very high values are measured with a Megger tester.

Wheatstone Bridge

- The Wheatstone bridge (or resistance bridge) circuit is used in a number of applications. Here we will use it to measure an unknown resistance.



$$V_1 = \frac{R_2}{R_1+R_2} V = V_2 = \frac{R_x}{R_3+R_x} V$$

$$\frac{R_2}{R_1+R_2} = \frac{R_x}{R_3+R_x}$$

$$R_2R_3 = R_1R_x$$

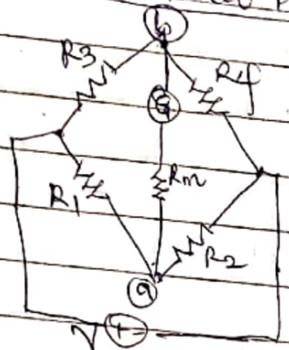
$$\boxed{R_x = \frac{R_3}{R_1} R_2}$$

If $R_1 = R_3$ & R_2 is adjusted until no current flows through the galvanometer, then $R_x = R_2$.

Example 1. Consider a balanced DC Wheatstone bridge with $R_1 = 500\Omega$ and $R_2 = 200\Omega$. Def. the unknown resistance when $R_2 = 125\Omega$

Example 2. A Wheatstone bridge has $R_1 = R_3 = 1K\Omega$. R_2 is adjusted until no current flows through the galvanometer. At that point, $R_2 = 3.2K\Omega$. What is the value of the unknown resistance? Ans $\approx 3.2K\Omega$

Unbalanced Wheatstone Bridge



To find the current through the galvanometer when the Wheatstone bridge is unbalanced?
Find the Thevenin equivalent (V_{TH} & R_{TH}) with respect to the galvanometer terminals. If R_m is the resistance of the galvanometer, the current through it under the unbalanced condition is?

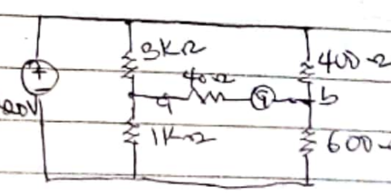
$$I = \frac{V_{TH}}{R_{TH} + R_m}$$

where $R_{eff} = \{R_1 \| R_2\} + \{R_3 \| R_4\}$

$$V_{eff} = V_a - V_b$$

$$V_a = \frac{R_2}{R_1+R_2} V \quad \& \quad V_b = \frac{R_4}{R_3+R_4} V$$

Example 3.



The circuit represents an unbalanced bridge. If the galvanometer has a resistance of 40Ω, find the current through the galvanometer.

Then, $Z_1 Z_3 = Z_2 Z_4$

$(R_1 + j\omega L_1) R_3 = (R_4 + j\omega L_4) R_2$

$R_1 R_3 + j\omega L_1 R_3 = R_2 R_4 + j\omega L_4 R_2$

Equating the real & imaginary

$\frac{R_1}{R_2} = \frac{R_4}{R_3}$ or $\frac{L_1}{L_4} = \frac{R_2}{R_3}$

Anderson Bridge

Used for measuring inductance over a wide range from few micro to several henrys.

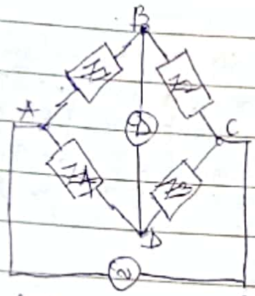
- It is a modified Maxwell-Wien bridge where an unknown inductance is measured in terms of a known capacitance and resistance.

AC BRIDGES

- Like measurement of resistance by a dc Wheatstone bridge, inductance and capacitance can also be measured in a similar way.
- The DC signal changes to an AC signal and a galvanometer is replaced by a vibrator galvanometer.

Condition of Balance

- There must be a balance of impedance in magnitude and phase.



When the bridge is balanced the potential at node B is equal to the potential at node D.

thus $\frac{Z_3}{Z_3+Z_4} = \frac{Z_2}{Z_1+Z_2}$

$Z_1 Z_3 = Z_2 Z_4$

$$\boxed{\frac{Z_1}{Z_2} = \frac{Z_4}{Z_3}}$$

Note for an ac network

$Z_1 = z_1 \angle \phi_1$

$Z_2 = z_2 \angle \phi_2$

$Z_3 = z_3 \angle \phi_3$

$Z_4 = z_4 \angle \phi_4$

Hence.

$Z_1 \angle \phi_1 \times Z_3 \angle \phi_3 = Z_2 \angle \phi_2 \times Z_4 \angle \phi_4$

$Z_1 Z_3 \angle \phi_1 + \phi_3 = Z_2 Z_4 \angle \phi_2 + \phi_4$.

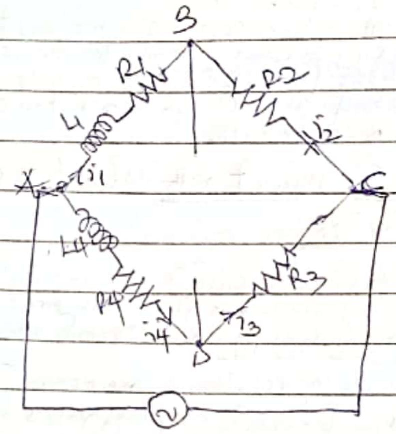
- thus for a balanced impedance bridge

i. $Z_1 Z_3 = Z_2 Z_4$

ii. $\phi_1 + \phi_3 = \phi_2 + \phi_4$

Inductance Measurement

(a) Maxwell's Inductance Bridge



Let $Z_1 \Rightarrow$ an unknown impedance

Z_3 can be impedance of equal phase which can be used on the adjacent, such that $\phi_1 = \phi_3$ or $\phi_2 = \phi_4$.

* If BC and DC have zero phase angle i.e pure resistors, such network is called Maxwell's ac bridge or L/L_4 bridge

* Otherwise, if an impedance with negative phase shift {phase angle} is used {capacitor} in the adjacent arm, such that $\phi_1 + \phi_3 = 0$, such is known as Maxwell's-Wien bridge or L/C bridge

$Z_1 = R_1 + jX_1 = R_1 + j\omega L_1$ — Unknown

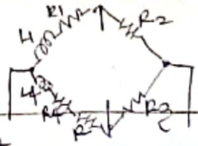
$Z_4 = R_4 + jX_4 = R_4 + j\omega L_4$ — Known

R_2 & R_3 $\Rightarrow$ Known pure resistor

δ = detector.

The bridge is balanced by varying L_4 and R_2 or R_3. Otherwise R_2 & R_3 are constant while one of R_4 & R_4 can be varied

Then, $Z_1Z_3 = Z_2Z_4$



$(R_1+j\omega L_1)R_3 = (R_4+j\omega L_4)R_2$

$R_1R_3 + j\omega L_1R_3 = R_2R_4 + j\omega L_4R_2$

Equating the real & imaginary

$\frac{R_1}{R_2} = \frac{R_4}{R_3}$ or $\frac{L_1}{L_4} = \frac{R_2}{R_3}$

Then $\boxed{L_1 = \frac{R_2}{R_3}L_4}$ or $L_4 = L_4\frac{R_1}{R_4}$ or $\frac{L_1}{R_1} = \frac{L_4}{R_4}$

Example: In the figure above, R_4 and L_4 if $R_1 = 32.7\Omega$, R_4 = unknown, $R_2 = R_3 = 100\Omega$ and R (in series with R_4) $= 136$, $L_1 = 47.8$ mH. Balance can be obtained by L_4 & R, and R_4 & L_4.

Soln

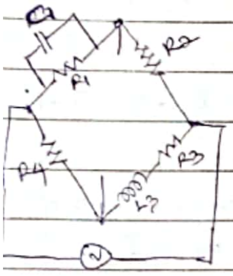
$R_1R_3 = R_2(R_4+R)$

$32.7 \times 100 = 100(136+R_4)$

$R_4 = 31.34\Omega$

$L_1/L_4 = R_2/R_3 \quad \therefore L_4 = L_1 = 47.8\,mH$

(b) Maxwell-Wien Bridge or LC Bridge



$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{-jX_C}$

$= \frac{1}{R_1} + j\frac{1}{X_C}$

$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C$

$Z_1 = \frac{R_1}{1+j\omega CR_1}$

$Z_2 = R_2$

$Z_3 = R_3 + j\omega L_3$

$Z_4 = R_4$

@ balance $Z_1Z_3 = Z_2Z_4$

$\frac{R_1(R_3+j\omega L_3)}{1+j\omega CR_1} = R_2R_4$

$R_1R_3 + j\omega L_3R_1 = R_2R_4 + j\omega CR_1R_2R_4$

$R_1R_3 = R_2R_4 \quad \therefore \boxed{R_3 = \frac{R_2R_4}{R_1}}$

and

$L_3R_1 = CR_1R_2R_4 \quad \boxed{L_3 = CR_2R_4}$

Example

Given a Maxwell Wien bridge with $R_1 = 1000\Omega$, $C = 0.5\mu F$, $R_2 = 600\Omega$ and $R_4 = 400\Omega$. Det. R_3 and L_3 @ balance

Soln

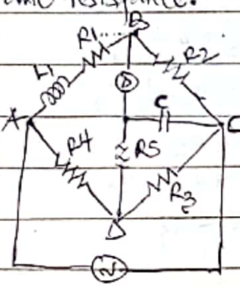
$R_3 = \frac{R_2R_4}{R_1} = \frac{600 \times 400}{1000} = 240\Omega$

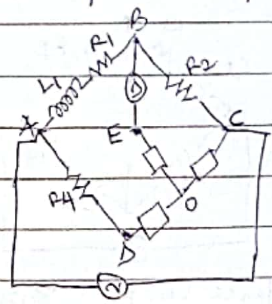
$L_3 = CR_2R_4 = 0.5\times10^{-6} \times 600 \times 400 = 0.12H$

Anderson Bridge

Used for measuring inductance over a wide range from few micro to several henrys.

- It is a modified Maxwell-Wien bridge where an unknown inductance is measured in terms of a known capacitance and resistance.





$Z_{OD} = \frac{R_3R_5}{(R_3+R_5)+\frac{1}{j\omega C}}$

$Z_{OC} = \frac{R_3(\frac{1}{j\omega C})}{(R_3+R_5)+\frac{1}{j\omega C}}$

$Z_{OE} = \frac{R_5(\frac{1}{j\omega C})}{(R_3+R_5)+\frac{1}{j\omega C}}$

Then

$Z_1 = R_1 + j\omega L_1$

$Z_2 = R_2$

$Z_3 = Z_{OC}$

$Z_4 = R_4 + Z_{OD}$

For balance, $Z_1Z_3 = Z_2Z_4$

$(R_1+j\omega L_1)Z_{OC} = R_2(R_4+Z_{OD})$

$(R_1+j\omega L_1)\frac{(R_3/j\omega C)}{(R_3+R_5+\frac{1}{j\omega C})} = R_2\left\{R_4 + \frac{R_3R_5}{R_3+R_5+\frac{1}{j\omega C}}\right\}$

$R_2R_3R_4 + R_2R_4R_5 - j\frac{R_2R_4}{\omega C} + R_2R_3R_5 = -j\frac{R_1R_3}{\omega C} + \frac{R_3L_1}{C}$

Equating the real and imaginary

$-j\frac{R_2R_4}{\omega C} = -j\frac{R_1R_3}{\omega C}$

$\boxed{R_1 = \frac{R_2R_4}{R_3}}$

$R_2R_3R_4 + R_2R_4R_5 + R_2R_3R_5 = \frac{R_3L_1}{C}$

$\boxed{L_1 = CR_2\left\{R_4+R_5+\frac{R_4R_5}{R_3}\right\}}$

Example

In the Anderson bridge above, let R_1 & L_1 be unknown

$R_2 = 1000\Omega$

$R_4 = 2000\Omega$

$R_3 = 2000$

$C = 1\mu F$

$R_5 = 200\Omega$

Soln

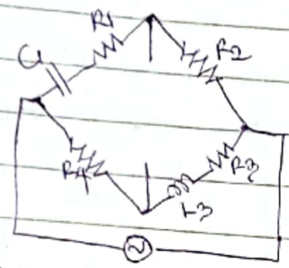
$R_1 = \frac{R_2R_4}{R_3} = \frac{1000 \times 2000}{2000} = 1000\Omega$

$L_1 = CR_2\left(R_4+R_5+\frac{R_4R_5}{R_3}\right) = \ldots$

$= 2.4H$

Hay's Bridge

It is also a modification of Maxwell-Wien bridge and used when the phase angle of the inductive impedance $\phi_m = \tan^{-1}\left(\frac{WL}{R}\right)$ is large.



$Z_1 = R_1 + \frac{1}{jWC_1}$
$= R_1 - j\frac{1}{WC_1}$
$Z_2 = R_2$
$Z_3 = R_3 + jWL_3$
$Z_4 = R_4$

Balance Condition. $Z_1Z_3 = Z_2Z_4$.

$\left(R_1 - j\frac{1}{WC_1}\right)\left(R_3 + jWL_3\right) = R_2R_4$

$R_1R_3 + jWL_3R_1 - j\frac{R_3}{WC_1} + \frac{L_3}{C_1} = R_2R_4$.

Separate the real and the imaginary

$R_1R_3 + \frac{L_3}{C_1} = R_2R_4$. - - (1)

$WL_3 - \frac{R_3}{WC_1} = 0$. - - (2)

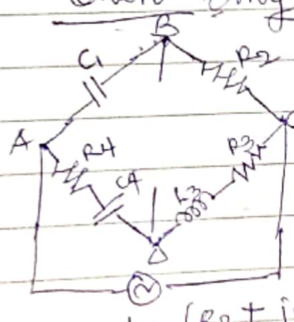
Solving the two equations simultaneously

$\boxed{L_3 = \frac{C_1R_2R_4}{1+W^2R_1^2C_1^2}}$ and $\boxed{R_3 = \frac{W^2C_1^2R_1R_2R_4}{1+W^2R_1^2C_1^2}}$

Example Solve given that $f = 50Hz$
R_1 & L_1 are unknown, $R_2 = 16800\Omega$, $R_3 = 1000\Omega$, $R_4 = 883\Omega$, $C = 0.38\mu F$. Find R_1 & L_1 for an Hay's bridge

Soln $W = 2\pi f = 2\times\frac{22}{7}\times 50 = 314.29$ rad/s
$W^2 = 314.29^2 = 98.724$
Apply the equations above
$R_1 = 210\Omega$
$L_1 = 6.38H$

Owen Bridge



@ balance
$Z_1Z_3 = Z_2Z_4$
$Z_1 = -j/WC_1$
$Z_2 = R_2$
$Z_3 = R_3 + jWL_3$
$Z_4 = R_4 - j/WC_4$.

$\frac{-j}{WC_1}\left(R_3 + jWL_3\right) = R_2\left\{R_4 - \frac{j}{WC_4}\right\}$

Separating the real and imaginaries

$R_3 = \frac{R_2C_1}{C_4}$

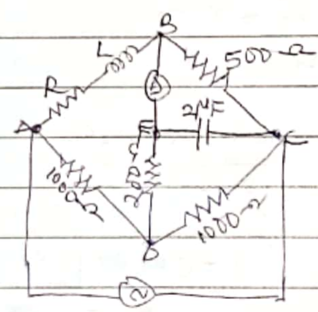
$L_3 = C_1R_2R_4$

This bridge is unaffected by the frequency of variation and waveform.

Exercise

1. An a.c. bridge is arranged as follows: The arms AB and BC consist of non-inductive resistance of 100Ω, the arms BE and CD of non-inductive variable resistance, the arm EC of a capacitor of $1\mu F$ capacitance, the arm DA of an inductive resistance. The a.c source is connected to A and C and the telephone reciever to E and D. A balance is obtained when the resistance of the arms CD and BE are 50Ω and 2500Ω respectively. Calculate the resistance and the inductance of the arm DA. What would be the effect of harmonics in the waveform of the alternating current source. [50Ω; $0.25H$]

2.

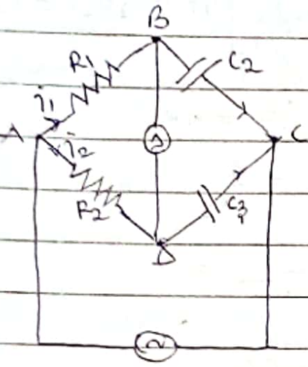


Determine the values of unknown resistance R and inductance L [500Ω; $1.5H$]

Capacitance Bridges

Measurement of capacitance can be obtained using De Sauty bridge method and Schering bridge method

De Sauty Bridge



Given that C_2 is unknown

$C_3 \Rightarrow$ Known

R_1 and R_2 are given

Balance is obtained by varying either R_1 or R_2 such that the potential @ B and D are equal

then $\quad i_1 R_1 = i_2 R_2$

$$\frac{-j}{wC_2} i_1 = \frac{-j}{wC_3} i_2$$

Solving the two equations

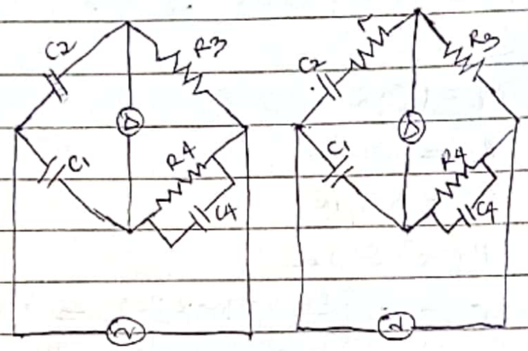
$$\frac{R_1}{R_2} = \frac{C_2}{C_3}$$

$$\boxed{C_2 = C_3 \frac{R_1}{R_2}}$$

It may be impossible to obtain perfect balance if the capacitors are not free of dielectric loss. A perfect balance is only possible if air capacitors are used.

Schering Bridge

- Useful in measuring the capacitance and dielectric loss of a capacitor. It works by comparing an imperfect capacitor C_2 in terms of a loss free capacitor C_1.



The imperfect capacitor C_2 can be represented by its equivalent loss-free capacitor C_2 in series with resistance r

$$Z_1 = -\frac{j}{wC_1}$$

$$Z_2 = r - \frac{j}{wC_2}$$

$$Z_3 = R_3$$

$$Z_4 = \frac{1}{(1/R_4) + jwC_4} = \frac{R_4}{1 + jwC_4R_4}$$

The balance condition is that, $Z_1 Z_3 = Z_2 Z_4$

$$\frac{-jR_3}{wC_1} = \left(r - \frac{j}{wC_2}\right)\left(\frac{R_4}{1 + jwC_4R_4}\right)$$

$$\frac{-jR_3}{wC_1}\left(1 + jwC_4R_4\right) = R_4\left\{r - \frac{j}{wC_2}\right\}$$

Separating the real and imaginaries

$$C_2 = C_1\left\{\frac{R_4}{R_3}\right\}$$

$$r = R_3\left\{\frac{C_4}{C_1}\right\}$$

Example: A loop capacitor is tested with a Schering bridge circuit. A balance is obtained with the capacitor under test in one arm, the succeeding arms kemf, a non-inductive resistor of 100 Ω, a non-reactive resistor of 309 Ω in parallel with a pure capacitor of 0.5 μF and a standard capacitor of 109 μF. The supply frequency is 50 Hz. Calculate the series capacitance and the equivalent resistance.

Soln

$C_1 = 109 pF$

$R_3 = 100 \Omega$

$C_4 = 0.5 \mu F$

$R_4 = 309 \Omega$

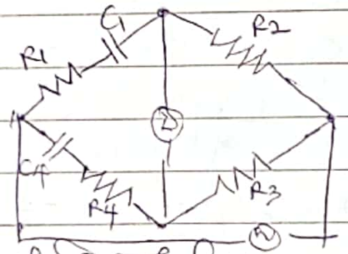
$C_2 = C_1 \left\{\frac{R_4}{R_3}\right\} = 109 \times \frac{309}{100} = 336.8 pF$

$r = R_3 \left\{\frac{C_4}{C_1}\right\} = 100 \left\{\frac{0.5 \times 10^{-6}}{109 \times 10^{-12}}\right\} =$

Wien Series Bridge / Wien Parallel Bridge

— They are simple ratio bridges used for audio-frequency measurement of capacitors over a wide range.

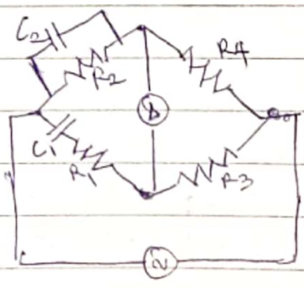
(a) Wien Series Bridge.



When the bridge is balance

$$R_1 = \frac{R_2 R_4}{R_3} \quad \text{and} \quad C_1 = C_4\left(\frac{R_3}{R_2}\right)$$

(b)



Wien parallel bridge can also be used to measure audio frequencies, though not as accurate as the norm of frequency meter.

Condition of balance

$$R_4\left\{R_1 - \frac{j}{wC_1}\right\} = R_3\left\{\frac{R_2}{1+jwC_2R_2}\right\}$$

$$R_4\left\{R_1 - \frac{j}{wC_1}\right\}(1+jwC_2R_2) = R_2R_3$$

Seperating the real and imaginary terms

$$R_1R_4 + R_2R_4\frac{C_2}{C_1} = R_2R_3$$

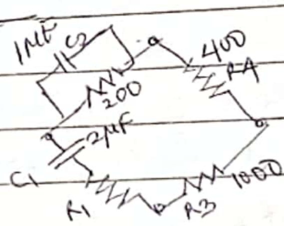
or $\frac{C_2}{C_1} = \frac{R_3}{R_4} = \frac{R_1}{R_2}$

and $wC_2R_2R_4 - \frac{R_4}{wC_1} = 0$

$$w^2 = \frac{1}{R_1R_2C_1C_2}$$

$$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

Example



Det. R_1 and the supply frequency when the bridge is balanced.

Soln

$$\frac{C_2}{C_1} = \frac{R_3}{R_4} - \frac{R_1}{R_2}$$

$$\frac{2}{1} = \frac{1000}{4000} - \frac{R_1}{200}$$

$R_1 = 200 \times 0.5 = 100 \Omega$

$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$ Hz

$= \frac{10^6}{2\pi\sqrt{100 \times 200 \times 1 \times 2}} = 796 Hz$