GET 201
Applied Electricity I

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Course Outline: Section A

- Electric Fields
- Charges
- Magnetic Fields
- B – H Curves
- Kirchoff's laws
- Superposition theorem
- Thevenin theorems
- Norton theorems
- Reciprocity
- RL, RC, RLC Circuits

Today's Goal

- Electric Fields
- Charges

Introduction

SI Unit

- As engineers, we deal with measurable quantities. Our measurement, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country where the measurement is conducted. Such an international measurement language is the International System of Units or Systeme Internationale (SI).

- The SI unit is based on the units for the six fundamental dimensions. Other dimensions are regarded as secondary because they are based and expressed in terms of the six fundamental dimensions.

Dimension	Unit	Symbol
Length	Meter	m
Mass	Kilogram	Kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol

Introduction

Charge: is an electrical property of the atomic particles measured in coulombs (C).

- The **Coulomb** is defined as the quantity of electricity which flows past a given point in an electric circuit when a current of one ampere is maintained for one second. In other words, one coulomb is one ampere second. ($1\ coulomb = 6.24 \times 10^{18}\ electrons\ i.e. 1/(1.602 \times 10^{-19})$)
- $\boldsymbol{Charge\ in\ Coulombs}$; $\boldsymbol{Q = It}$; where *I* is the current (A) and *t* is the time (s).

Example: If a current of 5A flows for 2 minutes, find the quantity of electricity transferred.

Force: **The newton** is defined as the force which, when applied to a mass of one kilogram, gives it an acceleration of one metre per seconds squared.

- **Force** in newton; ***F = ma***
- **Gravitational force or weight**; ***F = mg*** **; where** $\boldsymbol{g = 9.81 m/s^2}$

Example: A mass of 5000g is accelerated at $2\ m/s^2$ by a force. Determine the force needed.

Work: **The joule** is defined as the work done or energy transferred when a force of one newton is exerted through a distance of one metre in the direction of the force

- **Work done on a body**, in joules; ***W = Fs***

Introduction

Power:: Power is defined as the rate of doing work or transferring energy. The unit of power is the watt (W) where one watt is one joule per second.

- Power in watt; P = W/t ; where W is the work done or energy transferred, in joules, and t is the time, in seconds.
- Energy in joule; W = Pt

Example: A portable machine requires a force of 200N to move it. How much work is done if the machine is moved 200m and what average power is utilized if the movement takes 25s?

Term: A mass of 500kg is raised to a height of 6m in 30s. Find (a) the work done and (b) the power developed.

Electrical potential and e.m.f.: The unit of electric potential is the volt (v), where one volt is one joule per coulomb. One Volt is defined as the difference in potential between two points in a conductor which, when carrying a current of one ampere, dissipates a power of one watt, i.e.

$$volts = \frac{watts}{amperes} = \frac{joules/second}{amperes}$$
$$= \frac{joules}{ampere\ seconds} = \frac{joules}{coulombs}$$

A change in electric potential between two points in an electric circuit is called a **potential difference.**

Introduction

The electromotive force (e.m.f.) provided by a source of energy such as a battery or a generator is measured in volts.

Resistance and conductance: **One** ohm is one volt per ampere. It is defined as the resistance between two points in a conductor when a constant electric potential of one volt applied at the two points produces a current flow of one ampere in the conductor.

Resistance in ohms; $\boldsymbol{R = \frac{V}{I}}$

where V is the potential difference across the two points, in volts, and I is the current flowing between the two points, in amperes.

- The reciprocal of resistance is called **conductance** and is measured in siemens (S).

Conductance, in siemens; $G = \frac{1}{R}$

Example: Find the conductance of a conductor of resistance: (a.) 10Ω (b.) 5kΩ (c.) 100mΩ

Electrical power and energy: When a direct current of I amperes is flowing in an electric circuit and the voltage across the circuit is V volts; then

Power in watts; $\boldsymbol{P = VI}$

Introduction

Electric energy = $power \times time = VIt \ joules$

Although the unit of energy is the joules, when dealing with large amounts of energy, the unit used is the **Kilowatt-hour (kWh) where;**

1 kWh = 1000 watt-hour

= 1000 x 3600 watt seconds or joules

= 3 600 000 J

Examples: (a) A source e.m.f of 5V supplies a current of 3A for 10minutes. How much energy is provided in this time?

(b) An electric heater consumes 1.8 MJ when connected to a 250V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

An electrical/electronic system is a group of components connected together to perform a desired function. Example is a simple public address system, where a microphone is used to collect acoustic energy in the form of sound pressure waves and converts this to electrical energy in the form of small voltages and currents; the signal from the microphone is then amplified by means of an electronic circuit containing transistors/integrated circuit before it is applied to the loudspeaker.

Electric Fields

The space surrounding a charge can be investigated using a small charged body. This is similar to that applied to the magnetic field surrounding a current-carrying conductor. However, in this case the charged body is either attracted or repelled by the charge under investigation. The space in which this effect can be observed is termed **the electric field** of the charge and **the force** on the charged body is the **electric force**.

The lines of force can be traced out and they appear to have certain properties:

1. In an electric field, each line of force emanates from or terminates in a charge. The conventional direction is from the positive charge to the negative charge.
2. The direction of the line is that of the force experienced by a positive charge placed at a point in the field, assuming that the search charge has no effect on the field which it is being used to investigate.
3. The lines of force never intersect since the resultant force at any point in the field can have only one direction.

The **force of attraction or of repulsion** acts directly between two adjacent charges. All points on the surface of a conductor may be assumed to be at equipotential (same potential), and the lines of force radiate out from equipotential surfaces at right angles. The simplest case is that of the isolated spherical charge shown in Fig. 1. However, most electric fields exist between two conductors. The two most important arrangements are those involving parallel plates (as in a simple capacitor) and concentric cylinders (as in a television aerial cable). The resulting fields are shown in Fig. 2.

Electric Fields

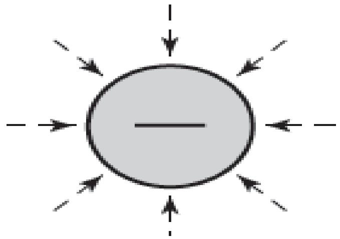

Fig. 1: Electric field about an isolated spherical charge

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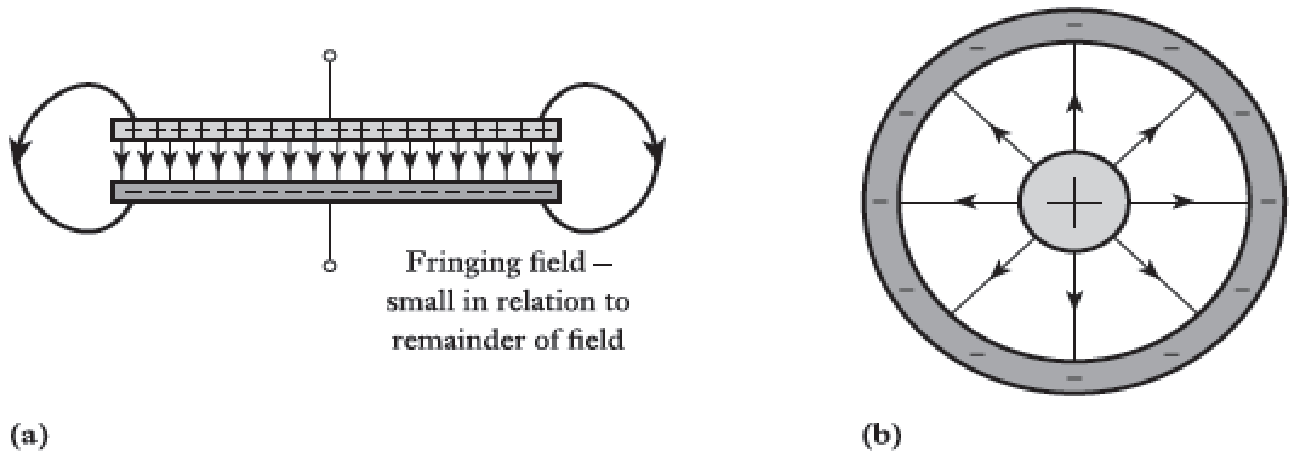

Fig. 2: Electric fields between oppositely charged surfaces. (a) Parallel plates; (b) concentric cylinders (cable)

It should not be overlooked that the space between the conductors needs to be filled with an insulator, otherwise the charges would move towards one another and therefore be dissipated. The **insulant** is called a **dielectric**.

Electric Field Strength and Electric Flux Density

- We can investigate an electric field by observing its effect on a charge. In the SI method of measurement this should be a unit charge, i.e. a coulomb. In practice this is such a large charge that it would disrupt the field being investigated.
- The magnitude of the force experienced by this unit charge at any point in a field is termed **the electric field strength** at that point (also known as electric stress). It can be measured in newtons per unit charge and represented by the symbol ***E***. (Since ***E*** can also represent e.m.f., we use a bold type for ***E*** when representing electric field strength and later we will meet ***D*** representing electric flux density.)
- The most simple field arrangement which we can investigate is that between parallel charged plates as shown in Fig. 3. Let us suppose that the plates are very large and that the distance between them is very small. By doing this, we can ignore any fringing effects of the type shown in Fig. 2 and assume that all the field exists between the plates. Let us also assume that there is free space between the plates.

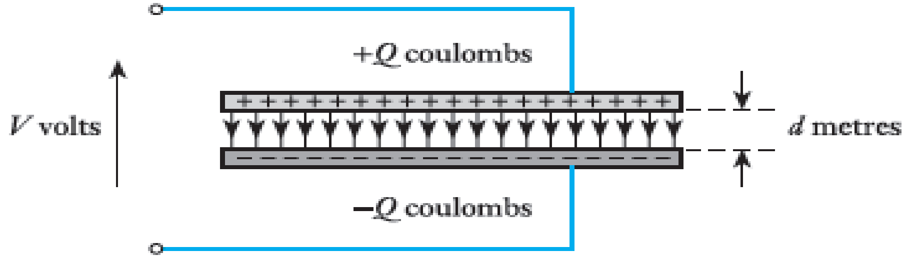

Fig. 3: A parallel-plate capacitor

Electric Field Strength and Electric Flux Density

- There is a potential difference of V volts between the plates. Therefore, the work in transferring 1 C of charge between the plates is V joules. But work is the product of force and distance, and in this case the distance is d metres. Therefore the force experienced by the charge is the electric field strength $\boldsymbol{E}$ given by

$$E = \frac{V}{d} \quad volts\ per\ metre$$

- **The total electric effect of a system** as described by the lines of electric force is termed the electric flux linking the system. **Flux** is measured in the same units as electric charge, hence a flux of Q coulombs is created by a charge of Q coulombs.
- **The electric flux density** is the measure of the electric flux passing at right angles through unit area, i.e. an area of $1m^2$. It follows that if the area of the plates in the capacitor of Fig. 3 is A then the electric flux density $\boldsymbol{D}$ is given by;

$$D = \frac{Q}{A} \quad coulombs\ per\ square\ metre$$

From above two expressions, we have;

$$\frac{Electric\ flux\ density}{Electric\ field\ strength} = \frac{D}{E} = \frac{Q}{A} \div \frac{V}{d} = \frac{Q}{V} \times \frac{d}{A} = \frac{Cd}{A}$$

Electric Field Strength and Electric Flux Density

- In electrostatics, the ratio of the electric flux density in a vacuum to the electric field strength is termed the *permittivity of free space* and is represented by ϵ_0 . Hence,

$$\epsilon_0 = \frac{\mathrm{Cd}}{\mathrm{A}} \quad \text{or} \quad \mathrm{C} = \frac{\epsilon_0 A}{d} \quad farads$$

Permittivity of free space Symbol: $\boldsymbol{\epsilon_0}$ Unit: **farad per metre (F/m)**

- The value of $\boldsymbol{\epsilon_0}$ can be determined experimentally by charging a capacitor, of known dimensions and with vacuum dielectric, to a p.d. of *V* volts and then discharging it through a ballistic galvanometer having a known ballistic constant *k* coulombs per unit deflection. If the deflection is θ divisions,

$$\boldsymbol{Q = CV = k\theta}$$

$$\therefore \ \boldsymbol{\epsilon_0 = \mathrm{C} \cdot \frac{d}{A} = \frac{k\theta}{V} \cdot \frac{d}{A}}$$

- From carefully conducted tests it has been found that the value of $\boldsymbol{\epsilon_0}$ is $8.85 \times 10^{-12} F/m$.
- Hence the capacitance of a parallel-plate capacitor with vacuum or air dielectric is given by;

$$\boldsymbol{C = \frac{(8.85 \times 10^{-12})[F/m] \times A[m^2]}{d[m]} \quad farads}$$

Relative Permittivity

- **The ratio of the capacitance of a capacitor** having a given material as dielectric to the capacitance of that capacitor with vacuum (or air) dielectric is termed the *relative permittivity* of that material (ϵ_r). See Table 1 for values of the relative permittivity of some important insulating materials. Note that some of them vary with frequency.

Relative Permittivity Symbol: ϵ_r Unit: **none**

- From above expression, it follows that if the space between the metal plates of the capacitor in Fig. 4 is filled with a dielectric having a relative permittivity ϵ_r,

Capacitance; $C = \frac{\epsilon_0 \epsilon_r A}{d} \quad farads$

$$= \frac{\left(8.85 \times 10^{-12}\right)[F/m] \times \epsilon_r \times A[m^2]}{d[m]} \quad farads$$

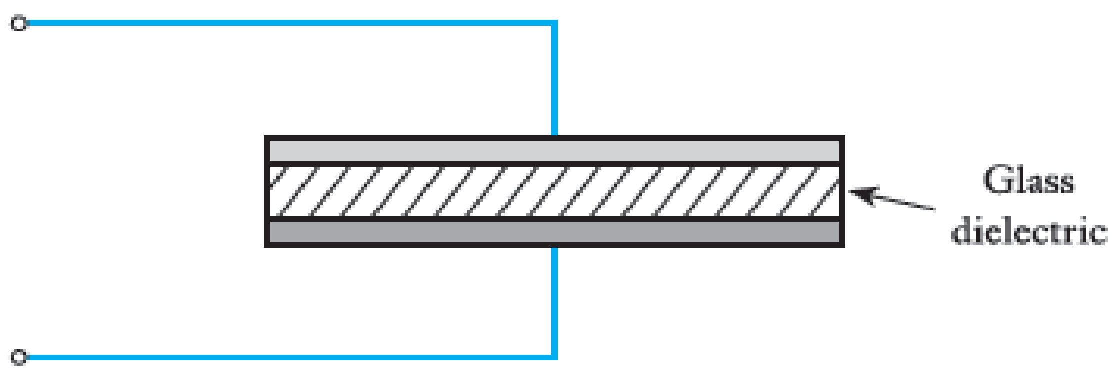

Fig. 4: A parallel-plate capacitor with a glass dielectric

Relative Permittivity

Material	Relative permittivity
Vacuum	1.0
Air	1.0006
Paper (dry)	2–2.5
Polythene	2–2.5
Insulating oil	3–4
Bakelite	4.5–5.5
Glass	5–10
Rubber	2–3.5
Mica	3–7
Porcelain	6–7
Distilled water	80
Barium titanate	6000+

Table 1 : Important insulating materials

And charge due to a p.d. of V volts is;

$$Q = CV = \frac{\epsilon_0 \epsilon_r A V}{d} \quad coulombs$$

$$\therefore \quad \frac{Electric\ flux\ density}{Electric\ field\ strength} = \frac{D}{E} = \frac{Q}{A} \div \frac{V}{d} = \frac{Qd}{VA} = \epsilon_0 \epsilon_r$$

$Let \qquad \boldsymbol{\epsilon_0 \epsilon_r = \epsilon}$

where $\boldsymbol{\epsilon}$ is the absolute permittivity

$$\therefore \quad Absolute\ permittivity\ \epsilon = \epsilon_0 \epsilon_r = \frac{C[farads] \times d[metres]}{A[metres^2]}$$

$$= \frac{Cd}{A} \ \ farads\ per\ metre\ (\mathbf{F/m})$$

Capacitance of a multi-plate capacitor

Suppose a capacitor to be made up of n parallel plates, alternate plates being connected together as in Fig. 5. Let A = area of *one* side of each plate in square metres, d = thickness of dielectric in metres and ϵ_r = relative permittivity of the dielectric

- Figure 5 shows a capacitor with seven plates, four being connected to A and three to B. It will be seen that each side of the three plates connected to B is in contact with the dielectric, whereas only one side of each of the outer plates is in contact with it. Consequently, the useful surface area of each set of plates is $6A$ square metres. For n plates, the useful area of each set is $(n-1)A$ square metres.

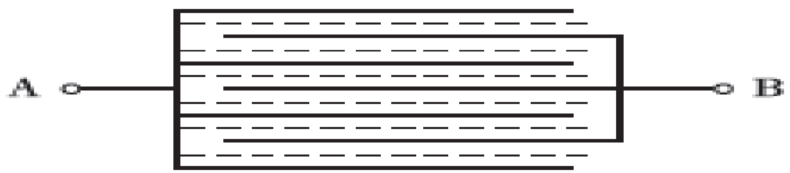

Fig. 5: Multi-plate capacitor

$$\boldsymbol{Capacitance = \frac{\epsilon_0 \epsilon_r (n-1)A}{d} = \frac{8.85 \times 10^{-12} \epsilon_r (n-1)A}{d} \quad farads}$$

Example: A capacitor is made with seven metal plates connected as in Fig 5 and separated by sheets of mica having a thickness of 0.3 mm and a relative permittivity of 6. The area of one side of each plate is 500 cm^2. Calculate the capacitance in microfarads.

Composite dielectric capacitors

- Suppose the space between metal plates M and N to be filled by dielectrics 1 and 2 of thickness d_1 *and* d_2 metres respectively, as shown in Fig. 6(a). Let Q = charge in coulombs due to p.d. of V volts and A = area of each dielectric in square metres, then $\boldsymbol{D} = Q/A$

which is the electric flux density, in coulombs per metre squared, in A and B.

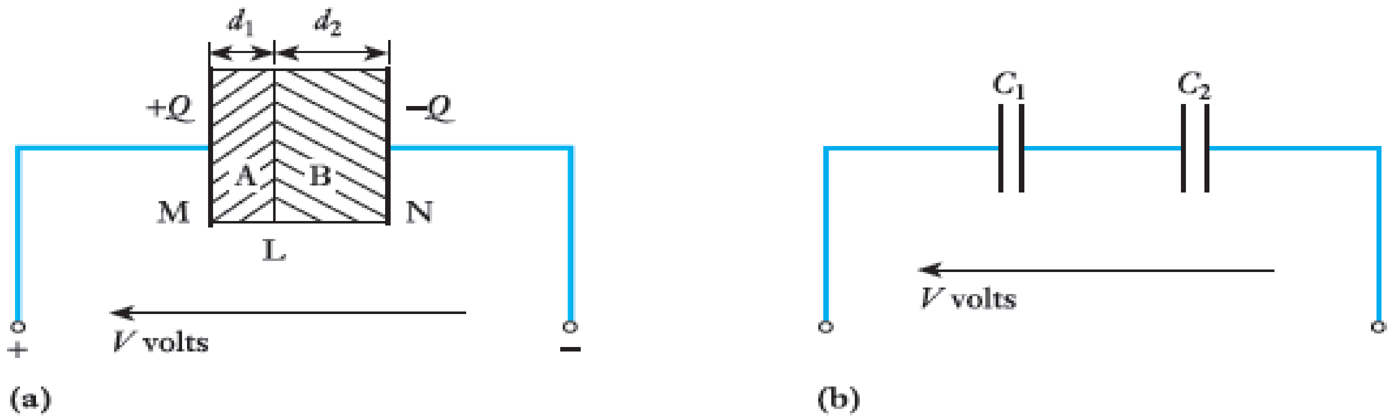

Fig. 6: Parallel-plate capacitor with two dielectrics

Let $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ = electric field strengths in 1 and 2 respectively; then if the relative permittivities of 1 and 2 are ϵ_1 and ϵ_2 respectively, electric field strength in A is

$$\boldsymbol{E_1} = \frac{\boldsymbol{D}}{\epsilon_1\epsilon_0} = \frac{\boldsymbol{Q}}{\epsilon_1\epsilon_0 \boldsymbol{A}} \quad \textit{and electric field strength in B is;} \quad \boldsymbol{E_2} = \frac{\boldsymbol{D}}{\epsilon_2\epsilon_0} = \frac{\boldsymbol{Q}}{\epsilon_2\epsilon_0 \boldsymbol{A}}$$

$$\textit{Hence} \quad \frac{E_1}{E_2} = \frac{\epsilon_2}{\epsilon_1}$$

Composite dielectric capacitors

i.e. for dielectrics having the same cross-sectional area in series, the electric field strengths (or potential gradients) are inversely proportional to their relative permittivities.

Potential drop in a dielectric is electric field strength × thickness

Therefore p.d. between plate M and the boundary surface L between 1 and 2 is $E_1 d_1$. Hence all points on surface L are at the same potential, i.e. L is an *equipotential surface* and is at right angles to the direction of the electric field strength. It follows that if a very thin metal foil were inserted between 1 and 2, it would not alter the electric field in the dielectrics. Hence the latter may be regarded as equivalent to two capacitances, C_1 *and* C_2, connected in series as in Fig. 6 (b), where

$$C_1 = \frac{\epsilon_1 \epsilon_0 A}{d_1} \quad and \quad C_2 = \frac{\epsilon_2 \epsilon_0 A}{d_2}$$

And total capacitance between plates M and N is; $\frac{C_1 C_2}{C_1 + C_2}$

Example: A capacitor consists of two metal plates, each 400 × 400 mm, spaced 6mm apart. The space between the metal plates is filled with a glass plate 5 mm thick and a layer of paper 1 mm thick. The relative permittivities of the glass and paper are 8 and 2 respectively. Calculate (a) the capacitance, neglecting any fringing flux, and

(b) the electric field strength in each dielectric in kilovolts per millimetre due to a p.d. of 10 kV between the metal plates.

Charging and Discharging Currents

Suppose C in Fig. 7 represents a capacitor of, say, 30 μF connected in series with a centre-zero microammeter A across a slider S and one end of a resistor R. A battery B is connected across R. If S is moved at a uniform speed along R, the p.d. applied to C, indicated by voltmeter V, increases uniformly from 0 to *V* volts, as shown by line OD in Fig. 8.

If *C* is the capacitance in farads and if the p.d. across C increases uniformly from 0 to *V* volts in t_1 seconds

$$Charging\ current = i_1 = \frac{Q\ [coulombs\ or\ ampere\ seconds]}{t_1[seconds]}$$

$$= \frac{CV}{t_1}\ amperes$$

i.e. charging current in amperes is equal to rate of change of charge in coulombs per second and is

C [farads] × rate of change of p.d. in volts per second

Since the p.d. across C increases at a uniform rate, the charging current, i_1, remains constant and is represented by the dotted line LM in Fig. 8.

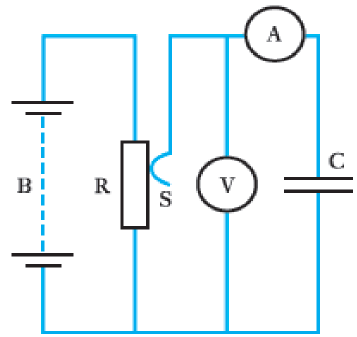

Fig. 7: Charging and discharging of a capacitor

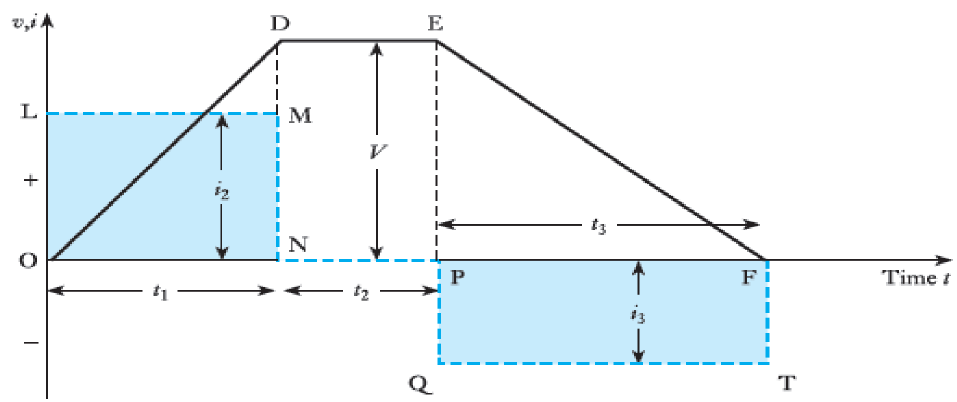

Fig. 8: Voltage and current during C&D of a Cap.

Charging and Discharging Currents

Suppose the p.d. across C to be maintained constant at *V* volts during the next t_2 seconds. Since the rate of change of p.d. is now zero, the current (apart from a slight leakage current) is zero and is represented by the dotted line NP. If the p.d. across C is then reduced to zero at a uniform rate by moving slider S backwards, the microammeter indicates a current i_3 flowing in the reverse direction, represented by the dotted line QT in Fig. 8. If t_3 is the time in seconds for the p.d. to be reduced from *V* volts to zero, then

$$Q = -i_3 t_3 \ \ coulombs$$

$$\therefore \ \ i_3 = \ -Q/t_3 = \ -C \times V/t_3 \ \ \text{amperes}$$

i.e. discharge current in amperes is equal to rate of change of charge in coulombs per second and is

C [farads] × rate of change of p.d. in volts per second

Since $Q = i_1 t_1 = -i_3 t_3$ (assuming negligible leakage current through C), areas of rectangles OLMN and PQTF are therefore equal.

In practice, it is seldom possible to vary the p.d. across a capacitor at a constant rate, so let us consider the general case of the p.d. across a capacitor of *C* farads being increased by d*v* volts in d*t* seconds.

Charging and Discharging Currents

- If the corresponding increase of charge is d*q* coulombs

$$dq = C \cdot dv$$

If the charging current at that instant is *i* amperes

$$dq = i \ \cdot dt$$

$$\therefore \qquad i \cdot dt \ = \ C \cdot dv$$

$$and \qquad i = C \cdot dv/dt$$

$$i = C \times rate\ of\ change\ of\ p.d.$$

If the capacitor is being discharged and if the p.d. falls by d*v* volts in d*t* seconds, the discharge current is given by

$$i = \frac{dq}{dt} \qquad or \qquad i = C \cdot \frac{dv}{dt}$$

Since d*v* is now negative, the current is also negative.

Growth and Decay

- The curves of the voltage across a capacitor during charging and discharging from the readings on a voltmeter connected across the capacitor have been derived. How the curves can be derived graphically from the values of the capacitance, the resistance and the applied voltage will now be considered. At the instant when S is closed on position, there is no p.d. across C. Consequently the whole of the voltage is applied across R and the initial value of the charging current = $I = V/R$.
- The growth of the p.d. across C is represented by the curve in Fig. 9. Suppose v to be the p.d. across C and i to be the charging current t seconds after S is put over to position a. The corresponding p.d. across R = $V - v$, where V is the terminal voltage of the battery. Hence

$$iR = V - v$$

$$and \quad i = \frac{V - v}{R}$$

If this current remained *constant* until the capacitor was fully charged, and if the time taken was x seconds, the corresponding quantity of electricity is

$$ix = \frac{V - v}{R} \times x \ \ coulombs$$

With a constant charging current, the p.d. across C would have increased uniformly up to V volts, as represented by the tangent LM drawn to the curve at L.

Growth and Decay

But the charge added to the capacitor also equals increase of p.d. × C which is

$$(V - v) \times C$$

$$Hence \quad \frac{V - v}{R} \times x = C(V - v)$$

$$and \;\; x = CR = the\ time\ constant, T, of\ the\ circuit$$

$$i.e. \quad T = CR \;\; seconds$$

The construction of the curve representing the growth of the p.d. across a capacitor is therefore similar to that for the growth of current in an inductive circuit. Thus, OA in Fig. 10 represents the battery voltage V, and AB the time constant T. Join OB, and from a point D fairly near the origin draw DE = T seconds and draw EF perpendicularly.

Join DF, etc. Draw a curve such that OB, DF, etc. are tangents to it.

From above expression it is evident that the instantaneous value of the charging current is proportional to $(V - v)$, namely the vertical distance between the curve and the horizontal line PQ in Fig. 9. Hence the shape of the curve representing the charging current is the inverse of that of the p.d. across the capacitor and is the same for both charging and discharging currents (assuming the resistance to be the same). Its construction is illustrated by the following example.

Growth and Decay

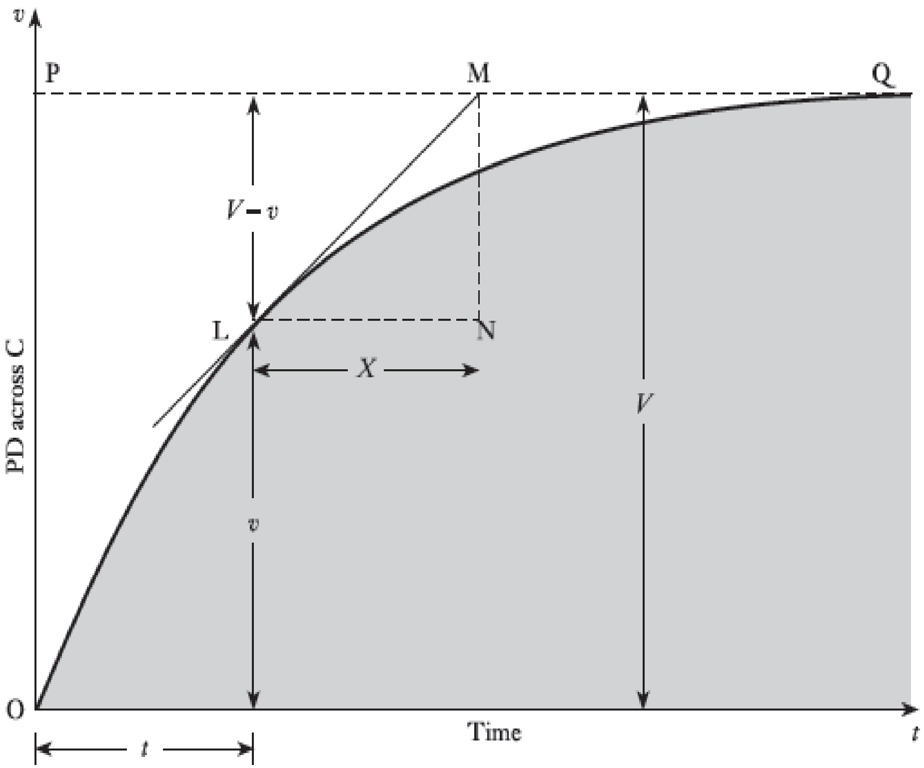

Fig. 9: Growth of p.d. across a capacitor in series with a resistor.

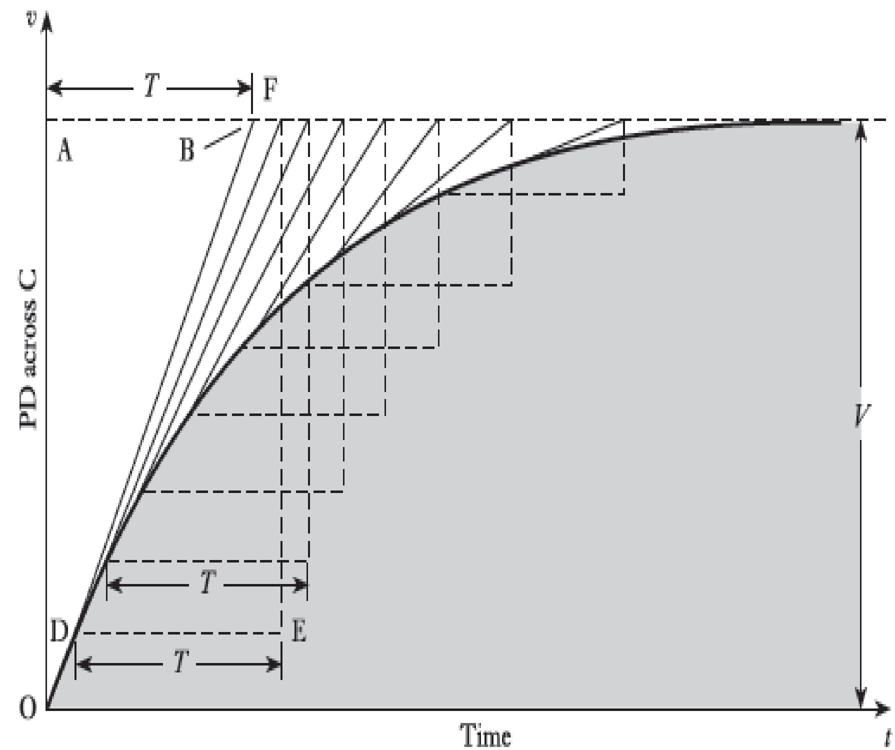

Fig. 10: Growth of p.d. across a capacitor in series with a resistor.

Example: A 20 μ F capacitor is charged to a p.d. of 400 V and then discharged through a 100 000 Ω resistor. Derive a curve representing the discharge current.

Analysis of Growth and Decay

Suppose the p.d. across capacitor C in Fig. 11, t seconds after S is switched over to position a, to be v volts, and the corresponding charging current to be i amperes, as indicated in Fig. 13. Also, suppose the p.d. to increase from v to $(v + dv)$ volts in dt seconds, then, from previous expression,

$$i = C\frac{dv}{dt}$$

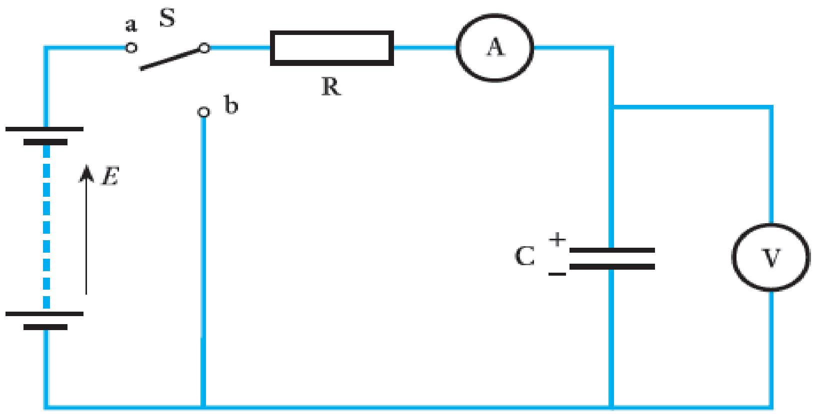

Fig. 11: Capacitor charged and discharged through a resistor

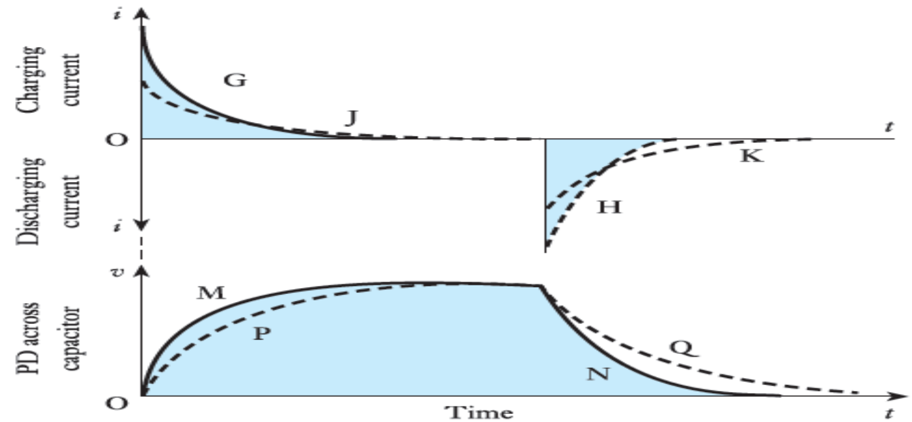

Fig. 12: Charging and discharging currents

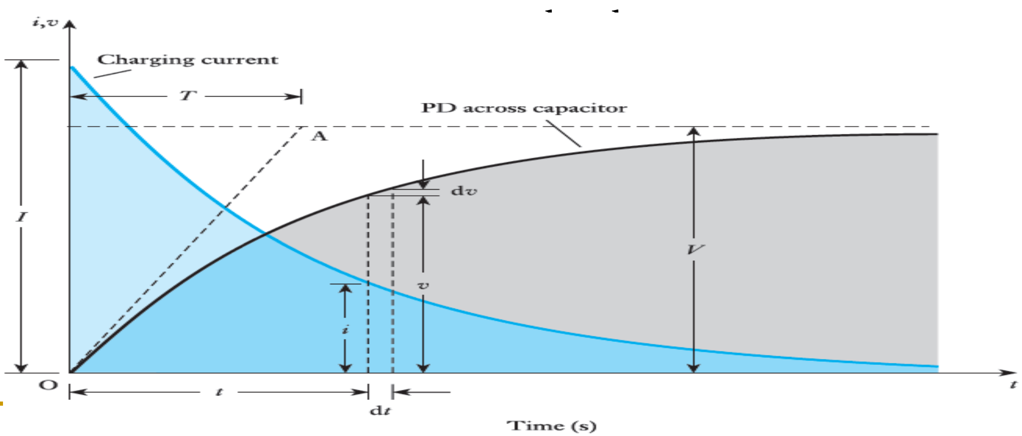

Fig. 13: Variation of current and p.d. during charging

Analysis of Growth and Decay

And corresponding p.d. across R is: $Ri = RC \cdot \frac{dv}{dt}$

But $V = p.d. across\ C + p.d. across\ R$

$$V = v + RC \cdot \frac{dv}{dt}$$

$$\therefore \qquad V - v = RC \cdot \frac{dv}{dt}$$

$$so\ that \qquad \frac{dt}{RC} = \frac{dv}{V - v}$$

$$Integrating\ both\ sides, we\ have: \qquad \frac{t}{RC} = -\ln(V - v) + A$$

$$where\ A\ is\ the\ constant\ of\ integration; when\ t =, v = 0,$$

$$\therefore \qquad A = in\ V$$

$$so\ that; \qquad \frac{t}{RC} = \ln \frac{V}{V - v}$$

$$\therefore \qquad \frac{V}{V - v} = e^{\frac{t}{RC}}$$

$$and \qquad v = V\left(1 - e^{-\frac{t}{RC}}\right) \quad volts$$

$$Also \qquad i = C \cdot \frac{dv}{dt} = CV \cdot \frac{d}{dt}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\therefore \qquad i = \frac{V}{R} \quad e^{-\frac{t}{RC}}$$

Analysis of Growth and Decay

At the instant of switching on, t = 0 and $e^{-0} = 1$,

$$\therefore \qquad Initial\ value\ of\ current = \frac{V}{R} = (say) I$$

This result is really obvious from the fact that at the instant of switching on there is no charge on C and therefore no p.d. across it. Consequently the whole of the applied voltage must momentarily be absorbed by R.

$$Substituting\ for \frac{V}{R}\ in\ above\ expression, we\ have\ instantaneous\ charging\ current\ as;$$

$$i = I\ e^{-\frac{t}{RC}}$$

If the p.d. across the capacitor continued increasing at the initial rate, it would be represented by OA, the tangent drawn to the initial part of the curve. If T is the time constant, namely the time required for the p.d. across C to increase from zero to its final value if it continued increasing at its initial rate, then;

$$Initial\ rate\ of\ increase\ of\ p.d. = \frac{V}{T}\ \ volts\ per\ second$$

But t follows from equation that at the instant of closing the switch on position a $\boldsymbol{v = 0}$, then

$$\boldsymbol{V = RC\,\frac{dv}{dt}}$$

$$\boldsymbol{Therefore\ initial\ rate\ of\ change\ of\ p.d.is;} \qquad \boldsymbol{\frac{dv}{dt} = \frac{V}{RC}}$$

$$\boldsymbol{Equating\ above\ equation, we\ have;\ \therefore \frac{V}{T} = \frac{V}{RC}; \qquad \therefore\ \ T = RC\ \ seconds}$$

$$\boldsymbol{Hence\ we\ can\ rewrite\ equations\ above\ as;\ \ v = V\left(1 - e^{-\frac{t}{T}}\right)\ and\ \ i = Ie^{-\frac{t}{T}}}$$

Discharge of a capacitor through a resistor

Having charged capacitor C in Fig. 11 to a p.d. of V volts, let us now move switch S over to position b and thereby discharge the capacitor through R. The pointer of microammeter A is immediately deflected to a maximum value in the negative direction, and then the readings on both the microammeter and the voltmeter (Fig. 11) decrease to zero as indicated in Fig. 14.

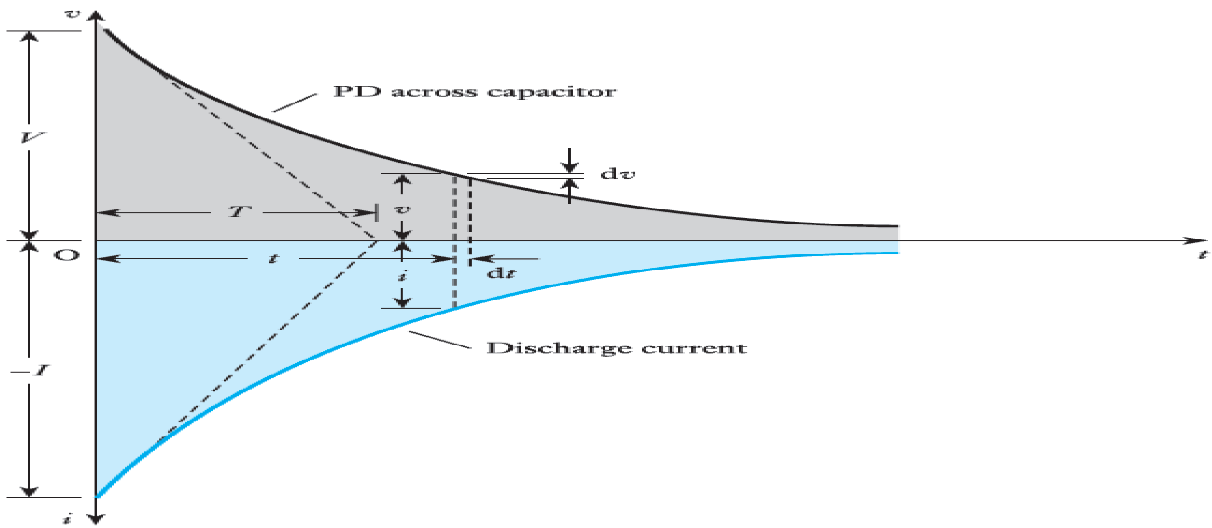

Fig. 14: Variation of current and p.d. during discharge.

Suppose the p.d. across C to be v volts t seconds after S has been moved to position b, and the corresponding current to be i amperes, as in Fig. 14, then $\boldsymbol{i = -\frac{v}{R}}$

The negative sign indicates that the direction of the discharge current is the reverse of that of the charging current.

$$Suppose\ the\ p.d.\ across\ C\ to\ change\ by\ dv\ volts\ in\ dt\ seconds, \therefore \quad \boldsymbol{i = C \cdot \frac{dv}{dt}}$$

Discharge of a capacitor through a resistor

Since dv is now negative, *i* must also be negative, as already noted. Equating above two equation, we have;

$$-\frac{v}{R} = C \cdot \frac{dv}{dt}$$

$$so\ that, \qquad \frac{dt}{RC} = -\frac{dv}{v}$$

$$Integrating\ both\ sides, we\ have;\ \frac{t}{RC} = -\ln v + A$$

$$when\ t = 0, v = V, \qquad so\ that\ A = \ln V\,.$$

$$Hence;\ \frac{t}{RC} = \ln V/v$$

$$so\ that;\ \frac{V}{v} = e^{\frac{t}{RC}}$$

$$and \qquad v = Ve^{-\frac{t}{RC}} = Ve^{-\frac{t}{T}}$$

$$Also; \qquad i = -\frac{v}{R} = -\frac{V}{R}\,e^{-\frac{t}{RC}} = -Ie^{-\frac{t}{T}}$$

$$\therefore \qquad -I\,e^{-\frac{t}{RC}}$$

$$where\ \ I = initial\ value\ of\ the\ discharge\ current = \frac{V}{R}.$$

Example: An $8\mu F$ capacitor is connected in series with a 0.5MΩ resistor across a 200 V d.c. supply. Calculate: (a) the time constant; (b) the initial charging current; (c) the time taken for the p.d. across the capacitor to grow to 160 V; (d) the current and the p.d. across the capacitor 4.0 s after it is connected to the supply.