Tilray Convertible Bond Valuation

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Question 1

< Tilray, 5% 1oct2023, USD (Conv.) Description >

Underlying Asset	Tilray Common Stock
Face Value	$100
Coupon rate	5%
Conversion rate	0.59735
Knock out Barrier	0
Call Barrier	$250
Issue date	2018/10/4
Maturity	2023/10/1
Call review date	After 2021/10/1
Coupon date	Semi-annually
Conversion date	Anytime
Dividend	0%
Interest rate	2.4%
Borrow rate	3%
Delta t	1/52
Initial stock price	145.5

Valuation

Question 1

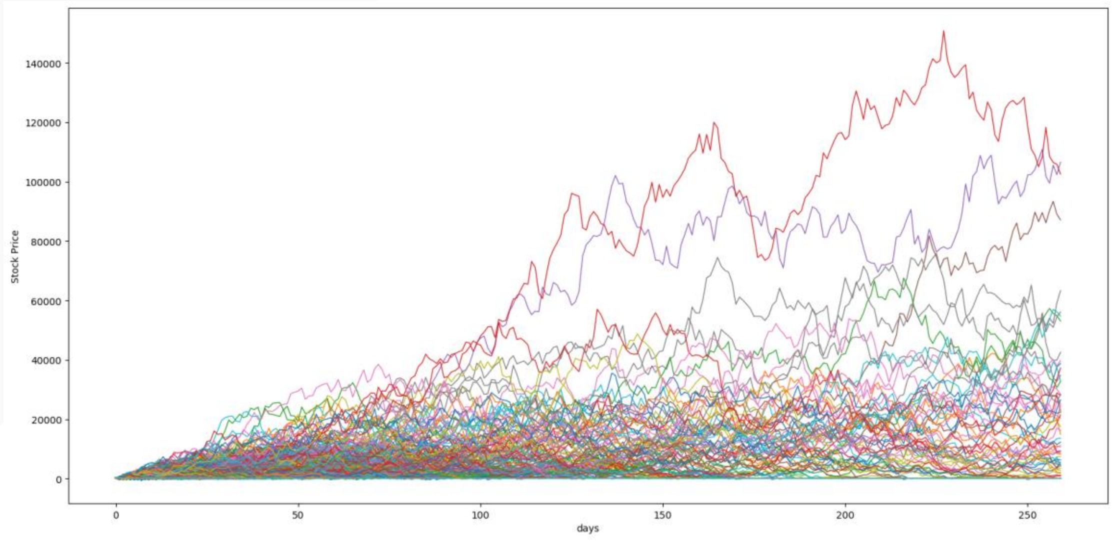
Constant Elasticity of Variance process(CEV)

- Physical : $\frac{dS}{S} = \mu\, dt + \frac{\omega}{\sqrt{S}} dW^P$

- Risk-neutral : $\frac{dS}{S} = (r - y)\, dt + \frac{\omega}{\sqrt{S}} dW^Q$

  - r (riskless rate) : 3%
  - y (share borrowing fee) : 2.4%
  - ω : 8.4

Stock paths under the CEV process



- Stock price hits 0
- Possible to check default probability

Valuation

Question 1

Implicit Finite Difference method(IFDM)

$$C_t + (r - y)C_s + \frac{1}{2}\omega^2 S C_{ss} - rC = 0$$

By using central difference scheme

$$a_j = -\left[sigma^2 * \left(S_j\right)^2 * \frac{\Delta t}{2(\Delta S)^2} + (r - y) * S_j * \frac{\Delta t}{2 * \Delta S}\right]$$

$$b_j = 1 + r * \Delta t + sigma^2 * \left(S_j\right)^2 * \frac{\Delta t}{(\Delta S)^2}$$

$$c_j = -\left[sigma^2 * \left(S_j\right)^2 * \frac{\Delta t}{2(\Delta S)^2} - (r - y) * S_j * \frac{\Delta t}{2 * \Delta S}\right]$$

$$d_j = V_j^{i+1}$$

$$\begin{pmatrix} b_0 & c_0 & 0 & \cdots & 0 \\ a_1 & b_1 & c_1 & \ddots & \vdots \\ 0 & a_2 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & c_{jmax-1} \\ 0 & \cdots & 0 & a_{jmax} & b_{jmax} \end{pmatrix} \begin{pmatrix} {V_0}^i \\ {V_1}^i \\ {V_2}^i \\ \vdots \\ {V_{jmax}}^i \end{pmatrix} = \begin{pmatrix} {d_0}^i \\ {d_1}^i \\ {d_2}^i \\ \vdots \\ {d_{jmax}}^i \end{pmatrix}$$

- Linear system
- Thomas algorithm or LU Decomposition

Valuation

Question 1

Result

- The number of stock node : 100
- The number of time node : 260(=52 x 5)
- Maximum value of stock node : 291(=145.5x2)
- Minimum value of stock node : 0
- **Theoretical Value : $94.65**

Q1 . Is this a good deal if Merrill Lynch is offering the bond at par(face value)?

"Theoretical value is $94.65, which is lower than offering price of Merrill Lynch. It is not good deal for us."

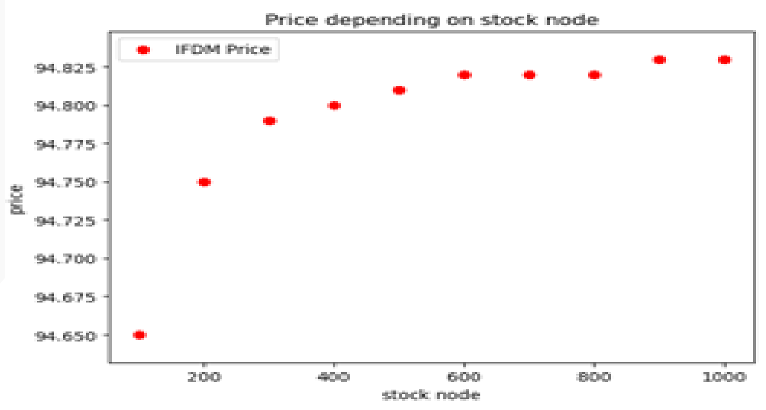
Valuation

Question 1

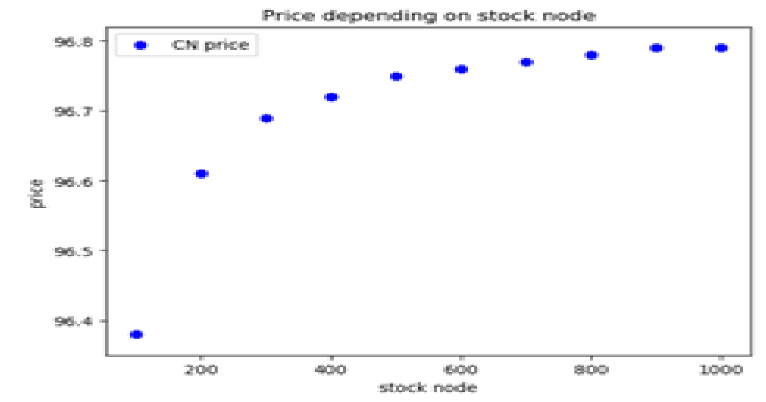
“Our solutions are not sensitive to the size of stock price grid.”

The number of stock node	IFDM Price	Crank-Nicolson Price
100	94.65	96.38
200	94.75	96.61
300	94.79	96.69
400	94.8	96.72
500	94.81	96.75
600	94.82	96.76
700	94.82	96.77
800	94.82	96.78
900	94.83	96.79
1000	94.83	96.79

IFDM price



Crank-Nicolson price



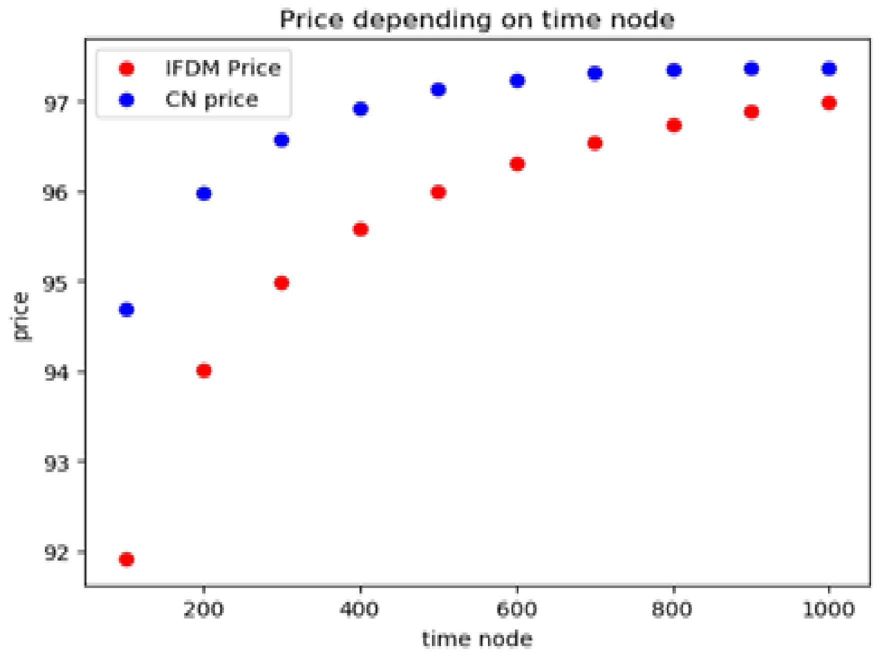
Valuation

Question 1

"When the number of time node go up, the price of each models converges to near $97."

The number of time node	IFDM Price	Crank-Nicolson Price
100	91.91	94.69
200	94.01	95.98
300	94.98	96.58
400	95.58	96.92
500	95.99	97.13
600	96.3	97.24
700	96.54	97.32
800	96.73	97.35
900	96.88	97.37
1000	96.99	97.37

IFDM/ Crank-Nicolson price



Q2. What are important real-world considerations that your valuation in Question 1 omitted to consider?

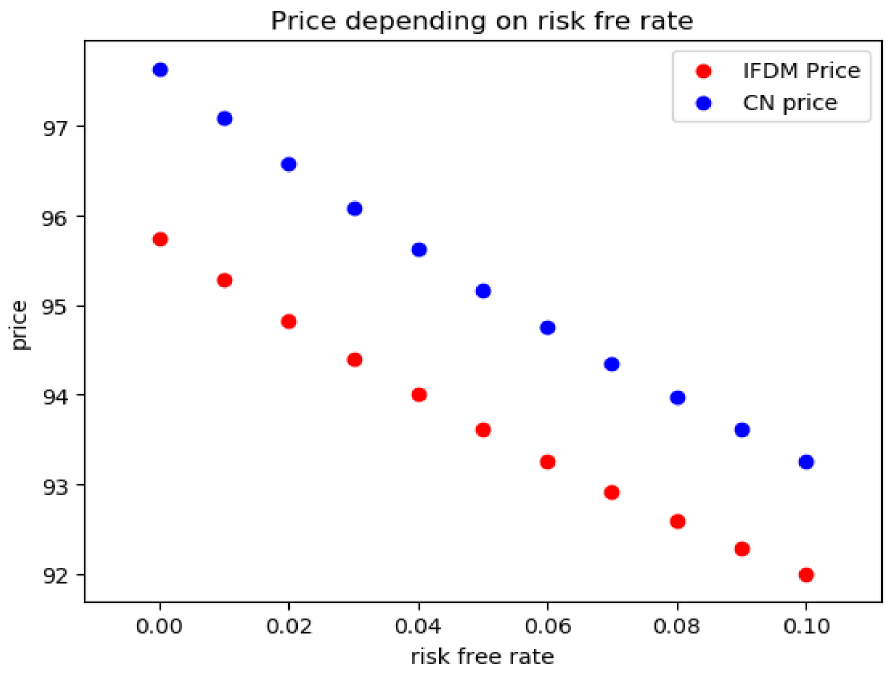
Variables to Consider	More valuable	Less valuable
Interest rate	Down	Up
Dividend rate	Down	Up
Volatility	Down	Up
Borrowing rate	Down	Up
The number of stock node	No correlation	
The number of time node	Up	Down

Real World Consideration

Question 2

About Interest Rates…

IFDM/ Crank-Nicolson price



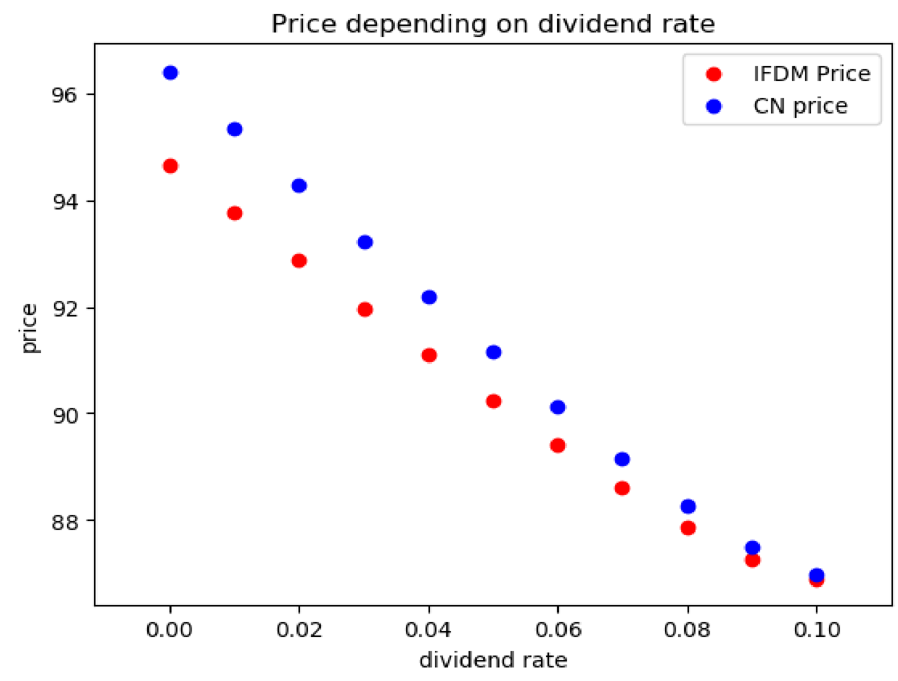
Interest rate	IFDM Price	Crank-Nicolson Price
0%	95.75	97.64
1%	95.28	97.1
2%	94.83	96.58
3%	94.4	96.08
4%	94	95.62
5%	93.61	95.17
6%	93.25	94.75
7%	92.91	94.35
8%	92.59	93.97
9%	92.28	93.61
10%	91.99	93.26

Real World Consideration

Question 2

About Dividend Rates…

IFDM/ Crank-Nicolson price



Dividend rate	IFDM Price	Crank-Nicolson Price
0%	94.65	96.38
1%	93.76	95.32
2%	92.86	94.27
3%	91.97	93.22
4%	91.1	92.18
5%	90.24	91.15
6%	89.41	90.14
7%	88.61	89.17
8%	87.88	88.27
9%	87.27	87.5
10%	86.91	86.98

Real World Consideration

Question 2

About Volatility…

CEV Process: $$\frac{dS}{S} = (r - y)\,dt + \frac{\omega}{\sqrt{S}}\,dW^{Q}$$

Acts as volatility!

Real World Consideration

Question 2

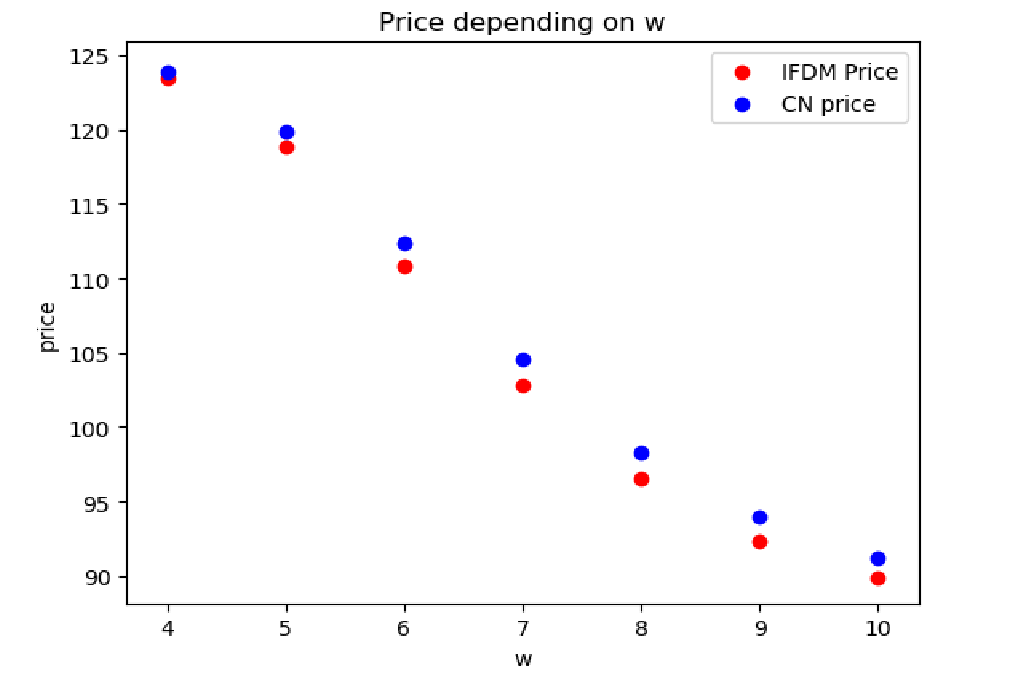
About Volatility…

CEV Process: $$\frac{dS}{S} = (r - y)\, dt + \frac{\omega}{\sqrt{S}}\, dW^Q$$

Acts as volatility!

Value of w	IFDM Price	Crank-Nicolson Price
4	123.44	123.88
5	118.83	119.82
6	110.83	112.34
7	102.8	104.59
8	96.55	98.34
9	92.4	93.98
10	89.91	91.23

IFDM/ Crank-Nicolson price

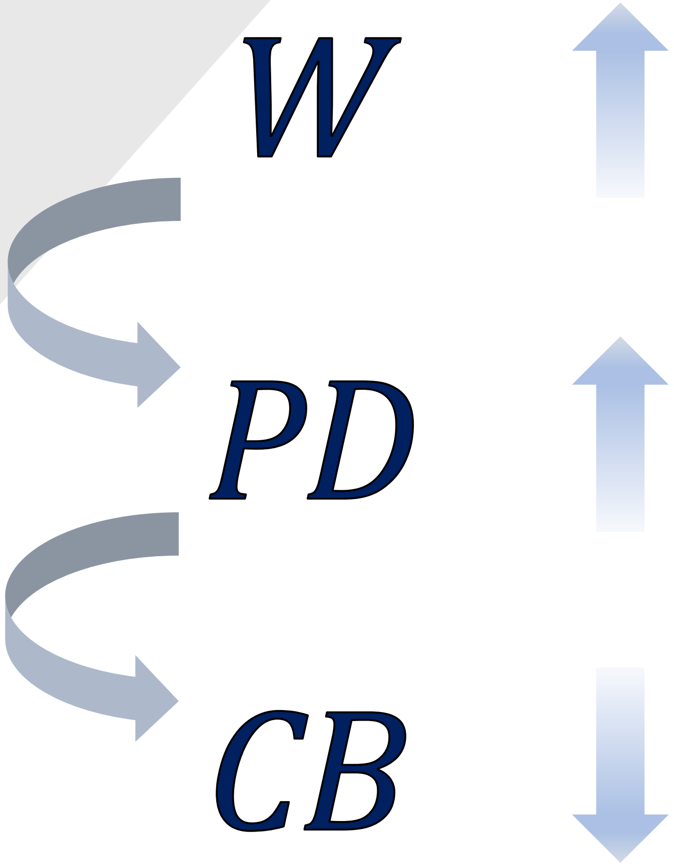


Real World Consideration

Question 2

When we simulated stock paths…

The value of w	Probability of default
0	0%
1	0%
2	0%
3	0.2%
4	2.6%
5	10.1%
6	19.7%
7	30.4%
8	41.2%
9	48.8%



Initial Hedge Ratio

Question 3

$$\textit{Delta formula}$$
$$(Bond_0(S_0+\Delta S) - Bond_0(S_0 - \Delta S))/2\Delta S$$
$$\Delta S = 2.91$$

	0	1	2	3
45	87.2212	87.3585	87.4973	87.6376
46	88.7233	88.859	88.9963	89.1352
47	90.2173	90.3513	90.4869	90.6243
48	91.7036	91.8357	91.9696	92.1052
49	93.1826	93.3127	93.4446	93.5784
50	94.6547	94.7827	94.9125	95.0442
51	96.1204	96.246	96.3735	96.503
52	97.5799	97.703	97.8281	97.9553
53	99.0337	99.1541	99.2767	99.4013

Q3 . Compute initial hedge ratio for the bonds, how many shares would you sell if you bought $100 face value?

"Initial hedge ratio is 0.50515,
I should sell 0.50515 shares per $100"

Scenario Testing

Question 4

Interpolation formula

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1} * (x - x_1)$$

y = target bond price, x = 78
y_1 = lower bond price, x_1 = 75.66
y_2 = higher bond price, x_2 = 78.57

	2018/10/4	2019/1/31
Stock price	$145.5	$78
Market price	$X	$83
Theoretical bond price	$94.65	$57.9490
The number of shares	50,515	50,515
Profit from stock price change	$3,409,762.5	

Q4 .

1. Stock has fallen to $78 and the bonds has fallen to 83% of face value. What are the bonds theoretically worth now?

"Theoretical bond price is $57.9490."

2. How much money did you make or lose, how do you interpret the market`s current valuation of the bonds?

"profit = $3,409,762.5,
mean: model price is undervalue or market price is overvalue."

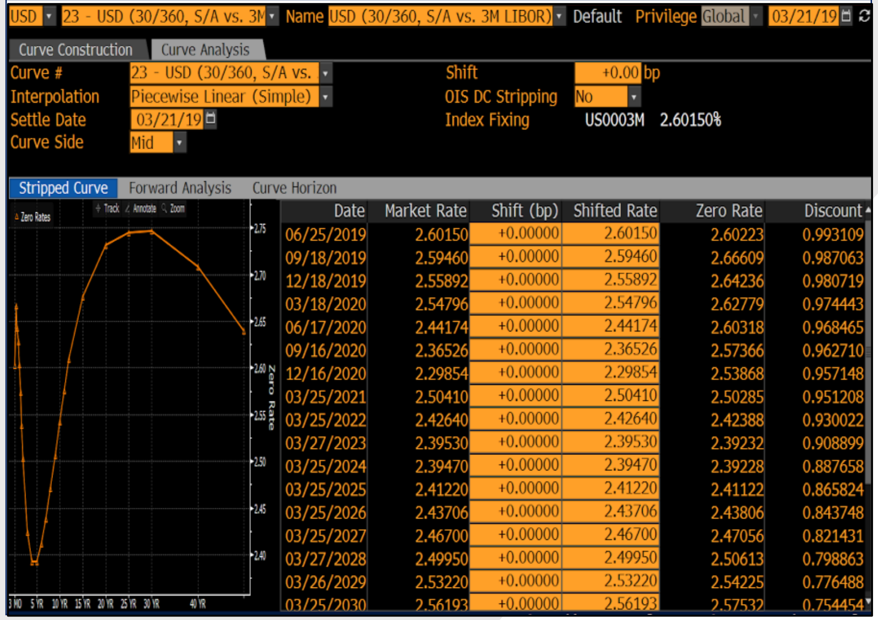
Model Verification

Question 5

Interest rate stochastic model

- ✓ $HO\&LEE\ model: dr = \ \theta(t)dt + \sigma dw$
- ✓ $Vasicek\ model:\ \alpha * (\mu - r) * dt + \sigma dw$

Interest rate term structure



Date	Market Rate	Shift (bp)	Shifted Rate	Zero Rate	Discount
06/25/2019	2.60150	+0.00000	2.60150	2.60223	0.993109
09/18/2019	2.59460	+0.00000	2.59460	2.66609	0.987063
12/18/2019	2.55892	+0.00000	2.55892	2.64236	0.980719
03/18/2020	2.54796	+0.00000	2.54796	2.62779	0.974443
06/17/2020	2.44174	+0.00000	2.44174	2.60318	0.968465
09/16/2020	2.36526	+0.00000	2.36526	2.57366	0.962710
12/16/2020	2.29854	+0.00000	2.29854	2.53868	0.957148
03/25/2021	2.50410	+0.00000	2.50410	2.50285	0.951208
03/25/2022	2.42640	+0.00000	2.42640	2.42388	0.930022
03/27/2023	2.39530	+0.00000	2.39530	2.39232	0.908899
03/25/2024	2.39470	+0.00000	2.39470	2.39228	0.887658
03/25/2025	2.41220	+0.00000	2.41220	2.41122	0.865824
03/25/2026	2.43706	+0.00000	2.43706	2.43806	0.843748
03/25/2027	2.46700	+0.00000	2.46700	2.47056	0.821431
03/27/2028	2.49950	+0.00000	2.49950	2.50613	0.798863
03/26/2029	2.53220	+0.00000	2.53220	2.54225	0.776488
03/25/2030	2.56193	+0.00000	2.56193	2.57532	0.754454

Q5 . If our initial model did not do a good job of telling us how many shares to sell in order to off-set the market risk of changes in the bond price, does this mean that our model was wrong?

"Yes, our model might be wrong."

- reason: Interest rate

- adjustment: Interest rate stochastic model or term structure