

# LLM

Exercise 1:  $y' = 1 (w \cdot x > 0)$

$$w' = w + \alpha (y - y')x \quad (\alpha > 0)$$

$$w'x = wx + \alpha (y - y')x \cdot x$$

assume  $y = 0 \quad y' = 1$

to train a point in one step  $\Rightarrow wx + \alpha (y - y')x \cdot x < 0$

$$\text{since } y - y' = -1 < 0 \Rightarrow (y - y')wx + \alpha (y - y')^2 x \cdot x > 0$$

$$(y - y')^2 = 1 \Rightarrow (y - y')wx + \alpha \cdot xx > 0 \Rightarrow \alpha > -\frac{(y - y')wx}{xx}$$

assume  $y = 1 \quad y' = 0$

to train a point in one step  $\Rightarrow wx + \alpha (y - y')xx > 0$

$$\text{since } y - y' = 1 > 0 \Rightarrow (y - y')wx + \alpha (y - y')^2 xx > 0$$

$$(y - y')^2 = 1 \Rightarrow (y - y')wx + \alpha xx > 0 \Rightarrow \alpha > -\frac{(y - y')wx}{xx}$$

so, as long as  $\alpha$  satisfy  $\alpha > \frac{-(y - y')wx}{xx}$ , we could train a point in one step.

Exercise 2.

$$w \cdot (2, 2) = 2 w \cdot (1, 1)$$

if  $w \cdot (1, 1) < 0$  then  $w \cdot (2, 2) = 2 w \cdot (1, 1)$  must smaller than 0.

so, it is not possible to find a weight  $w \in \mathbb{R}^2$  with  $w \cdot (1, 1) < 0$  and  $w \cdot (2, 2) > 0$ .

Exercise 3:

Assume we could train XOR use perceptron.

$$\text{then we have } \begin{cases} w \cdot (0, 0) = 0 \leq 0 \\ w \cdot (0, 1) = w_2 > 0 \quad (2) \\ w \cdot (1, 0) = w_1 > 0 \quad (3) \\ w \cdot (1, 1) = w_1 + w_2 < 0 \quad (4) \end{cases}$$

Base on equation (2) and (3), we have  $w_2 > 0$ ,  $w_1 > 0$ .

So,  $w_1 + w_2 > 0$ . However, in equation (4), we could see if we could train XOR use perceptron,  $w_1 + w_2 < 0$ . this contradicts with what we get from equation (2) and (3).

So, we could not train XOR use perceptions.