

# LLM

Exercise 1:  $y' = \underline{1} (w \cdot x > 0)$

$$w' = w + \alpha(y - y')x \quad (\alpha > 0)$$

$$w'x = wx + \alpha(y - y')x \cdot x$$

$$\text{assume } y=0 \quad y'=1$$

$$\text{to train a point in one step} \Rightarrow wx + \alpha(y - y')x \cdot x < 0$$

$$\text{since } y - y' = -1 < 0 \Rightarrow (y - y')wx + \alpha(y - y')^2 x \cdot x > 0$$

$$(y - y')^2 = 1 \Rightarrow (y - y')wx + \alpha \cdot xx > 0 \Rightarrow \alpha > -\frac{(y - y')wx}{xx}$$

$$\text{assume } y=1 \quad y'=0$$

$$\text{to train a point in one step} \Rightarrow wx + \alpha(y - y')x \cdot x > 0$$

$$\text{since } y - y' = 1 > 0 \Rightarrow (y - y')wx + \alpha(y - y')^2 x \cdot x > 0$$

$$(y - y')^2 = 1 \Rightarrow (y - y')wx + \alpha \cdot xx > 0 \Rightarrow \alpha > -\frac{(y - y')wx}{xx}$$

so, as long as  $\alpha$  satisfies  $\boxed{\alpha > -\frac{(y - y')wx}{xx}}$ , we could train a point in one step.

Exercise 2:

$$w \cdot (2, 2) = 2w \cdot (1, 1)$$

if  $w \cdot (1, 1) < 0$  then  $w \cdot (2, 2) = 2w \cdot (1, 1)$  must smaller than 0.

so, it is not possible to find a weight  $w \in \mathbb{R}^2$  with  
 $w \cdot (1, 1) < 0$  and  $w \cdot (2, 2) > 0$ .

Exercise 3:

Assume we could train XOR use perceptron.

then we have

$w \cdot (0, 0) = 0 \leq 0$ $w \cdot (0, 1) = w_2 > 0 \quad \textcircled{2}$ $w \cdot (1, 0) = w_1 > 0 \quad \textcircled{3}$ $w \cdot (1, 1) = w_1 + w_2 < 0 \quad \textcircled{4}$
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Base on equation  $\textcircled{2}$  and  $\textcircled{3}$ , we have  $w_2 > 0$ ,  $w_1 > 0$ .

so,  $w_1 + w_2 > 0$ . However, in equation  $\textcircled{4}$ , we could see if we could train XOR use perceptron,  $w_1 + w_2 < 0$ . this contradicts with what we get from equation  $\textcircled{2}$  and  $\textcircled{3}$ .

So, we could not train XOR use perceptrons.