Supplementary Material of "Evolutionary Bilevel Optimization based on Covariance Matrix Adaptation"

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***Abstract*—This is the supplementary material to the paper entitled "Evolutionary Bilevel Optimization based on Covariance Matrix Adaptation", written by Xiaoyu He, Yuren Zhou, and Zefeng Chen. This material contains two sections. The first section provides the effectiveness test for different components of BL-CMA-ES. The second section provides the comparison between BL-CMA-ES and a deterministic algorithm. Additional experimental results are also presented in this material.**

S-I. Effectiveness of Different Components of BL-CMA-ES

The proposed BL-CMA-ES has two main components: distribution sharing and elite preservation. The former is a key trait that distinguishes BL-CMA-ES from most existing BLEAs. The latter is a necessary part when applying CMA-ES to bilevel optimization due to its non-elitist framework. To investigate their effectiveness, we consider the following three variants of BL-CMA-ES:

- BL-CMA-ESv1. The distribution sharing mechanism is removed. The covariance matrix and $\boldsymbol{p}$-path of the lower-level CMA-ES are initialized to the $n \times n$ identity matrix and the n-dimensional vector $\mathbf{0}$, respectively. The initial mean value is uniformly sampled in the lower-level feasible region. Besides, the dimension of upper-level CMA-ES is reduced from $N+n$ to N. That is, this variant is exactly a nested CMA-ES with elite preservation.
- BL-CMA-ESv2. The elite preservation mechanism is removed. Technically, we remove Line 13 and Line 18 from Algorithm 3.
- BL-CMA-ESv3. Both the distribution sharing and elite preservation are removed from BL-CMA-ES.

BL-CMA-ES is compared with the above variants on the $(2+3)$-dimensional SMD problems. Tables S-I and S-II provide the experimental results in terms of solution accuracy and function evaluations, respectively. By comparing BL-CMA-ES and BL-CMA-ESv1, we find that removing the distribution sharing mechanism leads to a performance deterioration on almost all the problems. This demonstrates that the distribution sharing mechanism brings significant performance improvement. Whereas, as can be observed by comparing BL-CMA-ES and BL-CMA-ESv2, the effectiveness of elite preservation seems to be problem-dependent. On one hand, BL-CMA-ESv2 achieves very competitive solutions with less function evaluations on SMD1 to SMD6. On the other hand, it suffers from premature convergence on SMD8, SMD10, and SMD12. It shows that the elite preservation mechanism has certain benefits especially in dealing with relatively difficult problems. However, it is hard to say that the elite preservation makes no difference on simpler problems such as SMD1 to SMD6. It can be easily verified by comparing BL-CMA-ESv1 and BL-CMA-ESv3, where removing the elite preservation mechanism significantly weakens BL-CMA-ES1 on all problems. In summary, the distribution sharing mechanism plays a key role in BL-CMA-ES, while elite preservation is essential in handling relatively hard problems.

S-II. Comparison with Deterministic Algorithms

Although the evolutionary algorithm community has made considerable progresses in the theoretical analysis of CMA-ES, there is still no guarantee for the global optimality. Since BL-CMA-ES relies on CMA-ES to solve the lower-level problems, readers may wonder whether it is able to generate at least one feasible solution.

To address this question, this section empirically studies another BL-CMA-ES variant which uses CMA-ES at the upper level and a deterministic algorithm at the lower level. Specifically, we replace the lower-level CMA-ES in BL-CMA-ESv1 with the interior point (IP) method. We choose the implementation proposed in [1] which is available in the MATLAB *fmincon* package. This implementation consists of two main components: the trust region (TR) method and the sequential quadratic programming (SQP) method. TR firstly determines a local search region based on a self-adapted step size. Within this region, SQP then approximates the fitness landscape with a quadratic polynomial and finally minimizes it with Newton's method.

We choose IP as an alternative lower-level optimizer since it is similar to CMA-ES in two aspects. First, both IP and CMA-ES are second-order methods. IP learns the Hessian matrix of the objective function using Quasi-Newton techniques, while CMA-ES adapts the covariance matrix to the inverse of the Hessian matrix with the maximum-likelihood estimators. Second, both the mutation strength in CMA-ES and the step size in IP roughly determine a promising local region before conducting the subsequent estimations.

We denote the above variant as BL-CMA-ESv4. The $(2+3)$-dimensional SMD problems are chosen to test its performance. The experimental results in terms of solution accuracy are summarized in Table S-I. On 6 out of 12 problems, no significant difference can be found between BL-CMA-ESv4

TABLE S-I
MEDIAN AND IQR RESULTS OF THE ACCURACY (UACC AND LACC) ON THE SMD PROBLEMS. THE BEST RESULTS FOR EACH TEST INSTANCE, IN TERMS OF UACC, ARE SHOWN WITH DARK GRAY BACKGROUND.

		BL-CMA-ES	BL-CMA-ESv1	BL-CMA-ESv2	BL-CMA-ESv3	BL-CMA-ESv4
SMD1	UAcc	$1.00E-06(0.0E+0)$	$3.19E-03(5.7E-3)$ •	$1.00E-06(0.0E+0)$ †	$5.63E-03(9.2E-3)$ •	$1.00E-06(0.0E+0)$ †
	LAcc	$1.00E-06(0.0E+0)$	$2.07E-03(2.1E-3)$ •	$1.00E-06(0.0E+0)$ †	$3.13E-03(8.8E-3)$ •	$1.00E-06(0.0E+0)$ †
SMD2	UAcc	$1.00E-06(0.0E+0)$	$6.68E-01(6.8E-1)$ •	$1.00E-06(0.0E+0)$ †	$8.12E+00(6.6E+0)$ •	$1.00E-06(0.0E+0)$ †
	LAcc	$1.28E-06(1.4E-6)$	$8.08E-01(7.0E-1)$ •	$2.10E-06(3.2E-6)$ †	$8.30E+00(8.9E+0)$ •	$1.00E-06(0.0E+0)$ ∘
SMD3	UAcc	$1.00E-06(0.0E+0)$	$1.62E-02(3.9E-2)$ •	$1.00E-06(0.0E+0)$ †	$1.78E-02(2.6E-2)$ •	$1.83E-03(4.9E-3)$ •
	LAcc	$1.12E-06(6.9E-7)$	$2.08E-02(2.8E-2)$ •	$1.00E-06(7.6E-7)$ †	$3.62E-02(2.5E-1)$ •	$3.54E-04(3.6E-3)$ •
SMD4	UAcc	$1.00E-06(0.0E+0)$	$1.33E+00(1.2E+0)$ •	$1.00E-06(0.0E+0)$ †	$1.00E+01(7.4E+0)$ •	$4.38E+00(9.3E-1)$ •
	LAcc	$8.18E-06(2.0E-5)$	$1.98E+00(1.2E+0)$ •	$1.07E-05(4.9E-4)$ †	$1.11E+01(6.2E+0)$ •	$4.75E+00(1.0E+0)$ •
SMD5	UAcc	$1.00E-06(0.0E+0)$	$6.21E+00(5.8E+0)$ •	$1.00E-06(0.0E+0)$ †	$6.95E+01(5.5E+1)$ •	$1.00E-06(0.0E+0)$ †
	LAcc	$1.49E-06(1.8E-6)$	$6.58E+00(6.1E+0)$ •	$3.58E-06(8.0E-6)$ •	$7.20E+01(5.3E+1)$ •	$1.00E-06(0.0E+0)$ ∘
SMD6	UAcc	$1.00E-06(0.0E+0)$	$2.51E+00(3.9E+0)$ •	$1.00E-06(0.0E+0)$ †	$2.40E+00(5.2E+0)$ •	$2.68E+00(3.2E+0)$ •
	LAcc	$1.00E-06(0.0E+0)$	$7.08E-01(7.9E-1)$ •	$1.00E-06(0.0E+0)$ †	$5.70E+00(9.9E+0)$ •	$1.23E-01(3.5E-1)$ •
SMD7	UAcc	$4.91E-02(9.8E-2)$	$5.97E-01(4.3E-1)$ •	$3.69E-02(7.2E-1)$ †	$6.90E+00(2.8E+0)$ •	$1.00E-06(9.8E-2)$ †
	LAcc	$6.25E+01(2.4E+2)$	$9.43E-01(2.4E+2)$ †	$6.35E-01(2.1E+2)$ †	$3.82E+01(1.6E+2)$ †	$1.00E-06(2.4E+2)$ †
SMD8	UAcc	$1.00E-06(0.0E+0)$	$5.84E+00(7.3E+0)$ •	$5.56E+00(1.2E+1)$ •	$4.49E+02(1.5E+3)$ •	$1.00E-06(0.0E+0)$ †
	LAcc	$1.00E-06(0.0E+0)$	$7.95E+00(8.6E+0)$ •	$1.68E+01(3.4E+1)$ •	$4.52E+02(1.5E+3)$ •	$1.13E-04(3.1E-3)$ •
SMD9	UAcc	$1.00E-06(0.0E+0)$	$8.84E+00(1.7E+1)$ •	$1.00E-06(0.0E+0)$ †	$1.12E+02(2.2E+1)$ •	$1.52E-04(1.2E+0)$ •
	LAcc	$1.98E-06(3.1E-6)$	$1.01E+01(1.9E+1)$ •	$1.64E-06(2.6E-6)$ †	$1.19E+02(2.2E+1)$ •	$2.40E-05(1.2E+0)$ •
SMD10	UAcc	$1.60E+01(1.2E+1)$	$1.40E+01(6.9E+0)$ †	$1.60E+01(2.5E+0)$ †	$1.02E+02(2.8E+2)$ •	$1.60E+01(5.7E-1)$ †
	LAcc	$1.00E-06(8.8E-1)$	$1.14E+01(1.0E+1)$ •	$1.16E+00(2.1E+1)$ •	$1.38E+02(2.7E+2)$ •	$7.37E-03(1.4E-1)$ †
SMD11	UAcc	$2.66E-03(3.2E-3)$	$6.18E+00(3.3E+1)$ •	$5.39E+00(2.0E+1)$ •	$1.14E+02(2.5E+1)$ •	$2.20E-02(9.8E-2)$ •
	LAcc	$3.49E-03(5.5E-3)$	$6.46E+00(3.3E+1)$ •	$9.51E+00(3.1E+1)$ •	$1.15E+02(2.5E+1)$ •	$5.41E-03(2.8E-2)$ †
SMD12	UAcc	$1.00E-06(0.0E+0)$	$1.07E+00(1.0E+0)$ •	$1.27E-01(5.4E-1)$ •	$2.24E+00(1.1E+0)$ •	$9.88E-03(4.5E-2)$ •
	LAcc	$1.07E+01(1.6E+1)$	$3.99E+00(2.5E+0)$ †	$1.62E+01(1.3E+1)$ •	$6.17E+00(1.4E+0)$ †	$2.04E-02(7.3E-2)$ ∘

and BL-CMA-ES. However, BL-CMA-ES clearly outperforms BL-CMA-ESv4 on the other problems. For example, BL-CMA-ESv4 performs well on SMD1 but performs poorly on SMD3. Considering SMD3 is the same as SMD1 except for its lower-level multi-modality, the above observation may be resulted from the main difference between CMA-ES and IP: CMA-ES calculates the maximum-likelihood estimators of the covariance matrix through Monte Carlo sampling, while IP updates the Hessian matrix with finite-difference method. Due to this difference, we can conclude that CMA-ES is more robust than IP in dealing with multi-modal functions.

Another observation is that BL-CMA-ESv4 performs poorly in handling constraints, being inferior to BL-CMA-ES on SMD9, SMD11, and SMD12. It is known that, IP utilizes the barrier function in order to penalize constraint violations. It has an advantage that the introduced penalty term is differentiable. But it also keeps the feasible search region away from the constraint boundaries, causing potential inaccuracy at the lower level. Contrarily, BL-CMA-ES does not encounter this question due to its well-established constraint handling methods.

Moreover, as can be found in Table S-II, BL-CMA-ES possesses an overall advantage over BL-CMA-ESv4 in terms of the consumed function evaluations. Fig. S-1 further plots their running times on each problems. It seems that the running time of BL-CMA-ESv4 is quite sensitive to the problem types. For example, it requires much longer time in dealing with constrained problems such as SMD9, SMD10, SMD12. These results show that, with distribution sharing, using CMA-ES at the lower level is helpful in decreasing the computational burden.

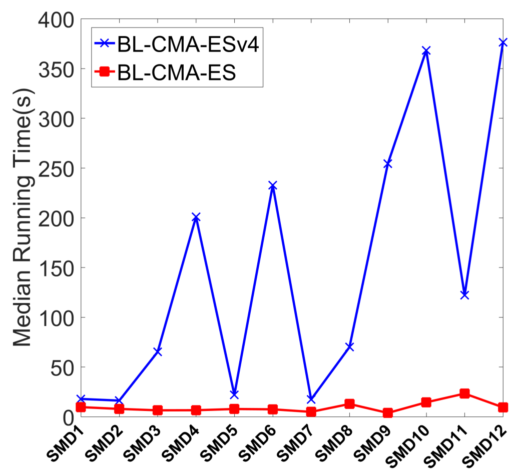


Fig. S-1. Median running time of BL-CMA-ES and BL-CMA-ESv4

REFERENCES

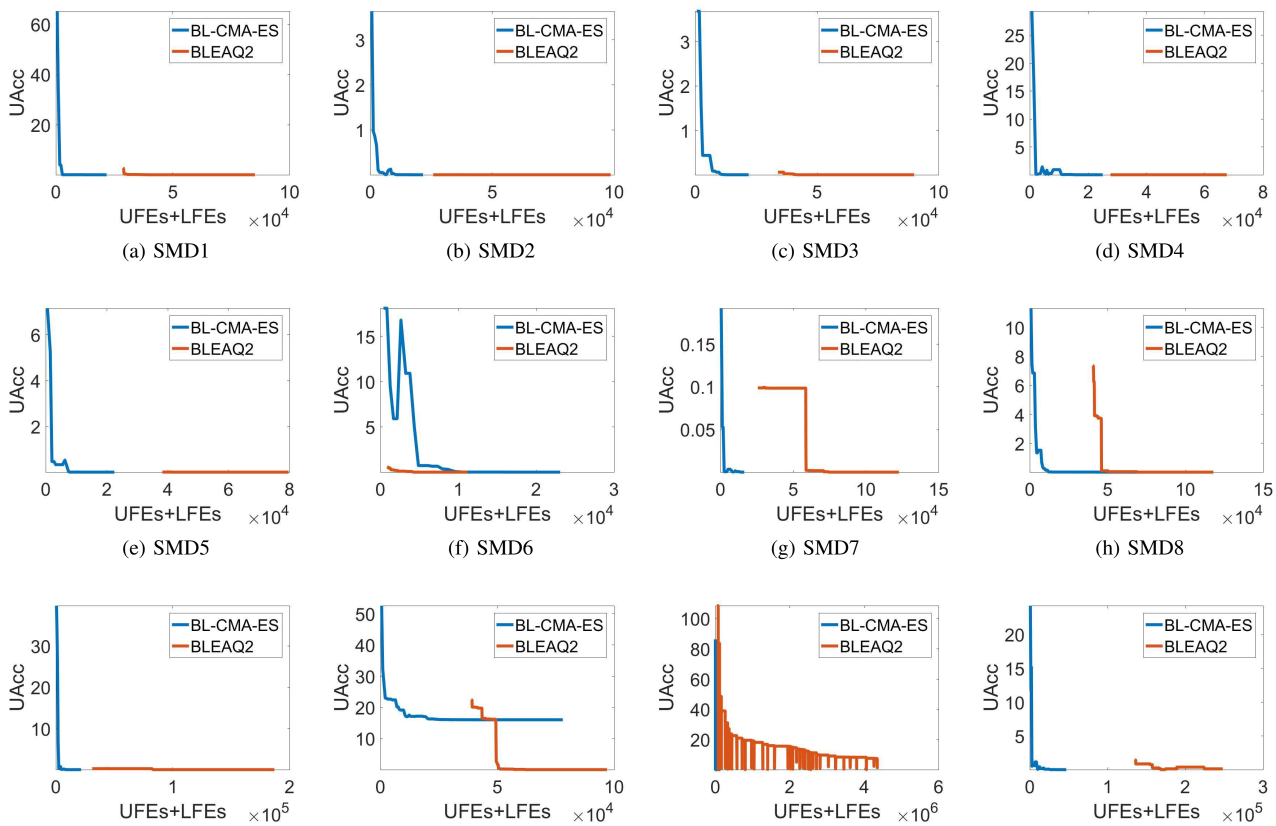
[1] R. H. Byrd, M. E. Hribar, and J. Nocedal, "An Interior Point Algorithm for Large-Scale Nonlinear Programming," *SIAM Journal on Optimization*, vol. 9, no. 4, pp. 877–900, Jan. 1999.

TABLE S-II
MEDIAN AND IQR RESULTS OF THE FUNCTION EVALUATIONS (UFES AND LFES) ON THE SMD PROBLEMS. THE BEST RESULTS FOR EACH TEST INSTANCE, IN TERMS OF UFES+LFES, ARE SHOWN WITH DARK GRAY BACKGROUND.

		BL-CMA-ES	BL-CMA-ESv1	BL-CMA-ESv2	BL-CMA-ESv3	BL-CMA-ESv4
SMD1	UFEs	$3.20E+02(3.5E+1)$	$6.97E+02(4.9E+2)$ ●	$3.12E+02(2.4E+1)$ †	$5.88E+02(3.6E+2)$ ●	$2.25E+02(4.2E+1)$ ○
	LFEs	$2.14E+04(1.6E+3)$	$6.53E+04(4.8E+4)$ ●	$1.88E+04(1.1E+3)$ ○	$4.85E+04(3.0E+4)$ ●	$4.25E+04(8.3E+3)$ ●
SMD2	UFEs	$3.14E+02(4.5E+1)$	$2.50E+03(2.5E+0)$ ●	$2.72E+02(5.6E+1)$ ○	$5.82E+02(2.7E+2)$ ●	$2.32E+02(3.8E+1)$ ○
	LFEs	$2.10E+04(3.9E+3)$	$2.04E+05(2.7E+3)$ ●	$1.77E+04(2.5E+3)$ ○	$4.65E+04(2.3E+4)$ ●	$4.09E+04(7.2E+3)$ ●
SMD3	UFEs	$3.39E+02(3.9E+1)$	$8.25E+02(4.7E+2)$ ●	$3.28E+02(2.8E+1)$ †	$5.82E+02(2.9E+2)$ ●	$6.95E+02(5.2E+2)$ ●
	LFEs	$2.15E+04(2.3E+3)$	$7.51E+04(4.2E+4)$ ●	$1.94E+04(2.0E+3)$ ○	$4.60E+04(2.4E+4)$ ●	$1.62E+05(1.2E+5)$ ●
SMD4	UFEs	$3.69E+02(4.9E+1)$	$2.50E+03(2.5E+0)$ ●	$3.52E+02(6.0E+1)$ ○	$6.27E+02(3.3E+2)$ ●	$2.50E+03(2.0E+0)$ ●
	LFEs	$2.35E+04(2.8E+3)$	$1.93E+05(2.3E+3)$ ●	$2.16E+04(3.4E+3)$ ○	$4.76E+04(2.5E+4)$ ●	$4.95E+05(5.6E+3)$ ●
SMD5	UFEs	$3.64E+02(3.4E+1)$	$2.50E+03(2.5E+0)$ ●	$3.48E+02(5.6E+1)$ †	$6.60E+02(3.9E+2)$ ●	$2.33E+02(4.4E+1)$ ○
	LFEs	$2.20E+04(1.8E+3)$	$1.95E+05(2.7E+3)$ ●	$2.03E+04(2.3E+3)$ ○	$4.97E+04(3.0E+4)$ ●	$5.66E+04(1.1E+4)$ ●
SMD6	UFEs	$4.48E+02(2.9E+1)$	$2.50E+03(2.5E+0)$ ●	$4.00E+02(6.0E+1)$ ○	$6.54E+02(2.5E+2)$ ●	$2.50E+03(4.0E+0)$ ●
	LFEs	$2.47E+04(1.7E+3)$	$1.97E+05(4.7E+3)$ ●	$2.07E+04(3.3E+3)$ ○	$4.85E+04(1.8E+4)$ ●	$5.83E+05(1.6E+4)$ ●
SMD7	UFEs	$4.73E+02(3.9E+2)$	$2.50E+03(2.5E+0)$ ●	$3.32E+02(7.2E+1)$ ○	$6.21E+02(2.7E+2)$ †	$2.39E+02(3.8E+2)$ ○
	LFEs	$2.36E+04(7.8E+3)$	$2.04E+05(1.9E+4)$ ●	$1.63E+04(3.4E+3)$ ○	$4.74E+04(2.5E+4)$ ●	$4.31E+04(8.2E+4)$ ●
SMD8	UFEs	$1.44E+03(2.5E+2)$	$2.50E+03(3.0E+0)$ ●	$3.68E+02(5.6E+1)$ ○	$5.37E+02(3.2E+2)$ ○	$7.03E+02(2.7E+2)$ ○
	LFEs	$7.03E+04(1.1E+4)$	$1.86E+05(2.8E+3)$ ●	$2.32E+04(3.2E+3)$ ○	$3.97E+04(2.3E+4)$ ○	$1.81E+05(6.2E+4)$ ●
SMD9	UFEs	$3.22E+02(4.9E+1)$	$2.50E+03(4.0E+0)$ ●	$3.08E+02(4.4E+1)$ †	$6.63E+02(3.4E+2)$ ●	$2.20E+03(1.4E+3)$ ●
	LFEs	$2.02E+04(2.8E+3)$	$2.07E+05(3.0E+3)$ ●	$1.86E+04(1.7E+3)$ ○	$5.42E+04(2.7E+4)$ ●	$4.92E+05(3.0E+5)$ ●
SMD10	UFEs	$1.59E+03(9.7E+2)$	$2.50E+03(4.0E+0)$ ●	$1.41E+03(1.2E+3)$ †	$5.58E+02(2.6E+2)$ ○	$2.50E+03(4.0E+0)$ ●
	LFEs	$7.11E+04(3.4E+4)$	$2.06E+05(2.6E+3)$ ●	$6.00E+04(4.3E+4)$ †	$4.03E+04(1.8E+4)$ ○	$6.58E+05(6.7E+3)$ ●
SMD11	UFEs	$2.51E+03(4.0E+0)$	$2.50E+03(3.0E+0)$ †	$3.60E+02(2.8E+1)$ ○	$6.63E+02(2.1E+2)$ ○	$7.67E+02(3.3E+2)$ ○
	LFEs	$1.27E+05(8.7E+3)$	$2.45E+05(1.3E+4)$ ●	$2.21E+04(2.1E+3)$ ○	$6.24E+04(2.1E+4)$ ○	$2.03E+05(8.7E+4)$ ●
SMD12	UFEs	$9.64E+02(1.4E+2)$	$2.50E+03(2.5E+0)$ ●	$5.36E+02(1.4E+2)$ ○	$7.62E+02(3.3E+2)$ ○	$2.50E+03(3.0E+0)$ ●
	LFEs	$4.92E+04(7.0E+3)$	$2.26E+05(6.0E+3)$ ●	$3.07E+04(5.3E+3)$ ○	$6.21E+04(2.7E+4)$ ●	$6.75E+05(4.3E+3)$ ●

TABLE S-III
MEDIAN AND IQR RESULTS OF THE FUNCTION EVALUATIONS (UFES AND LFES) ON THE TP PROBLEMS. THE BEST RESULTS FOR EACH TEST INSTANCE, IN TERMS OF UFES+LFES, ARE SHOWN WITH DARK GRAY BACKGROUND.

		BL-CMA-ES	NBLEA	BLEAQ	BLEAQ2
TP1	UFEs	$1.72E+03(5.3E+2)$	$8.76E+02(2.5E+2)$ ○	$5.98E+02(9.5E+0)$ ○	$6.35E+02(2.2E+2)$ ○
	LFEs	$7.68E+04(1.9E+4)$	$1.54E+04(4.5E+3)$ ○	$1.60E+03(1.7E+2)$ ○	$7.98E+03(1.8E+3)$ ○
TP2	UFEs	$1.69E+03(6.0E+2)$	$5.97E+02(1.6E+2)$ ○	$4.02E+02(1.3E+2)$ ○	$3.88E+02(3.6E+2)$ ○
	LFEs	$7.13E+04(2.6E+4)$	$1.05E+04(2.9E+3)$ ○	$1.32E+03(5.4E+2)$ ○	$3.87E+03(3.3E+3)$ ○
TP3	UFEs	$2.50E+03(5.0E+0)$	$8.51E+02(2.5E+2)$ ○	$2.88E+02(9.3E+1)$ ○	$6.31E+02(2.5E+2)$ ○
	LFEs	$1.10E+05(1.7E+4)$	$1.50E+04(4.4E+3)$ ○	$9.15E+02(2.7E+2)$ ○	$4.23E+03(1.2E+2)$ ○
TP4	UFEs	$1.09E+03(1.7E+2)$	$9.60E+02(4.7E+2)$ †	$6.07E+02(3.0E+0)$ ○	$9.84E+02(3.9E+2)$ †
	LFEs	$4.97E+04(7.1E+3)$	$2.58E+04(1.3E+4)$ ○	$3.11E+03(4.7E+2)$ ○	$1.23E+04(5.6E+3)$ ○
TP5	UFEs	$1.41E+03(1.1E+3)$	$7.74E+02(2.3E+2)$ ○	$3.72E+02(9.7E+1)$ ○	$8.25E+02(2.3E+2)$ ○
	LFEs	$6.21E+04(4.1E+4)$	$1.36E+04(4.0E+3)$ ○	$1.42E+03(3.3E+2)$ ○	$5.98E+03(1.6E+3)$ ○
TP6	UFEs	$5.70E+02(1.5E+2)$	$8.19E+02(2.0E+2)$ ●	$5.97E+02(2.5E+0)$ †	$8.85E+02(2.0E+2)$ ●
	LFEs	$3.09E+04(6.0E+3)$	$1.44E+04(3.6E+3)$ ○	$1.98E+03(2.7E+2)$ ○	$6.98E+03(1.7E+3)$ ○
TP7	UFEs	$2.46E+03(8.0E+2)$	$9.34E+02(3.1E+2)$ ○	$4.99E+02(1.7E+2)$ ○	$7.28E+02(1.4E+2)$ ○
	LFEs	$1.09E+05(3.4E+4)$	$2.57E+05(5.7E+4)$ ●	$2.64E+04(6.6E+3)$ ○	$5.12E+04(5.1E+3)$ ○
TP8	UFEs	$1.50E+03(6.1E+2)$	$5.20E+02(2.9E+2)$ ○	$3.79E+02(8.4E+1)$ ○	$5.70E+02(5.1E+2)$ ○
	LFEs	$6.40E+04(2.2E+4)$	$9.18E+03(5.1E+3)$ ○	$1.33E+03(2.8E+2)$ ○	$6.26E+03(5.1E+3)$ ○
TP9	UFEs	$1.06E+03(5.5E+1)$	$5.92E+02(1.5E+2)$ ○	$6.71E+02(3.7E+2)$ ○	$5.97E+02(1.6E+2)$ ○
	LFEs	$5.92E+04(3.0E+3)$	$3.02E+05(3.3E+4)$ ●	$3.79E+04(7.9E+3)$ ○	$3.56E+04(3.3E+3)$ ○
TP10	UFEs	$2.26E+03(8.5E+1)$	$9.50E+02(8.9E+2)$ ○	$1.95E+03(9.2E+2)$ †	$1.17E+03(2.5E+2)$ ○
	LFEs	$1.06E+05(4.4E+3)$	$6.33E+05(4.2E+5)$ ●	$1.64E+05(3.8E+4)$ ●	$1.61E+05(1.8E+4)$ ●



(a) SMD1 (b) SMD2 (c) SMD3 (d) SMD4

(e) SMD5 (f) SMD6 (g) SMD7 (h) SMD8

(i) SMD9 (j) SMD10 (k) SMD11 (l) SMD12

Fig. S-2. Evolutionary trajectories of UAcc on the $(2+3)$-dimensional SMD problems

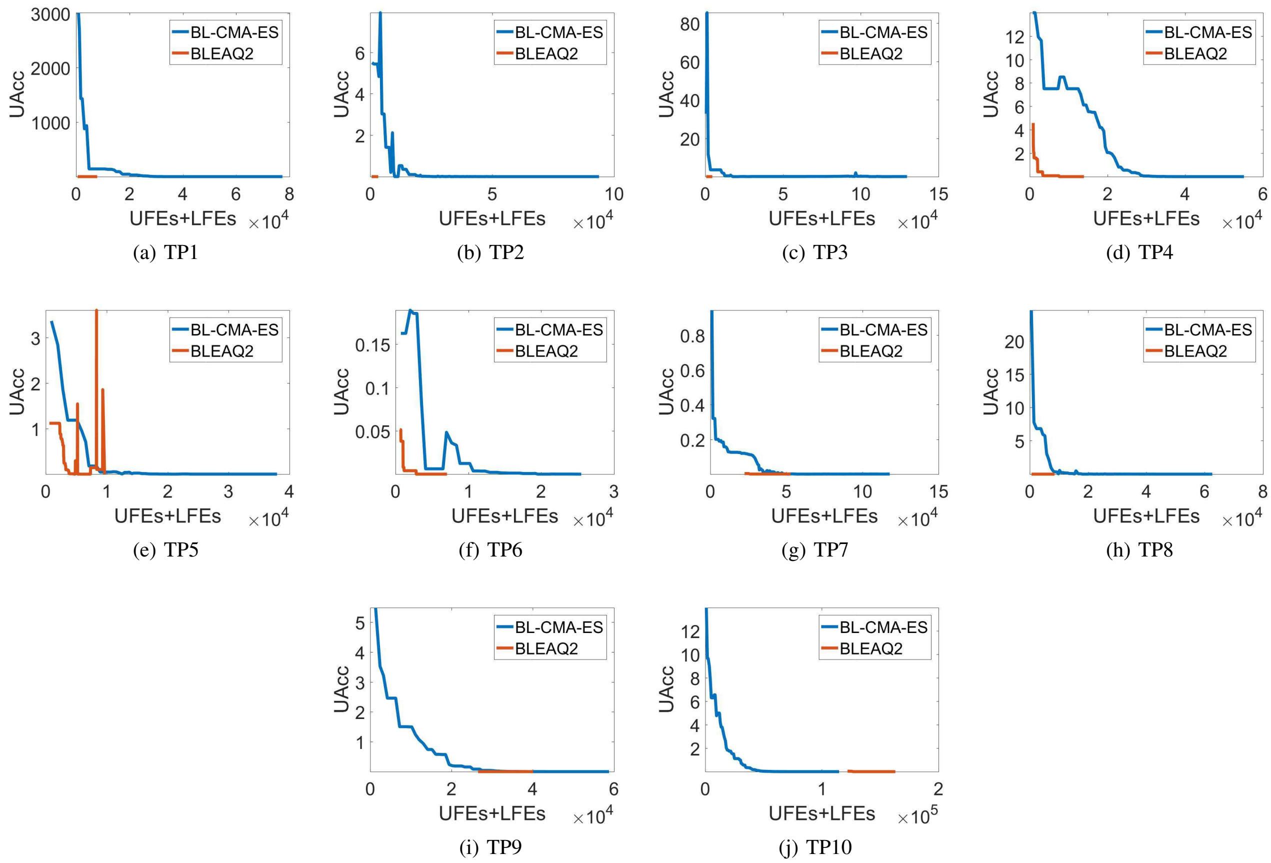


Fig. S-3. Evolutionary trajectories of UAcc on the TP problems

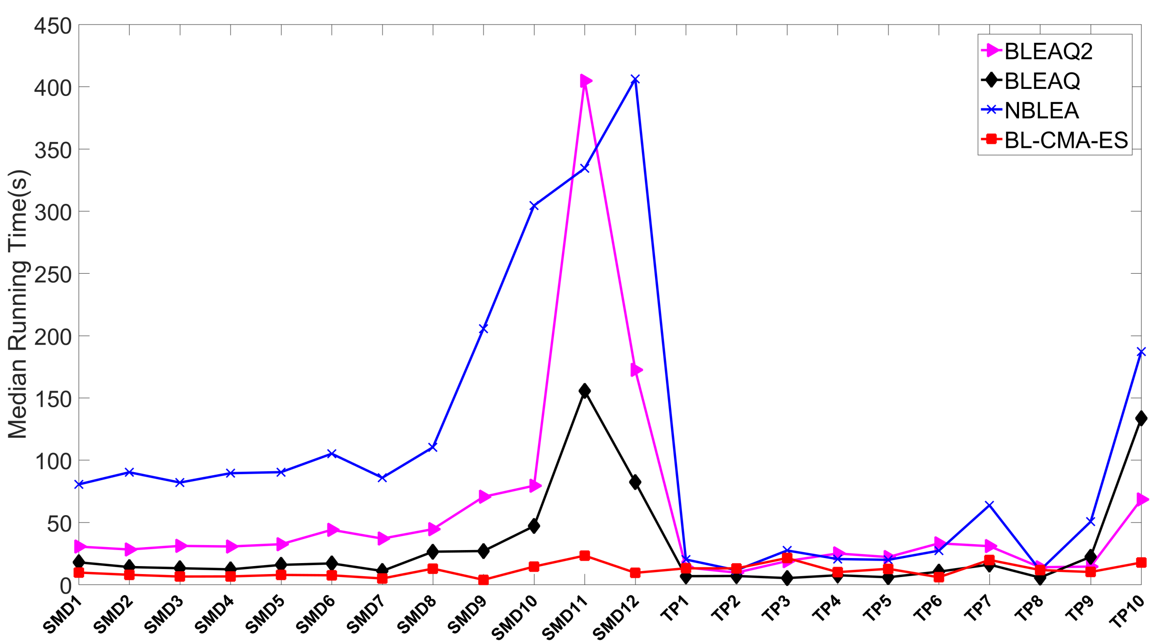


Fig. S-4. Median running time on all test problems

TABLE S-IV
MEDIAN AND IQR RESULTS OF THE FUNCTION EVALUATIONS (UFES AND LFES) ON THE $(5+5)$- AND $(10+10)$-DIMENSIONAL SMD PROBLEMS. THE BEST RESULTS FOR EACH TEST INSTANCE, IN TERMS OF UFES+LFES, ARE SHOWN WITH DARK GRAY BACKGROUND.

$N=5, n=5$		BL-CMA-ES	BLEAQ2
SMD1	UFEs	$7.25E+02(2.6E+1)$	$1.68E+03(5.2E+2)$ ●
	LFEs	$7.57E+04(4.1E+3)$	$1.51E+05(2.5E+4)$ ●
SMD2	UFEs	$6.97E+02(7.4E+1)$	$2.05E+03(1.0E+3)$ ●
	LFEs	$7.67E+04(6.6E+3)$	$1.32E+05(3.9E+4)$ ●
SMD3	UFEs	$7.30E+02(5.5E+1)$	$2.21E+03(5.5E+2)$ ●
	LFEs	$7.26E+04(4.3E+3)$	$1.90E+05(3.1E+4)$ ●
SMD4	UFEs	$7.23E+02(4.9E+1)$	$2.01E+03(6.4E+2)$ ●
	LFEs	$7.67E+04(4.4E+3)$	$8.91E+04(1.5E+4)$ ●
SMD5	UFEs	$9.06E+02(1.4E+2)$	$2.10E+03(8.9E+2)$ ●
	LFEs	$8.55E+04(1.1E+4)$	$1.60E+05(4.5E+4)$ ●
SMD6	UFEs	$8.39E+02(6.6E+1)$	$2.14E+03(8.6E+2)$ ●
	LFEs	$8.12E+04(4.9E+3)$	$2.41E+04(5.6E+3)$ ○
SMD7	UFEs	$1.23E+03(1.6E+2)$	$2.41E+03(8.9E+2)$ ●
	LFEs	$7.47E+04(1.2E+4)$	$1.81E+05(6.9E+4)$ ●
SMD8	UFEs	$3.50E+03(8.4E+2)$	$4.91E+03(2.5E+3)$ ●
	LFEs	$2.49E+05(4.6E+4)$	$3.50E+05(1.7E+5)$ ●
SMD9	UFEs	$8.45E+02(1.6E+2)$	$4.39E+03(5.2E+3)$ ●
	LFEs	$7.99E+04(1.4E+4)$	$2.80E+05(3.3E+5)$ ●
SMD10	UFEs	$3.50E+03(5.0E+0)$	$4.02E+03(3.8E+3)$ †
	LFEs	$2.10E+05(3.0E+3)$	$2.35E+05(1.3E+5)$ †
SMD11	UFEs	$3.50E+03(1.5E+2)$	$7.77E+03(6.0E+3)$ ●
	LFEs	$1.93E+05(1.3E+4)$	$2.65E+05(2.2E+5)$ ●
SMD12	UFEs	$3.50E+03(5.5E+0)$	$5.03E+03(4.0E+3)$ ●
	LFEs	$2.15E+05(6.7E+3)$	$2.25E+05(1.3E+5)$ †
$N=10, n=10$		BL-CMA-ES	BLEAQ2
SMD1	UFEs	$1.38E+03(9.1E+1)$	$4.78E+03(2.4E+3)$ ●
	LFEs	$2.01E+05(8.9E+3)$	$6.69E+05(1.6E+5)$ ●
SMD2	UFEs	$1.29E+03(1.2E+2)$	$5.03E+03(2.0E+3)$ ●
	LFEs	$1.98E+05(1.0E+4)$	$5.90E+05(2.3E+5)$ ●
SMD3	UFEs	$1.42E+03(5.5E+1)$	$7.94E+03(2.5E+3)$ ●
	LFEs	$1.66E+05(1.1E+4)$	$8.35E+05(1.4E+5)$ ●
SMD4	UFEs	$1.44E+03(1.6E+2)$	$2.91E+03(2.7E+3)$ ●
	LFEs	$1.97E+05(1.7E+4)$	$2.32E+05(5.1E+4)$ ●
SMD5	UFEs	$1.88E+03(2.9E+2)$	$6.73E+03(2.9E+3)$ ●
	LFEs	$2.25E+05(2.6E+4)$	$6.34E+05(2.1E+5)$ ●
SMD6	UFEs	$1.68E+03(8.7E+1)$	$4.15E+03(3.3E+3)$ ●
	LFEs	$2.25E+05(1.2E+4)$	$1.14E+05(5.5E+4)$ ○
SMD7	UFEs	$2.81E+03(1.5E+3)$	$7.45E+03(2.8E+3)$ ●
	LFEs	$2.11E+05(9.7E+4)$	$8.59E+05(2.4E+5)$ ●
SMD8	UFEs	$5.00E+03(6.5E+0)$	$8.46E+03(5.9E+3)$ ●
	LFEs	$6.86E+05(3.5E+4)$	$8.60E+05(4.7E+5)$ ●
SMD9	UFEs	$1.80E+03(5.0E+2)$	$2.94E+03(2.5E+3)$ ●
	LFEs	$2.23E+05(4.0E+4)$	$4.00E+05(1.3E+5)$ ●
SMD10	UFEs	$5.01E+03(5.0E+0)$	$4.04E+03(3.3E+3)$ †
	LFEs	$3.92E+05(1.4E+4)$	$5.25E+05(2.7E+5)$ ●
SMD11	UFEs	$5.00E+03(6.0E+0)$	$8.66E+03(4.1E+3)$ ●
	LFEs	$3.50E+05(9.5E+3)$	$5.79E+05(1.3E+5)$ ●
SMD12	UFEs	$5.01E+03(7.0E+0)$	$5.26E+03(3.1E+3)$ †
	LFEs	$4.07E+05(1.6E+4)$	$4.92E+05(1.5E+5)$ ●