Reimann Solver in Geoclaw

Synthesis Paper

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1 Introduction

2 Wave Propagation Algorithm (WPA)

The order one dimensional (1D) wave propagation method is given by eqaution (1)

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}(\mathcal{A}^+\Delta Q_{i-\frac{1}{2}}^n + \mathcal{A}^- Q_{i+\frac{1}{2}}^n) \tag{1}$$

where Q_i^n is a numerical approximation to $\frac{1}{\Delta x}\int_{C_i} q(x,t^n)dx$, $\Delta x = (x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}})$, $\Delta t = (t^{n+1} - t^n)$, $\mathcal{A}^{\pm}\Delta Q_{i\pm\frac{1}{2}}^n$ are fluctuations determined by the to the Riemann Problems at cell interfaces at $x_{i\pm\frac{1}{2}}$. The net updating contributions from the rightward and leftward moving waves into the grid cell C_i from the right and left interface are respectively given by $\mathcal{A}^+\Delta Q_{i-\frac{1}{2}}^n$ and $\mathcal{A}^- Q_{i+\frac{1}{2}}^n$[1].

The second order accuracy is obtained by taking the correction terms into account as shown described in equation (2)

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}(\mathcal{A}^+\Delta Q_{i-\frac{1}{2}}^n + \mathcal{A}^- Q_{i+\frac{1}{2}}^n) - \frac{\Delta t}{\Delta x}(\tilde{F}_{i+\frac{1}{2}}^n - \tilde{F}_{i-\frac{1}{2}}^n) \tag{2}$$

where $\tilde{F}_{i\pm\frac{1}{2}}^n$ are second order correction terms determined the waves in the Riemann problems at $x_{i\pm\frac{1}{2}}$.

3 Shallow Water Equations (SWE)

The 1D SWE are given in equations (3) and (4) below.

$$h_t + (hu)_x = 0 \tag{3}$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghb_x \tag{4}$$

where $h(x,t)$ is the fluid depth, $u(x,t)$ is the vertically averaged horizontal fluid velocity, g is the gravitataional constant, $u(x,t)$ is the vertically averaged horizontal fluid velocity, and $b(x)$ is the bottom surface elevation [1].

4 Reimann Problem for Wet/Dry States

5 Numerical Examples

References

[1] David L. George. Augmented Riemann solvers for the shallow water equations over variable topography with steady states and inundation. 227(6):3089 – 3113, 2008.

[2] Randall J. LeVeque, David L. George, and Marsha J. Berger. Tsunami modelling with adaptively refined finite volume methods. 20:211 – 289, May 2011.