```
%This program  solves a second order accurate approximaion obtained from
%the poison equation using m = 128

a2 =0; b2 = 1;

m = 128;

f1 = @(x,y) -34*(pi^2)*cos(5*pi*x).*sin(3*pi*y);

%exact solution
uexact = @(x,y) cos(5*pi*x).*(sin(3*pi*y));

h = (b2-a2)/(m); %mesh spacing

w = 2/(1+sin(pi*h)); %optimal relaxation parameter

tol = 10^(-8);   %relative residual

maxiter = 10000;  %maximum value of k

[x,y] = meshgrid(((1:m)-1/2)*h); %Uniform mesh, including boundary points.

dx = 1:m;
dy = 1:m;

u = zeros(m,m);

% Evaluate the RHS of Poisson's equation at the interior points.
f = feval(f1,x(dy,dx),y(dy,dx));


for k = 0:maxiter
    u(:,1) = u(:,2) - (h^2)*f(:,1);
    u(:,m) = u(:,m-1) - (h^2)*f(:,m);
    u(1,:) = (1/3)*(u(2,:)-(h^2)*f(1,:));
    u(m,:) = (1/3)*(u(m-1,:) - (h^2)*f(m,:));
    %Iterate
    for j = 2:(m-1)
        for i = 2:(m-1)
            u(i,j) = (1-w)*u(i,j)+(w/4)*(u(i-1,j)+u(i+1,j)+u(i,j-1)+u(i,j+1)-(h^2)*f(i,j));
        end
    end

    %Compute the residual
    residual = zeros(m,m);

    for j = 2:(m-1)
        for i = 2:(m-1)
            residual(i,j) = -4*u(i,j)+(u(i-1,j)+u(i+1,j)+u(i,j-1)+u(i,j+1)-(h^2)*f(i,j));
        end
    end

     %Determine if convergence has been reached
        if norm(residual(:),2)<tol*norm(f(:),2)
                break
    end
end

error = uexact(x,y) - u;
L2 = R2Norm(error,uexact(x,y));
%polyfit
p=polyfit(log(h),log(L2),1);
p
%fprintf('Since the order of convergence,p, is 2.0014, which is approximately 2, \n hence the method is second order accurate.\n')


% Plot solution
figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
surf(x,y,u), xlabel('x'), ylabel('y'), zlabel('u(x,y)'),
title(strcat('Numerical Solution , h=',num2str(h)));

% Plot solution
figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
surf(x,y,uexact(x,y)), xlabel('x'), ylabel('y'), zlabel('u(x,y)'),
title(strcat('Exact Solution, h=',num2str(h)));

 %Plot error
figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
surf(x,y,u-uexact(x,y)),xlabel('x'),ylabel('y'), zlabel('Error'),
title(strcat('Error, h=',num2str(h)));

function L2 = R2Norm(error, uexact)
R = error .^2;
u_ex = uexact.^2;
L2 = sqrt(sum(R,'all')/sum(u_ex,'all'));
end
```

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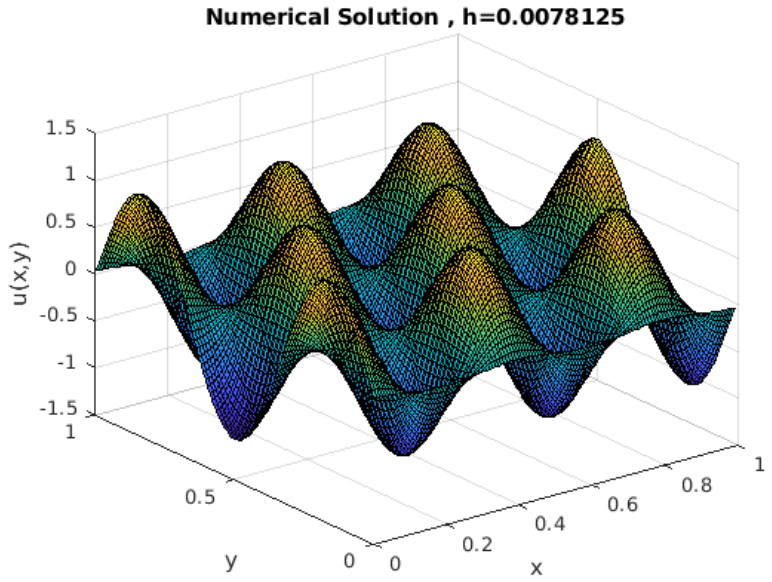
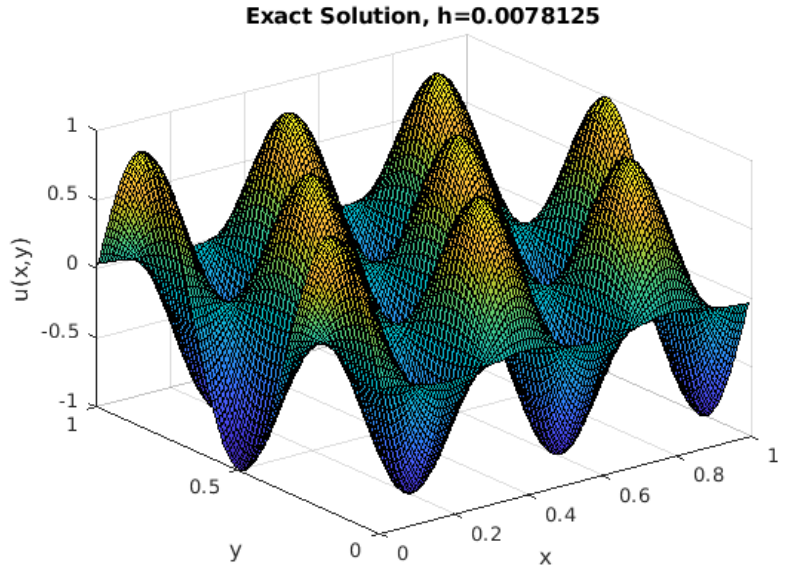
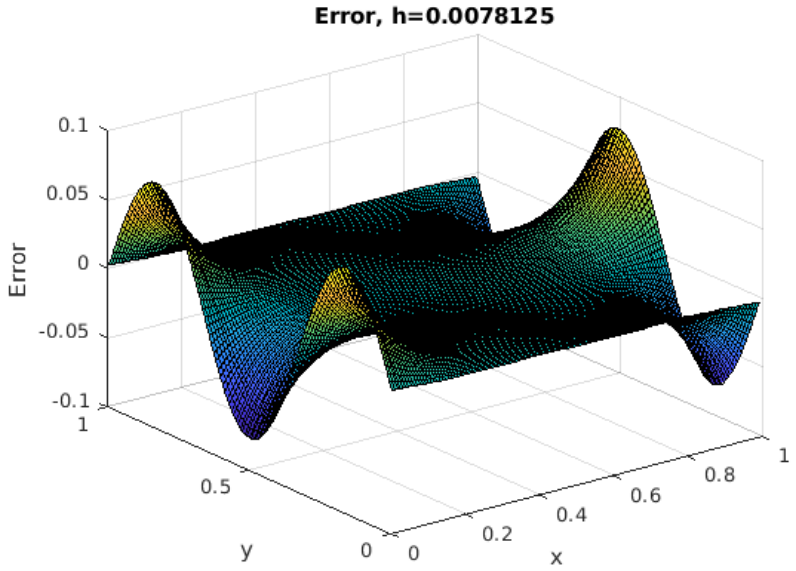
Warning: Polynomial is not unique; degree >= number of data points.

p =

    0.6812         0
```







Published with MATLAB® R2020a