Contents


```
% Script for testing fd2poisson over the square [a,b]x[a,b]
a1 = 0; b1 = 1;
m = (2^7)-1;
h = (b1-a1)/(m+1);

[x,y] = meshgrid(a1:h:b1); %Uniform mesh, including boundary points.

% Laplacian(u) = f
f = @(x,y) 10*pi^2*(1+cos(4*pi*(x+2*y))-2*sin(2*pi*(x+2*y))).*exp(sin(2*pi*(x+2*y)));
% u = g on Boundary
g = @(x,y) exp(sin(2*pi*(x+2*y)));

% Exact solution is g.
uexact = @(x,y) g(x,y);

tol = 10^(-8);

idx = 2:m+1;
idy = 2:m+1;

% Compute boundary terms, south, north, east, west
ubs = feval(g,x(1,1:m+2),y(1,1:m+2));     % Include corners
ubn = feval(g,x(m+2,1:m+2),y(m+2,1:m+2)); % Include corners
ube = feval(g,x(idy,m+2),y(idy,m+2));     % No corners
ubw = feval(g,x(idy,1),y(idy,1));         % No corners

% Evaluate the RHS of Poisson's equation at the interior points.
f1 = feval(f,x(idy,idx),y(idy,idx));

% Adjust f for boundary terms
f1(:,1) = f1(:,1) - ubw/h^2;             % West
f1(:,m) = f1(:,m) - ube/h^2;             % East
f1(1,1:m) = f1(1,1:m) - ubs(idx)/h^2;    % South
f1(m,1:m) = f1(m,1:m) - ubn(idx)/h^2;    % North

b = reshape(f1,m*m,1);

I = eye(m);
ze = zeros(m,1);
e = ones(m,1);
T1 = spdiags([ze -2*e ze],[-1 0 1],m,m);
S1 = spdiags([e e],[-1 1],m,m);
T2 = spdiags([e -2*e e],[-1 0 1],m,m);
S2 = spdiags([ze ze],[-1 1],m,m);

D2x = (1/h^2)*(kron(I, T1) + kron(S1,I));
D2y = (1/h^2)*(kron(I, T2) + kron(S2,I));

A = D2x +D2y;


%Using the conjugate gradient.
u = CG(A,b,tol);

%Reshape u for plotting
u = reshape(u,m,m);

% Append on to u the boundary values from the Dirichlet condition.
u = [ubs;[ubw,u,ube];ubn];
```

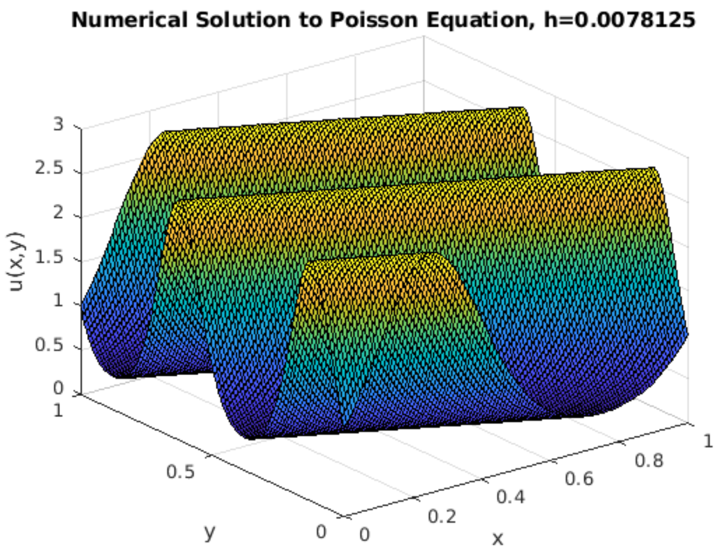
```
346   5.4475e-03
```

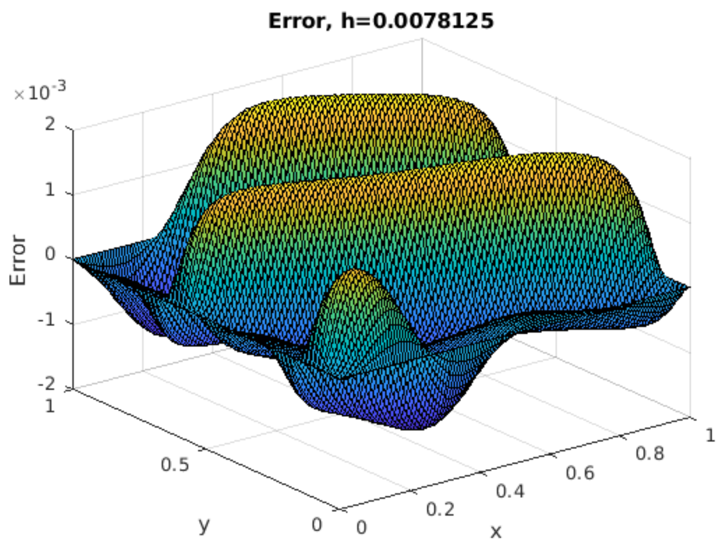
Plot solution

```
figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
surf(x,y,u), xlabel('x'), ylabel('y'), zlabel('u(x,y)'),
title(strcat('Numerical Solution to Poisson Equation, h=',num2str(h)));
%Plot error
figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
surf(x,y,u-uexact(x,y)),xlabel('x'),ylabel('y'), zlabel('Error'),
title(strcat('Error, h=',num2str(h)));

t2   = zeros(1,3);
t1   = [];
t4   = zeros(1,3);
t3   = [];

for ii = 1:3
    %Using the conjugate gradient.
    u = CG(A,b,tol);

    %Reshape u for plotting
    u = reshape(u,m,m);
```

```
    % Append on to u the boundary values from the Dirichlet condition.
    tic
    u = [ubs;[ubw,u,ube];ubn];
    gedirect = toc;
    t2(ii) = gedirect;

    h = (b1-a1)/(m+1);
    w = 2/(1+sin(pi*h)); %optimal relaxation parameter
    tic
    [usor,x,y] = fd2poissonsor(f,g,a1,b1,m,w);
    gedirect = toc;
    t4(ii) = gedirect;

end
t1   = [t1,t2];
t3   = [t3,t4];
fprintf('The number of iterations the CG code takes to converge, k = 346 with norm(rk) = 5.4475e-03 in an average time of %d\n',mean(t1));
fprintf('SOR method takes an average time of %d\n',mean(t3))
fprintf('Comparing the timing between CG and SOR, its seen that CG converges in a short time faster than SOR method.\n')
```

```
346   5.4475e-03
346   5.4475e-03
346   5.4475e-03
The number of iterations the CG code takes to converge, k = 346 with norm(rk) = 5.4475e-03 in an average time of 5.683333e-04
SOR method takes an average time of 2.355237e-01
Comparing the timing between CG and SOR, its seen that CG converges in a short time faster than SOR method.
```





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