LU Decomposition

Chapter 3 - Linear Systems

- · Chapter 3.1 Linear Algebra Renéw
- · Using the LU decomposition to compute.

 the matrix inverse.
 - The A is now and we find an non-norm B such that BA = I. then AB = I and B is unique

Proof: Show A x = 0 has exactly one solution: columns of A x = 0 A are linearly BA x = Bo = 0 independent. I x = 0 A exists,

 $Ax = x_1 col_1(A) + x_2 col_2(A) + \cdots + x_n col_n(A) = 0$

· Columns of A are linearly independent

T(x) = $A \times is$ "one-tu-one" and

. We say A is invertible

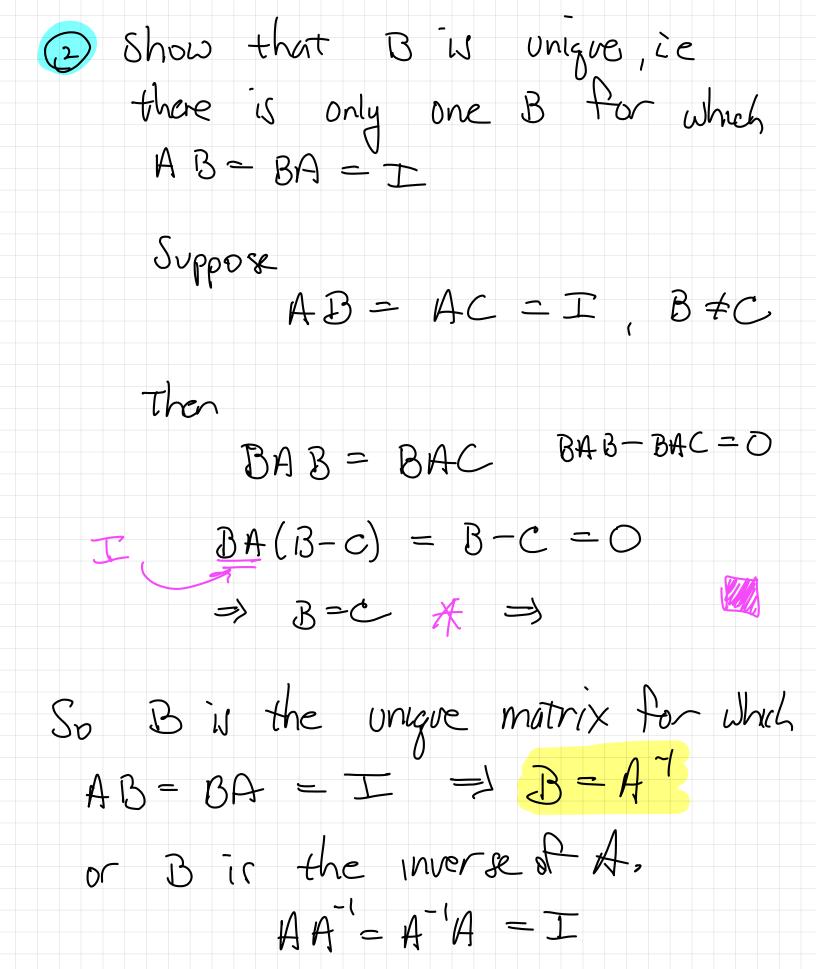
Show that iA BA = I, then AB = I

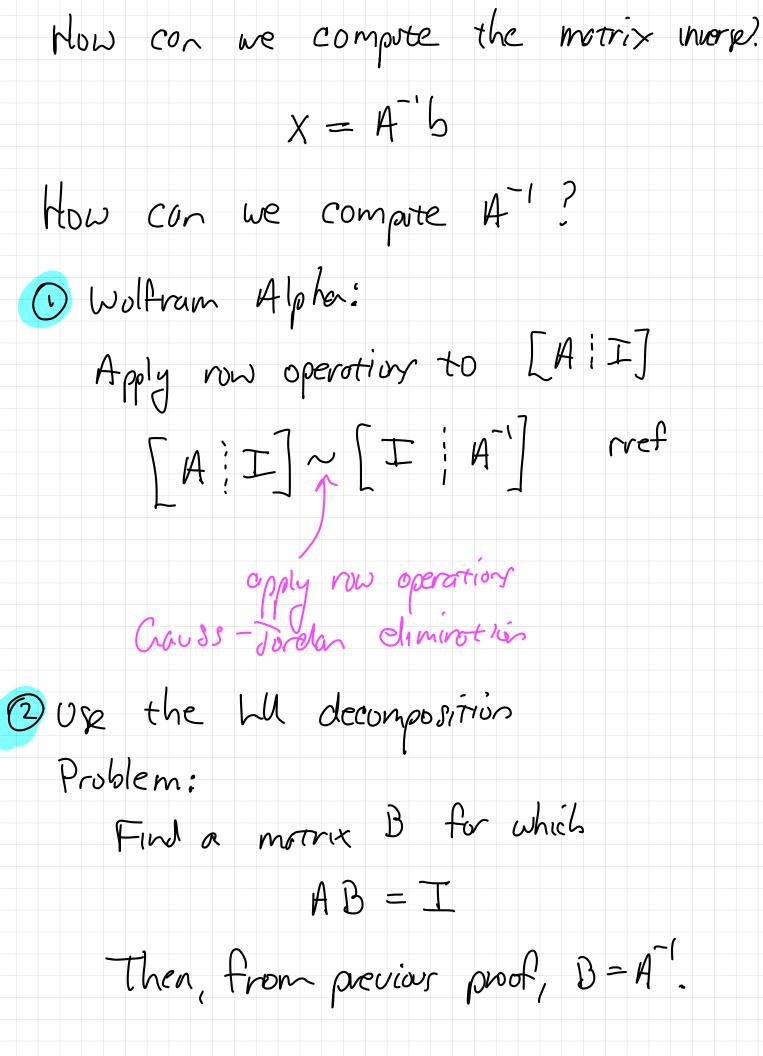
A(BA)x = Ax (Since BA = T)

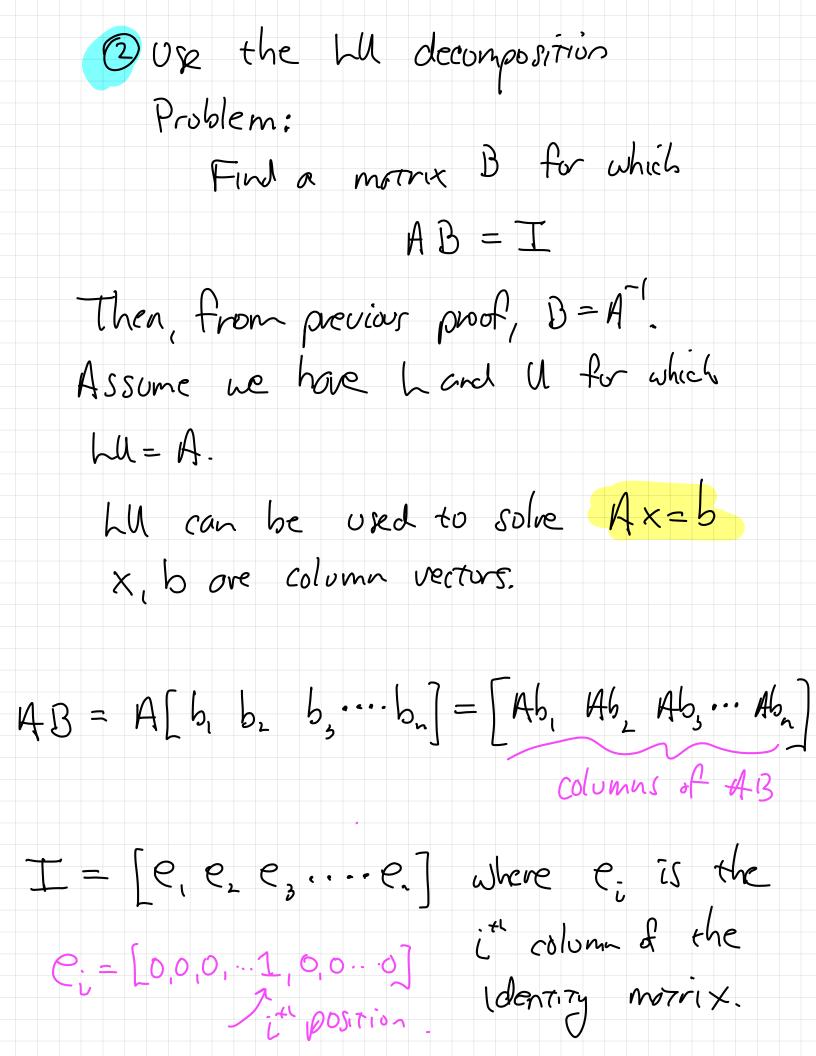
(AG)Ax = Ax (associetuity)

(AB-T)Ax = 0

Tree for ony x, so we must have $AB-I=0 \Rightarrow AB=I$







 $AB = A[b_1 \ b_2 \ b_3 \cdots b_n] = [Ab_1 \ Ab_2 \ Ab_3 \cdots Ab_n]$ $T = [e, e_2, e_3, \dots e_n]$ columns of AB Equating columns of AB and I we get: $Ab_1 = C_1$ $Ab_{\perp} = C_{\perp} \qquad \text{Incar}$ $Ab_{3} = C_{3} \qquad \text{Solves}.$ $Ab_n = b_n$ Idea: factor first: A=LU~3/8 N Then solve Abi = ei using LU 1 forward solve: n^2 3 2n for 1 back solve: n^2 0 and Abi e total cost of computing A^{-1} : $\frac{2}{3}n^3 + 2n = \frac{8}{3}n^3$

Lo compute marrix inerse 3n -0ps. X: mutiply A'b: O(n2) ops $X: \frac{8}{3}n^3 + n^2$ x = A'b is 4x the cost of solving Ax=b directly.

X= INU(A) *b (i) \$\$ To solve Ax = 6 requires χ : $\frac{2}{3}n^3 + 2n^2 =$ Forward + back Solve X = Ab (i) \$ $\chi = A^{n_1 - 1/3} b$