8(b).

The directions P_k Satisfy $P_{k+1}^T A P_k = 0$, iff P_{k+1} is A-Conjugate to P_k, with A a Symmetric positive definite matrix.

Suppose A is positive Symmetric definite, let P_k be ~~an~~ eigen vector of the matrix A, with Corresponding eigen value λ_k then

$$A P_k = \lambda_k P_k \quad \text{for } k = 1, \ldots, n$$

So since $A \in \mathbb{R}^{n \times n}$ and is symmetric positive definite then $\exists$ n eigen vectors $P_1, \ldots P_n$ which are mutually orthogonal i.e. $(P_i, P_j) = 0$ for $i \neq j$, ~~hence~~

therefore

$$P_{k+1}^T A P_T = (P_{k+1}, A P_k)$$

$$= (P_{k+1}, \lambda P_k)$$

$$= \lambda (P_{k+1}, P_k)$$

Since k is a positive Integer, so $k+1 \neq k$ therefore;

$$P_{k+1}^T A P_k = \lambda (0) = 0$$

Hence directions P_k satisfy $P_{k+1}^T A P_T = 0$ Since they are mutually orthogonal, hence A-Conjugate to each other.