What? How? Why?

1. Solve non-linear scalar equations
(Scalar linear eqns are too simple!

$$3x = 7$$
)

Harder: $\cos(x) = x$

$$x = \cos^{-1}(x)$$

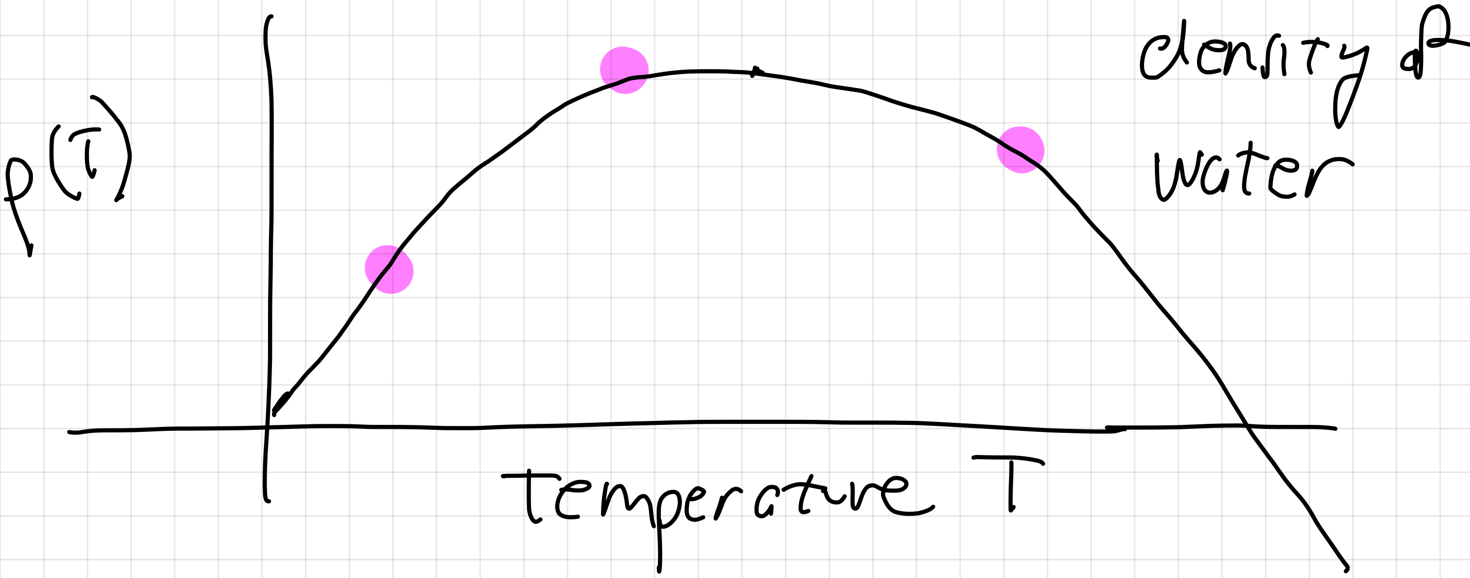
2. Linear systems with lots of equations! 100 variables in 100 unknowns

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{100,1} = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{100,2} = b_2$$
$$\vdots$$
$$a_{100,1} x_1 + a_{100,2} x_2 + \cdots a_{100} x_{100} = b_{100}$$

Convert to a matrix Equation:

$$Ax = b \Rightarrow \text{solve for } x$$

③ Data fitting 'or' "Interpolation"



$\rho(T)$ = density

How do we determine the curve through 3 points?

$$p(x) = ax^2 + bx + c$$

How do we find (a, b, c)?

Interpolation problem

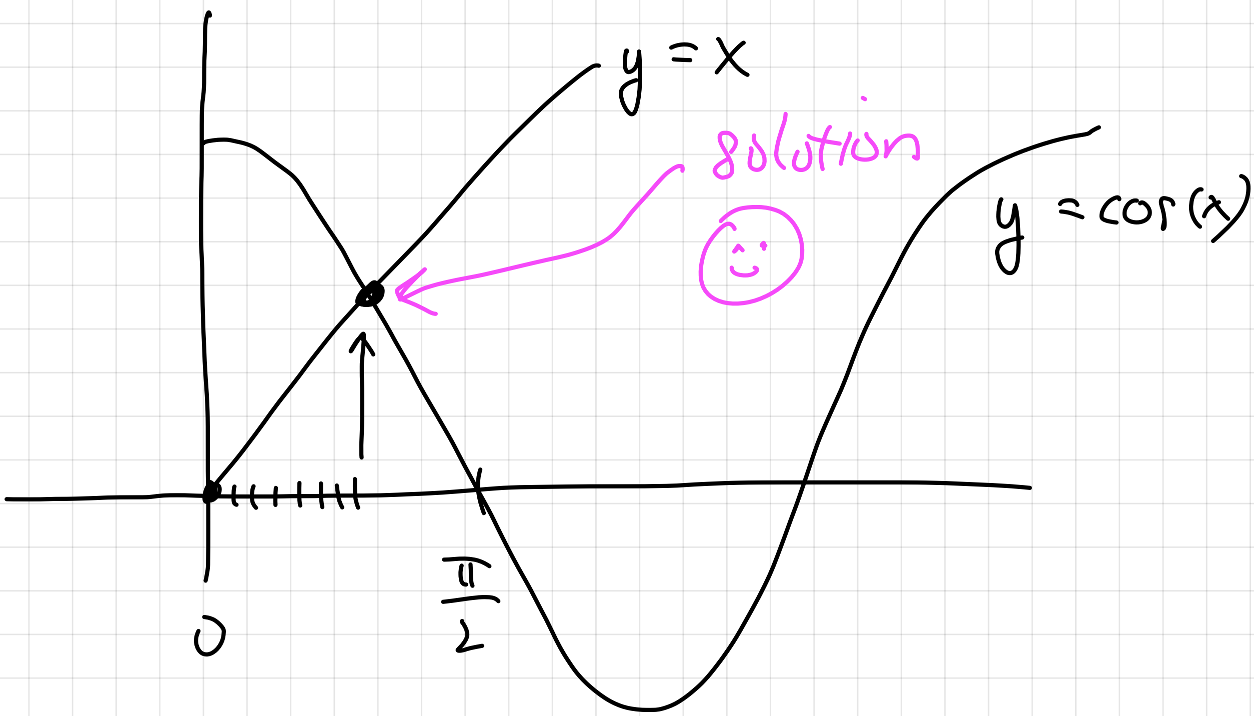
④ Evaluate Definite integrals:

$$\int_0^1 \frac{1}{1+x^2}\,dx = ?$$

How? Part I

Use Numerical methods!

1. $\cos(x) = x$



Solution in $[0, \frac{\pi}{2}]$

root finding methods?
$f(x) = 0$?

- Generate sequence of "guesses"
- Bisection method
  Generate a sequence of intervals that are smaller and smaller and that contain the solution.
- Newton's method $x_0, x_1, x_2, x_3 \longrightarrow$

2. Linear Systems

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{n1} \\ a_{21} & a_{22} & \cdots & a_{n2} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$m = n \gg 1$$

"much bigger than 1"

$\Rightarrow$ $m, n \sim 1000,\ 1000000, \ldots$

$$Ax = b$$

To solve: $A^{-1}Ax = A^{-1}b$

$$x = A^{-1}b$$

☺

$\Rightarrow$ but how do we get A^{-1} ??

$\Rightarrow$ Use Gaussian Elimination

$\Rightarrow$ Use "LU decomposition"

$A = LU =$ 

3. Interpolation?

$$p(x) = ax^2 + bx + c$$

Known data points: (x_0, y_0), (x_1, y_1), (x_2, y_2)

Want:

$$p(x_0) = \underline{\underline{a}}\,x_0^2 + \underline{\underline{b}}\,x_0 + \underline{\underline{c}} = y_0$$

$$p(x_1) = \underline{\underline{a}}\,x_1^2 + \underline{\underline{b}}\,x_1 + \underline{\underline{c}} = y_1$$

$$p(x_2) = \underline{\underline{a}}\,x_2^2 + \underline{\underline{b}}\,x_2 + \underline{\underline{c}} = y_2$$

$$\begin{bmatrix} x_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

- Linear system! Use LU decomposition

- Other approaches: Lagrange polynomials

How? Part II

How do we get actual numbers!

Code the numerical algorithm: using programming tools:

recommend

- Matlab
- Python
- C/Fortran
- Julia
- R
- C++/C#/Java

1. $\cos(x) = x$

Convert to a root finding problem

Solve $f(x) = \cos(x) - x = 0$

Start with a good guess
$x_0 \in [0, \frac{\pi}{2}]$

$$x_0 = 1$$

for $k = 0, 1, 2, 3 \ldots$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

end

Algorithm?

Questions?

- what is that weird formula -
- what is x_k? terms in the series

$$x_0, x_1, x_2, \ldots x_k, \ldots$$

- How do we know this works?
- When do we stop?

See code example